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Source: *Proceedings of the American Mathematical Society*, Vol. 62, No. 1 (Jan., 1977), pp. 165-166

Published by: [American Mathematical Society](#)

Stable URL: <http://www.jstor.org/stable/2041968>

Accessed: 14/01/2011 09:50

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FIBRATIONS OVER A CWh-BASE

ROLF SCHÖN

ABSTRACT. This note provides a short argument for the known fact that the total space of a fibration has the homotopy type of a CW-complex if base and fiber have.

1. **NOTATION.** $F \rightarrow E \rightarrow B$ is a (Hurewicz) fibration. A CWh-space is a space having the homotopy type of a CW-complex. The following result is due to Stasheff [9, Proposition (0)].¹

2. **THEOREM.** E is a CWh-space if F and B are.

PROOF. We replace the inductive construction of [9] by the CW approximation theorem [8, p. 412] that is due to Whitehead [11]: to the topological space E there exists a CW-complex X , called ‘CW-substitute for E ’ in [10, p. 97], and a weak homotopy equivalence $f: X \rightarrow E$. We make f into a fibration by taking the associated mapping path fibration $q: P_f \rightarrow E$, see e.g. [8, p. 99]. Then q is a weak homotopy equivalence too, and P_f is a CWh-space. Therefore pq is a fibration with a CWh-fiber by 3 below. Hence q induces a genuine homotopy equivalence between the fibers of pq and p and is therefore a fiber homotopy equivalence by [3, 6.3].

3. **PROPOSITION.** F is a CWh-space if E and B are.

PROOF. Compare [9, Corollary (13)]. By coglueing homotopy equivalences, see [4, (1.2)] or [5, (8.7)], the pullbacks of the horizontal rows in the following diagram are homotopy equivalent.

$$\begin{array}{ccccc}
 * & \longrightarrow & B & \longleftarrow & E \\
 \downarrow & & \downarrow & & \downarrow \\
 PB & \longrightarrow & B & \xleftarrow{p} & E \\
 \uparrow & & f \uparrow & & \uparrow \\
 PZ_p & \longrightarrow & Z_p & \xleftarrow{i} & E
 \end{array}$$

f is the standard factorization of p over its mapping cylinder Z_p , $PZ_p \rightarrow Z_p$, $PB \rightarrow B$ are the fibrations of paths starting from a point $b \in Z_p$, resp.

Received by the editors March 8, 1976 and, in revised form, July 19, 1976.

AMS (MOS) subject classifications (1970). Primary 55F05, 54E60.

¹ As it is remarked in [7, p. 27] Stasheff’s proof is not correct, but can be patched.

$f(b) = * \in B$, and the other arrows are obvious. The upper pullback is the fiber F (over $*$), the lower one is the space of the paths on the CWh-space Z_p starting from b and ending in $E \subset Z_p$, and is therefore a CWh-space by [6].

4. REMARK. If we assume that F and E are CWh-spaces, then the following is true: (a) B is not a CWh-space in general. Fiber and total spaces of Example 2.4.8 of [8, p. 77] are contractible, but the base space, the “Warsaw circle”, is not contractible, because it has the nonvanishing Čech homotopy group $\check{\pi}_1(B) \cong \mathbf{Z}$ [2, §6]. (b) the loop space ΩB is a CWh-space, because it is homotopy equivalent to the fiber of the inclusion $F \rightarrow E$ [10, 2.56], and by delooping homotopy equivalences, see [1], B is a CWh-space too, if it is path-connected and has a numerable, null homotopic covering.

REFERENCES

1. G. Allaud, *De-looping homotopy equivalences*, Arch. Math. (Basel) **23** (1972), 167–169. MR **46** #8217.
2. R. Ciampi and G. De Cecco, *Gruppi d'omotopia di Čech*, An. Univ. Bucureşti Mat.-Mec. **22** (1973), no. 2, 87–101. MR **50** #14739.
3. A. Dold, *Partitions of unity in the theory of fibrations*, Ann. of Math. (2) **78** (1963), 223–255. MR **27** #5264.
4. R. Brown and P. R. Heath, *Coglueing homotopy equivalences*, Math. Z. **113** (1970), 313–325. MR **42** #1120.
5. K. H. Kamps, *Kan-Bedingungen und abstrakte Homotopietheorie*, Math. Z. **124** (1972), 215–236. MR **45** #4412.
6. J. W. Milnor, *On spaces having the homotopy type of a CW-complex*, Trans. Amer. Math. Soc. **90** (1959), 272–280. MR **20** #6700.
7. J. P. May, *Classifying spaces and fibrations*, Mem. Amer. Math. Soc. **1** (1975), issue 1, no. 155. MR **51** #6806.
8. E. H. Spanier, *Algebraic topology*, McGraw-Hill, New York, 1966. MR **35** #1007.
9. J. D. Stasheff, *A classification theorem for fibre spaces*, Topology **2** (1963), 239–246. MR **27** #4235.
10. R. M. Switzer, *Algebraic topology-homotopy and homology*, Springer-Verlag, New York, 1975.
11. J. H. C. Whitehead, *A certain exact sequence*, Ann. of Math. (2) **52** (1950), 51–110. MR **12**, 43.

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