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Mathematics Applied to Dressmaking

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DRESSMAKING can raise interesting questions in both geometry and topology. My own involvement began in Bangkok, where I once bought a dress-length of some rather beautiful Thai silk. Unfortunately when I got home all the dressmakers claimed it wasn't long enough to make a dress. It became clear that I had to either abandon the project or make the thing myself.

Now, I had never made a dress before, nor any garment for that matter, and so I thought it would be amusing to try designing it from scratch. Fools rush in where angels fear to tread. In my innocence I chose a simple sleeveless summer dress with a princess line; or in layman's language it just consisted of two panels, one at the front and one at the back, sewn together at the sides. What I had not realized was that, the simpler the dress, the more accurately it has to fit. As a precaution I first tried a mock-up made out of an old sheet. And a good thing, too, because the result was hopeless: when she tried it on it hardly fitted anywhere. I slowly began to realize that I did not yet understand the basic mathematical problem of how to fit a flexible flat surface round a curved surface. So back to the drawing board to do a little differential geometry.

I was particularly intrigued by the negative curvature at the small of the back. How does one make a dress, not only to sit smoothly, but also to follow smoothly the natural twisting movements of the torso without wrinkling? If the skin can do it, why not the dress? To induce negative curvature in a flat surface, one must either stretch the perimeter of shrink the middle. For example, plant leaves produce their negative curvature by extra growth around the perimeter. In dressmaking there are two standard methods of achieving negative curvature, which we shall discuss. One is by cutting the material on the cross, which in effect stretches the perimeter, and the other is by making seams and darts, which in effect shrinks the middle.

Cutting 'on the cross' or 'on the bias' merely means arranging things so that the fibres of the warp and weft go diagonally relative to the body instead of going horizontally and vertically. Therefore, if the dress is pulled tight horizontally and vertically, then the material will be stretched diagonally relative to the fibres. Now, it is a characteristic property of woven material that it cannot be stretched *along* the fibres but it can be stretched *diagonal* to them. And if it is stretched along both diagonals at the same time then it will form a surface of negative curvature like a quadric, such as $xy = z$ (see Fig. 1). In a quadric the fibres are in fact straight lines (as in a plane), but they are no longer parallel to one another, and consequently they must have been pulled slightly apart round the perimeter (or pushed slightly closer together in the middle, or both). Incidentally, it is a classical theorem of projective geometry that any non-planar surface containing two intersecting families of lines is a quadric. Of course, I am not suggesting that a dress is a quadric, but where it has negative curvature the weave must be distorted in the same way as in a quadric, either pulled apart round the perimeter or bunched together in the middle.

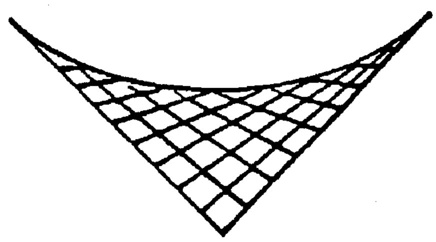

FIG. 1.

Cutting on the cross can be most effective in full skirts, swathed dresses, or soft rolled collars, but if it is used in fitted dresses then it suffers from three disadvantages: first, the stretching round the perimeter thins and weakens the material, and pulls on the seams. Secondly, if the material resists the stretching elastically then the dress will tend to cling to the body (although this is sometimes interpreted as an advantage rather than a disadvantage). Thirdly, when the torso twists, the unstretchability of the diagonal-lying fibres tends to make the dress go taut in rather unsightly diagonal folds. If we want the dress to remain graceful under movement this last criticism is the most serious, and therefore it is preferable to have darts.

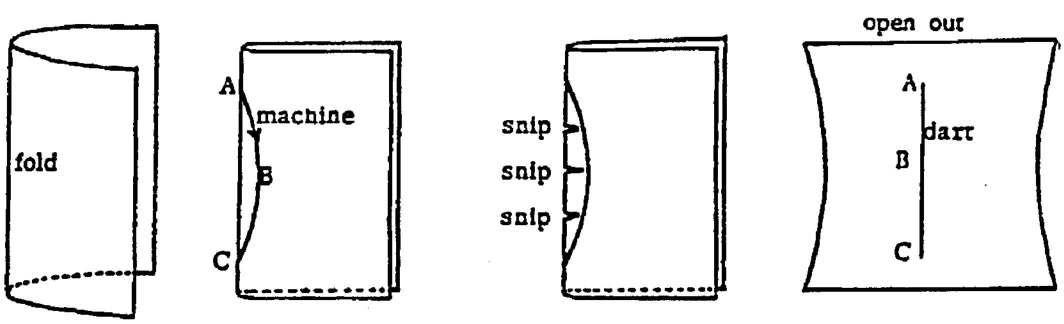

FIG. 2.

To make a dart you merely fold the material along the desired dart-line, with the outside inside, machine along an arc ABC (see Fig. 2), and make a few snips to allow the curvature to go negative the way you want. But this immediately raises several awkward questions:

1. In which direction should the darts go?
2. How many should there be?
3. How deep should they be?
4. Where should they begin and end?
5. What shape should they be?

Let us try and derive answers to these questions from basic principles in the case of a princess-line dress. The beauty of the princess line is the continuous flow from neck to hem. Clearly this calls for vertical darts so as not to interrupt the flow, giving the answer to Question 1. The next question is, how many darts? Here there is a conflict between aesthetics and mathematics, because aesthetic simplicity demands as few as possible, whereas accurate fitting of the curvature demands as many as possible. In particular, central darts would be undesirable in a princess line because that would destroy the visual attraction of the single panels at front and back. The simplest solution is to have two darts in each panel. Therefore, if we include the side seams we have six vertical lines to play with, for creating the desired curvature.

Next we proceed to Question 3, how deep should the darts be? We shall use an approximate model to establish a simple but effective principle, as follows. The first approximation is to assume that the cross-section at the hips is a circle of radius r, and that at the waist is a smaller circle of radius r–x (see Fig. 3).

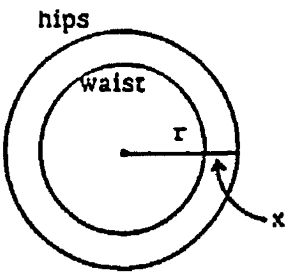

FIG. 3.

Hence the hip measurement is $2\pi r$ and the waist measurement is $2\pi(r-x)$. Therefore we need to achieve a reduction of $2\pi x$ in the perimeter* at the waist, and we can achieve this by means of our six vertical lines. Since 2π is approximately equal to 6, we have to achieve an average reduction of approximately x at each dart and side seam. Therefore the maximum depth of the dart from the fold to the stitch line is $\frac{x}{2}$, since the folded material is double thickness. This simple pinciple of halving the depth not only answers Question 3, but also suggests a practical method for solving the more difficult Questions 4 and 5, as follows.

Consider the darts in the back panel. Place a straight-edge against the back of the lady, and then the two points of contact where it touches the shoulder-blade and the bottom will determine the two points where the dart should begin and end. To obtain the shape of the dart, you merely take the shape of the lady relative to the straight-edge, and halve its depth (Fig. 4). The resulting quantitative shape will, of course, depend upon the individual, but the qualitative features are the same for nearly everybody, although seldom found in any shop-made dresses. The first important feature to notice is that at both ends of the dart the machine stitching should be *tangential* to the fold-line, whereas in most ready-made garments it is usually at an angle of about 10° or more, because this is easier to make. One only has to reflect for a moment to realize that the end of a dart is a cusp catastrophe, and, unless the stitching is tangential, the end will pucker so that it cannot be ironed flat. The tangentiality also implies the second important feature, that there must be two points of inflexion on the stitch line, where it changes from curving to the right to curving to the left, or vice versa.

* Did you know that π is called π because it is the first letter of the Greek word for perimeter, περιμετρος?

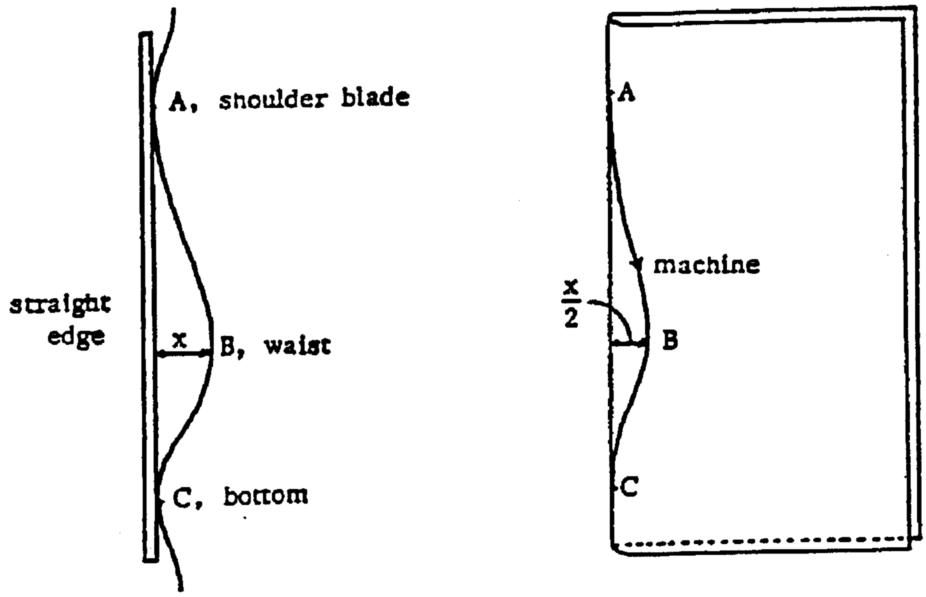

FIG. 4.

The third important feature to notice is the vertical asymmetry of the dart, because the main bulge is concentrated at the waist. A difference of a few millimetres away from the true (or desired) shape can dramatically alter the tailored look of a dress. Meanwhile the front darts are of a totally different shape, because this time they begin at the nipple and end at the stomach, as shown in Fig. 5. As before, the beginning and end should be tangential, but the most important feature to notice this time is the angle at B (suitably snipped), if it is desired that the dress should fit snugly under the bust.

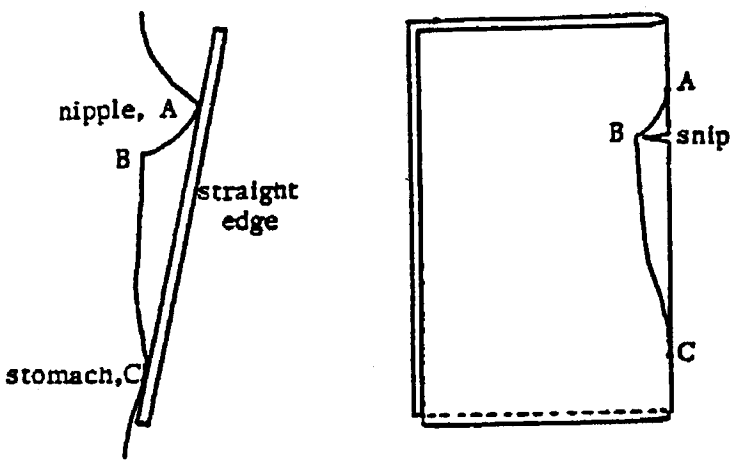

FIG. 5.

I experimented with this dart-making principle on another old sheet, and was gratified to find that it fitted like a glove. Of course, it should have fitted like a dress. So when I came to make the silk itself I modified the design slightly to allow for a more flexible movement of the torso, and put in a couple of extra darts at the back to improve the negative curvature.

Now came the problem of attaching the lining. This is the point where the instructions in a bad pattern can drive you up the wall for lack of decent mathematical notation. For instance, what does 'the inside of the lining' mean? Does it mean the nice side next to the skin, or the nasty side with the frayed edges of the seams on? Let us label the sides in sequence, starting at the outside:

1 = the outside of the dress
2 = the inside of the dress
3 = the nasty side of the lining next to 2
4 = the nice side of the lining next to the skin

First make the dress and the lining separately. To attach them together, you slip the lining over the dress so that 4 is next to 1, machine round the neck, and then poke the lining through the neck-hole as in Fig. 6. Then, *and only then*, do you attach the lining to the dress at the arm-holes (and along the side zip).

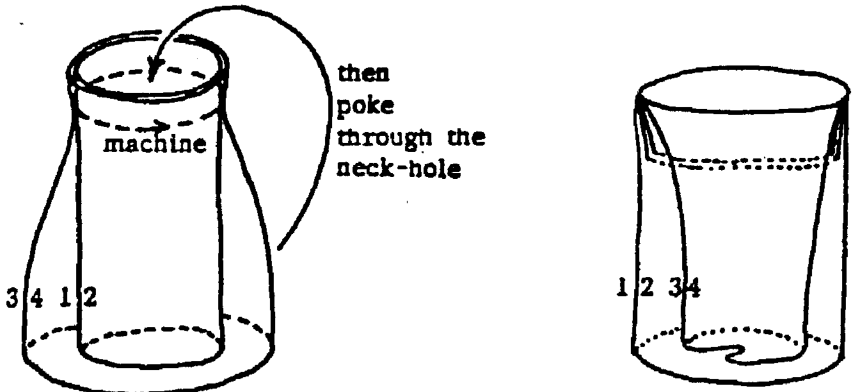

FIG. 6.

I stress this point because I once saw an innocent-looking instruction in an American dress pattern, saying:'Slip the lining over the dress, machine round the neck and armholes, and then turn it inside out'. In fact it was shown to me with wry amusement by the very lady who had written the instruction, and who had lost her job as a result of that one sentence. For the dress-pattern firm had been inundated by furious letters from thousands of frustrated dress-makers, struggling in vain to turn their ruined dresses inside out, and vowing never again to buy that firm's patterns. She was a mathematics graduate and she said to me, 'You're a topologist—just prove to me why its impossible to turn the damn thing inside out'. I confess it took me a sleepless night before I found the explanation. Of course, it seems obvious, when you have the dress in front of you, that it cannot be done, but that is not a mathematical proof. Intuitively I felt that, in addition to the geometric obstruction inherent in the unstretchability and incompressibility of the material, there must be a deeper topological obstruction, analogous to the impossibility of unknotting a knot. To my surprise I managed to establish the following result, which seemed at first to prove the opposite.

Theorem. Let M be an unknotted orientable surface in R^3 of genus 3 with two boundary components (as in Fig. 7). Then there is an isotopy of M back onto itself that interchanges the two sides of M.

Here *genus 3* means the three vertical holes in the surface, and the *two boundary components* mean the two circular rims at the top and bottom. An *isotopy* means a continuous movement of the surface, allowing bending, stretching and shrinking, but not allowing tearing or gluing; and the words *back onto itself* means that at the end of the

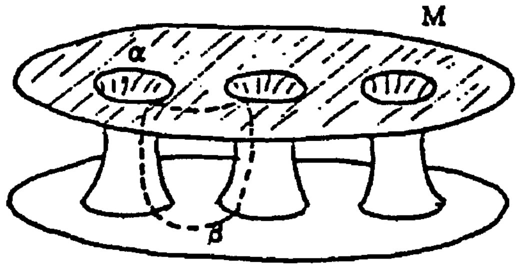

FIG. 7.

movement the surface occupies exactly the same position that it did at the beginning, except that individual points on the surface have been moved around. The surprising feature is the reversal of the sides. A similar isotopy is illustrated in Fig. 6, but that is easier to understand because there the surface is only of genus 1. In Fig. 7 if we identify the top of M with the dress, the bottom of M with the lining, and the three holes with the neck and armholes, then the theorem seems to imply that it is topologically possible to turn the dress inside out, after all. It was only after some thought that I realized the isotopy would also move the curve α into the curve β, because α can be spanned by a disk on only one side of the surface, while β can be spanned on the other. Therefore if the isotopy were applied to the dress, then the identity of the neck and arm holes would be lost. Hence the correct topological obstruction is: there is no side-reversing isotopy of the *pair* (M, α).

Returning to the silk dress, it took me longer than anticipated and eventually I had to stay up all night to get it finished in time. I remember once getting a jacket made in a couple of days in Hong Kong, and during the fitting being astonished to catch a glimpse of the tailor's assistant sitting cross-legged high up on the cutting bench at the back of the shop. I tried to imagine what it must be like — so I sat cross-legged up on the kitchen table all night, sewing away, drinking black coffee, and listening to the nocturnal disc jockeys. When morning came she tried it on, and formally pronounced it, at last, a success.

NOTE *by Lady Zeeman*

The length of cloth brought back from Bangkok, and declared by all to be 'insufficient for a dress', was ivory-coloured Thai silk, with two bands of delicate colour, one dove-grey and the other *café au lait*, woven into either end. From this, Sir Christopher designed and made a princess-line dress, doing all the cutting, machining and hand-finishing himself. It had a scoop neckline, no sleeves, and fell to just below the knee, with an eight-inch zip hidden in the side seam. It was lined throughout, and skimmed the figure in a manner both comfortable and elegant. It was a great pleasure to wear as well as being very pleasing to the eye. Unfortunately, the figure outgrew the dress in time, and it was given away. But a long evening skirt of five panels, in chestnut-coloured velvet with cream flowers, also designed and made by Sir Christopher for his wife, is still in frequent use.

Acknowledgements
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