FRITZ HIRZEBRUCH (1927-2012)

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Oberwolfach, 29 May, 2012
Hirzebruch’s influence, especially on surgery theory

- Hirzebruch worked in many areas of mathematics: singularities, topology, complex manifolds and algebraic geometry.

- Name lives on:
  - the Hirzebruch surfaces,
  - the Hirzebruch signature theorem,
  - the Hirzebruch $L$-genus,
  - the Hirzebruch-Riemann-Roch theorem,
  - the Atiyah-Hirzebruch spectral sequence,
  - the Hirzebruch modular surfaces
  - ...

- His work had enormous influence, not least in surgery theory!
The Hirzebruch signature theorem

- The signature of a closed oriented $4k$-dimensional manifold $M$ is defined by
  $\tau(M) = \text{signature}(H^{2k}(M), \text{intersection pairing}) \in \mathbb{Z}$.

- **Theorem** (H., 1953) The signature of $M$ is
  $\tau(M) = \langle \mathcal{L}_k(M), [M] \rangle \in \mathbb{Z} \subset \mathbb{Q}$
  with $[M] \in H_{4k}(M)$ the fundamental class, and
  $\mathcal{L}_*(M) \in H^{4*}(M; \mathbb{Q})$ the $\mathcal{L}$-genus, a $\mathbb{Q}$-coefficient polynomial
  in the Pontrjagin classes $p_i(M) \in H^{4i}(M)$.
- The coefficients in the $\mathcal{L}$-genus are determined explicitly by
  the Bernoulli numbers, starting with
  $\mathcal{L}_1(M) = p_1(M)/3 \in H^4(M; \mathbb{Q})$.
- Princeton 1970 lecture of Hirzebruch:
  The signature theorem: reminiscences and recreation
The Milnor exotic spheres

- Milnor discovered the exotic spheres in 1956 by observing that the Hirzebruch signature theorem failed for 3-connected 8-dimensional manifolds with non-empty boundary \((M, \partial M)\), i.e. that in general

\[ \tau(M) - \langle \mathcal{L}_2(M), [M] \rangle \not\in \mathbb{Z} \subset \mathbb{Q} \]

- Princeton 1996 lecture of Milnor: Classification of \((n - 1)\)-connected \(2n\)-dimensional manifolds and the discovery of the exotic spheres describes the discovery.

- The Hirzebruch signature theorem plays a central role in the 1962 surgery classification of exotic spheres by Kervaire and Milnor, giving the simply-connected \(4k\)-dimensional surgery obstruction.
Differentiable manifolds and quadratic forms

- Hirzebruch 1960 lecture
  *Zur Theorie der Mannigfaltigkeiten*
  gave the first $E_8$-plumbing construction of an exotic 7-sphere.

- 1962 book with Koh
  *Differentiable manifolds and quadratic forms*
  Still the best introduction to the relationship of manifolds and quadratic forms!

- Hirzebruch’s 1967 Bourbaki seminar
  *Singularities and exotic spheres*
  describes the Brieskorn construction of exotic spheres as links of singularities, which was informed by Hirzebruch’s work on the topology of singularities.
The Hirzebruch signature theorem in Browder-Novikov theory I.

**Theorem** (B., 1962) Let $X$ be a $4k$-dimensional Poincaré complex. For $k \geq 2$ and $\pi_1(X) = \{1\}$ $X$ is homotopy equivalent to a closed $4k$-dimensional manifold if and only if there exists a $j$-plane vector bundle $\nu$ over $X$ such that the fundamental class $[X] \in H_*(X) \cong H_{n+j}(T(\nu))$ is represented by a map $\rho : S^{n+j} \to T(\nu)$ such that the Hirzebruch signature formula holds

$$\tau(X) = \langle \mathcal{L}(-\nu), [X] \rangle \in \mathbb{Z}.$$

This converse of the signature theorem proved in Browder’s 1962 paper

Homotopy types of differentiable manifolds
The Hirzebruch signature theorem
in Browder-Novikov theory II.

- The Hirzebruch signature formula plays a similar role in Novikov’s 1964 paper *Homotopically equivalent smooth manifolds.*

- The difference between a signature and the evaluation of the $\mathcal{L}$-genus as the surgery obstruction to making a homotopy equivalence of simply-connected $(4k - 1)$-dimensional manifolds homotopic to a diffeomorphism.
Hirzebruch and the Novikov conjecture

- The 1969 Novikov conjecture started as a question about non-simply-connected analogues of the Hirzebruch signature theorem.

- See Volume I of the Proceedings of the 1993 Oberwolfach conference on Novikov conjectures, index theorems and rigidity for the background.
Hirzebruch in Edinburgh

- 1958, International Congress of Mathematicians, at which Hirzebruch was a plenary speaker.
- 2003, Hodge100 conference
- 2009, Atiyah80 conference
- Reminiscences of the Fifties
  Video of Hirzebruch lecture on Atiyah
- 2010, Honorary Fellow of the Royal Society of Edinburgh
- Aspects of quadratic forms in the work of Hirzebruch and Atiyah
  Slides of lectures given in 2010 in Edinburgh and Bonn by A.R.
Hirzebruch in Edinburgh, September, 2010
Hirzebruch-related links

- Max Planck Institute for Mathematics, Bonn
- Wikipedia Biography
- MacTutor Biography
- Simons Foundation Video
- Simons Foundation Photo Archive