LXV.

JOHANN BENEDICT LISTING.

[*Nature, February* 1, 1883.]

ONE of the few remaining links that still continued to connect our time with that in which Gauss had made Göttingen one of the chief intellectual centres of the civilised world has just been broken by the death of Listing.

If a man's services to science were to be judged by the mere number of his published papers, Listing would not stand very high. He published little, and (it would seem) was even indebted to another for the publication of the discovery by which he is most widely known. This is what is called, in Physiological Optics, *Listing's Law.* Stripped of mere technicalities, the law asserts that if a person whose head remains fixed turns his eyes from an object situated directly in front of the face to another, the final position of each eye-ball is such as would have been produced by rotation round an axis perpendicular alike to the ray by which the first object was seen and to that by which the second is seen. "Let us call that line in the retina, upon which the visible horizon is portrayed when we look, with upright head, straight at the visible horizon, the horizon of the retina. Now any ordinary person would naturally suppose that if we, keeping our head in an upright position, turn our eyes so as to look, say, up and to the right, the horizon of the retina would remain parallel to the real horizon. This is, however, not so. If we turn our eyes straight up or straight down, straight to the right or straight to the left, it is so, but not if we look up or down, and also to the right or to the left. In *these* cases there is a certain amount of what Helmholtz calls 'wheel-turning' (*Raddrehung*) of the eye, by which the horizon of the retina is tilted so as to make an angle with the real horizon. The relation of this 'wheel-turning' to the above-described motion of the optic axis is expressed by Listing's law, in a perfectly simple way, a way so simple that it is only by going back to what we might have thought nature should have done, and from that point of view, looking at what the eye really does, and considering the complexity of the problem, that we see the ingenuity of Listing's law, which is simple in the extreme, and seems to agree with fact quite exactly, except in the case of very short-sighted eyes." The physiologists of the time, unable to make out these things for themselves, welcomed the assistance of the mathematician. And so it has always been in Germany. Few are entirely ignorant of the immense accessions which physical science owes to Helmholtz. Yet few are aware that he *became* a mathematician in order that he might be able to carry out properly his physiological researches. What a pregnant comment on the conduct of those "British geologists" who, not many years ago, treated with outspoken contempt Thomson's thermodynamic investigations into the admissible lengths of geological periods!

Passing over about a dozen short notes on various subjects (published chiefly in the Göttingen *Nachrichten*), we come to the two masterpieces, on which (unless, as we hope may prove to be the case, he have left much unpublished matter) Listing's fame must chiefly rest. They seem scarcely to have been noticed in this country, until attention was called to their contents by Clerk-Maxwell.

The first of these appeared in 1847, with the title *Vorstudien zur Topologie.* It formed part of a series, which unfortunately extended to only two volumes, called *Göttinger Studien.* The term Topology was introduced by Listing to distinguish what may be called qualitative geometry from the ordinary geometry in which quantitative relations chiefly are treated. The subject of knots furnishes a typical example of these merely qualitative relations. For, once a knot is made on a cord, and the free ends tied together, its nature remains unchangeable, so long as the continuity of the string is maintained, and is therefore totally independent of the actual or relative dimensions and form of any of its parts. Similarly when two endless cords are linked together. It seems not unlikely, though we can find no proof of it, that Listing was led to such researches by the advice or example of Gauss himself; for Gauss, so long ago as 1833, pointed out their connection with his favourite electromagnetic inquiries.

After a short introductory historical notice, which shows that next to nothing had then been done in his subject, Listing takes up the very interesting questions of Inversion (*Umkehrung*) and Perversion (*Verkehrung*) of a geometrical figure, with specially valuable applications to images as formed by various optical instruments. We cannot enter into details, but we paraphrase one of his examples, which is particularly instructive:—

"A man on the opposite bank of a quiet lake appears in the watery mirror perverted, while in an astronomical telescope he appears inverted. Although both images show the head down and the feet up, it is the dioptric one only which:—if we could examine it:—would, like the original, show the heart on the left side; for the catoptric image would show it on the right side. In type there is a difference between inverted letters and perverted ones. Thus the Roman V becomes, by inversion, the Greek Λ; the Roman R perverted becomes the Russian Я; the Roman L, perverted and inverted, becomes the Greek Γ. Compositors read perverted type without difficulty:—many newspaper readers in England can read inverted type. * * * The numerals on the scale of Gauss' Magnetometer must, in order to appear to th

Listing next takes up helices of various kinds, and discusses the question as to which kind of screws should be called right-handed. His examples are chiefly taken from vegetable spirals, such as those of the tendrils of the convolvulus, the hop, the vine, &c., some from fir-cones, some from snail-shells, others from the "snail" in clock-work. He points out in great detail the confusion which has been introduced in botanical works by the want of a common nomenclature, and finally proposes to found such a nomenclature on the forms of the Greek δ and λ.

The consideration of double-threaded screws, twisted bundles of fibres, &c., leads to the general theory of paradromic winding. From this follow the properties of a large class of knots which form "clear coils." A special example of these, given by Listing for threads, is the well-known juggler's trick of slitting a ring-formed band up the middle, through its whole length, so that instead of separating into two parts, it remains in a continuous ring. For this purpose it is only necessary to give a strip of paper one *half*-twist before pasting the ends together. If three half-twists be given, the paper still remains a continuous band after slitting, but it cannot be opened into a ring, it is in fact a trefoil knot. This remark of Listing's forms the sole basis of a work which recently had a large sale in Vienna:—showing how, in emulation of the celebrated Slade, to tie an irreducible knot on an endless string!

Listing next gives a few examples of the application of his method to knots. It is greatly to be regretted that this part of his paper is so very brief; and that the opportunity to which he deferred farther development seems never to have arrived. The methods he has given are, as is expressly stated by himself, only of limited application. There seems to be little doubt, however, that he was the first to make any really successful attempt to overcome even the preliminary difficulties of this unique and exceedingly perplexing subject.

The paper next gives examples of the curious problem:—Given a figure consisting of lines, what is the smallest number of *continuous* strokes of the pen by which it can be described, no part of a line being gone over more than once? Thus, for instance, the lines bounding the 64 squares of a chess-board can be drawn at 14 separate pen strokes. The solution of all such questions depends at once on the enumeration of the points of the complex figure at which an odd number of lines meet.

Then we have the question of the "area" of the projection of a knotted curve on a plane; that of the number of interlinkings of the orbits of the asteroids; and finally some remarks on hemihedry in crystals. This paper, which is throughout elementary, deserves careful translation into English very much more than do many German writings on which that distinction has been conferred.

We have left little space to notice Listing's greatest work, *Der Census räumlicher Complexe* (Göttingen *Abhandlungen*, 1861). This is the less to be regretted, because, as a whole, it is far too profound to be made popular; and, besides, a fair idea of the nature of its contents can be obtained from the introductory Chapter of Maxwell's great work on Electricity. For there the importance of Listing's Cyclosis, Periphractic Regions, &c., is fully recognised.

One point, however, which Maxwell did not require, we may briefly mention. In most works on Trigonometry there is given what is called *Euler's Theorem* *about polyhedra*:—viz. that if S be the number of solid angles of a polyhedron (not self-cutting), F the number of its faces, and E the number of its edges, then

$$S + F = E + 2.$$

The puzzle with us, when we were beginning mathematics, used to be "What is this mysterious 2, and how came it into the formula?" Listing shows that this is a mere case of a much more general theorem in which corners, edges, faces, *and regions of space*, have a homogeneous numerical relation. Thus the mysterious 2, in Euler's formula, belongs to the two regions of space:—the one enclosed by the polyhedron, the other (the *Amplexum*, as Listing calls it) being the rest of infinite space. The reader, who wishes to have an elementary notion of the higher forms of problems treated by Listing, is advised to investigate the modification which Euler's formula would undergo if the polyhedron were (on the whole) ring-shaped:—as, for instance, an anchor-ring, or a plane slice of a thick cylindrical tube.