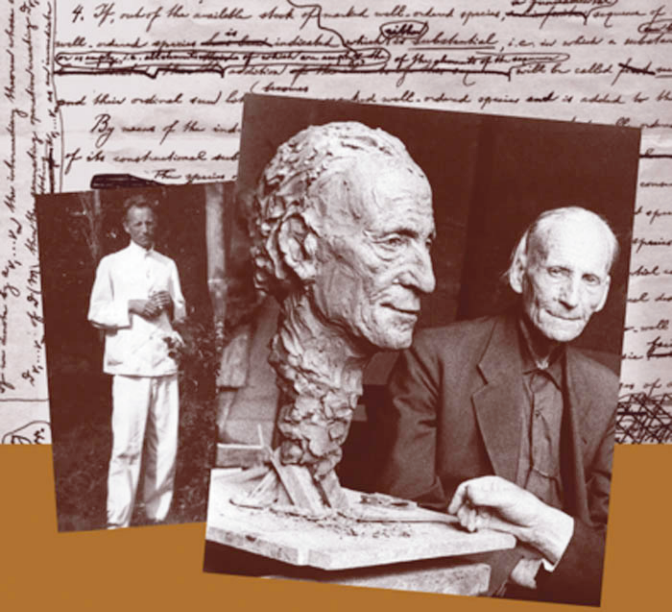

Mystic, Geometer, and Intuitionist

The Life of L. E. J. Brouwer

Volume 2: *Hope and Disillusion*

Dirk van Dalen

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The Life of L. E. J. Brouwer

1881–1966

Volume 2

Hope and Disillusion

DIRK VAN DALEN

Department of Philosophy

Utrecht University

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DEDICATION

Dedicated to the Mathematical Research Institute at Oberwolfach– the organization and all members of staff– in appreciation of its major role in re-establishing mathematics in a free Europe, its role in promoting international co-operation, and for its generosity and encouragement, from which the present book also benefited

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PREFACE

> Faire abstraction du monde d'objects (ce qui est nécessaire pour travailler dans les mathématiques intuitionistes) n'est possible qu'en éprouvant la vie comme un rêve.
> Brouwer.

In the twenties Brouwer had reached the undisputed status of master topologist and mysterious revolutionary. He had given topology a new lease of life; his innovations and his almost magical intuition determined to a large extent the course of topological research in the first half of the century. It had surprised the professional world that this genius had turned his back on traditional mathematics and started a thorough revision of the foundations of his subject. In volume 1 we have sketched the basic ideas and principles of his intuitionism; in this volume we will concentrate largely on the reception (and rejection) of his programme. Looking dispassionately at the turbulent years that followed after 1920, we can but conclude that Brouwer did not seek the conflict. His papers were technical and almost dull; if there was any provocation at all, it was in the subject matter, not in surreptitious attacks.

It has been Brouwer's misfortune that his name usually came up in conjuction with Hilbert's. In itself that was nothing to be ashamed of; after all, David Hilbert was the man who succeeded Poincaré as the unchallenged master of mathematics. Unfortunately a comparison between Brouwer and Hilbert was not particularly instructive or clarifying. It is difficult to find two mathematical geniuses of such totally opposite character. The Dutchman was the artistic, unfettered scientist, with an unbridled lust for travelling and friendships, while at the same time practicing meditation and entertaining austere moral and political convictions. A man who felt at home in salons as well as in the company of esoteric revivalists, who could be totally absorbed in mathematics, but who could equally well leave the subject alone; who had a reputation for defending lost causes, and who refused to sacrifice the moral issue to personal comfort.

The great Göttinger was a strong single-minded specialist, who devoted his life to the cultivation of mathematics and physics. A man whose mind never left mathematics alone, who nimbly jumped from one area to another to perform miracles. A man with just two goals in life: mathematics and Göttingen, goals that were largely attained by his sheer mathematical intellect and his talent for leadership. The following chapters will show that a conflict between the two was not a predetermined matter. One may well conjecture that without the First World War, there would not have been a conflict. As the reader will see, the *Grundlagenstreit* greatly exasperated Hilbert, and the exchange of the twenties was not always kept within the confines of scholarly courtesy, but without the political complications there would not have been the fateful act that brought Einstein to the exclamation 'I would never have thought that Hilbert was capable of such emotional outbursts.'

Both men would have coexisted peacefully with now and then a scientific exchange on foundational matters. Hilbert's Programme would have been killed by Gödel, so to speak, before a serious conflict could have erupted.

The documents used in the present volume come mostly from sources mentioned in volume 1. There is, however, a certain amount of new material from various archives, to mention the major sources: the archives of Bieberbach, Courant, Doetsch, Hilbert, Hk de Vries, St.John's College Cambridge, Heidelberg University, Nationaal Archief Den Haag.

Since the publication of the first volume a number of relevant historical studies have appeared; the following old and new texts have been useful in providing information on various topics: [Blauwendraat 2004], [Georgiadou 2004], [Groot 1946], [Hesseling 2003], [Hübner 2002], [Johnson 1979, Johnson 1981], [Knegtmans 1998], [Maas 2001], [Menger 1979, Menger 1994], [Urysohn1951], [Schroeder-Gudehus 1966], [Segal 1986], [Siegmund-Schultze 2001].

The references in this volume to Brouwer's publications do not match the list of volume 1.

The correspondence of Brouwer will in due time be published, the work is carried out as part of the Brouwer project in Utrecht. A similar project for Brouwer's unpublished manuscripts is envisaged.

To the long list of those who supported me with information and advice, I am happy to add the names of S.M.P. Brouwer-Euwe, Felix Browder, T. Corkery, Jonathan Harrison, Hans Huebner, C.B.J. Jonkers, Hans Ewald Keszler, R. van der Laan-van Heemert, M.S.van Leeuwen-Boomkamp, Eckart Menzler-Trott, Agnes van Noort, Volker Remmert, Bert Schweizer, Paul van Ulsen.

I hardly need repeat what I said about the cooperation with my wife in volume I, she was again in every respect my partner in this joint project.

Over the years my indebtedness to the Mathematical Institute at Oberwolfach has been accumulating. I started by attending the annual meetings on geometry, these were followed by meetings on logic, but gradually the institute became a friendly retreat where I could concentrate on the biography of Brouwer, away from the distraction called university. Like many before and after me, I came to appreciate the institute as the ideal place for exchange of ideas, research, and reflection. I am most grateful to the Oberwolfach establishment and the ever supportive staff members, who managed to turn the institute into a mathematician's home away from home.

Utrecht, June 2005

CONTENTS

12

THE FATHERS OF DIMENSION

12.1 The two Russians

For the next, and final, episode in Brouwer's topological career we have to go back to 1923. Now forty-two years old, Brouwer had acquired a reputation in mathematics that few could match. With his glorious past he had become in a way the wizard of topology, a man with an apparent direct access to the mysteries in which topology had been, and still was, veiled. That this man had given up his first place in topology, to save mathematics from the sterility that was threatening it, only gave greater intensity to his aureole of unselfish redeemer of mathematics. Like Moses he gave up his prominent role at the court of mathematics, in order to lead his people out of the affliction of Egypt into the land of intuitionism flowing with milk and honey.

That few were prepared to give up the 'flesh pots of Egypt', for a prolonged journey through the desert, did not matter. The determined radical reformer often ignores this realistic point.

Brouwer's critique of existing mathematics, his acknowledgement of fundamental uncertainty, fitted well in the mood of the times. The world—and in particular Germany—had lost its point of reference; the war had left behind a new awareness of social imperfections and of the inadequacies of the institutions of the older generation. In the wake of the war an exodus of royalty had taken place. The central powers were reduced in size, stability, and international status. Germany, the primary home of mathematical foundational activity, and the country where Brouwer's ideas could count on vivid interest, was in an exceptional state of confusion. The republic was struggling for recognition, for authority, and for survival. And the academic world was not being very sympathetic towards the new state. The consequences of a lost war, multiplied by the treaty of Versailles, were considerable; there was the trauma of the loss of national identity, and the all too apparent infringement of the national integrity. The French occupied the Ruhr in January. There was galloping inflation—from day to day the mark sunk further; on July 25 the exchange rate had dropped to a terrifying $ 1 to 600,000 Mark. And the political instability was disturbing—a certain Hitler was putting his party organization in order, and in November his Coupe in Munich just barely miscarried.

In this uncertain world thousands of scientists worked, and tried to live under the notice 'business as usual'. This was not Brouwer's style; he was heavily involved in the struggle for the rehabilitation of German scientists and scientific organization, and he quietly carried on his research for a rebuilding of mathematics under

the laws of his intuitionism. The year 1923 saw the last of his efforts to finish off the Denjoy affair. He had started a new life after the war, and the rebuilding of mathematics along constructive lines occupied him almost exclusively. But in spite of Brouwer's farewell to topology, other than as a subject of editorial and general interest, the topic was not to leave him alone. The twenties brought him a certain degree of pleasure as it furnished the recognition that was due to him for his penetrating insights, but it also plunged him into one of the most bitter fights of his career. It all had to do with a minor slip of the pen (*Schreibfehler*) in 1913.

In 1923 the German Mathematics Society, *DMV*, held its annual meeting from 20 to 25 September 1923 in Marburg. The meetings of the *DMV* mirrored in a way the state of mathematics; new trends became visible in the list of topics. The 1923 meeting showed a pronounced presence of topology, represented by the speakers Alexandrov, Urysohn, Schoenflies, and Furch. There were furthermore many mathematicians among the participants with a first-hand expertise in topology, such as Bieberbach, Brouwer, Carathéodory, Dehn, Reidemeister, Schreier, Vietoris, and Wilson. The post-war development of topology had persisted, and finally the pioneering works of Brouwer found their worthy successors. Two young Russians, Alexandrov and Urysohn, had taken up the thread of Brouwerian style topology, and they shared a strong and natural talent for the subject. Both owed their topological education to Egorov and Lusin. In 1913 Alexandrov, the scion of an upper middle class family, enrolled at Moscow University at the age of seventeen. Having been taught by an outstanding mathematics teacher, he already knew most of the first year mathematics; he started to read the mathematics texts for himself, and good fortune brought him in contact with the writings of Cantor. From that moment, he was under the spell of set theory and topology. One of his fellow students was M. Suslin, who left his stamp on significant parts of set theory although he died, only 25 years old, in 1919 of typhus. Alexandrov took as a guide in mathematics, the mathematician Egorov, who introduced him to such topics as measure theory, Hilbert spaces, and Baire's discontinuous functions. By 1915 Alexandrov had, in answer to a problem set by Lusin, determined the cardinality of Borel sets, and introduced the operation A (called thus after its inventor by Suslin). Lusin later called sets obtained by the A-operation analytic sets, claiming that 'A' stood for 'analytic'.[1]

At the time of Alexandrov's first results in descriptive set theory (comprising the theory of Borel sets, analytic sets, the Baire hierarchy, Borel measure, etc.) the young student Urysohn entered the stage. Urysohn was two years younger than Alexandrov, but he soon proved a match for him in topology.

The mathematical career of the two was interrupted for a few years by various kinds of activities (for example, Alexandrov spent a great deal of time in artistic circles), and of course by the Revolution. Fortunately both survived the period of

[1] Cf. [Alexandrov 1979].

civil war, and in 1920 they returned to the study of mathematics in Moscow. From then on things went fast.

In 1921 Urysohn finished his dissertation and was subsequently appointed as a lecturer. At the instigation of Egorov he turned towards topology. The problems that Egorov pointed out to him were right in the heartland of contemporary topology: what is, topologically speaking, a curve or a surface? The question may seem strange, in particular since 'everybody knew what a curve (surface) was'. But the usual definitions usually described curves by equations, and thus made use of notions which were not prima facie topological. What one needed was a definition that used only topological notions, something like 'a curve is a set such that . . . ', where in the definition only notions from (basic) topology, such as, point, neighbourhood, continuous, . . . , are used.

In the course of his investigations Urysohn came upon the notion of dimension. In view of the topological nature of curves and surfaces, this was not so unexpected; Euclid had already stated that 'the extremities of surfaces are lines'.

In Alexandrov's memoirs there is a description of the birth of Urysohn's dimension theory:

> One morning in August [1921], Urysohn and I were both at the Burkovo dacha and went to swim in the Klyaz'ma. During our bath Urysohn told me about his definition of dimension at which he had just arrived and then began to expound at great length the basic propositions of dimension theory. I was thus present at the conception of one of the finest chapters in topology: Urysohn's dimension theory.[2]

Soon Alexandrov and Urysohn started to put their ideas on paper, the first result being their famous '*Mémoire sur les espaces topologiques compacts*', published many years later by the Dutch Academy of Sciences.[3]

The two young mathematicians were sufficiently enterprising to see that beyond Moscow there was a whole world waiting to be discovered. And where better could gifted beginners go than to Göttingen, the capital of the mathematical world? One might object that beginning topologists would do well to go to Amsterdam and sit at the feet of the old wizard of the subject. But Brouwer had more or less said farewell to the subject; although he had published some 18 papers on topology after the war, this activity was more an aftershock of a big earthquake, than the beginning of a new life. Moreover, Brouwer was a loner, and in Göttingen Alexandrov and Urysohn would find a whole group of eager researchers. There might have been other options—Paris, Berlin—but in the mathematical culture of the time Göttingen was the perfect choice. It was a mathematical wonderland for them. During the summer semester of 1923 they met all the great mathematicians: Klein, Hilbert, Courant, Emmy Noether, ... Indeed, Göttingen went out of its way to welcome these first two Soviet mathematicians that had crossed the border.

[2] [Alexandrov 1979], p. 296.

[3] [Alexandroff and Urysohn 1929].

Alexandrov, in his memoirs, describes his experiences; he went to the lectures of Hilbert, Courant, Landau and—best of all—Emmy Noether. Hilbert was lecturing on intuitive geometry, his lectures were presented in 'an inspiring way, with a large number of individual remarks, always interesting, often witty, and sometimes profound.'[4] It is interesting to read that Alexandrov was not impressed by the presentation itself—'Hilbert spoke badly and could not even draw the simplest figure. Once he wanted to draw an ordinary rectangular parallelepiped. He tried to do so without success, and finally he turned angrily on his assistant (that was Bernays that summer). Bernays got up and (also without much sparkle) drew the ill-starred parallelepiped.' When Hilbert, with Cohn-Vossen, prepared the publication of these lectures, he asked Alexandrov to provide an appendix on topology. But Alexandrov did not oblige. Instead, he wrote a short monograph '*Einfachste Grundbegriffe der Topologie*' (The simplest basic notions of topology) , which became as popular as the Hilbert–Cohn-Vossen book.

In Göttingen, Emmy Noether was the greatest influence on Alexandrov and Urysohn. She was already acknowledged as the mother of the new algebra, and she was lecturing on the theory of ideals. This term was the beginning of a lasting friendship.

Urysohn and Alexandrov gave a number of talks on topology in Göttingen, and Hilbert was sufficiently impressed to invite them to submit some papers to the *Mathematische Annalen*. They did so, but Ostrowski, who was asked to handle the papers, failed to take the appropriate steps to get them published. Apparently he considered them unsuitable for the *Annalen*, and so they just gathered dust for a year. When Emmy Noether, in 1924, discovered that the papers had been disregarded, she told Hilbert and insisted that the manuscripts should be sent right away to Brouwer for referee reports. Ostrowski was taken to task by Hilbert and the papers were handled without delay. It is plausible that Brouwer's help was called in after all.[5]

In August 1923 Urysohn and Alexandrov made a trip to Norway; it was the end of term, they went on a long walking tour along the coast and fiords. In order to save their shoes, they walked barefoot when possible.

12.2 The definition of dimension

They returned to Göttingen and ended their visit to Germany at the meeting of the *DMV* in Marburg. There Alexandrov and Urysohn lectured respectively on 'Investigations from the theory of point sets' and 'Theory of general Cantorian curves'.[6] In Moscow Urysohn had continued his research in dimension theory, and the results were presented by Maurice Fréchet to the French Academy for publication in the *Comptes Rendus*, where they appeared in 1922 and 1923.[7] During

[4] [Alexandrov 1979], p. 298.

[5] [Alexandrov 1979], p. 300. Blumenthal apparently proposed that Brouwer should check the proof sheets after the papers were accepted, cf. p. 460.

[6] *Jahresber. d. Deutschen Math. Vereinigung* 1924, p. 68, 69 (italics).

[7] [Urysohn 1922, Urysohn 1923].

his stay in Göttingen someone must have told Urysohn about an earlier paper of Brouwer on the same subject. He duly looked up Brouwer's 1913 paper on natural dimension, read it, and discovered a mistake. He mentioned this in his lecture in Marburg, which was not attended by Brouwer; of course the news reached Brouwer, be it that Urysohn approached him privately, or that a participant at the meeting told him. Brouwer was shocked—a ten-year-old basic paper, and nobody so far had found anything wrong with it! Here Brouwer was confronted with the consequences of his decision to publish his dimension paper in '*Crelle*' and not in the *Mathematische Annalen*: the paper had been systematically overlooked. In fact Brouwer's results were not used for further research, or only discussed in passing in seminars.

Brouwer, who had not thought about the dimension notion since the publication in 1913, asked Urysohn to send him an exposition of the alleged mistake.

At the Marburg meeting Brouwer lectured in the 'foundations' section. He gave a talk on the negative consequences of his intuitionistic programme, 'The role of the principle of the excluded middle in mathematics, in particular in the theory of functions'.[8] This talk had already been presented at the Flemish Congress for Science and Medicine in Antwerp, one month before, where for the first time Brouwer demonstrated his Brouwerian counterexamples in public.[9]

Brouwer's talk was wedged in between those of Behmann (Algebra of logic and decision problems) and Fraenkel (New ideas on the founding of analysis and set theory). The *Jahresbericht* of 1923 contained brief reports of the lectures at the Marburg meeting. At the end of the report on Behmann's lecture, an objection of Brouwer to the use of 'etc.' and 'finite number' is mentioned. The speaker and the chairman (Schoenflies), the report goes on, refuted these objections, as the notions played no role in the lecture. Brouwer was not altogether pleased with this detail. In the next volume he protested in a letter to the editor that his remarks had been misinterpreted.[10] One may guess that the remarks were provoked by Behmann's somewhat irrelevant statement that 'Thanks to the efforts of the symbolic logicians (Frege, Peano, Russell) we now know, that in the first place the whole of mathematics is represented as a collection of purely logical facts'. In fact, Brouwer only wanted to point out that the contents of Behmann's lecture had no foundational consequences, precisely because of the use of the above notions. And, so Brouwer concluded, his remarks were refuted neither in the lecture, nor by the speaker or the chairman. Was the matter important enough for a reaction, one wonders? Brouwer, in any case, thought so; he could not appreciate the implicit suggestion that his contribution to the discussion was silly and irrelevant.

[8] *Die Rolle des Satzes vom ausgeschlossenen Dritten in der Mathematik, insbesondere in der Funktionentheorie* [Brouwer 1923a].

[9] See p. 487 In a sense the Unreliability paper of 1908 contained Brouwerian counterexamples. The application of Brouwer's technique to realistic mathematical problems followed in 1923.

[10] [Brouwer 1925c].

In order to put the topic of this section in the proper perspective we now have to retrace our steps to the beginning of the twentieth century.[11]

The grand master of mathematics, Poincaré, had in the nineteenth century dominated the development of mathematics, and his innovations in many areas have largely determined the shape of mathematics as we know it. But not only was he a great and deep scholar, he also had an admirable gift for popularization, a very rare talent indeed. He published a large number of essays, mostly in the *Revue de Métaphysique et de Morale*, that were subsequently collected into small paperbacks. These books were an immense success. What made these books so extraordinary was that Poincaré explained things that could be understood by high school students, but that were equally well of use to professional mathematicians and physicists, who would draw inspiration from Poincaré's ideas and views.

Brouwer, for one, had been a devoted reader of Poincaré's philosophical essays, as is testified by his dissertation, where Poincaré is quoted ten times. This influence of Poincaré is best seen in the section on mathematics and the world (i.e. physics) and that on mathematics and logic.[12]

The topic of the present chapter, dimension, was also presented by Poincaré in one of his semi-popular expositions. He had already considered the question of the dimension of our (physical) space in 1895; that particular approach was based on the group theoretical approach of Sophus Lie, in the tradition of the *Riemann–Helmholtz Raumproblem*. For our account of the dimension theory this approach is not really relevant; much more so is a renewed attack of Poincaré on the question 'why has space three dimensions?' From 1903 onwards the idea of a 'cut' appeared in his considerations. Some similar ideas had already been expressed by Euclid, who stated that a point is the end of a line, a line the boundary of a plane, and a plane the boundary of a solid body. Roughly speaking, we assign the dimension 0 to points and the dimension 1 to a line because if we cut it in a point we get two pieces. Similarly, if you cut a plane along a line, you get two pieces, etc. In 1903 Poincaré basically restricts these arguments to physical space, but in 1912 in the paper 'Why has space 3 dimensions?'[13] he considers mathematical space and its objects, and states his famous definition:

> A continuum has n dimensions when it is possible to divide it into several parts by means of one or more cuts which are themselves continua of $n-1$ dimensions. The continuum of n dimensions is thus defined by the continuum of $n-1$ dimensions; this is a definition by recursion.

The 'one or more cuts' are necessary: a circle for example, which one would certainly wish to be 1-dimensional, needs two cuts before two pieces are obtained. Poincaré's paper is a beauty, full of sound insight into geometry and physics.

[11] My account of the dimension episode makes use of original documents, of Freudenthal's comments in the Collected Works of Brouwer, and of the outstanding papers of Dale Johnson, [Johnson 1979, Johnson 1981]. The reader who wishes to learn more about the topic is urged to consult these publications.

[12] Cf. p. 80 ff.

[13] *Dernière Pensées* [Poincaré 1912], p. 488.

As he died that same year, it is idle speculation to guess what Poincaré would have done with this new purely mathematical definition. It is worth noting that at the time that Poincaré submitted his paper to the *Revue de Métaphysique et de Morale*, Brouwer's proof of the invariance of dimension had already been published, and that Lebesgue's second paper on that topic had already been presented to the *Academie des Sciences*.[14]

In Poincaré's paper there is no mention of this fundamental fact; as Dale Johnson puts it: 'The attitudes of Poincaré and Brouwer, the two greatest topologists at the beginning of our century, towards the problem of proving dimensional invariance were very different. While the former apparently thought that this problem was not very important, the latter thought that it was highly important, urgently requiring solution'.[15] The importance of the invariance problem is indeed a consequence of one's faith in the correct choice of notions. If one believes that dimension is essentially a topological notion, i.e. independent of our knowledge of the real line, plane, space, etc., and that topology is the science that studies properties invariant under topological transformations (i.e. continuous transformation without cutting or pasting), then Brouwer's invariance theorem is the supreme test for the notion of dimension. On the other hand, if one thinks that it is too early to say whether we have the right notions, and a certain amount of adjustment is still to be expected, then the invariance of dimension is nice, but not the last word. Maybe, this was roughly what distinguished Poincaré and Brouwer.

Before Poincaré's last paper, Frederich Riesz and René Baire had already investigated the topic of dimension, however, without success.[16] As their work is not relevant for our story, we will move on to Brouwer.[17]

The birth of modern dimension theory took place in 1913 in one of Brouwer's last papers before the First World War. This paper, with the cryptic title 'On the natural notion of dimension', was published in the *Journal für die reine und angewandte Mathematik*, also known as *Crelle's journal*, or simply '*Crelle*' (after its founder). The choice of this journal remains a matter of conjecture; although it was one of the better journals, it certainly did not cater to the specialists in topology. And, as a matter of fact, Brouwer's paper was totally overlooked by all and sundry. Why, is hard to say. If only Brouwer would have sent his paper to the *Mathematische Annalen* the history of dimension theory might have taken a totally different course. It is difficult to ascertain why Brouwer sent it to *Crelle* instead of the *Mathematische Annalen*; one reason might be that Brouwer wanted to avoid a leak that would give away the result, and might cause more priority problems. The reader may recall that in 1913 Lebesgue had not yet produced a proof of his paving

[14] 27.III.1911.

[15] [Johnson 1981], p. 105.

[16] Cf. [Johnson 1981].

[17] There are two more interesting definitions, Fréchet's *dimension types* and Hausdorff's *dimension*. They are interesting, and up to a point, fruitful notions, but they fall short of being 'natural'. Hence we will not consider them (cf. [Johnson 1981]). Note, however, that Hausdorff's dimension has suddenly become important in the theory of fractals.

principle, although he had informed Brouwer by mail of a new proof, to be published in the *Bulletin de la Société Mathématique de France*.[18] So if Brouwer's proof of the principle appeared in 1913, one could say in all fairness that Lebesgue had been given ample time to make good his claims.

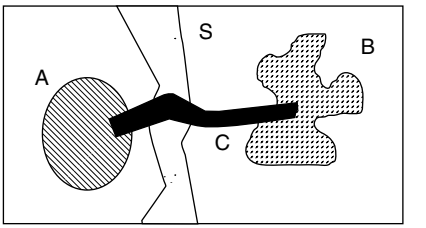
A letter from Hellinger, editor of Crelle, has been preserved, in which Brouwer is promised immediate publication.[19] So, maybe Brouwer was in a hurry to get the paper published before others would catch on to Poincaré's ideas.

Brouwer's paper started with a discussion of Poincaré's proposal. The most obvious shortcoming was that some objects were assigned a dimension that one clearly would not want them to have, e.g. consider a double cone (like a diabolo), one cut of just one point separates it into two parts (to be precise, the removal of one point—a 0-dimensional set—produces two parts), so it should have a dimension 1. *Quod non.*

Furthermore, Poincaré had used the word 'continuum' in his definition. In order to develop the notion of dimension 'out of nothing' Brouwer therefore replaced the word 'continuum' by 'normal set in the sense of Fréchet'; which is a set with the property that any two of its points are contained in a closed connected subset.

Brouwer also objected to the use of 'one or more' in Poincaré's definition, pointing out that a finite number of m-dimensional manifolds can be joined to form an $(m+p)$-dimensional manifold.[20]

He then proceeded by giving a definition of separation: a set S separates two sets A and B if every closed connected set[21] C meeting A and B, intersects S. In the definition S, A and B are closed subsets of a set P.



Next he went on to define 'n-dimensional':

> The expression: 'P has the general dimension degree n', in which n denotes an arbitrary natural number, now will express that for every choice of [closed sets] A and B there exists a separating set S, which has the

[18] Footnote in [Brouwer 1913b], p. 151. Lebesgue's proof appeared in 1921.

[19] Hellinger to Brouwer 21.XI.1912.

[20] The simplest example is: the sets of the rationals and of the irrationals are 0-dimensional, but their union—the real line—is 1-dimensional

[21] Brouwer was using the modern notion of 'connected'. This was not an obvious thing in 1913, and questions about the use of the notion have been raised by Urysohn and Menger. Whenever in the sequel 'connected' is used in the text without additional specifications, the modern notion is meant.

> general dimension degree $n-1$, but not for every choice of A and B there exists a separating set which has a general dimension degree less than $n-1$. Furthermore the expression: 'P has general dimension degree zero, resp. infinite' means that P has no part that is a continuum, resp. that for P neither zero, nor any natural number can be found as its general dimension degree.[22]

Brouwer's definition is, like Poincaré's, inductive ('recurrent' as he called it); it avoids the 'double cone' difficulty because all points (subsets A and B) are treated on an equal footing.

It is not too far fetched to guess that Poincaré, could he have returned to the topic, would have hit on the same or a similar improvement. Speculations of this sort are always tricky; it is nonetheless pertinent to point out that Brouwer's geometrical intuition was better adapted to the infinite variety of possible pathologies than Poincaré's intuition, which felt more comfortable with the natural aspects of geometry. So Brouwer may have been in a better position to handle the intricacies of dimension.

Brouwer, of course, realized that it is one thing to introduce a new general notion, but that the proof of the pudding is in the fact that the notion extended the old one, so he went on to put this definition to the test. That is to say, he set out to prove that the traditional dimension of our space is indeed 3 according to the new definition (or rather that R_n has general dimension degree n).

An interesting detail, more of a methodological than of a topological nature, is that Brouwer rephrased the definition in the form of a two-person game. Let the players be called $\forall$ and $\exists$.[23] The players move alternately, and we will denote the moves by $i_\forall$, $i_\exists$.

Here are the moves of both players:

$1_\forall$—$\forall$ chooses two disjoint closed subsets A_1 and B_1.
$1_\exists$—$\exists$ chooses a closed separating set S_1.
$2_\forall$—$\forall$ chooses two disjoint closed subsets A_2 and B_2 of S_1.
$2_\exists$—$\exists$ chooses a closed separating set S_2 in S_1. Etc.

$\exists$ wins if a separating set S_h is reached which does not contain a continuum (we would say, is totally disconnected); otherwise $\forall$ wins. We say that $\exists$ has a winning strategy if, no matter what the moves of $\forall$ are, $\exists$ wins. If $\exists$ has a winning strategy such that each game (sequence of moves) ends after at most n moves (of each) then the original set has dimension at most n. If there is no winning strategy for $\exists$ to win

[22] We have replaced Brouwer's π, ρ, ρ', π_1 by P, A, B, S. Note that 0 is not considered a natural number. The problematic and controversial aspects of this definition will be considered in detail in the sequel (see p. 454 ff.)

[23] *Abélard* and *Héloise* in Wilfrid Hodges' terminology. The presentation of the game is somewhat updated.

in at most $n-1$ moves, then the dimension is exactly n (i.e. $\forall$ can choose his moves in such a way that n moves are required for $\exists$ to win). Brouwer's reformulation of the definition of dimension in terms of a game is remarkable because it is (to my knowledge) the first such reformulation of an inductive definition.[24] Here is an example of how the game works for the dumb-bell:

Two solid balls are connected by a line. If $\forall$ chooses closed sets in the left and right hand balls, then $\exists$ can separate them by a point; however, $\forall$ can play a better game—if he chooses two points in the right hand ball, then $\exists$ has to separate them by, for example, a plane section, then $\forall$ can choose two points in this disc, which $\exists$ will separate by a line, and finally $\forall$ may choose two points in the line, which $\exists$ separates by a point. This game takes 3 steps, and indeed, $\exists$ has a winning strategy for a 3-step game, but not for a 2-step game, so the general dimension degree is 3. Far from considering the game definition an exotic toy, Brouwer used it in the following soundness proof.

Brouwer had already observed that the new notion was no good unless one could prove the *dimension theorem*,[25] which states that n-dimensional Euclidean space has general dimension degree n. He had immediately realized that here Lebesgue's tiling principle was the thing to use, and so he wrote down a proof, which was—by the way—short and elegant. He used the occasion to indicate in a footnote the gap in Lebesgue's 1911 proof, and to give a slick proof of the tiling principle by means of his mapping degree. In a footnote he pointed out that, since the dimension (dimension degree, as he called it) was a topological invariant, the invariance of dimension now became an immediate corollary.

At this point the matter rested for some 10 years, until, in 1923, Brouwer to his surprise met a young man who had independently discovered the definition of dimension, and Urysohn was equally surprised to find out that someone (and not just any old topologist) had long ago given a definition of dimension. There must have been some measure of consolation in his quick discovery of an 'irreparable mistake', as he called it.

Brouwer might have switched from topology to intuitionism, but that did not mean that he was willing to acquiesce in the criticism of Urysohn. Being very meticulous in his work, Brouwer was not prone to serious errors, so he felt that he was basically right. It could not be denied that Urysohn's remarks seemed to imply that Brouwer's claim to the right notion of dimension, including the 'soundness proof', was forfeited. So a reaction was in order. As soon as he returned from the Marburg meeting, he consulted his notes, and almost immediately sent a card to Urysohn from Zandvoort, where he spent a short holiday at the seaside. Only Brouwer's draft remains—Urysohn never received the card.[26] It ran as follows:

[24] Cf. [Aczel 1977].

[25] [Brouwer 1913b], p. 148.

[26] He replied (27.XII.1923) that he blamed the loss of this card on the university rather than on the Russian mail. Letters addressed to his house had always reached him, but recently 3 letters, addressed to the university, had gone astray.

> Saturday 29.IX.23 from Zandvoort, pension John Bückmann, written to Dr.P. Urysohn, Mathematisches Seminar der Universität, Moskau, concerning the note in pencil (at the separation definition) in the margin of my personal copy (*hand exemplaar*) of *Über den natürlichen Dimensionsbegriff*. This pencilled note, that clarifies everything, must date back many years; it is very well possible, that it has been made as a result of a remark by a colleague (in that case probably Weyl, Gross or Rosenthal). I shall try to ascertain this and also check, if the note has not already been added as an Erratum to a later publication.
>
> (signed) L.E.J. Brouwer.

Notwithstanding, his conviction that—in view of the correction—his dimension notion was impeccable, he was somewhat worried about Urysohn's handling of the matter. After all, a beginning mathematician does not every day catch a famous mathematician nodding. Brouwer was himself no stranger to this vice. A letter to Bieberbach, one of the editors of the *Jahresberichte*,[27] which routinely published reports of the annual meeting of the German mathematicians, sheds light on Brouwer's worries:

> Furthermore I would like to request the following (which I beg you to pass on to Gutzmer, in case it concerns not you but Gutzmer): In Marburg a Russian, Mr. Urysohn from Moscow gave a talk (which I did not attend), in which he presented my general dimension theory as being untenable on principle. This certainly was unjustified, for my paper in question has, in the course of the years, not only been studied and checked in numerous seminars, but also by very critical and penetrating mathematicians like Weyl, Rosenthal, Veblen and Alexander,[28] without yielding anything but a few omissions, such as occur in any paper, that don't damage the body in the least and the correction of which occurs automatically to the reader: and so the paper has so far always been cited without objections, and now Urysohn doubtlessly has misunderstood it.—Now, if the mentioned Marburg talk or a report of it should be submitted to the *Jahresberichte*, I would like to ask and to advise you to submit the manuscript in question to my inspection (taking into account the abovementioned, and to avoid unpleasant polemics) before sending it to the printer.

It is likely that the letter to Bieberbach predates the card to Urysohn, otherwise Brouwer would probably have mentioned the 'minor correction in the margin' referred to in the card to Urysohn. Brouwer always maintained that the mistake discovered by Urysohn was a minor slip of the pen, one of those little details that any serious reader would automatically correct. In view of the fact that—in spite of all foundational preoccupations—topology had become second nature to him, it

[27] Only Brouwer's handwritten and undated copy survives, but it must have been written between the meeting at Marburg and Urysohn's letter with details.

[28] Even though the paper was apparently discussed in certain circles, it found no follow up in the mathematical literature.

is quite plausible that Brouwer was somewhat amazed at the fuss made about such a little slip in the definition. The above letter also shows that in Brouwer's view the natural dimension paper was not an overlooked item. He had of course sent out reprints to his brethren in topology, and the above-cited mathematicians may very well have read the paper, but *grosso modo* it remained relatively unnoticed until Urysohn brought it to the attention of qualified mathematicians. From a letter to Schoenflies it appears that Brouwer had inserted the corrections in handwriting in the reprints that he had sent out, but which he seemed unable to recover when they would have been helpful.

Anyway, it is evident that right after the Marburg conference Brouwer had difficulty recollecting the precise details as to what happened with the 1913 paper and its corrections. As we will see, he soon found enough evidence to convince himself that he had not overlooked the point raised by Urysohn. Bieberbach apparently complied with Brouwer's request and sent him Urysohn's manuscript and Brouwer did not find any objectionable remarks in it (certainly the published account of the talk contained no reference to Brouwer's oversight[29]) for he wrote to Bieberbach:

> Enclosed, Urysohn's report is returned, with warm thanks. My name is not mentioned in the citations. I hope to have a discussion with Mr. Urysohn (who was in Marburg, probably under the fresh influence of the smear campaign (*Hetze*) led by Hilbert against me), for the time being I would have only one request to you to inform me, should Mr. Urysohn in a possible correction of his report insert my name afterwards, ...

Clearly Brouwer did not wish to take risks; as we have already seen, he was somewhat over-sensitive to real or imagined slurs on his professional status, and he did not want to be ridiculed by a newcomer on account of an ordinary lapse in exactness. He wanted to be prepared if it should come to a public debate, and so without qualms he used his status to get first hand information on a possibly looming battle.

It must have been a relief for him to find out that Urysohn was not after a cheap victory; to some it might have been tempting to correct the high and mighty Brouwer, but Urysohn had no inclination of the sort, nor did he need the doubtful publicity involved in a bit of one-upmanship.

Brouwer may have been right about Urysohn being somewhat prejudiced against him; the remark on Hilbert's campaign against himself is evidence that is not easy to overlook. Brouwer was all too familiar with the goings on at the major universities in Germany (in particular at Göttingen), to remain unaware of an unkind reception of his foundational views. It is unlikely that it was 'all in the mind'.

Urysohn, who had not received Brouwer's card of 29 September, reacted to Brouwer's request for details, which Brouwer had made in Marburg. In a letter of October 24, in which he clearly and precisely pointed out where Brouwer's proof of the 'dimension theorem' failed: 'In Marburg you have called upon me to impart to you the objections which I have made in my lecture to your proof in

[29]JDMV 1923, p. 69 italics.

Crelle's journal.' Urysohn's letter gives a perspicuous and to the point exposition of his objections. It contained a clever counterexample to some claims in Brouwer's proof, Brouwer quoted it in his note in the *Proceedings of the Amsterdam Academy*.[30] Urysohn's letter would have alarmed anybody less well-versed in topology than Brouwer. Criticizing Brouwer's definition of 'connectedness', Urysohn warned, 'I will show below that your proof remains defective under *any* definition of these notions.' The conclusion being that (i) the notion of dimension was not properly defined, (ii) whatever the corrections were, the proof of "dimension degree of R^n is n" could not be saved.'

Brouwer must have been in some confusion how to react. He finally replied in December, apologizing for the delay and explaining that he had hoped to get a reaction to his card of September, so that an exchange of views might be more fruitful. He enquired if Urysohn had received the card at all, and he closed the letter on an unusually complimentary note:

> I have the impression, that the method sketched there[31] leads to an extremely valuable insight into the structure of continua, after trivialities have been published by the dozen in this field during the last years.

For those readers who think that Brouwer is laying it on a bit thick, it must be pointed out that Urysohn was indeed an exceptionally talented mathematician; during the short period of his mathematical activity (only 3 years) he produced a wealth of results and insights. To this day his name is connected with important notions and results in topology. Brouwer simply recognized a young genius when he met one (or read his work).

In the ensuing correspondence Brouwer and Urysohn promised to exchange reprints; it is characteristic of the hardships of the time that Urysohn could not afford to pay the price for the usual number of reprints of the *Comptes Rendus* notes, so he promised Brouwer to copy them for him by hand.[32]

In a letter of 22 January 1924,[33] Brouwer set himself to discuss the dimension paper in some detail. He began by repeating the content of the lost card:

> After returning home from Marburg, your objection raised there, became immediately clear to me by consulting my personal copy of the paper *Ueber den natürlichen Dimensionsbegriff*, where I have an old note in the margin of p. 147, lines 17–20, according to which this passage is 'to be brought into conformity with page 150 at *)'.[34] It was this note in the margin that my card, mailed from Zandvoort, referred to.

[30] [Brouwer 1924b].

[31] I.e. in the *Comptes Rendus* notes of 1922.

[32] The copies are still in the Brouwer Archive.

[33] Brouwer's copy (handwritten).

[34] '*mit S. 150 bei *) in Uebereinstimmung zu bringen*'.

He then entered into Urysohn's objections; in the first place he remarked that the notion of domain (*Gebiet*)[35] in his older papers had been used in a way that fixed the meaning in the natural dimension paper of 1913 in the second place, he agreed that the key to the gap in the proof was the separation definition. Here Brouwer recounted the history of the paper and its sequel:

> As to the origins of the oversight (*Versehen*) of p. 147,[36] my records of that time make it probable that in the manuscript of the paper there had originally not been an explicit definition of separation, like e.g. in my paper 'Proof of the invariance of dimension', published in Math. Ann. 71, and that such a definition was inserted rather thoughtlessly, after a co-reader of the proofs had pointed out the absence to me. When, not long after the publication of the paper, the oversight became clear, a quick correction must have remained forthcoming, because I expected soon the publication of the paper, on the same topic, promised by Lebesgue and mentioned on p. 151, and I was convinced that this paper would require a rejoinder, which would accommodate in a natural way the required correction. When subsequently the paper promised by Lebesgue kept us waiting year after year, the matter disappeared in the course of the years gradually from the realm of my thoughts, and without your interpellation I would perhaps never have thought of it again. I have now also, as a result of your remarks, studied the statement of Lebesgue, published with a delay of 10 years (and not, as agreed, in the Bull. de la Soc. Math., but in the second volume of Fundamenta Mathematicae, and seen that it, exactly as I expected already 10 years ago, calls for a rejoinder on my part, because the proof of Lebesgue of the auxiliary theorem of p. 150 of 'On the natural notion of dimension', only offers a botched up form of my proof of the same theorem. I hope that this rejoinder will appear soon. It will contain (mentioning your priority) at the same time the correction of my old slip. [...]
>
> To be sure, my own investigations are oriented since some years in another direction, but my interest in topology has remained and I consider you as one of the few, who could here open up new perspectives.

Within a month Brouwer mailed a manuscript, containing the correction to Urysohn.[37] What, in fact, was this mysterious 'oversight'? This is spelled out in Brouwer's revised paper,[38] which provided an argument based on a seemingly trivial correction. The cause of all problems and confusion was the erroneous insertion of the adjective 'closed' in the separation definition.

Consider three disjoint closed sets A, B and S. S is said to separate A and B if every closed connected set meeting A and B also meets S. By deleting the last

[35] A domain is an open, connected set, see e.g. [Brouwer 1910] p. 169.

[36] [Brouwer 1923d].

[37] Brouwer to Urysohn, 19.II.1924.

[38] [Brouwer 1923d].

occurrence of 'closed' the standard separation notion is obtained. What Brouwer claimed in the above letter, was that the separation definition was added at the proof-reading stage, and that the insertion of 'closed' was a slip made in haste. We will return to this slip later. In the published corrections Brouwer used a somewhat different version of separation: S separates A and B if S determines a domain containing A, but not B.

Both corrections were noted by Urysohn in his letter of 24 October. In fact, the modern reader will find Urysohn's formulation of the separation definition more congenial.

The old definition stated that any closed connected set 'running from A to B' must intersect S (A and B disjoint). By dropping the 'closed' from the definition, S has to block more 'connecting' sets, so the notion of separation becomes stronger. In other words, it becomes easier for $\exists$ to win the dimension game. We therefore speak of a weaker dimension.

Indeed, Urysohn's example (reproduced in [Brouwer 1924b]) exhibits a separating set that meets all closed connected 'intersecting' sets, but not all 'connecting' sets which are merely connected.[39]

The alternative separation definition does not need this barrier between A and B: A and B are separated if there is an open connected set S such that $A \subset S$ and $B \cap S = \emptyset$.[40]

This letter is followed by a long silence on the part of Urysohn. So long indeed that Brouwer started to worry: did Urysohn get the letter; was Urysohn not convinced? One should keep in mind that Brouwer's reputation was at stake. He had become the paragon of exactness, as one may infer from a letter from Carathéodory to Hilbert in 1912: 'You know how often, if you make an exception for Brouwer, in this part of mathematics people are sinning.'[41] With his high standards of exactness, he did not hesitate to publish his own corrections. But it was more than he could bear to seeing the crowning paper of his topological days reduced to a 'nice try'. The more so, as he was certain that the criticized point was no more than a slip of the pen, a detail that every well-informed reader would correct in passing. When Urysohn's reaction remained forthcoming, he took minor preventive action. At the Marburg meeting Urysohn had made it known that a comprehensive treatment of the notion of dimension (and much more) was to appear in *Fundamenta*, so any criticism that had been communicated privately to Brouwer could be expected to be made public in this paper. And so Brouwer wrote to the editor of *Fundamenta Mathematicae*, Sierpinski, that the substantial paper, submitted by Urysohn, might contain a criticism of Brouwer's dimension paper that was based on a 'small isolated error' in the definition of a technical notion, and that a

[39] Cf. [Johnson 1981], p. 173.

[40] In Brouwer's formulation 'S separates A and B if S determines an open connected set S' such that $A \subseteq S'$ and $B \cap S' = \emptyset$. The modern formulation runs as follows: 'S separates A and B if the complement of S is the disjoint union of two open sets A' and B', such that $A \subseteq A'$ and $B \subseteq B'$.' Cf. [Hurewicz and Wallman 1948].

[41] Carathéodory was discussing topological papers of his for the *Annalen*. 5.V.1912.

correction of this definition answered all objections of Urysohn.[42] He conjectured that the unreliable Russian postal system might be the reason for the breakdown of the communication between Urysohn and him:

> Being convinced that the editorial board of the Fundamenta Mathematicae, just like me, wants to prevent avoidable polemics, I am informing you for the present that, except for the above mentioned correction, my memoir 'Ueber den natürlichen Dimensionsbegriff' is perfectly in order. I hope therefore that you will be so kind to see that the work of Urysohn which you must publish, contains no unfounded criticism.

It turned out, however, that Brouwer's intervention with Sierpinski was not necessary. Urysohn's reply showed that he took Brouwer's point:[43]

> I admit that your new definition[44] is the only one that is connected with the remaining contents.[45]

He added that he failed to see why Brouwer called this notion of separation 'the usual one',[46] for in publications before the natural dimension paper separation was only used for manifolds—'where the two existing notions are identical'.

The matter may appear somewhat confusing, but one can visualize a separation of (say) two points x and y by a circle, and more generally by a set that is connected. Now 'the more connected' this set is, the harder it is to get from x to y. So if you cannot get from x to y past a weakly connected set (a thin wall) you can certainly not get from x to y past a strongly connected set (a thick wall) and so if two sets (or points) are already separated by a weakly connected set, they are strongly separated. Without following Urysohn's criticism in detail let us remark that the crucial point was the notion of *separation* and hence also that of *connected set*. As the episode of Urysohn's critique (and the later controversy with Menger) rests on the question 'was Brouwer aware of the correct notion of connectedness?', we will take a somewhat closer look at the matter.[47]

The first idea that comes to mind when considering connected sets in the plane is that of 'arcwise' connectedness, i.e. a set is (arcwise) connected if any two of its points can be connected by an arc or a path (i.e. continuous image of the closed interval [0,1]). This idea goes back to Weierstrass, who used connection by a polygonal line. The next step towards a more abstract notion was taken by Jordan, who

[42] In the letter to Sierpinski (25.III.1924), Brouwer cited a specific place in his paper on the Jordan curve theorem ([Brouwer 1910], p. 170, line 9), where one of the basic terms is introduced. Its role becomes clear in his correction, e.g. [Brouwer 1924c].

[43] Brouwer to Sierpinski 28.III.1924. 'After waiting for more than three months I finally received a sign of life from Urysohn, which makes my letter of 25 March superfluous.'

[44] of separation.

[45] Urysohn to Brouwer 20.III.1924.

[46] 1923 D2, footnote 11.

[47] We follow Freudenthal's exposition in [Brouwer 1976].

defined a connected closed set as one that could not be split into two non-empty disjoint closed sets.[48]

Schoenflies adopted the same definition in his *Bericht* of 1908 (without the 'non-empty' clause), and he gave separate definitions for open and closed connected sets. As late as in 1918 Carathéodory followed this example in his lectures on real functions.

The present notion, which says that a set A in a topological space is connected if it cannot be written as the disjoint union of two open (or closed) sets in the relative topology of A, was formulated by Lennes,[49] and (almost certainly) independently by Brouwer in the same year.[50] There is also an elegant completely general definition of connectedness in Brouwer's letter to Engel.[51]

Hausdorff's book,[52] the bible for whole generations of topologists, spelled out the modern definition, and so the notion is generally ascribed to him, but Brouwer would consistently refer to Lennes.[53] In a private note (in the handwriting of Cor Jongejan) he remarked:

> I only started to quote Lennes after I observed that the definition of connectedness which was in the old days considered obvious, was blown up into a discovery by a later generation.

Anyway, there is no doubt that Brouwer knew the general notion of 'connectedness'. When Karl Menger suggested in the late twenties that Brouwer in 1913 did not know Lennes' paper (and hence the above notion), Brouwer even produced letters showing that he had refereed the paper of Lennes that was a sequel to the 1911 paper.[54]

Actually, virtually unknown to the topologists of the day, Frigyes (Friederich) Riesz had already given the correct definition in a long paper, which was unfortunately published in an inaccessible place (1906).[55] So, as far as one can judge, he should also be credited with the modern notion.

The notion of connectedness was really important for the definition of dimension, since both the notions of 'separation' and 'domain' were based on it, as Urysohn had seen at once.

[48] [Jordan 1893], p. 25.

[49] [Lennes 1911] We know that Brouwer had seen Lennes' definition in the subsequent publication of Lennes in 1912.

[50] [Brouwer 1911], p. 308. 'Eine innerhalb κ abgeschlossene Punktmenge [π] soll *zusammenhängend* heißen, wenn sie nicht in zwei innerhalb κ abgeschlossene [(oder relative abgeschlossene)] Teilmengen zerlegen lässt.' The insertions are Brouwer's, made in the margin of his private copy.

[51] A set X is connected if for any assignment of neighbourhoods U_p to points p in X and for any two points p and q there is a finite sequence $p = p_0 = \ldots = p_n = q$, with $p_i \in U_{p_{i+1}}$. Brouwer to Engel, 9.III.1912, CW II, p. 149.

[52] *Grundzüge der Mengenlehre*, 1914.

[53] Cf. Freudenthal in [Brouwer 1976], p. 487.

[54] Blumenthal to Brouwer 3.XII.1912, 12.II.1912 cf. [Brouwer 1976], p. 487.

[55] [Riesz 1960].

In his manuscript of the revised dimension paper Brouwer had explained to Urysohn that he had indeed meant by 'connected' and 'domain' the correct notions all the time, and that the fateful word *abgeschlossen* (closed) was a slip of the pen, and that he always had the intended concept of separation in mind, cf. p. 455.

For a punctilious man like Brouwer the matter of the oversight presented an awkward situation, to say the least. He must have felt that outsiders would and could consider his claims as 'fiddling the books of history'. Freudenthal, in his edition of Brouwer's topological work, has carefully sorted out the evidence, and although Brouwer's case would probably have a hard time in a court of law (for after all, a devious person could fake the historic evidence), it looks strong enough to be adopted as the correct one.

In the first place, Brouwer had corrected his reprints in handwriting by inserting 'connected in the sense of Lennes', in the margin of the definition in question. In his private copy the word *abgeschlossen* is struck out with pencil, and commented 'to be deleted conform footnote *) p. 150' and finally, Brouwer had added a note in the proofs of Schoenflies' new edition of the *Bericht* (1913) on p. 382: 'similarly the investigations of Brouwer in *Math. Ann.* 70, p. 161–165 and *Journ. f. Math.* 142, 146–152 (in the latter one the word 'closed' on p. 147, line 18 has to be deleted, according to a communication of Brouwer)'. That particular part of the proof sheets of Schoenflies' Bericht is still in the Brouwer Archive and it shows a handwriting that fits his style of writing in the early teens of the century. The new edition of Schoenflies' *Bericht* did not contain this footnote. The reason for this is unclear, perhaps Schoenflies thought the note irrelevant (and he would have a point), perhaps the printer missed it. It had escaped Brouwer's eye, for only after Urysohn's intervention did he find out that it had not been adopted.

Urysohn, evidently, was sufficiently satisfied to drop the matter and to accept Brouwer's views and hence his priority for the definition of dimension. It should be pointed out that Brouwer's and Urysohn's definitions are in general not equivalent. There is also a technical distinction: Brouwer gave a *global* definition of dimension, i.e. for a space in its totality, whereas Urysohn gave a global and a *local* one (where dimension is defined in a point x, i.e. by applying the separation procedure in arbitrarily small neighbourhoods of x).[56]

The, for Brouwer so desirable, peace was suddenly disrupted when on June 21 a registered letter from Göttingen was forwarded to Brouwer, who was at that moment conducting gymnasium examinations in the country (the traditional, time consuming but useful, voluntary task of Dutch professors). It contained an urgent message from Urysohn, and a proof sheet of a paper that Urysohn had submitted to Hilbert for the *Annalen*. It was entitled '*Über den natürlichen Dimensionsbegriff*', and contained an exposition of the mistake in Brouwer's eleven year old paper with the same title. The note leaves the impression that Brouwer had utterly failed the goal he had set himself. Hilbert must certainly have felt a certain quiet amusement, seeing his (by now) arch-enemy stumble, especially where Brouwer had always shown

[56]Cf. [Johnson 1981], p. 178.

little patience with the weaknesses of others, such as Lebesgue, Engel, and Schoenflies.

But it is always easier to attack the weaknesses of a person you don't know personally, than those of a man you have come to appreciate and like, so Urysohn, somewhat embarrassed, wrote Brouwer that he had already submitted this small note to the *Annalen* in July (thus before the Marburg meeting), and since then had completely forgotten about it. So when he suddenly received the proofs he was at a loss:

> It is completely unclear to me what I should do with it, perhaps the 'supplement added in the proof' will satisfy you.

The supplement ran as follows:

> In the preceding claims I have obviously based myself on the assumption that one sticks to the definition of dimension of Vol. 142 of Crelle's Journal. Now, Mr. Brouwer has since then published a correction,[57] where he indeed changes the notion of separation on which the notion of dimension is based. *In that way* the proof has been corrected completely, and I would like to stress particularly that, as I have been informed, the necessity of such a modification of the definition of separation had been known already for a long time to Mr. Brouwer, and that its publication has not been carried out due to a lapse. Nonetheless, I believe that the above lines may be useful, since Mr. Brouwer, in his Correction, has not indicated *why* the old definition has to be rejected.
>
> Göttingen, 21 June 1924.

Brouwer acted immediately; he elaborately explained to Urysohn that publication would be unwise:[58]

> Many thanks for sending me the proofs of your forgotten small note for the Annalen and for seeking my advice on it. It is my opinion, that in both our interests, the publication of this note should absolutely be refrained from. For the publication of an oversight, which had escaped the notice of scholar B by scholar A, is only then compatible with the dignity of scholars, if either the oversight can only be grasped by means of an extensive exposition of new discoveries of A, or if all consultation between both parties involved has become impossible (e.g. on political grounds or because of the death of B). In each other case such a publication creates a suspicion that either A allows himself to be carried away by imprudent ambition, resp. deliberately wants to offend B, or that B did not want to acknowledge his oversight to A, resp. has refused public acknowledgement, at least to fullest extent. Fortunately, none of the above circumstances exists in the present case, rather the contrary, in every respect.

[57] [Brouwer 1924c, Brouwer 1923d].

[58] Letter of 24.VI.1924.

It would be useful, he continued, to publish your counterexample, but that would find a natural place in a note in which I am going to exhibit the evidence of my early correction. He added that he hoped that Urysohn would agree with the retraction of the note, saying that the matter with the editorial board and the publisher of the *Mathematische Annalen* could easily be arranged by himself, being after all an editor.

Actually, Urysohn's letter and the proof sheet did not really catch Brouwer unawares. It is not unlikely that the Alexandrov and Urysohn manuscripts were, after all, passed on to Brouwer for advice. The small note mentioned above must also have passed through Brouwer's hands, since Brouwer contacted Blumenthal, before he had received the letter of Urysohn, asking permission to deal with Alexandrov and Urysohn directly in the name of the editorial board, and Blumenthal replied on 14 June that the decision was not his; Hilbert had to be consulted about the matter. On the same day, in a letter, he had suggested that Hilbert authorize Brouwer to discuss the matter of the dimension note directly with Urysohn, and that on account of the alleged error the paper might after all have to be rejected. He also suggested that it would be wise to send Brouwer the already accepted manuscripts of Alexandrov and Urysohn so that he could if necessary make corrections—'for this single example makes me apprehensive.' In his letter to Brouwer he carefully added that Brouwer could only correspond with Urysohn in his own name; since he was personally involved in the matter of the definition of dimension, it would not be fitting to act in an editorial capacity. Hilbert's reaction has not been preserved, but it is likely that he gave Blumenthal his fiat. So when Brouwer wrote to Urysohn, he was not violating any editorial guidelines. He must have felt confident—on account of Urysohn's letter—that an agreement would be forthcoming; indeed he informed Blumenthal that Urysohn was about to agree that his paper was not suitable for publication. Should some more persuasion be necessary, he wrote, then Blumenthal could tell Urysohn that Brouwer would pay the bill for the now useless proofs, and that the editorial board 'would obviously follow in this matter the judgement of the only involved and only qualified member'. We see that Brouwer was not totally adverse to some muscle-flexing; the cause justified this, as Brouwer had already refuted Urysohn's claim that Brouwer's mistake was beyond repair; moreover Urysohn had accepted Brouwer's explanations. In Urysohn's words, 'I have received your card of the twenty second and your letter of the twenty fourth of this month; I am very grateful to you that you have given me so kindly and extensively your opinion. I completely agree with everything that you have written.' It would certainly have been a serious blot on Brouwer's topological record if Urysohn had been right. So this episode in the history of dimension ended well. It remains a bit of a mystery why Hilbert did not assign the papers of Alexandrov and Urysohn to Brouwer right away. One would expect that in a well-run editorial board manuscripts would be handled by the recognized experts, and certainly in a tricky subject like topology.

In the Brouwer–Urysohn correspondence there is a particular–interesting letter, which plays no role in the discussion, but which sheds more light on the confusion of notions. On 14 June Brouwer wrote:

> Perhaps the enclosed variant to the passage in Crelle's journal 42, between p. 149, l.2 bottom and p. 150, line 10 bottom, by means of which the proof, is adapted to the separation definition found on p. 147, (thus without the deletion of the word 'closed') will interest you. [...]
>
> This variant, which I recently found among my papers from the years 1912 and 1914, is most probably communicated in the correspondence that I pursued at the time with Schoenflies, Gross and others about, among the things, dimension. I will see if perhaps the other parties have filed this correspondence better than I have. My own interest has remained turned away a full 9 years from these topics, and as an archivist I have always been a failure.
>
> I consider, by the way, as before the separation definition without the word 'closed', as the proper and more fruitful one for dimension theory.

The enclosed part of a proof was submitted two days later to the *Mathematische Zeitschrift*, and published that same year.

The upshot of this message was that Brouwer had at an earlier period investigated another notion of separation (weak separation), which led to another (strong) concept of dimension. Freudenthal, who had no access to the Brouwer–Urysohn correspondence, noted in his comments on the dimension episode, 'The fine distinction between weak and strong connexion may appear as an *a posteriori* implantation—such subtleties in the stone age of topology! Yet, Brouwer was subtle.'[59] The above letter proved Freudenthal right. From the argument of the paper in *Crelle* and Brouwer's use of the correct notion of connectedness, one may well conclude that he indeed had the correct (weak) dimension notion in mind, and the 'slip' was really a slip. The fact that Brouwer discussed the above strong dimension is probably a partial explanation of the confusion, and also a demonstration of his undiminished topological powers. Both dimension notions are mentioned again in [Brouwer 1924b] and [Brouwer 1928d].

Urysohn did not react to Brouwer's alternative notion (at least no written evidence is extant).[60]

When Urysohn received Brouwer's letter concerning the note for the *Mathematische Annalen*, he suddenly recalled that in the manuscript of his paper '*Mémoire sur les multiplicités Cantoriennes*', submitted to *Fundamenta Mathematicae*, a similar

[59] [Brouwer 1976] p. 551.

[60] For a thorough analysis of the various notions the reader is referred to Freudenthal's commentary in the Collected Works II, p. 548 ff. and [Johnson 1981] p. 171 ff.

reference to Brouwer occurred, and so he hurried to ask Sierpinski to replace the reference by a revised one.[61] The original reference to Brouwer was stern in tone:

> I lately learned about a paper of M. Brouwer (*Über den natürlichen Dimension begriff*, ... where he proposes to solve this question by a method which seems (at first sight, at least) very close to mine. Now, the proof of M. Brouwer contains an error which it seems to me, cannot be corrected, and which undermines all his results. I refer for the details to a supplement to be found at the end of the first part of the present mémoire.[62]

Urysohn pointed out to Sierpinski[63] that although the remark was justified with respect to the original paper, it would be out of place after Brouwer's correction.[64]

> Analyzing my 'Brouillon', I have seen that in the introduction to my 'Mémoire sur les multiplicitées Cantoriennes' I have written about Brouwer's 'Natural notion of dimension' an observation, in which I wrote roughly the following: 'Now the proof of this theorem contains an error which, it seems to me, cannot be corrected.' Although this observation is justified with respect to the old formulation of Brouwer's paper in his correction[65] he changes the *definition of the notion of dimension*, it seems to me that the publication of my observation would not be appropriate. Therefore I permit myself to beg you to modify this observation: if possible to replace it by the observation below, if it is already impossible on technical grounds, to delete it at least altogether. Here is the text of the desired observation: the memoir was already finished when I got to know the paper 'Über den natürlichen Dimensionsbegriff', published by Brouwer in 1913 in the Journal für [die reine und angewandte] Mathematik (v.142, p. 146). I hope to return to the definition of Brouwer and mine.

The letter must have reached Sierpinski, since Menger at some later date acquired a handwritten copy. The first proofs of Urysohn's paper do not contain the desired correction, so either Sierpinski thought the matter of marginal importance, or the proofs were ready before he could inform the printer.

The correspondence between Brouwer and Urysohn (often in combination with Alexandrov) continued right up to the death of Urysohn. The two Russians left Göttingen first for Bonn to visit Hausdorff. After repeated fruitless visits to the Dutch consulate in Cologne, they finally got a visa for an extended trip to Holland and continued their journey. The two stayed in Blaricum with Brouwer. From this

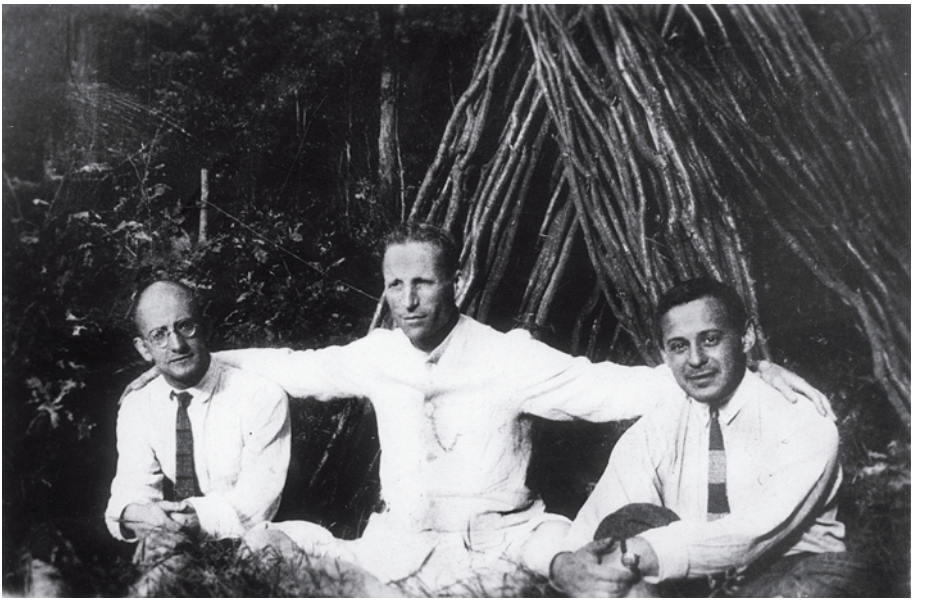
[61] This request was made in a letter mailed from Göttingen, so it was not written under the supervision of Brouwer. Urysohn to Sierpinski 27.VI.1924.

[62] Copy in the Menger Archive.

[63] Urysohn to Sierpinski 27.VI.1924 (Menger Archive)

[64] [Brouwer 1923d].

[65] [Brouwer 1924b].



Alexandrov, Brouwer and Urysohn in the garden at Brouwer's hut (1924). (Brouwer Archive)

visit there is a pictorial witness: a photograph of Brouwer sitting between his two young friends in the garden.[66]

Brouwer was so impressed by the two young visitors that he insisted that they should come back in October and spend the year in Holland.

Unfortunately Brouwer had a previous engagement in Göttingen,[67] so that he had to leave the two Russians behind. Urysohn and Alexandrov tried to make the best of it, and they went as proper tourists with Cor Jongejan and a lady friend of hers to Amsterdam. From Amsterdam they sent a postcard to Brouwer, lamenting his absence. The Russians must have made a somewhat curious impression on the Dutch, as appears from the fact that they were kicked out of the Rijksmuseum on the grounds that they did not wear a jacket.

From Amsterdam they moved on to Paris where they met Fréchet. Their next stop was in Brittany, in Batz, a small fishing village at the end of the world. Urysohn reported their experiences in a letter:[68]

Hochgeehrter und lieber Herr Professor;

[66] See Collected Works II, p. 453.

[67] He was invited for a talk on the foundations of mathematics; it was one of the exchanges in the Brouwer–Hilbert debate, commonly known as the *Grundlagenstreit*.

[68] Urysohn and Alexandrov to Brouwer, 29.VII.1924.

> Only now we finally got down to writing a letter. In Paris we walked around every day from 9 in the morning until 10 at night[69]—for apart from the city and the museums there was still the police headquarters, which gave us troubles, and the German consulate, where we asked for a transit visa for the return journey etc. After 4 days we got so tired,[70] that we made the decision to postpone the continuation of Paris until the return journey (Urysohn), resp. until eternity (Alexandrov). We have come here the day before yesterday, and it took us a whole day before we could find a quiet place at the coast. [...] With our best wishes for you and the two ladies. Paul Urysohn Paul Alexandrov
> Le Batz (Loire Inférieure), Pension de famille 'Le Val Renaud'.

Alexandrov and Urysohn had in the meantime contemplated Brouwer's invitation to spend a year in Amsterdam. Being exceptionally bright, they could have spent a year almost anywhere, in particular Göttingen must have been on their mind. For any young mathematician that place offered glimpses of mathematical heaven. It says something for Brouwer that they decided to come to Amsterdam. Their curricula vitae and a formal letter of acceptance were enclosed with their letter.

Brouwer answered by return post, reporting about the reception of his talk,[71] and asking Urysohn to add his signature to the preceding letter.

The letter was immediately answered by Urysohn and Alexandrov, and the missing signature was provided.[72] The letter is written in a jocular tone, and it closes with the promise to tell Brouwer all about France at the meeting of the German Mathematics Society in Innsbruck.

A week later they sent a picture postcard from the Pointe du Raz—'Many warm greetings from the place here at the picture, which we climbed and swam around in all directions'.

Returning from Cap Finisterre to Batz they moved into a small cottage at the sea shore, where they alternatingly pursued their research and went swimming. On August 15 and 16 the sea started to get rougher and 'our swims became more and more interesting', as Alexandrov put it. On the 17th Urysohn had thought out the proof of his famous metrization theorem and he managed to finish writing down the first page of the paper '*Zum Metrizationsproblem*'. At 5 o'clock the two went down for their customary swim.

The dramatic consequences were recorded in Alexandrov's 'Pages from an autobiography':[73]

[69] With the greatest pains: Paris is even more horrible than I could ever have thought. [Urysohn's note]

[70] and Alexandrov had cursed so much and has become so unbearable [Urysohn's note]

[71] See p. 491.

[72] Alexandrov and Urysohn to Brouwer, 4.VIII.1924.

[73] [Alexandrov 1980]

When we got into the water, a kind of uneasiness rose up within us; I not only felt it myself, but I also saw it clearly in Pavel. If only I had said, 'maybe we shouldn't swim today?' But I said nothing … After a moment's hesitation, we plunged into a not very large shore wave and swam some distance into the open sea. However, the very next sensation that reached my consciousness was one of something indescribably huge, which suddenly grabbed me, and this sensation was accompanied by the rather absurd but quite precisely formulated thought: had this wave come to me all the way from Venezuela to no useful purpose here? A moment later I came to myself on the shore, which was covered with small stones—it was the shore of a bay, separated from the open sea by two rocks between which we had had to swim as we made our way to the open sea. I had been thrown over by a wave, right across these rocks and the bay. When I was on my feet, I looked out to the sea and saw Pavel at those same rocks, already in the bay, in a half-sitting position. I immediately swam up to him. At that time I saw a large group of people on the shore. (It was a Sunday, and many people from various places had come to Batz to admire the sea.) After swimming to Pavel, I put my right arm around him above his waist, and with my left arm and my legs I began to paddle to shore with all my might. This was difficult, but no one came to my assistance. Finally, when I was already quite near the shore, someone threw me a rope, but within a few moments I reached land. Then eyewitnesses told me that the same great wave that had thrown me across the bay had struck Urysohn's head against one of the two rocks and after that he had begun to roll helplessly on the waves in the bay. When I pulled Pavel to the shore and felt the warmth of his body in my hand, I was in no doubt that he was alive. Some people then ran up to him, and began to do something to him, obviously artificial respiration. Among these people, there happened to be, as I was later told, a doctor, who apparently directed the attempts at life-saving. I do not know and did not know then how long they continued, it seemed like quite a long time. In any case, after some time I asked the doctor what the condition of the victim was and what further measures he proposed undertaking. To this the doctor replied 'What do you want that I should do with a corpse'.[74]

As I now remember, the only thought that entered my mind when I heard these words was that the word 'fasse' is the 'présent de subjonctif' form of the verb 'faire' and that our French teacher at my school had often asked us for this form and for the subjunctive in general.

Some more time passed, and I went into my room and finally dressed. (Until then I had remained in my swimming clothes.) Pavel Urysohn lay on his bed covered by a sheet there were flowers at the head of the bed. It was here that I thought for the first time about what had happened. All

[74] 'Que voulez vous que je fasse avec un cadavre?'

> my experiences, all my consciousness, with such distinctness and clarity. All this merged into a single awareness of how good, how exceptionally good, things had been for each of us, only about an hour ago.
>
> And the sea raged. Its roaring, its crashing, its bubbling, seemed to fill everything. The next day, I sent telegrams to Brouwer, and to my brother Mikhail Sergeevich, in Moscow, whom I asked to tell the Urysohn family about what had happened. That same evening I received in reply a telegram from Brouwer with the words 'Appelez-moi où vous voulez.' I asked Brouwer to come to Göttingen, where I planned to stop for a few days on my way to Moscow.
>
> The funeral was on 19 August. In the belief that it would accord with the wishes of Urysohn's father, I asked a rabbi to perform the funeral rites. As far as the funeral itself is concerned, I remember the huge number of people who came to it, the pile of living flowers on the new grave, and the noise of the sea, which could be heard even in the cemetery. On 20 August I left Batz and after stopping in Paris for a day I arrived in Göttingen on the 22nd, where Brouwer, Courant and Emmy Noether awaited me. Hilbert and Klein asked me to come and see them. This was my last meeting with Klein. He died in the summer of the following year.

The death of Urysohn was a shock for everybody who had known him; many mathematicians who had met him during his short presence on the European scene of mathematics sent their condolences to Alexandrov.

Brouwer, in particular, suffered from the terrible blow. He had in their all too brief acquaintance come to see Urysohn as a precious gift from heaven, as the predestined person to carry on topology from where he had left it.

The little time Alexandrov, Urysohn and Brouwer were together was spent with intense discussions on Urysohn's recent work (and undoubtedly Alexandrov's work as well). They discussed in particular Urysohn's dimension theory and his big memoir for the *Fundamenta*. Knowing Brouwer's principles were historical credit and detail were concerned, it was no surprise that considerable attention was paid to these aspects of the paper. The conversations confirmed Brouwer's impression, created by the correspondence; here was the spiritual son he had not even dared to hope for. Indeed, all reports of Urysohn's personality bear witness to a genuine geniality and to an exceptional, sympathetic character.

Long after Urysohn's death, Brouwer expressed his feeling in a moving letter to Paul's father.[75]

> Dear Mr. Urysohn,
>
> I thank you for your kind and trusting letter. I believe I can understand your feelings, precisely because of the deep and almost mystical impression Paul left in my mind. He must have united in a rare way the best from you and from his mother's nature, and indeed in such a mature

[75] Brouwer to Urysohn Sr. 14.II.1925.

way, that already during his lifetime his soul almost seemed to dream and to float over the earth. This particular impression put itself, while we were together, alongside my admiration for his considerable mathematical achievements, which I felt for a long time. In particular for the powerful and surprising new life that he infused into the scientific subject topology, cultivated in the past by me.

It almost has the appearance as if there were a transcendental causal connection between his superterrestrial state of mind and his short meteor-like corporal existence, and as if death had been for him more a waking up than a falling asleep.

May you, heavily tried father, in awareness and in certainty of the epic beauty of his short earthly course of life, find the solace that I wish you so much with all my heart.

In case you should want to go this summer to the resting-place of your dear deceased, and I can be of any assistance in making this journey possible, please dispose yourself of me without any reservation.

I remain in warmest sympathy
Yours sincerely
L.E.J. Brouwer

Although the paths of Urysohn and Brouwer joined only briefly, Urysohn made a lasting impression and Brouwer with his characteristic impetuosity set himself the task to protect the memory and reputation of Urysohn.

12.3 The Viennese connection

At the time that Urysohn was working out his ideas on dimension, another young man was turning his mind to the same problems. Karl Menger, an Austrian student and son of the famed economist Carl Menger, had the good fortune to be in his first year at the University of Vienna when Hans Hahn joined the faculty.[76] Hahn, born in 1879 in Vienna, studied mathematics in Strassburg, München and Vienna, where he got his Ph.D. in 1902. After a sequence of short appointments at and visits to Göttingen, Vienna, Innsbruck and Czernowitz he was drafted into the army, where he sustained serious injuries and was subsequently decorated and discharged in 1916.

After a five year term as a professor in Bonn, he returned to Vienna where he stayed until his death in 1934.

Hahn was a versatile mathematician with wide ranging interests. He published in a great number of areas, among others variational calculus, function theory, set theory and topology, real functions, Fourier series and foundations of mathematics. His book *Reelle Funktionen* became the standard work for a generation of mathematicians. Hahn's philosophical interests led him to take part in some of the sessions of the Vienna Circle. One of his first actions in Vienna was the introduction

[76] [Mayrhofer 1934].

of a mathematics seminar for students. The first topic was the theory of curves. Menger fell immediately for this fascinating subject. He started to work out his ideas and within a week presented Hahn with a proposed solution. Encouraged by Hahn, he pursued his ideas further and in June 1921 he handed in a short note with the basic notions of curves, including the notion of one-dimensionality.

Overtaxing himself in a situation where the consequences of the war were still noticeable in the form of lack of heating and proper food, he fell ill and had to spend more than three semesters in a sanatorium. Nonetheless he finished his first paper at the end of 1921. In the preceding fall of 1921 Menger felt so worried about the state of his health and the prospects for future research and publications that he decided to deposit his main results with the Austrian Academy, remembering that his father once told him that one of the roles of academies was to act as trustees of people's ideas by accepting sealed envelopes and testifying later to their contents and the date of their deposition.[77] Menger's document contained the discussion of a number of basic concepts of topology, their definitions and properties.[78]

A copy, authenticated by a notary, is in the Brouwer Archive. The document contained under the heading I. *On the dimensionality of Continua (Draft)*[79] the following items:

§1. The notion of curve
§2. Theorems about curves
§3. The notion of surface
§4. A continuum is called n-dimensional if D is $(n-1)$-dimensional[80]

Menger's definition of dimension, although equivalent to that of Urysohn, had a very appealing form. It did not make use of any form of separation, but directly made the induction step by reducing a neighbourhood to its boundary. Here is Menger's definition: A set is at most n-dimensional if any of its points has arbitrary small neighbourhoods with at most an $(n-1)$-dimensional boundary. It is a routine matter to define 'has dimension n'.

The note ends with the remark that the definition of curve, surface and dimension were already given by him in April 1921, as could be confirmed by Hans Hahn and Menger's fellow student, Otto Schreier.

The paper 'On the dimensionality of point sets', that was to become the first published account of Menger's theory of curves and dimension, was submitted to the *Monatshefte für Mathematik und Physik*, the Austrian journal for mathematics and physics, and it was read by Hans Hahn, one of the editors and definitely

[77] [Menger 1979], p. 251.

[78] Sealed document No. 778 (1921) Subsequently published in Menger 1929, part 1.

[79] *Über die Dimensionalität von Kontinuen.* ('*Zur Theorie der Punktmengen*' in [Menger 1929], the change of title is not explained).

[80] D is assumed to be the boundary of the intersection of a sufficiently small neighbourhood with the continuum.

the most competent referee one could wish for. Hahn discovered a mistake which Menger could not repair (for good reasons: Sierpinski had already provided a counter-example), and although Menger in a letter to Hahn[81] suggested a solution by changing the definition of 0-dimensional, the paper was not published in its present form.

At the time of the writing of his paper, Menger was still unaware of Brouwer's dimension paper of 1913. This explains why Menger erroneously ascribed to Brouwer the viewpoint that 'dimensionality is a property belonging to certain point sets derived from the possibility to map them one-one and continuous onto certain other ones.' In the days before the general notion of dimension was made explicit, geometric dimension was considered only in the traditional setting of Euclidean spaces, and Brouwer's invariance of dimension must be viewed in this light. Menger's above characterization would rather apply to Baire's work.

Menger's poor health kept him in the sanatorium until April 1923; roughly at that time he became aware of Brouwer's 1913 paper.[82] Later in the year he found out that Urysohn (who was virtually unknown at the time) had presented a talk at the Marburg conference in September; he hastened to get hold of the *Comptes Rendus* notes and was just in time to insert a reference to Urysohn in the proofs of his first published paper.

Upon publication of this paper 'On the dimensionality of point sets, part I', he mailed reprints to various mathematicians, and got in exchange a number of reprints from Fréchet, Brouwer, and the Warsaw School. Brouwer obviously was interested because of the close similarity with his own and Urysohn's work.

The first contact between Brouwer and Menger must have been in late February–early March 1924, for there is a letter of March 12 in which Menger thanks Brouwer for sending him a reprint of the 1913 paper, adding that:

> In 1921, when I tried to define the notion of curve and of dimension, I was in the first year of my study at the university and I was not aware at all of your paper, dear Professor, in the *Journal f. d. reine und angew. Math.* 142, in which the definition is essentially anticipated.

In June 1924 Brouwer had communicated his own paper 'Remarks on the notion of natural dimension' to the Amsterdam Academy, in which he took the Menger–Urysohn definition into account and showed that it was equivalent to the natural dimension for locally compact metric spaces, a result that was already mentioned in Urysohn's letter of 24.X.1923, as Brouwer pointed out in a footnote. Brouwer sent a copy to Menger, who replied:[83]

> I cannot, dear Professor, thank you enough for your attention that you paid to my little note, and also for your kind letter and the mailing of your

[81] Menger to Hahn, 15.II.1922.

[82] In my description of the role of Menger in the development of dimension theory, I am relying on Menger's reminiscences, Menger 1979, the surviving correspondence and printed material.

[83] Menger to Brouwer 13.XI.1924.

> paper which I had already read with great interest in the Proceedings of 28.6.1924.

He went on to point out that for wider classes of spaces the MU- and N-dimension were not equivalent.[84]

Menger's interests were by no means restricted to topology, and he shared an interest with his teacher in foundational matters. He told Brouwer that:

> Recently I had to present a talk on research on the foundation of mathematics in a privatissimum of the epistomologist Prof. Schlick. It may well have been the first time here in Vienna that an extensive presentation of intuitionism was offered. The report was followed by a long discussion. It would be very fortunate for me to obtain in a few months time instruction from your lectures, on these fundamental questions which touch me in my innermost being.

Fate had plunged Brouwer once more into topology; it definitely was not by his wish that he had to divide his attention between his mission in life—intuitionism—and his one-time love—topology. But the sudden appearance, first of Urysohn and Alexandrov and then of Menger left him no choice but to return to the fields of his first glory and to attend to some unfinished business.

12.4 The scientific legacy of Urysohn

During his life, cut short by the tragic accident, Urysohn had been incredibly creative and productive. The papers and manuscripts of the topological genius allowed Alexandrov to publish two volumes of collected works of almost one thousand pages.[85] At the time of his death only a small part had appeared in print. We have already come across Urysohn's *Comptes Rendus* notes, and the manuscripts that he had submitted to the *Mathematische Annalen* (p. 444). In addition, at the time of Urysohn's death there was a substantial manuscript in the hands of Sierpinski, the editor in chief of *Fundamenta Mathematicae*.

The personal and scientific impact of Urysohn on Brouwer had been tremendous—a brilliant scientists and a soul mate who had miraculously appeared. Terrible as the shock of Urysohn's death must have been for Brouwer, it was nothing compared with its impact on Alexandrov. He had lost his closest friend, a fellow mathematician who shared his soul with him. Reading the surviving correspondence one gets the impression that Alexandrov had lost the will to live. In one of Brouwer's characteristic, deeply empathetic letters he encouraged and comforted the despairing young man:[86]

> I have received both letters, and I am in my thoughts continually with you. Yet I would not pray, in accordance with your statement, that you will not have a long life. In the first place, as it is not because of objective

[84] MU-dimension = Menger–Urysohn dimension, N-dimension = natural (i.e. Brouwer's) dimension.

[85] [Urysohn1951].

[86] Brouwer to Alexandrov, 31.VIII.1924.

> events, but only for the illumination of our sense of duty and for the sake of strength to bear the trials that are imposed on us, that we may pray. In the second place, because our earthly existence was given us only for the purification of our soul from the original sin of fear and desire, and that it is only in accordance with the time required for the satisfaction of *this* purpose, that the span of life of the righteous is measured. Just because of that the death of the righteous person has for himself always the characteristic of fulfilment, a release and a salvation, and we should after his death offer him further our love, but not our compassion, in particular not when his transition to death was a light one. And for the mourning surviving friends and relatives the following holds: each sorrow has for the heart that feels it, its ennobling meaning and in the days of sorrow it is often easier than in the days of joy, to become aware of God's presence, because sorrow, to be born in peace, forces dematerialisation. May this also be so for you!

Almost immediately after the heartbreaking message from Alexandrov, Brouwer had decided to see that justice was done to Urysohn's unpublished work. For a man with his meticulous editing habits, this was not just a matter of sending manuscripts to the printer after a cursory inspection of the text. He was known for his punctilious treatment of papers, his own as well as those of others. The page proofs of his own papers that have been preserved show a rich variety of corrections, concerning both the formulation and the mathematical content. Mostly these corrections were of a cosmetic nature, improving the text. It was only natural that he and Alexandrov should act together in this enterprise. Both were more than competent where the content of the papers of Urysohn was concerned; Alexandrov wholeheartedly shared Brouwer's sentiment; for him this work of love for his deceased friend was a matter of course. As guardians of Urysohn's scientific estate there was much to do, sorting out the manuscripts and representing the scientific interests of the deceased. Their first challenge was the monograph-sized paper for the *Fundamenta Mathematicae*, the so-called *'memoir'*. In a way this was a touchy project, mainly because the two first-rate topologists who had taken it upon themselves to supervise the editing of this fascinating manuscript were in their own research so close to the subject matter that extra care had to be exercised to keep the necessary distance. For Brouwer the matter was particularly sensitive, as Urysohn's original text contained a rather unflattering characterization of Brouwer's dimension paper. Brouwer was, however, not the man to adopt the safe procedure, and to back out. He had no doubt at all that he was the right man for the task.

Urysohn's criticism of Brouwer's dimension paper was concentrated in a particular footnote which explicitly claimed that Brouwer in 1913 had failed to establish the right notion of dimension (see p. 462). Urysohn had written the memoir unaware of Brouwer's earlier definition of dimension. He discovered Brouwer's paper just in time to insert a last minute footnote, as appears from a letter to Sierpinski.[87]

[87] Urysohn to Sierpinski 27.VI.1924.

The first page proofs of the memoir still contained the old footnote, see p. 462, but the next proof took Urysohn's wishes into account. The formulation, however, is different from the one in Urysohn's letter; a reference to Menger is appended:

> I mention moreover the work of Menger (Monatshefte für Math. u. Phys. 23 (1923)), where the point of departure is more or less the same as the one of my notes cited in the present memoir, and which has only come to my notice in the spring of 1924.

Indeed Urysohn learned about Menger's work, as appears from a letter written in March 1924:[88]

> As it appears, we have almost simultaneously found the definition of dimension: mine I have found in July 1921, presented to the Moscow Mathematical Society in October 1921. Two announcements have appeared in the Paris Comptes Rendus (1922, Vol. 175, pp. 440, 481); the first part of my major publication is at present in print at the Fundamenta Mathematicae.

This, apparently was the only contact between the two new masters of dimension theory.

As we have seen, p. 456, Brouwer had already contacted Sierpinski about the memoir. In March, uncertain of Urysohn's views, Brouwer had approached the editor in chief of the *Fundamenta Mathematicae*, asking him to keep an eye on possibly unjustified comments on Brouwer's definition of dimension—an unmistakable reference to the footnote. A possible argument between Brouwer and Urysohn had been smoothly averted, when Urysohn had accepted Brouwer's explanation. Unfortunately Urysohn had failed to correct his comments in the memoir, and he had informed Sierpinski in haste, scarcely a month before his visit to Brouwer, that a correction of the footnote was in order.

Once Alexandrov and Brouwer had agreed to edit Urysohn's scientific *Nachlass*, interested parties had to be informed. In the case of the *Mathematische Annalen* there was no real problem. Brouwer was the editor who handled the papers of Alexandrov and Urysohn, and it was completely natural that the matter could be settled by Brouwer and Alexandrov. As for the *Fundamenta Mathematicae*, the editor had to be informed of the new arrangement concerning the corrections of the proofs. On 11 September Alexandrov wrote to Sierpinski that Brouwer was so favourably impressed with Urysohn, that all correspondence on the subject of Urysohn's papers would be handled jointly by the two of them.[89]

It seems that Sierpinski was pleased to accept the arrangement, for Brouwer referred in a letter to information from Sierpinski:[90] the memoir was scheduled to appear before the autumn of 1925 in two volumes of *Fundamenta Mathematicae*, the introduction and the first two chapters in Volume 7, and the rest in Volume 8. He

[88] Urysohn to Menger 22.III.1924.

[89] Alexandrov to Sierpinski 11.IX.1924.

[90] Brouwer to Alexandrov 24.X.1924.

advised Alexandrov to leave the matter alone for the moment. But he also foresaw the possibility of difficulties:

> In the meantime it seems that we should, alas, take into account the possibility, that Kuratowski will have on his own authority declared the introduction of Paul's memoir ready for printing (although such an act without the authority of Paul's heirs should appear incomprehensible to me), that thus about note 3, which criticizes me, nothing could be done. Finally the proofs sent to me do not look as if they are ready for printing at all: they still contain many annoying printing errors.

Almost all of the following correspondence is concerned with the proofs and corrections. There seemed to have been one long struggle to get the proof sheets, to convince Sierpinski to grant more time to insert corrections, both of a material character and of a purely typesetting nature. Sierpinski was probably not happy at all to have two excellent topologists go over the proofs with a fine-tooth comb, and so we see a running battle over access to proofs, the right to insert corrections and the like.

A week after Brouwer's letter Alexandrov tried to get Sierpinski's cooperation[91]

> Shortly before his death Paul Urysohn has talked at length with Brouwer about his own papers and in particular his *memoir*. In the course of the exchange various questions concerning the final formulation were discussed and joint decisions were taken accordingly. Therefore it is very important that the proof sheets of the memoir, until the moment that they are marked with the decision 'imprimatur!', are read by Professor Brouwer, so that, if it were only in appendices, we can enter such changes as Paul Urysohn would have entered himself, had he been alive.

Without speculating on the possible state of mind of a deceased person, one can say that, if anybody, Alexandrov knew Urysohn's mind; he had worked with him over the past years, and he was in an excellent position to understand Urysohn's work, not just the polished results but also the thoughts and deliberations that had gone before. So it is not unreasonable to trust him in this matter. As for Brouwer, he had a singular gift to fathom a kindred spirit. So there was no pair better equipped for this task than Alexandrov and Brouwer.

The task of supervising the editing of the *memoir* was not an easy one. Printing was in an advanced stage, and apparently the proof-reading was done by local staff in Warsaw, so it seemed almost hopeless to expect that the publisher would welcome more proof-reading, and hence more corrections.

A week later Alexandrov, who had returned to Moscow, where he was living with Urysohn's family (and actually was thrown out of the apartment together with them), dropped a harmless looking bombshell. He wrote Brouwer that he was about to approach Sierpinski, and he included the text of the letter.[92] In this letter he

[91] Alexandrov to Sierpinski 20.X.1924.

[92] Alexandrov to Brouwer 20.X.1924.

confessed a painful oversight: although he and Urysohn had together gone over the final version of the text, they had forgotten to replace the expression 'se propose a résoudre' by 'résout', see p. 462. As the old formulation contained 'a certain offensive meaning' for Brouwer, a meaning which had escaped them at the time, he had begged Sierpinski to change the formulation 'as required by scientific truth and by the wishes of the author as expressed in his last days.'

In his letter to Brouwer, Alexandrov cautiously expressed his hope that things would work out, 'I believe that the revision of the flawed footnote 7) in the introduction is not totally hopeless.' Brouwer could not draw much comfort from this kind but rather lame reassurance. Some two weeks later he himself wrote to Sierpinski; thanking him for his cooperation, he went on to say that there were a number of corrections and emendations, made necessary by the exchange of ideas between Urysohn, Alexandrov and Brouwer, shortly before the death of Urysohn—'in a form that only the author could have recognized [...], a form which Alexandrov and I have determined in accordance with the last notes and the last verbal statements of the deceased, for which we accept full responsibility'. He added that one of the corrections concerned himself, and that Alexandrov had written about it.

A complicating factor on Brouwer's side was that he was still recovering from a serious illness. In October he mentioned his health problems, and Alexandrov was worried accordingly. Brouwer's health was again in a poor state, in letters to Alexandrov cod liver oil is mentioned, and a recommendation of the family doctor to spend the Christmas vacation in the Engadin (Switzerland). Needless to say, Brouwer refused to go; there were too many things he had to do. His ill health no doubt was real, but keeping Brouwer's medical history in mind, one can hardly doubt that Urysohn's death and the subsequent pressure of Urysohn's posthumous publications made things worse. It looked as if his not uncommon nervous breakdowns had surfaced again.

On 13 October Brouwer replied to an earlier letter of Alexandrov saying that he was still in bed. 'My recuperation progresses smoothly, but not very fast, and my doctor declares moreover that I have to be careful for a long time and that I will have to spare myself. I have not been able to extract a diagnosis with a scientific medical name: he spoke of "flu with complications."

In a touching letter Alexandrov replied, expressing his worries that the mysterious illness might have been a flu combined with pneumonia, which in his opinion could easily develop into tuberculosis. 'I am very much worried about your weak lungs.... You should therefore not only take care, but also make your food as intensive as possible, to put the whole organism in order, otherwise it will not work. You should take great quantities of butter, eggs, milk, cream, etc., and *cocoa*, I think that now meat is indispensable to you. ... Please keep me continuously informed of the news of your health.'[93] The last part of the advice was not self-evident for a vegetarian like Brouwer! Alexandrov's advice to take cod-liver oil ('oleum jecoris Aselli')

[93] Alexandrov to Brouwer 20.X.1924.

was, however, dead on target; Brouwer adored any medication that came straight from Mother Nature's drugstore.

Alexandrov's letter also contained a German translation of the poem *Worüber singt der Wind?* of Alexander Block, a poem that summed up the sad emotions of Alexandrov.

> Warte, mein alter Freund, und dulde, dulde,
> Das Dulden wird nicht lange dauern, nur schlafe fester;
> Es wird ja alles doch vergehn,
> Es wird ja niemand was verstehn,
> Weder dich verstehn, noch mich,
> Noch das, was von dem Winde
> Uns vorgesungen ist …

> So much was sung us by the wind. It was really as if between the lines of life a divine song was sung, intertwined. I recall how we landed at 4 o'clock in the morning for the first time at the Norwegian coast at Christiaansand. The sun had not yet risen, we stood in this cool morning air and we gazed at the rocks of the shore, at the motionless, totally quiet sea, at the blazing red sky. So we stood for a long time, silent, until the sun had come up, and still longer. We did not speak a word, but we both felt in the same way the eternal infinity of nature. But even in daily life, in the most ordinary aspects, this other thing was present, that what we felt and understood, and what separated us. This haze of eternity … it surrounded our joy, thus perhaps it lasted so briefly.

It was actually neither the first nor the last time that 'tuberculosis' was mentioned, nor the first time that Brouwer's vegetarian diet was deplored. On 16 February 1921 Van Eeden wrote in his diary: 'I went home with Brouwer. He said that he had been ill, and I understood that it was the beginning of tuberculosis. I loved him so much, he was friendly and so warm-hearted. If only he would eat meat.' Van Eeden's fears may have been somewhat exaggerated, but he was a medical man and he definitely knew how a tuberculosis patient looked! The conclusion one may safely draw, is that Brouwer's ascetic way of life and the nervous tensions took a heavy toll.

On 20 October Brouwer wrote Alexandrov that his health was improving and that he hoped to resume his lecturing the next Friday. 'I feel as if I have come slowly to the light from a dark abyss,' and on 3 December he sent a card to Alexandrov with a reassuring message, 'My health is now really satisfactory; I live indeed most carefully (the cod liver oil is faithfully taken)'. That Christmas Brouwer stayed home. His family doctor had wanted to send him to Engadin, but Brouwer argued successfully that the backlog of tasks he would find on returning from Switzerland was a greater danger for his health, than a Christmas in Laren. Brouwer was grateful for the attention of his young friend. There was also good news concerning the grant. The Rockefeller Foundation had approved the arrangements for Alexandrov's stay in Amsterdam, where he would receive $180 per month from the grant.

The exchange of letters must have been quite frequent in the fall, although only a few have survived. There is a note of Brouwer's in which a few points of a letter of 7 November to Alexandrov are written down; one of these says 'vision in my dream of Urysohn: "Yes, yes, yes, yes, – yes, of course" about my plan to have a new typesetting of the imperfect text'. Far from being a cheap trick to impress Alexandrov, it shows that the memoir occupied a prominent place in his thoughts.

A few days earlier he had written to Sierpinski, stressing the importance of allowing more time for a proper proof-reading.[94] After thanking him for the proofs of the introduction, he expressed his hope that

> there would possibly be time to make changes and corrections in the introduction, that had become necessary by the exchange of ideas and that had taken place in the last weeks of the life of the author, between him, his friend Alexandrov and me. In this hope I will send you shortly the corrected proofs of the introduction in the form that only the author could recognize at the day of his death, a form which Alexandrov and I have established from the last notes and the last verbal statements of the deceased, and for which we accept the full responsibility.
>
> Among the changes there is one which concerns me personally, and about which Alexandrov has written you a letter, which he will send through me as an intermediary and which I will get registered myself to-day.
>
> In the proofs which I will return to you I have done my best to correct typographic errors which were left in fairly large numbers and some of which are not all that innocent.

On 17 November Brouwer again turned to Sierpinski, explaining that certain corrections could only be made in the final proof.[95] There were for example, he wrote 'passages [...] thoughtlessly written at the time, and recognized as absurd by the author shortly before his death—which, if they were printed in their actual form, would gravely damage his scientific memory ...'. For this reason he begged Sierpinski to suspend all printing, to send all proof-sheets to him, and to have new proof-sheets made after his corrections, including those pages which had gone through their final printing. Any necessary costs would be gladly paid by him. One would not have to be extremely sensitive to resent a letter of this sort. No editor likes to be told in a patronizing way, what to do. The next day Brouwer reported to Alexandrov that he had written in unmistakable terms to Sierpinski, about the poor editing and correcting. He urged Alexandrov to support him. Thus Alexandrov duly informed Sierpinski of his surprise and displeasure, to find out that the introduction and the first chapter had been printed *without enabling him to read* the text. The fact that Kuratowski had read the text did not mean that there was there-

[94] Brouwer to Sierpinski, 4.XI.1924.

[95] Brouwer to Sierpinski 17.XI.1924.

fore no reason for Brouwer and himself to read the proofs. He strongly appealed to Sierpinski to redress the situation.[96]

Sierpinski reacted swiftly and firmly.[97] Urysohn, he wrote, had asked for one correction (see p. 462) *if possible*, and he clearly did not expect to see any proofs. 'Therefore the editor of the Fundamenta Mathematicae considered himself entitled to start the printing of the memoir of Paul Urysohn, without sending any proofs, except for that single correction. Paul Urysohn never proposed any more modifications of his memoir.'

What had started as a simple duty to honour Urysohn's memory, and a service to a well-known mathematical journal, thus started to take the form of another conflict. That was exactly how Brouwer saw it.[98]

> Now it also occurs to me, that for the fortifying of our position (it indeed is like a real war; our position assumes the role of a fortified camp, in the wall of which, as a consequence of your fateful letter to Sierpinski, a breach was shot, which we have to close with all our might) it were very important to add the following passage ...

In view of the position taken by Sierpinski, Brouwer thought it best that Alexandrov would make it clear to Sierpinski that Brouwer and he would take no important decision without mutual consultation, and that hence it was obvious that Alexandrov had to see the proofs. Moreover, Sierpinski's claim that Urysohn had not asked for proofs, seemed in conflict with the fact that the first galley proof of the memoir had reached Urysohn in Le Batz. Anyway, would any author of a memoir of more than three hundred pages voluntarily forego the proof-reading?

Should it come to the worst, then Brouwer could, as an 'ultimate medicamentum heroicum, withdraw in name of the family, the paper from the Fundamenta.' That would probably teach the editors a lesson.

In December another complicating factor arose—Brouwer, with his refined instinct for possibly hidden insults, had discovered that the uncorrected version of the manuscript implied a reading with drastic personal consequences. Namely, Urysohn had characterized Brouwer's notion of separation as 'the old notion of cut' (*coupure*). This, he was certain, would immediately tell every reader that Brouwer (i) missed the essential point in his definition of dimension, (ii) tried to cover up his lapse. To what extent Brouwer's fears were justified is hard to say. Urysohn's formulation was unfortunate, and went back to the period before his exchange with Brouwer, and indeed an acute reader might doubt Brouwer's explanation. So, Brouwer foresaw a disaster, unless he could repair the formulation. In view of the fact that Urysohn had fully accepted Brouwer's view, and in view of the discussions in Blaricum (where, one may be certain, Brouwer once more pointed out the true history of his oversight), there is little doubt that the unfortunate passage was overlooked in July 1924. But for Brouwer the dimension matter began to border on an

[96] Alexandrov to Sierpinski 24.XI.1924.

[97] Sierpinski to Brouwer 25.XI.1924.

[98] Brouwer to Alexandrov 21.XI.1924.

obsession. In his letter to Alexandrov he forcefully argued his case.[99] He spoke of 'Sierpinski and Kuratowski tainting Paul and the two of us'. Somehow the matter was settled, for on 15 January 1925 he wrote 'The peace with the Warsaw people, also towards me, has been completely re-established'.

Peace or no peace, the end of the problems had not been reached. On February 17 Brouwer informed Alexandrov that a new disaster had taken place. Brouwer had returned the galleys 8 and 9 to *Fundamenta Mathematicae*, but the printing shop had in the meantime, 'on account of the small stock of type (just enough for a few galley sheets)' destroyed the type for galley 8, as Sierpinski informed me in an otherwise most polite and apologetic letter.' So galley 8 had to be set again, and—whereas galley 9 was impeccably set—'contained, of course, again the normal quantity of Kuratowski errors.'

The last surviving letter to Sierpinski on the matter of editing Urysohn's memoir is the one of May 24, 1925. In it Brouwer conveyed the wish of the family of Urysohn to preserve the original manuscript of chapters 4 through 6, which they considered 'the culminating part of the scientific activity of the deceased,' as 'a relic'. So he begged Sierpinski in the name of the Urysohn family to cede that part of the manuscript. A copy 'carefully made under my direction (in which moreover some posthumous wishes of the deceased were taken into account)' was offered in exchange. The correspondence does not say if Sierpinski honoured the request of the family.[100]

Urysohn's memoir appeared without further problems in the *Fundamenta Mathematicae*. It was a substantial piece of topology, altogether 236 pages; the paper dealt with a rich variety of topics, including dimension theory and indecomposable continua. The modern reader, accustomed to Bourbaki-style presentation, would find the paper a bit long winded here and there, but this opus of the twenty-five-year-old topologist contains the fundamental theorems of dimension theory.

The editing of Urysohn's memoir took a heavy toll on Brouwer, both emotionally and clerically. In addition to this, Brouwer was also handling the newer papers of Alexandrov and Urysohn for the *Mathematische Annalen*, complete with correcting formulations, streamlining proofs, etc. All of this took place at the time that he was occupied with the preparation and publication of his finest intuitionistic work: the bar theorem, fan theorem and the continuity theorem.

Did Brouwer actually 'falsify the text', as Menger later suggested? It is highly unlikely; after all he did not act without Alexandrov's approval, and Alexandrov had been present and taken part in the discussion between Brouwer and Urysohn in July 1924. Those who have known Alexandrov, know that he would not be a party to shady practices of this sort, and he was certainly not one to be pressured into such an act.

It is an altogether different question whether it was wise to take part in the editing. Although his motives were totally unselfish, and based on his admiration

[99] Brouwer to Alexandrov 24.XII.1924).

[100] So far no copies of the manuscript in whatever form have been found.

and love for this sudden topological prodigy, less kind tongues could easily spin a tale of text manipulation. Actually only one such case occurred before long, and we will meet it in a later chapter. It is a pity that Brouwer and Alexandrov did not add an editorial comment to the memoir.

Brouwer's factual influence on the text is hard to assess. He probably, with his editorial routine, did some polishing of the formulations, and added an occasional useful reference. The most damaging accusation would of course be that of systematic self-promotion. Without access to the original manuscript there is little one can say. Brouwer is mentioned a number of times in footnotes (on references), but these are all harmless 'useful information for the reader'. There is no reference beyond generally recognized facts. Yet, the editing of the memoir would become the subject of some most unpleasant altercations.

The correspondence with Alexandrov contains regular references to a planned stay in Amsterdam. In the letter of February 17 arrangements for Alexandrov's stay in Amsterdam are mentioned. The Rockefeller Foundation had put a grant at Alexandrov's disposal and it had taken some ingenuity to get permission for this stay from the Soviet authorities. Brouwer left no stone unturned, wrote letters and asked his friend and colleague Mannoury to write a letter to the Soviet authorities. Mannoury duly did so; he asked the Volkscommissariat for Education to grant Alexandrov permission to come to Holland to collaborate with Brouwer. He signed his letter as 'Professor of Mathematics at the University of Amsterdam' and 'Member of the K.P.H.' (Communist Party of Holland).[101]

Brouwer had also enlisted the help of his colleague Pannekoek, the astronomer, who had an international reputation as a Marxist theoretician. Pannekoek, in turn had asked a Dutch engineer, Rutgers, to put pressure on the authorities.[102] The organization of Alexandrov's stay in Amsterdam began to take shape. For Brouwer it was important to arrange the details, as he wanted to make the most of not only Alexandrov's presence, but also that of a number of other visitors, to wit Menger, Vietoris, Kerékjártó, and Wilson.[103] A special set-theoretical lecture series would only be worthwhile if all the visitors could attend. Brouwer strongly advised his visitors to come and live in Laren or Blaricum; he preferred to have them around, so that he could organize meetings and seminars at home, without travelling to Amsterdam. In his letter of March 15, 1925 he discussed the housing arrangements with Alexandrov; it had apparently been agreed that Alexandrov was going to live in Laren. Was it not a good idea, he asked, if Menger also found himself rooms in Laren, or the neighbouring towns of Hilversum or Bussum? 'I hesitate to make this suggestion myself to him, because I would accept thus certain imponderable obligations, from which I want to safeguard myself as long as I don't know Menger

[101] Mannoury to Lunatscharsky 4.XI.1924.

[102] Rutgers had offered his expert services to the Soviet government, and in time he became an appreciated courier of Lenin, with quite responsible assignments for the promotion of the world revolution.

[103] Wilfrid Wilson was a British topologist, who took his doctorate with Brouwer in 1928.

personally'. Alexandrov would not be bound by any obligations if he arranged temporary housing for Menger. Brouwer would, of course, if necessary look for suitable rooms for Menger. One should not read too much into these lines; Brouwer had always been extremely sensitive in his contacts, being completely in the dark with respect to Menger, he felt he could not commit himself as he did towards Alexandrov.

Menger, in fact, had already contacted Alexandrov; in an undated letter from Alexandrov to Menger an earlier letter of Menger is mentioned. Alexandrov's letter must have been written in the first months of 1925 (perhaps even earlier). It is an enthusiastic welcoming of a fellow topologist, expressing admiration for the parallel efforts of Menger and his friend Urysohn in the area of dimension theory—'of course you will be cited in the 'Mémoire sur les multiplicités Cantoriennes, and the relevant footnote has already been sent to the editors; Brouwer and I are completely of the same opinion on this matter.' He looked forward to working with Menger in Brouwer's seminar, believing that this would be useful to both of them. In fact, he bade Menger to come to Holland a bit later in order to make their stays overlap for a longer period. He looked forward, he wrote, to making the acquaintance with Menger from whom he expected many interesting things.

Menger had entered Brouwer's life through correspondence, and Brouwer had immediately recognized Menger's contribution to dimension theory. Indeed, in his sequel to the dimension paper of 1913 ([Brouwer 1924b], [Brouwer 1924h] is an abridged version in which Menger is not mentioned) he referred to [Menger 1923] and spoke of the *Menger–Urysohn dimension* (*MU*-dimension).

In 1925 things were brightening up for Brouwer and his Urysohn project. Already in January Alexandrov had informed him that the peace with the Warsaw mathematicians had been completely restored. Of course, Brouwer had enough to grumble about, but he was no longer hampered by editorial interference. In the spring his topological admirers started to arrive, and the stage was set for a period of intensive study and research in modern topology.

13

PROGRESS, RECOGNITION, AND FRICTIONS

> If I still have the time and energy, I will myself show the mathematical world that not only geometry but also arithmetic can point the way to analysis, and certainly a more rigorous one. If I cannot get this done, then they who come after me, will do it, and they will also recognise the incorrectness of all those inferences with which the at present so-called analysis operates.
>
> L. Kronecker.

13.1 The first skirmishes in the foundational conflict

To an outsider the foundational situation in mathematics after the crisis of the set theoretic *paradoxes* may have seemed quiet and unproblematic. It was almost as if a moratorium had been called on foundational research. In the early years of the century great men had proposed various remedies for the sudden outbreak of paradoxitis in the wake of Cantor's monumental innovations. Poincaré, Russell, Hilbert, Brouwer each had their ideas for saving mathematics. In this line Brouwer came last, and after his contribution to the foundational debate no new enterprises were undertaken until after the First World War. It would be a mistake, however, to assume that everyone had returned to 'business as usual'. Hermann Weyl's concern about the foundations, for example, had been (re)awakened in 1910, as appears from a letter to his Dutch friend Mulder, 'lately I have been thinking a lot about the foundations of set theory, and there I arrive at views that deviate rather from those of Zermelo, and which are in a sense close to the, here much ridiculed, position of Borel–Poincaré.'[1]

Hilbert had not given up his interest in the foundations, and he lectured regularly on the foundations of mathematics.[2] He also had a number of researchers around him, who worked on various issues. The best known collaborator of the early years was Ernst Zermelo, a singularly gifted mathematician, who is now mostly known for his proof of the well-ordering theorem and the axiomatization of set theory. Zermelo was in Göttingen until he left in 1910 for Zürich. Under Hilbert's influence the philosophers Edmund Husserl and Leonard Nelson were also appointed for some time in Göttingen. Hilbert managed to get a steady stream of young researchers to work on his ideas. Among these Paul Bernays

[1] Weyl to Mulder 24.VII.1910.

[2] See [Sieg 1999]

became his principal assistant for his own research. Bernays was born in London in 1888; he possessed many talents, among which those for music (he was an accomplished pianist), mathematics and philosophy stand out. After studying in Berlin and Göttingen, he obtained his doctorate in 1912 from Edmund Landau. In the same year he got his 'habilitation' under Zermelo. He followed Zermelo to Zürich, where he stayed until in 1917 Hilbert called him to Göttingen. As a matter of fact, it was Polya who, on a walk in the mountains, had called Hilbert's attention to the promising young man. A most fruitful collaboration followed, without Bernays' competent and patient toil, Hilbert's programme might perhaps not have materialized.

Suddenly, after the World War First, the foundations of mathematics came into focus again. In 1918 Hermann Weyl published his monograph *Das Kontinuum*, Hilbert's Axiomatic thinking (*Axiomatisches Denken*) appeared in the *Mathematische Annalen*, and Brouwer had launched his new constructive programme in the first Begründung-paper. Hilbert's paper contained the text of a lecture in Zürich the year before; it was one of those expository masterpieces for which Hilbert was famous. It consisted of a brief tour of the foundations, with views of mathematical problem areas, including physics. The significance and scope of the axiomatic method was discussed in broad terms (still a long way from the later proof theory). The paper was free of personal attacks; it contained just a slightly depreciative observation on Kronecker and Poincaré.[3] It contains an interesting change of view on 'Hilbert's dogma'. He mentioned 'the problem of solvability in principle of each mathematical problem', whereas in his famous Paris Lecture of 1900 spoke of 'this axiom of the solvability of every problem.'[4]

Brouwer continued his Begründung series with the second paper, which dealt with the topology of the plane and the theory of measure [Brouwer 1919a]. Just like its predecessor it was dry as dust, scholarly to the point of dull. In retrospect, one might say that Brouwer carefully steered away from any possible controversy. It will remain a matter of conjecture, in how far he followed a deliberate policy; did he fear to upset Hilbert, whom he greatly admired? Generally speaking Brouwer was well-informed about the mood in Göttingen, and it is not wholly impossible that he foresaw Hilbert's disapproval. He had already disappointed Hilbert by turning down the offer of a chair in Göttingen (cf. section 8.4), and it is not certain how this influenced Hilbert's view of his Dutch admirer. One should bear in mind, however, that it was not a grave sin to turn down a flattering offer in order to cash in at one's own university. Hilbert was himself no stranger to the game; in 1919 he turned down an offer from the university at Bern 'after the ministry has granted all my wishes.'[5] Nonetheless, it is one thing to improve one's own position

[3]Hilbert claimed that Kronecker rejected set theory on the grounds of the paradox of the set of all sets. This is rather a misjudgement of Kronecker's motives; moreover Kronecker died before the paradoxes surfaced.

[4]Cf. [Gray 2000] p. 248. Brouwer was indeed quick to point out this shift [Brouwer 1919b].

[5]There are certain indications that Hilbert's independent position during the war had made him no friends among his colleagues. Born wrote in a letter to Hilbert (27.VIII.1919) that he guessed that

through 'position bargaining', and another thing to be used in the game. Whatever Brouwer had in mind, the result was a series of non-provocative, scholarly papers, hardly read by the mathematical community. The first paper that addressed a larger audience was 'Intuitionistic set theory', published in the *Jahresbericht* and in the proceedings of the Dutch academy ([Brouwer 1919b], [Brouwer 1922]). The paper contained a number of non-technical, conceptual remarks that put some meat onto the skeleton of his intuitionistic mathematics. In particular he equated Hilbert's dogma with the principle of the excluded third, and gave an argument for their rejection. Hilbert is explicitly mentioned in the paper. None of this seemed to provoke Hilbert, but when in 1921 (cf. section 8.8) Hermann Weyl sided in his spectacular paper 'On the new foundational crisis of mathematics' with Brouwer, it was more than Hilbert could take. Weyl's paper is one of the all-time beauties; formulated in a sparkling, literary style, it gave convincing reasons for joining Brouwer's intuitionism. None of this could be called derogative or insulting. Perhaps this was one of the things that irked Hilbert: not only had his prize student left him for another, but he had done so with great charm and wit![6]

Yet, the developments could not have taken Hilbert by complete surprise. Weyl had addressed the Göttingen Mathematical Society on May 11 of 1920. It is true that Hilbert had missed the lecture because he was not informed in time. But he must have heard what Weyl had to say. Indeed he had ordered Bernays and Kneser to draft an extensive report. As he wrote to Weyl, 'there are all kinds of similar lines of thought, as I also have pursued in my lecture courses of the last semesters, even though our fundamental tendencies seem to be very different. I am very curious about the development of your ideas.' There is no trace here of exasperation; on the contrary, Hilbert seemed somewhat pleased with some similarities that he noticed.[7] Admittedly, there is a slight possibility that Weyl's talk dealt with his older ideas. The title of the talk was 'The Continuum', so he might have kept his powder dry, and stuck to the less controversial material. This, however, seems highly unlikely. Weyl was at that time fully committed to Brouwer's programme and he had already lectured on the topic in Zürich, and prepared the manuscript of the New Crisis paper. Perhaps his talk in Göttingen did not contain the provocative statements that are so characteristic for the idealistic revolutionary in the prime of life. Some time after this letter, the daring paper of Weyl appeared, and Hilbert apparently realized that there was a challenger at the gate. He quickly took the offensive and used an invitation of the University of

Hilbert's action was largely determined by his relationship with the 'Göttingen' colleagues, who mostly subscribed to the *Alldeutschtum*. In view of the treatment by this group during the war, Hilbert's threat to leave Göttingen might not have been empty talk.

[6] See section 8.8.

[7] Since there is no elaboration of this remark, one can only make an educated guess as to what these similarities were. A possible candidate could be Weyl's treatment of quantified statements as 'judgement abstracts'. In Hilbert's writings a similar phenomenon surfaces in the distinction between finitary, meaningful statements and quantified, ideal statements, cf. [Hilbert 1926], p. 174. In Hilbert's papers no reference to Weyl's ideas is to be found. There is, however, a reference in [Hilbert, D. 1934, Hilbert, D. 1939].

Copenhagen to lecture, among other subjects, on the foundations of mathematics. On 18 February Niels Bohr, Hjelmslev, and four other members of the faculty had submitted a proposal to award David Hilbert an honorary doctorate. The academic machinery handled the matter quickly and efficiently, and already on March 16 the King authorized the faculty to award the doctorate. At a meeting of the faculty on that same day the faculty carried through the honorary doctorate ceremony of Hilbert. With characteristic modesty the great man described his work with the words, 'My scientific work is in the field of algebraic invariants, higher arithmetic and analysis; it occupies itself moreover with the foundations of geometry and the axiomatization of the physical and mathematical sciences.'[8] The doctorate was not the sole purpose of Hilbert's visit he also gave three lectures on the following topics: 'Nature and mathematical cognition', 'Axiomatic theory and consistency', and a discussion session.[9] Later that year Hilbert repeated the lectures on the foundations in Hamburg on July 25-27.[10] The Copenhagen and Hamburg lectures were enthusiastically received by eager audiences. It must indeed have been fascinating to hear a living legend speak about matters of life and death for mathematics.

It is, however, a dangerous matter to publish a lecture for a mixed audience, as a specialist paper. One tends to slip in details and side remarks that did well before a sympathetic audience, but that do not lend themselves for a scholarly readership. Maybe Hilbert did not realize this, maybe he hoped that readers in their study would react as the audience in the lecture hall. Whatever he may have had in mind, the modern reader is struck by the querulous pedantic style. As a matter of fact all Hilbert's logical papers, but one, between 1918 and 1931 were reports of lectures. And indeed all of these published lectures have this same emotional, partly indignant, partly sulking, but always aggressive tone.

His colleagues and students must have been surprised to see the brilliant expositor who had enthralled the mathematical world with his epochal contributions, turn into an aggressive, argumentative preacher. Hilbert always had a reputation for a certain brusque wit; visitors to the Göttingen mathematics seminar were often confronted with Hilbert's caustic Baltic irony, in which tactfulness took second place to truth. But in the foundational lectures the wit was bordering on rudeness. Hilbert's counter-revolution opened in 1921, with his Copenhagen–Hamburg lectures and the subsequent report 'New founding of mathematics', [Hilbert 1922]. This opening volley starts with five pages of criticism of the revolutionaries Weyl and Brouwer, with additional shots at Poincaré and Kronecker. As to be expected in a talk for a non-specialized audience, none of the points of critique are elaborated or convincing. Weyl's predicative approach, including his objections to the vicious circle, is answered perfunctorily: Weyl's' introduction of the vicious circle

[8] Festschrift udgivet af Københavns Universitet. November 1921, p. 114.

[9] *Natur und mathematisches Erkennen* (March 14), *Axiomenlehre und Widerspruchsfreiheit* (15 and 17 March). I am indebted to Sigurd Elkjaer for the information and documentation on Hilbert's Copenhagen lectures and doctorate.

[10] See [Hesseling 2002] for details.

in mathematics is waved aside as artificial: there is no evidence that the class of sets of natural numbers is susceptible to the same paradoxical phenomena as the universe of sets. And, understandably, Hilbert appeals to the axiomatic method: the continuum can be introduced axiomatically. Instead of offering the required consistency proof (which, by the way, would have to reckon with Weyl's objection), Hilbert notes that this axiomatic approach corresponds roughly to our intuition. As he was to find out in his subsequent talks and papers, the consistency of analysis was not the answer to Weyl's observations, but the problem itself. Weyl, he said, when speaking of 'the inner instability of the foundations, upon which the structure of the empire rests.'[11] and the 'impending dissolution of the State of analysis', is seeing ghosts. Hilbert's view of the work of Weyl and Brouwer was simple and drastic,

> What Weyl and Brouwer do, amounts in principle to striding along the former path of Kronecker: they try to provide a foundation of mathematics by jettisoning everything that seems inconvenient to them and by setting up a dictatorship of prohibitions à la Kronecker. This means, however, to chop up and mutilate our science, and we risk to lose a large part of our most valuable treasures, if we follow such reformers. Weyl and Brouwer outlaw the general notion of irrational number, of function, even of the number theoretic functions, the Cantorian numbers of higher number classes, etc.; ... and even the logical Tertium non datur.[12]

Serious allegations indeed. But to what extent did Hilbert's accusations do justice to the revolutionary duo? He could point to Weyl's paper and its restrictions on quantification,[13] but certainly not to Brouwer's foundational views. Indeed a large part of Brouwer's first Begründung paper dealt with Cantor's second number class, that is to say with Brouwer's constructive version of it. Nor did Brouwer's or Weyl's approach rule out the totality of irrational numbers (in Brouwer's terminology 'a species'). Reading the philosophical part of the Hamburg lecture, one is tempted to think that Hilbert rather felt a duty to answer the challenges of Hermann Weyl, than that he deemed a mathematical, foundational analysis of the papers of Brouwer and Weyl worthwhile. His formulations betray a degree of vexation that is unusual in mathematical discourse. The fact that two leading mathematicians, one of them his favourite student, dared challenge the mathematical tradition, including his own views, must have shocked him. The tone of this paper, and most of the later ones, can hardly be explained by the issue itself. In mathematics it is an old custom to refute mathematical claims by mathematical arguments. Blasting opponents is and was certainly not common practice. There must have been a powerful emotional urge behind Hilbert's reaction, partly the result of his impression that the barbarians were at the gates, partly the resentment of the older, established professor who is confronted with insubordination and desertion. In his words,

[11] Cf. I, p. 323.
[12] [Hilbert 1922], p. 159.
[13] Cf. p. 320 ff.

> Brouwer is not, as Weyl thinks the revolution, but only the repetition of an attempted coup (*Putsch*) by old means, which in its day had been attempted with more vigour, but which failed utterly, and which now, where the power of the state is so well armed and strengthened by Frege, Dedekind and Cantor, is all the more from the beginning doomed to failure.

The modern reader may think the talk about revolutions and coups (*Putsch*) an amusing exercise in metaphorical language, but for the German readers it was not so in 1922. The German nation was thoroughly unstable and coups were part of the harsh reality. In 1920 Berlin had its '*Kapp Putsch*' and Munich had been the scene of a left-radical revolution in 1919, the same year that the Spartacist coup took place in Berlin. So Hilbert's rhetoric was not the abstract reference of an intellectual to some misfortune of the old Romans, but the voice of the statesman-soldier who, in desperate times, follows in Cincinnati's footsteps. The reader should, by the way, keep in mind that Hilbert's political metaphor was a reaction to Weyl's 'new crisis' paper, in which the revolution was proclaimed.[14]

Not even Kronecker and Poincaré were spared Hilbert's displeasure, 'Kronecker coined the slogan: the Lord created the integers, everything else is the work of man. Accordingly, he outlawed–the classical prohibiting dictator–what was no integer for him.' Poincaré gets off with a lighter reprimand, but Hilbert remained adamant: 'His objection that this principle [i.e. complete induction] could not be proved, then by complete induction, is not justified and will be refuted by my theory.'[15]

It tells something about Hilbert's faith in his new programme that he was willing to make these strong claims. On the one hand, it shows that he trusted the tools that he was designing, on the other hand a certain naiveté cannot be denied. To be fair, there was little or no experience in the more sophisticated parts of logic; before Gödel hardly anybody had an idea about the limits of what was provable. Of course, Poincaré and Brouwer had opposed Hilbert's claims, but rather on philosophical than on technical grounds.

The remainder of Hilbert's paper consisted of a general exposition of his programme He sketched how a formalization of number theory would be obtained. There is an explicit formulation of the separation of the material into (i) mathematics proper (*die eigentliche Mathematik*), which is a collection of provable formulas (in the formal system), and (ii) *metamathematics*, which consists of contentual inferences about mathematics proper, and which served to show the consistency of it. The term metamathematics appears here for the first time in print. And a few pages earlier the term 'proof theory' (*Beweistheorie*) was coined for the study of formal proofs in a formal system, an important step, as it explicitly mentioned formal proofs as objects of study.

[14]Cf. p. 322,323.

[15]Cf. [Poincaré 1905], Les derniers efforts des Logisticiens, [Poincaré 1908] (livre II, Ch. IV, V) [Brouwer 1907], p. 172.

Before moving on, let us note that Hilbert mentioned in his attacks Weyl and Brouwer in that order. In view of Weyl's new crisis paper he may have viewed Weyl as the ringleader even though Weyl gave the credit for the revolution to Brouwer.

With the lectures and papers of Weyl and Hilbert the *Grundlagenstreit* (foundational conflict) had started in earnest. Weyl had certainly provoked his old teacher, and the mathematical world saw Hilbert's counter-offensive as the proper reprimand for a person who dared to challenge the supreme commander of mathematics.

Neither Brouwer nor Weyl answered Hilbert's attack in kind, both men revered the grand old man (who, by the way, was only sixty) too deeply to upbraid him for a spell of impetuosity.

The discussion immediately drew the attention of the mathematical community, and the words 'revolution' and 'Putsch' became bywords. Adolph Kneser, for example, quoted Hilbert with apparent approval in his paper on Kronecker:[16] 'As a matter of fact, as Hilbert has said very aptly referring to a similar new movement, it was not a revolution, but a coup (*Putsch*) which had to collapse under the general strike of the analytical workers (*analytische Arbeiter*).' Kneser apparently expected a similar breakdown for the intuitionistic revolution.

In 1923 Brouwer broke with his habit of producing scholarly, technical expositions. He published a paper in the *Jahresbericht*, which was the result of lectures at conferences. He lectured, first in Antwerp at the Flemish conference of science in August, and a month later at the annual meeting of the DMV in Marburg, on September 21, 'On the significance of the theorem of the excluded third in mathematics, in particular in function theory.'[17]

A paper with the same title appeared in the *Journal für die reine und angewandte Mathematik*, the journal that also had published Brouwer's dimension paper, and also in Dutch in the *Wis- en Natuurkundig Tijdschrift*, Brouwer must have had his reasons for not submitting it to his favourite journal, the *Annalen*. In the first section the principle of the excluded third[18] is discussed. For finite systems, Brouwer acknowledged the applicability of PEM: for properties deduced with the help of PEM 'always the certainty exists that, given a sufficient amount of time, one can arrive at the empirical confirmation.'[19]

Now, from the fact that many objects and mechanisms of the outer world can be viewed as 'finite discrete-system for certain known parts ruled by certain laws of temporal concatenation', one may conclude that the laws of logic, including PEM, are applicable, notwithstanding the fact that a complete empirical confirmation is mostly a priori materially impossible. In view of this experience, logic and PEM were granted an a priori character. One particular historical consequence of this phenomenon was that PEM was applied in all of mathematics, without taking

[16] [Kneser 1925].

[17] [Brouwer 1923e, Brouwer 1923c], translated in [Heijenoort 1967].

[18] Abbreviated as PEM, *principle of the excluded middle*.

[19] [Brouwer 1923e], p. 2.

into account that the results did in general defy theoretical or empirical confirmation. Already in his 'Intuitionistic set theory' of 1919,[20] Brouwer had observed that PEM, as well as Hilbert's dogma, was an unjustified generalization of the mathematics of finite systems. The present paper contained a more elaborate foundational analysis of this phenomenon.

At the end of the first section Brouwer very briefly mentioned the formalist consistency programme. Considering Brouwer's earlier criticism, along the lines of Poincaré, the comments are surprisingly mild: one need not despair of reaching the ultimate goal of a consistency proof. He added, however, that such a proof would not amount to much:

> nothing of mathematical value will thus be gained: an incorrect theory, even if it cannot be inhibited by any contradiction, that would refute it, is none the less incorrect, just as a criminal policy is none the less criminal even if it cannot be inhibited by a court that would curb it.'[21]

A rather mild *badinage*, that would not upset any modern reader. But did it upset Hilbert (if he saw it at all)? It would not have done much to endear Brouwer to him. In fact, Brouwer had formulated his objections to the formalist programme more pointedly in his dissertation:[22] 'from the consistency of the axioms the existence of the accompanying mathematical system does not automatically follow', and 'it has not been proved anywhere that if a finite number has to satisfy a system of conditions, for which the consistency can be proved, that this number indeed exists.'

The paper does not, however, contain any personal attacks; in particular, Hilbert's name is not mentioned. The significance of PEM for mathematical practice is shown by means of 'Brouwerian counterexamples', that is, examples showing that the validity of some mathematical property would imply the solvability of a particular open problem for which no evidence exists. The standard open problem is the presence of a specific finite sequence of decimals in the expansion of π: 'if in the decimal expansion a sequence 0123456789 occurs, then ...'[23]

Brouwer listed a number of classical theorems which could not be accepted in the intuitionist framework:

– the ordering of the continuum has the trichotomy property[24]
– every set is either finite or infinite
– a continuous function on a closed interval has a maximum.

[20] [Brouwer 1919b].

[21] In [Fraenkel 1923], p. 239, Brouwer is quoted slightly different: 'For intuitionists, however, "consistency" has by no means the same meaning as "existence", no more than a crime which cannot be detected by prescribed means of investigation stops being a crime.'

[22] [Brouwer 1907], p 138 (footnote), p. 141, 142.

[23] Cf. I.10.4, p. 392. Brouwer had picked a hard, although elementary, problem. Only in 1998 was a sequence 0123456789 found in the decimal expansion of π [Borwein 1998]. Needless to say the demise of a single counterexample does not weaken the intuitionist cause—there are enough new counterexamples.

[24] The trichotomy property asserts the comparability of any two elements: $x \leq y \vee y \leq x$. Brouwer's favourite formulation was 'the continuum cannot be ordered' [Brouwer 1950, Brouwer 1992].

– the theorem of Heine–Borel
– the theorem of Bolzano–Weierstrass.

Brouwer's lecture in Marburg was attended by a good many mathematicians, and the audience must have been surprised, to say the least, by the consequences of Brouwer's criticism.

Hilbert's Hamburg paper will have reassured most mathematicians that nothing was wrong with classical mathematics that could not easily be set right. The general impression was that Brouwer and Weyl would not recover from this blow, as is implicit in Fraenkel's letter to Brouwer of April 18, 1923: 'It was, among other things, most interesting for me to observe the active life of intuitionism which had already been pronounced dead in many quarters.'[25] It is no exaggeration to say that Fraenkel's books and papers were most influential in disseminating the facts about progress in the foundations of mathematics, in particular he reported in a neutral way on what was going on in intuitionism. The expanded version of his famous 'Introduction to set theory'[26] of 1919 was under preparation when Fraenkel met Brouwer, and Brouwer was sufficiently impressed to assist Fraenkel with the proof-reading of the second edition (1923). Fraenkel's exposition helped to put oil on the troubled waters. On the one hand, it took intuitionism seriously and lent it the respectability that Hilbert's talk had tried to deny it; on the other hand, it put the reader at ease—this revolution was not the only way to save mathematics. The mathematician Konrad Knopp voiced a widespread feeling in a letter to Fraenkel, 'I am glad to see in it a confirmation of my feeling that the "destruction of the foundations" by Brouwer is by no means so disastrous as it seemed to be.'[27] Fraenkel himself had not given up his neutrality, he was well aware of the foundational virtues of Brouwer's intuitionism, but, like his fellow mathematicians, he was loath to exchange the fleshpots of Egypt for the desert of intuitionism:[28]

> After this excursion to philosophical grounds, we must go on to another group of dangerous revolutionaries, which is exclusively made up of mathematicians.
>
> This group is more dangerous, because the attack is carried out with much sharper, partly mathematically finely whetted arms, but also insofar as here it is not just a small border correction, aimed at the exposed province of set theory, at the cost of the mathematical empire, but the attack is carried into domains of this empire that are most flourishing and imagined as most safe.
>
> If it ultimately succeeds, then there will remain of present-day mathematics, apart from tightly bordered impregnable areas (in particular arithmetic in the narrow sense), only ruins, from which indeed through the

[25] Cf. p. 396; see [Dalen, D. van 2000] for more on Fraenkel and intuitionism.

[26] *Einleitung in die Mengenlehre*, [Fraenkel 1919].

[27] Knopp to Fraenkel 2.I.1924.

[28] [Fraenkel 1923], p. 164.

> labour of generations new, somewhat inhabitable (and not comparable in comfort to the old one) housing can be constructed.

Fraenkel's appreciation for Brouwer's ideas competed with an understandable attachment to his first love—Cantorian set theory. The author is, he wrote, 'emotionally and in view of the factual successes of science, convinced that the far reaching intuitionistic amputations are not necessary'; at the same time he recognized 'the present justification of a part of the intuitionistic objections.'[29]

After Hermann Weyl, Fraenkel was probably the best informed man where Brouwer's intuitionism was concerned. He had in fact attended Brouwer's lectures in Amsterdam, and discussed the foundational problems with Brouwer in private.[30] He was certainly not afraid to speak his mind, as one can see in his discussion of Hilbert's second paper [Hilbert 1923], where he points out that a particular argument of Hilbert, pertaining to PEM, 'touches on the final part of an older essay of Brouwer.'[31]

If there were mathematicians who thought that Hilbert had silenced the opposition, Brouwer's Marburg lecture shattered that hope. Indeed, Brouwer pursued in relative quiet the founding of his intuitionistic mathematics. In Amsterdam he was virtually on his own, his first intuitionistic PhD student, Maurits Belinfante, had defended his dissertation, 'On infinite series',[32] on December 12, 1923. This dissertation treated the theory of infinite series from the intuitionistic point of view.[33] Brouwer developed in 1924 his new insights further in a number of papers [Brouwer 1924, Brouwer 1924d, Brouwer 1924f, Brouwer 1924e, Brouwer 1924a]. In 1924 the relationship of Hilbert and Brouwer had not reached the point where contact became impossible. No direct comments of Brouwer on the situation have been preserved; he had no difficulty acknowledging the greatness of the mathematician who had stimulated his research and recognized his qualities; he bore his dissatisfaction in silence. As Weitzenböck wrote to Hermann Weyl 'As far as Hilbert's matters are concerned, Brouwer for the time being shrugs his shoulders. I believe that he is somewhat annoyed that Hilbert ignores his things.'[34] Slowly and imperceptible the old relationship, which was characterized by Brouwer's outspoken admiration for the great mathematician, and the appreciation of an established authority for a gifted younger colleague, began to assume the Saul-David pattern. Brouwer, who was generally well-informed about the atmosphere in Göttingen, could not dismiss the rumours that his intuitionistic programme estranged him

[29] [Fraenkel 1923], p. 173.

[30] See. p. 396.

[31] [Fraenkel 1923], p. 239.

[32] [Belinfante 1923]

[33] Belinfante had not made use of Brouwer's new results, such as the fan theorem and the continuity theorem. From a modern point of view, one could say that he belonged to the Bishop school *avant la lettre*.

[34] Weitzenböck to Weyl 16.IV.1923.

from Hilbert. In a letter to Bieberbach Brouwer spoke of the 'smear campaign conducted by Hilbert' against him.[35]

Brouwer had occasion to feel personally the winter of Hilbert's discontent when he was invited to give a lecture in Göttingen before the Mathematical Society on July 22, 1924. The lecture was probably a sequel to his Marburg lecture; it bore the title 'Consequences of the intuitionistic position in mathematics.'[36] Brouwer addressed a large audience; Hilbert, against his custom preserved an icy silence during the lecture, but when Brouwer had finished, he rose saying 'With your methods most of the results of modern mathematics would have to be abandoned, and to me the important thing is not to get fewer results but to get more results.'[37] He then sat down to an enthusiastic applause. And that was all Hilbert contributed to the discussion. Another member of the audience attacked Brouwer's constructive position on existence with the words 'You say that we can't *know* whether in the decimal representation of π ten nine's occur in succession—maybe we can't know, but God knows.' Objections of this sort, which illustrate the fact that education and old habits make it easy to miss the point, were quite commonplace in the early days of intuitionism; obviously, Brouwer had a ready reply, 'I do not have a pipeline to God.' The occasion left Brouwer with a poor impression; after the talk he visited the seventy-five-year old Felix Klein and had a long talk with him.[38] In a letter of August 1 he wrote to Alexandrov 'In Göttingen there was a lot of interest for my subjects—I had an audience of roughly 150 persons, but the discussion remained rather superficial. [...] With Klein I had a discussion of several hours. He is completely lucid and (much more than the rest of the Göttingers) mathematically and personally broadly oriented and interested.'[39] One may read a mild disappointment in this letter. Already people were taking sides in the formalist–intuitionist discussion. Constance Reid reports that the majority of the Göttingen mathematicians did not see the point of Brouwer's criticism. In the words of Hans Lewy,

> It seems that there are some mathematicians who lack a sense of humour or have an over-swollen conscience. What Hilbert expressed there seems exactly right to me. If we have to go through so much trouble as Brouwer says, then nobody will want to be a mathematician. After all, it is a human activity. Until Brouwer can produce a contradiction in classical mathematics, nobody is going to listen to him.
>
> That is the way, in my opinion, that logic has developed. One has accepted principles until such time as one notices that they may lead to contradiction and then he has modified them. I think this is the way it will always be. There may be lots of contradictions hidden somewhere;

[35] Brouwer to Bieberbach 1924, undated. See p. 452.

[36] *Konsequenzen des intuitionistischen Standpunktes in der Mathematik*.

[37] [Reid 1970], p. 184.

[38] Since Blumenthal, Brouwer, and Klein were at the time entangled in the unpleasant Mohrmann affair, there was no shortage of topics, see also p. 628.

[39] In the same letter he told of his strongly favourable impression of Kneser and Neugebauer.

> and as soon as they appear, all mathematicians will wish to have them eliminated. But until then we will continue to accept those principles that advance us most speedily.[40]

Lewy's view was doubtlessly the prevailing one among the working mathematicians, for whom foundational considerations were a luxury option. Only the most crude criterion, viz. consistency, appealed to them.

The finer arguments of Brouwer and Weyl, and even of their icon, Hilbert, must have been wasted on the mathematicians of the day. The modern distinctions based on effectiveness, complexity, degree of abstraction and the like, of which we know so much more than our forebears, have equipped us with a sensitivity for the finer traits of mathematics, that one could not expect in the twenties. It is all the more surprising that mathematicians with an extraordinary vision, such as Weyl and Brouwer, could see beyond the first necessities. Nonetheless, Hilbert's 'we want more theorems' has a hollow ring that even at that time must have struck the more perceptive minds as dubious. After all, one does not want just *more* theorems, but rather good theorems, beautiful theorems, useful theorems. ... The history of mathematics abounds with disputes over the right theories, the right theorems, ... To be fair, Hilbert would probably agree, but his reaction shows that nice catch phrases often get the better at public occasions.

The foundational discussion, soon to become a conflict, certainly had its serious observers, such as Bernays and Fraenkel, but there was also a measure of loyalty involved of the sort 'right or wrong, my country.' Hilbert, in particular, could inspire a large measure of loyalty; Brouwer was by nature a loner; his following remained modest to the point of marginal.

In the middle of Brouwer's hectic life, being occupied with his new intuitionism, the *Grundlagenstreit*, and dimension theory, a tragic event took place in his personal life. His close, and perhaps only, intimate friend, Carel Adama van Scheltema, died. In March Carel took a fall that did not seem serious, but in April he suddenly suffered from dizzy spells, and on 6 May 1924 he closed his eyes after days of excruciating pain. A neglected concussion was conjectured to be the cause. On 9 May the socialist poet prince was carried to his grave in an impressive ceremony.[41]

Brouwer had lost not just a friend, but his spiritual brother, the man who had watched over Brouwer's mind, who had been his mentor and who, in spite of a difference in political outlook, had this magical and mystical rapport with him. In a terse homage he remembered his friend:

> It was the reflection in your eyes, the inflowing grip of your hand, the warm engulfing of your voice, the peaty smell of your overcoat. It was the wild wealth of your dream life, the confusing abundance of your imagination.

[40] [Reid 1970], p. 184.

[41] [Drost 1952], p. 26.

> But around you roams the power of determinacy that cannot be rejected, which you sensed and had to acknowledge; and you were determined, and you wanted to understand, and to be a figure. You have understood much and you have become a figure. And a part of your flourishing rhythm became generally accepted.[42]

13.2 Consolidation and entrenchment

The second half of the twenties was probably the most dramatic period of Brouwer's life, it contained episodes and moments of great exhilaration, but it ended in gloom and disaster. The threads of his various activities all came together in this short period, and the sheer complexity of it would have been enough to undo a lesser spirit. As it was, Brouwer had to pay the price at the end—the juggler had too many glittering balls in the air. We will deal with Brouwer's activities one-by-one, but the reader has to keep in mind that Brouwer was all the time playing simultaneously at a large number of boards.

There was his intuitionism that had to be fostered, topology that required his attention, the editorial business of the *Mathematische Annalen*, the fight against the boycott of Germany, faculty politics, teaching, the *Grundlagenstreit*, the management and administration of his wife's pharmacy, significs, private life, etc.

During the twenties a steady stream of papers of Brouwer,[43] and later also of Heyting, started to build up a body of intuitionistic mathematics. Of course, this body could in no way match the existing body of traditional mathematics, but that was a point that did not worry Brouwer. In the first place, this was just 'work to be done', and in the second place large parts of traditional mathematics could, with proper care, be 'rescued.' Heyting's thesis and the function theoretic work of Belinfante were unmistakably the first steps towards a rebuilding of mathematics along intuitionistic lines, and one had to hope and trust that more would follow. When Brouwer delivered the laudation at the occasion of Heyting's PhD defence in 1925, he asserted that the day had come that intuitionistic mathematics had proved to be viable, and to encompass the central parts of mathematics. He was convinced that his mathematics would eventually be largely accepted.[44]

Brouwer's personal research was more along fundamental lines. In particular he sought to fathom the nature and the consequences of the new notions that he had introduced. He had come to see the incredible complexity of the notion of choice sequence, and tried to analyse the notion further.

In his first Begründungs paper the notion of choice sequence was left almost completely open. He spoke of 'choices of arbitrary digit-complexes.'[45] It may seem surprising that Brouwer used linguistic entities, such as 'sign', 'digit-complexes',

[42] [Brouwer 1929a].

[43] 1921-1; 1923-4; 1924-7, 1925-2; 1926-2; 1927-3; 1928-2; 1929-1 (not counting the multiple versions).

[44] Interview A. Heyting.

[45] *Ziffernkomplex*. In [Brouwer 1925b] Brouwer used '*Nummer*'.

'numeral' in his definition, as mathematics was for him independent of language. It was only in his 1947 paper 'Guidelines of intuitionistic mathematics' that he spelled out his intention: 'the word *symbol*' (*Zeichen*) . . . must be understood in this definition in the sense of *mental symbols*, consisting in previously obtained mathematical objects of our thinking (*denkbaarheden*).' In his post-war publications, e.g. [Brouwer 1954], Brouwer spoke of 'sequence of natural numbers.'

Little to nothing, however, was said in 1918 about the choice process and its limitations. Only in 1925 did Brouwer add some comments, albeit only in a footnote:[46] 'Including the character of its freedom of continuation, which can narrow down after each choice (possibly until its complete determination, each time, however, according to a spread law).' By itself this footnote is not all that illuminating, after all a spread law, say the underlying tree of the spread, can at a certain point exhibit a path that has no branchings. This was not, however, what Brouwer had in mind. He wanted to allow the possibility that, although many numerical choices could be made, the subject might just stick to a law.

In one of his reprints a large number of corrections and additions has been inserted; one of these reads 'The *arbitrariness* of these "restricting-clauses" associated to a finite choice sequence, under preservation of the possibility of continuation, assigns to this choice sequence, and hence also to all its continuations a new arbitrariness.'

One might well wonder what good higher order restrictions may do, and what kind of restrictions Brouwer had in mind. Apart from the obvious restriction to a fixed law for all future choices Brouwer discussed only one example; it occurs in a letter of June 26, 1924 to Heyting: the completely free choice sequences. In that case there is namely a second order restriction: there will be no future restrictions on the numerical choices.[47] The matter of the correct version of the notion of choice sequence kept Brouwer occupied; in 1942 the higher-order restrictions were explicitly mentioned in print,[48] but subsequently doubts set in; in the Cambridge lectures[49] the higher order restrictions are still mentioned, but with a caveat: 'But at present the author is inclined to think this admission[50] superfluous and perhaps leading to needless complications.'[51] And a few years later he made a complete U-turn:

> However, this admission is not justified by close introspection, and moreover would endanger the simplicity and vigour of further developments.[52]

[46] [Brouwer 1925b].

[47] These sequences were later independently introduced by Kreisel as 'lawless sequences', cf. [Kreisel 1968]. See also [Troelstra 1982].

[48] [Brouwer 1942c].

[49] Cambridge Lectures 1946-1950, [Brouwer 1981].

[50] of higher-order restriction

[51] [Brouwer 1981], p. 13.

[52] [Brouwer 1952b], p. 142. Indeed, higher-order restrictions in general destroy not only the simplicity, but put the continuity principle in danger. See [Atten, M. van and D. van Dalen 2002].

So as far as higher-order restrictions are concerned, Brouwer had in 1952 made the full circle.

Intuitionism was viewed by some as a threat to mathematics, an infectious disease that had to be stopped. Now, almost a century later, one has difficulty understanding all the excitement. In order to get a realistic view of the 'strategic' situation in the twenties, we have to review the development of foundational ideas and methods very briefly. As pointed out before, hardly anybody was aware of an intuitionistic programme, before 1920. Only after Brouwer's talks at meetings (in particular of the German Mathematics Society) and Hermann Weyl's 'new crisis' paper in 1921, people started to take intuitionism seriously. Some considered it as a phenomenon that merited attention, simply because honest foundational ideas, such as those of Kronecker, Cantor, Poincaré, Russell, Brouwer and Weyl held promises or warnings for mathematics. Some considered it as an idiosyncrasy of a few mathematicians with a hang-up about paradoxes and the like, and, finally, some considered Brouwer and Weyl a danger for mathematics and mathematicians. Among the latter Hilbert figured prominently. Already in the late nineteenth century Hilbert had run across opposition to his spectacular innovations. He had shown by abstract means that a system of invariants had a finite basis (nowadays known in algebra as *Hilbert's basis theorem)*, in such a way that the number of elements could not actually be determined. The grand old man of constructivism, Kronecker, had been sternly critical of abstract mathematics, including Hilbert's contributions. Kronecker actually formulated a programme for a reduction of mathematics (that is to say, algebra, number theory and analysis) to natural numbers. Kronecker's undisputed authority as a mathematician lent extra force to his attacks at 'modern' mathematics, and his shadow fell over mathematics long after his death. In a personal interview with Hilbert, the older man had told him outright that everything in algebra could be replaced by 'the discrete and singular', a remark that Hilbert never forgot.

In continuation of his beautiful work in geometry, Hilbert had started to study the axiomatization of arithmetic (and set theory) in the hope and expectation to obtain consistency results that would remove the last doubts from mathematics. His first basic paper in this area was presented at the International Conference of Mathematicians in Heidelberg, 1904. It contained a, not wholly flawless, axiomatic study of the natural numbers and a consistency proof for a small fragment of arithmetic. The actual details of the system and the proof are not that important; what really counts is that Hilbert gave a combinatorial, metamathematical, consistency proof, that is, he applied mathematical techniques to a formal system. This paper[53] remained an isolated effort to carry the consistency tradition over from geometry to, say, arithmetic and analysis. As we have seen it came under a heavy attack in Brouwer's dissertation, but it remains to be seen to which extent Hilbert was aware of the fact. It is a matter of speculation whether he had a copy of Brouwer's dissertation (written in Dutch). Perhaps Brouwer sent him a copy at the time that

[53]Hilbert 1904, reprinted in the later versions of *Grundlagen der Geometrie.*

he was writing his papers on Lie groups, or maybe after his walks with Hilbert in the dunes at Scheveningen,[54] where, as he wrote in his paper 'Intuitionistic reflections on formalism',[55] 'An oral communication of the first insight [the necessity to handle the contentual study of formal systems by means of 'the intuitionistic mathematics of the set of natural numbers'] to Hilbert had taken place in the fall of 1909 in several discussions.' The researches of Sieg[56] have shown that Hilbert did not abandon the foundations of mathematics after 1904. He lectured repeatedly on foundational topics in the years before he started to present his new programme in public. An account of the development of his ideas over the years would provide a welcome historical clarification.

Interest in foundational matters had markedly increased in the post-war years. In addition to the fundamental papers of Brouwer, Hilbert and Weyl there had already been a number of more or less expository papers on foundational topics, but generally speaking these were not terribly exciting, or even well-informed. For example, Felix Bernstein extensively discussed Finitism in Mathematics, and lumped Poincaré, Richard, Borel, Lindelöf and Brouwer together as finitists.[57]

After the first manifestations of Brouwer and Weyl, Hilbert had returned to his formalist founding of mathematics. A perspicuous formulation of Hilbert's new ideas was given by Paul Bernays at the annual meeting of the German Mathematical Society in September 1921.[58] He reported on the conceptual progress that Hilbert had made since 1904. In the 1904 talk no serious attention was paid to the more refined metamathematical problems, e.g. a syntactical consistency proof was carried out by (a tacit) induction, without any discussion whether this constituted a reduction to a more basic theory. In the meantime, possibly recalling the discussions with Brouwer, Hilbert had grasped the significance of the metamathematics, i.e. the mathematics employed in the consistency proofs. The problem was to justify (or to show the consistency) of a mathematics dealing with infinite objects, such as sets of natural numbers, real numbers, and the like. These objects and their theories were called, for obvious reasons, transcendent. Now Hilbert proposed to handle the mathematics of these transcendent theories by simple means, 'to justify those transcendent assumptions in such a way that only *primitive intuitive insights* (*Erkenntnisse*) *will be employed*.'[59] In this way 'A sharply outlined, comprehensible programme has taken the place of the earlier, indeed obscure, indications.'[60]

Bernays quite correctly observed that if one wants to base metamathematics on primitive intuitive notions and objects, one has to take into account Brouwer's

[54] See p. 127.

[55] [Brouwer 1928b].

[56] Cf. [Sieg 1999, Sieg 2000]

[57] [Bernstein 1919], p. 63-78.

[58] [Bernays 1922], submitted 13.X.1921. Translations of many of the key publications of this period can be found in [Mancosu 1998]. See also [Heijenoort 1967].

[59] [Bernays 1922], p. 11.

[60] Ibid. p. 10

considerations, and to question the applicability of PEM. Hilbert, said Bernays, wanted to exploit the constructive features of mathematics rather than give them up—'More fundamental than the contact with symbolic logic, is the fact of Hilbert's enterprise touching the constructive theories of Weyl and Brouwer.'[61]

The paper of Bernays is a cogent and objective exposition of what was to become 'Hilbert's programme'. The problems of the programme are duly mentioned, and clearly explained. For example the problem that a consistency proof will doubtlessly require mathematical induction. But in contrast to Poincaré and Brouwer he does not view that as an insurmountable obstacle. With Hilbert, Bernays is convinced that one only needs a weak form of induction. That is to say, one where only simple combinatorial properties come into play, such as 'if the sign + occurs in a concrete proof, then one finds, reading through the proof, a place where it occurs for the first time.' There seemed to be a feeling that if the subject matter is simple (here sequences of symbols), then one only needs simple applications of induction; generally speaking, a simple question will find a simple solution. Indeed, the whole 'pre-Gödel' era of Hilbert's programme is characterized by a contagious optimism: just a few technical problems to be solved and we are done. In retrospect one can only conclude that Hilbert and his collaborators had an overly rosy view of the project.

It is curious that Hilbert could not see, even for the sake of the argument,[62] the point of Brouwer's programme; after all, one would expect that a mathematician should have no difficulty to appreciate that questionable means of proof could not yield reliable results. Admittedly, it was not easy to unravel the foundational ideas behind Brouwer's early papers, but Weyl's presentation should have done much to compensate for that shortcoming. Moreover, one would expect some discussions on the topic in Hilbert's Göttingen. But with Weyl in Zürich and Brouwer in Amsterdam, who would have the courage and insight to play the devil's advocate? Bernays could have done it, but he was not the person to stand up to Hilbert, even as an exercise in purely academic argument.

The whole conflict that started with Weyl's manifesto and Hilbert's reply is marked by an unwillingness of both parties, Brouwer and Hilbert, to contemplate the virtues of the other side; from the people involved only Hermann Weyl managed to take a moderate position between the parties. While Brouwer considered Hilbert's formalism an enterprise doomed to failure, even if it succeeded (see p. 488), Hilbert saw Weyl and Brouwer as iconoclasts of the worst kind.

Somehow Hilbert managed to miss the point of the new intuitionistic mathematics. To mention two of Hilbert's examples—the status of the irrationals and of the number theoretic functions—it is certainly true that in the first intuitionis-

[61] [Bernays 1922], p. 15.

[62] That is to say, Bernays pointed out in his writings, cf. [Bernays 1922], that Hilbert did not lack in appreciation for constructive metamathematics. For some reason Hilbert just did not see, or did not wish to see, his intellectual debt to Brouwer and Weyl.

tic period, beginning with the dissertation, the irrational numbers were not always recognized as a set, but from 1919 onwards the irrationals had found a place in constructive mathematics.[63] The rejection of the general notion of number theoretic function too, was a thing of the past. Brouwer's choice sequences were precisely designed to allow the set (spread) of *all* numerical functions (infinite number sequences). Of course, many a traditional theorem failed to make it to the intuitionistic practice, and this was what annoyed Hilbert. Brouwer, for his part, had once and for all abjured the axiomatic method; that is to say, in its extended sense, not as a convenient tool for daily practice, but as a basic tool for capturing and creating the true mathematics. He had summed up his objections in his dissertation and they had not been met by the proponents of axiomatics, not even by the best principled ones, including Hilbert.[64] And so he saw no reason to change his views. From the available material one may infer that Brouwer was reasonably well informed about the views and prospects of the formalists; in the dissertation he showed a profound knowledge of the contemporary foundational situation (with the exception of Frege's work) and in the twenties the real formalist innovations were of a technical rather than philosophical nature—no reason for Brouwer to contemplate drastic changes in his position.

The *Grundlagenstreit* is a typical example of one of those scientific tragicomedies, where both parties run around the stage, shouting but not listening. In particular Hilbert acted as a headmaster dealing with two protesters who threaten to set the building on fire. Indeed, Hilbert viewed Weyl and Brouwer as real threats to the survival of mathematics; being the leader of contemporary mathematics, he felt it his personal duty to save mathematics and to thwart the efforts of the evil pair. And he made this perfectly clear in his lectures and publications (cf. page 485).

From 1922 onwards Hilbert and Brouwer exchanged regular blows like legendary knights of the past. In 1922 Hilbert addressed the *Deutsche Naturforscher Gesellschaft* in Leipzig with a paper titled, 'The logical foundations of mathematics.'[65] This paper constituted real progress with respect to the preceding one; it contains the idea to reduce mathematics (arithmetic) to a small fragment, i.e. finitistic mathematics. Furthermore it introduced the τ-operator, a forerunner of the ε-operator. This paper was totally uncontroversial, it abstained from diatribes against third parties.

While Hilbert was defending mathematics in his lectures and papers, a good deal of the painstaking research in proof theory was done in Göttingen by his modest, even shy, assistant, Paul Bernays. From 1917 onwards Bernays had worked in Göttingen on various topics in mathematics. But gradually his sole task became the study of Hilbert's metamathematics. It is no exaggeration to state that without him (or a suitable replacement) the project would have come to nought, the intricacies of logic and proof theory, which nowadays are more or less routine, had

[63]The terminology is anachronistic; Brouwer started to use the name 'intuitionism' for his programme after the First World War.

[64][Brouwer 1907], Ch.3.

[65][Hilbert 1922].

to be explored, and all consequences had to be painstakingly checked. Eventually Bernays moved to Zürich and became his own master, somewhat to his surprise, but in the twenties he was the quiet man behind Hilbert's grand schemes. The relation between the two was unmistakably that of master and servant. The historian of mathematics, Dirk Jan Struik, told an anecdote illustrating the relation between Hilbert and Bernays; Struik was once spending some time in Göttingen with a Rockefeller stipend, and one day he asked Bernays to accompany him on a Sunday morning walk, 'All right', said Bernays, 'but I first have to get permission from *Herr Geheimrat*.'[66] The collaboration between Hilbert and Bernays culminated in a two volume treatise, the famous *Foundations of Mathematics*,[67] finished after Bernays had to leave the country in 1933.

Paul Bernays was indeed one of the most saintly persons one can imagine; he was wholly incapable of harming anybody—a man of an absolute and unassailable integrity. During the Brouwer-Hilbert conflict he managed to preserve good relations with both adversaries, and also in later years Brouwer and he were on quite cordial terms. There is no doubt that Bernays appreciated the force of the arguments from both sides, unhindered as he was by personal antagonism, or a 'save mathematics' complex.

Both parties in the conflict were in 1922 busily investigating their respective projects. Brouwer had not yet achieved his great breakthrough, but he was making an inventory of the damage sustained by classical mathematics. He knew that a good part of traditional mathematics had to be revised and that another part had to be chucked overboard. From 1921 onwards he had lectured on the theory of real functions (a practice he kept up until 1929), and this provided ample opportunity to test out the destructive consequences of his new intuitionism.

In spite of Hilbert's attacks there was not yet an open conflict. Actually the two adversaries, Brouwer and Hilbert, hardly ever met during the foundational debate. Whereas in certain disciplines and certain debates the opponents got together to try to decide the matter in a personal exchange of thought (think of the Bohr-Einstein debate), Brouwer and Hilbert probably did not see what could be gained from a personal discussion. The *Grundlagenstreit* was conducted like some sort of a match: the players would in turn address some conference or seminar, present the progress in their own project and return to their respective desks.

We have already seen that Brouwer unfurled his banner on German soil in 1923 at the Marburg meeting of the DMV.[68] And a year later he addressed the Göttingen mathematicians.[69] Hilbert had lectured on his new programme in Copenhagen (1921), Hamburg (1921) and Leipzig (1922)

[66] [Rowe 1989a].

[67] *Grundlagen der Mathematik I* (1934), *II* (1939).

[68] See p. 487. Brouwer's Nauheim lecture predates the *Grundlagenstreit*; it is by all standards a totally innocuous academic treatise on effective procedures, without programmematic claims. Nobody could possibly take exception to it. Therefore I will not view it as part of the foundational conflict.

[69] See p. 491.

The next episode in the *Grundlagenstreit* took place in Münster; the *Westfälische Mathematische Gesellschaft an der Wilhelm-Universität* organized a Weierstrass-week from June 2 to 6, 1925. The meeting offered a select group of speakers: Mittag-Leffler, Bieberbach, Hilbert, Perron, Koebe, Knopp and Weyl.[70]

Hilbert presented at the Weierstrass meeting his famous talk 'On the infinite and the founding of mathematics',[71] a lecture that presented his progress in proof theory and which also contained an attempt to solve the continuum problem. Again he used the occasion to point out the futility of the objections from various quarters. He pointed out that he was going to provide the ultimate security of the mathematical method and he demonstrated (probably genuine) astonishment at 'the old objections, that were believed to have been renounced long ago, which reappeared in a new cloak. So at present something like this is put forward: even if the introduction of a notion were possible without danger, i.e. without yielding contradictions, and this could be shown, then this would not yet establish its justification.' One hears a faint echo of Brouwer's words. But where Brouwer saw the justification of theories in mathematical constructions, Hilbert saw it in the ultimate success of the practice of these notions. 'Indeed, success is necessary, it is here the highest authority (*Instanz*), to which everybody bows.'[72] The clarification of the notion of infinite was for Hilbert something that transcended the sciences by far: it was necessary for 'the honour of human understanding itself.' Indeed, Hilbert shows himself here fully modern, possibly more progressive than his younger audience; he was prepared to uphold set theory to the ultimate consequences, 'From this paradise that Cantor gave us, nobody will be able to expel us.'[73]

The comments on the notion of infinite are quite balanced, and give credit to the two basic notions: potential and actual infinite. One can even hear an echo of Brouwer: 'in analysis we have only to deal with the infinitely small and the infinitely large as limit notions, as something in growth, arising, generated (*etwas Werdendem*, *Entstehendem*, *Erzeugtem*) . . .'[74]

There is also a rather remarkable reprimand for the finite-number-of-steps people: 'The mathematical literature finds itself, if one takes notice of it, inundated by absurdities and lack of thought, mostly due to the infinite. Thus, for example, if in the sense of a restrictive condition it is required that in exact mathematics only a finite number of steps is allowed in a proof—as if somebody ever could succeed in carrying out infinitely many steps.' No doubt Hilbert meant well, but this example looks somewhat like the teacher who scolds a pupil who has just given the right answer, because he might have thought of the wrong answer.

[70] Curiously enough Brouwer was in Münster on June 1 and 2. At least that was what Brouwer announced in a letter to Gerda Holdert, 14.V.1925: 'I have to be in Münster on June 1,2.' He may or may not have gone to Münster after all, but it remains a curious coincidence.

[71] *Über das Unendliche und die Begründung der Mathematik*

[72] [Hilbert 1926], p. 163.

[73] Ibid, p. 170.

[74] The fact that Hilbert accepted the modest constructive approach: potential infinite for *cardinalities* in the style of Brouwer 1907, does of course not imply that he would accept, or even contemplate, choice sequences.

The paper was, on the whole, rather more expository than polemic; it did not call the adversaries by name (Weyl, anyway, no longer figured in the debate) and it radiated a spirit of confidence and enthusiasm, as for example expressed by the words 'And what we experienced twice, once when it concerned the paradoxes of the infinitesimal calculus, and then when it concerned the paradoxes of set theory, that cannot happen to us a third time and will never happen again.' It is a spirit of progress and of the ultimate security of mathematics that is within reach: 'Our proof theory which is sketched here, can not only secure the foundations of the mathematical sciences, but I believe that it opens up a route by which general problems of a fundamental nature that fall under the domain of mathematical thought can be treated, that could not be got at in the old days.' Hilbert saw mathematics already as the supreme judge of fundamental issues, and, he added, 'Even the claims of the recent so-called "intuitionism"—modest as they may be—will in my opinion get their authorization from this tribunal.'

The mathematical-logical content was also more substantial than that of earlier papers. Whatever the shortcomings of the paper—again the text of a lecture—were, it leaves with the reader even today a strong impression. It is the inspiring talk of a general to his troops in the face of a dangerous enemy. It radiates strong confidence and it contains not just the pep talk, but an outline for concrete foundational work and progress. If one would like to make a critical remark at all, it would be that as before the content consists of sketches, expectations and promises. After this paper the exchanges were suspended for a while.

Hilbert's paper is indeed rich in ideas, it contains for example the notion of 'finite attitude of thought' (*finite Einstellung des Denkens*). The idea had crossed Hilbert's mind already in 'The logical foundations of mathematics' (1922); it is explicitly mentioned in his next paper [Hilbert 1923], p. 160:

> Our thinking is finitary;'[75] while we are thinking, a finitary process takes place. This truth, acting by itself, is more or less used in my proof theory in such a way, that the experience of this contradiction should automatically bring with it the choice from among the infinitely many objects. In my proof theory it is therefore not claimed that the selection of an object among infinitely many can always be carried out, but rather that one can act, without the risk of an error, as if the choice had been made.

Hilbert did not make here excessive claims about the effectiveness of this finitary attitude.

In 'On the infinite' the fundamental ideas of the notion of 'finitary' is further worked out. Finitary arithmetical statements are for example 2 + 5 = 7 or $12 > 3$, and even $n + m = m + n$, or $\forall mn(m + n = n + m)$.

The latter statement roughly means, if we take for m and n concrete numbers, then a finitary check will establish the truth. However, $\exists x A(x)$ for some finitary statement $A(x)$, for example, is no longer finitary. It involves an infinitary opera-

[75]Hilbert used the German '*finit*' instead of '*endlich*'. Following Kleene, the term 'finitary' has been adopted in the foundations of mathematics.

tion, namely unbounded search. A finite set can be checked element by element, albeit highly idealized. But for an infinite set no such good fortune can be expected. One consequence of this insight is that finitary statements cannot be negated, or more precise: if A is finitary, then $\neg A$ need not be so. Hilbert formulated the case of the existential statement as 'In general an existential statement, of the form: there is a number with such and such a property, has from the finitary point of view only a meaning as a *partial statement*'. There are striking parallels between Hilbert's ideas about finitary statements and Hermann Weyl's earlier 'judgement abstracts'. More historical research will be required to clear up the relation. In the sense of 'who published first', Weyl's priority is clear, but it is very well possible that Hilbert had been discussing the issue in his lectures. Although Weyl's new crisis paper was known to Hilbert, there is no reference to Weyl in Hilbert's infinity paper.

Hilbert concluded quite correctly that if we restrict ourselves to the realm of finitary statements—'as we should'—then 'those laws of logic, which mankind has been using all the time, since it started to think, and which indeed Aristotle has taught, are not valid.' But, he continued, 'one could set oneself to formulate valid logical laws for the domain of finitary statements; but that would not serve us, as indeed we do not want to forego the use of the simple laws of Aristotelian logic, and nobody, though he might speak with the tongues of angels, will keep man from negating arbitrary statements, to formulate partial judgements and to apply the Tertium non datur.'

It is clear that Hilbert had moved quite a bit in the direction of the intuitionists; given the basic finitary nature of the operation of our thinking, he agreed that the logic of finitary statements is really the appropriate one. In this sense he actually was a sub-intuitionist. However, he took the pragmatic view of mathematics, and sought to salvage the rest by logic. The solution he had in mind is imaginative: he compared the situation to the old geometrical practice of interpreting the projective plane (or space) in terms of the affine one. Given the affine plane, one defines points at infinity in terms of pencils of parallel lines. The old points of the affine plane are called *real points* and the new points *ideal points*.

In this sense one obtains ideal statements from the finitary ones by unrestricted quantification. Hilbert then went on to formulate his new version of the consistency proof:

> For there is one condition, just one, but indeed an absolutely necessary one, with which the method of the ideal elements is tied up, and this is the consistency proof: the extension by means of the adjunction of ideals is in fact only permissible if it causes no contradiction in the old domain, that is, if the relations for the old objects that result at the elimination of ideal objects are always valid in the old domain.[76]

[76] [Hilbert 1926], p. 179. A more modern version would run roughly as follows: if a finitary statement is provable in the extended system, then it is already provable in the finitary system (the extension is *conservative*)

The great innovation of the paper was the introduction of the choice operator, ε, the embodiment of the 'thus far in the mathematical literature most challenged so-called axiom of choice.' The ε-axiom is in Hilbert's formulation $A(a) \rightarrow A(\varepsilon A)$. It in fact eliminates the $\exists$-quantifier: $\exists x A(x) \rightarrow A(\varepsilon A)$. This indeed was a brilliant stroke; by getting rid of the quantifiers one could concentrate on much simpler formulas. However, the complications now shifted to the ε-operator, and these difficulties were just as complicated as the old ones. This was something that further research would indeed show. In sophisticated areas, such as proof theory, free lunches are rare.

In September Hilbert repeated the Infinity lecture in Copenhagen, where it found universal acclaim. Courant reported that 'your Münster lecture in Copenhagen came as a real bombshell and was received from all quarters with the greatest enthusiasm.' He added, 'It did me also an exceptionally great pleasure, to see that finally the outsiders begin to understand that with your theory the situation is saved (*dass mit Ihrer Theorie das erlösende Wort gesprochen ist*). I believe, that your lecture will really have an enormous influence in the most diverse directions.'[77]

13.3 The Riemann volume

In the middle of the *Grundlagenstreit* there is an intermezzo that has nothing to do with foundational issues, but which widened the gulf separating Brouwer and Hilbert. The issue involved the old bogeyman, the *Conseil*, and the boycott of German scholars.

Mathematics knows a number of men of unusual genius, whose life and career was tragically cut short, at an early age. Among these was the nineteenth century mathematical genius, Bernhard Riemann. He died of consumption at the age of forty, at the height of his mathematical career. It is no exaggeration to say that his ideas transformed function theory, differential geometry and topology. He also had a tremendous and inspiring influence in the foundations of mathematics through his monograph *Ueber die Hypothesen welche der Geometrie zu Grunde liegen* (1867), which dominated the discussions in geometry far into the twentieth century under the name of the *Riemann–Helmholtz Raumproblem*.

As a tribute to Riemann's genius, the *Mathematische Annalen* had decided to dedicate a volume to Riemann's memory at the occasion of the commemoration of his hundredth birthday in 1926. The usual plans were made by the editorial board, the main issue being whom to invite for a contribution. This seemingly innocuous question raised however problems that from our vantage point of view seem irrational and childish. In 1924, when the matter was raised, German scientists were still the outcasts of the international scientific community and in reaction to the official boycott proclaimed and enforced by the *Conseil international des Recherches*, (cf. chapter 9) the German scientists had answered with a counter-boycott. Professional organizations were wary, to say the least, of contacts with parties from the Allied countries. Efficiently as ever, a network of organizations and persons

[77] Courant to Hilbert 10.IX.1925.

had been put together for the purpose of protecting and furthering the interests of German science.

There was a number of organizations that occupied themselves with the promotion and protection of German science and scientists; the two that concern us here were the *Reichszentrale für naturwissenschaftliche Berichterstattung* under Karl Kerkhof and the *Notgemeinschaft der Deutschen Wissenschaft*, directed by F. Schmidt-Ott. The *Notgemeinschaft*, founded on 30 October 1920, was a corporation of the five academies of the *Kaiser-Wilhelm Gesellschaft* and the universities, institutes for technology, etc.[78] It was essentially a scientific and not a political enterprise. All the same it was a reaction to the boycott, and its members may have fostered anti-*Conseil* sentiments, but its main business was the support of actual scientific activities. It is interesting to note that the two deputy presidents were chosen from the exact sciences: W. von Dyck, the mathematician from Munich, and Fritz Haber, the famous chemist from Berlin. The *Reichszentrale für naturwissenschaftliche Berichterstattung*, an official organization for the promotion and support of the exact sciences, had a strong political role, that is to say, it occupied itself intensely with the foreign relations of German science.

The head of this bureau was Dr. Karl Kerkhof, a man with an impressive international network.[79] In the years of the Weimar Republic, resentment ran high in certain academic circles; almost every occasion was used to indulge in self-pity and in anti-*Conseil* sentiment. We have seen that the *Conseil* and its various branches did not excel in the virtue of brotherly love, and they abhorred the sinner just as much as his sin. Even worse, the allied wrath was equally aimed at sinners and innocents. In 1924 the recollections of the war and the subsequent peace treaty of Versailles was fresh enough to turn the average German into a nationalist. Under these circumstances the matter of inviting foreign scholars to contribute to a Riemann volume became an apple of discord.

Brouwer, when invited to submit a paper for the Riemann volume, replied that he would be happy to do so; he added that he agreed with the proposed list of contributors, with one exception: Painlevé.[80] This was not a wholesale refusal of French contributors, indeed the second Frenchman on the list, Hadamard, was perfectly acceptable to Brouwer. To underscore his objections he went on to list a selection of Painlevé's offensive characterizations of Germans and more in particular German scientists.[81] In his eyes Germany would make itself the laughing stock of the world if the man who, in his functions of president of the *Académie des Sciences*, had heaped such insults on German science, were invited to contribute to the Riemann volume. He invited Blumenthal to send copies of his Painlevé quotes to the editorial board, so that none of the members would even contemplate the possibility of this invitation.

[78] Cf. [Schroeder-Gudehus 1966] p. 199 [Schmidt-Ott. 1952].

[79] See p. 340.

[80] Brouwer to Blumenthal 1.XI.1924.

[81] Cf. p. 338.

Blumenthal, a man of peace, did not know how to handle this letter, so he asked another managing editor, Carathéodory, to reply to Brouwer.[82] Carathéodory admitted in his letter to Brouwer,[83] that he shared Brouwer's opinion. although he wondered 'if the nonsense, that was put together in all countries, should be recalled all the time, then it would not have been necessary to stop the shooting.' He did not see why the Riemann volume would need any French contributors at all, but if one wanted a French author, then Painlevé was precisely the man to invite. Only a man with his mathematical, but even more, political reputation could take part in the Riemann volume without getting the yapping pack of barbarians after him.[84] Painlevé seemed to have mellowed considerably towards the old enemy, as was reported by the physicist Nernst, who had told that Painlevé had not so long ago offered to lecture in Berlin. Even the minister of foreign affairs had welcomed Painlevé, but some Berlin colleagues had successfully resisted this overture. 'You see that, at least according to this report, Painlevé seems to have forgotten the words that you charge him with.'

Blumenthal was in an awkward predicament; it is more than likely that he was not a free agent in this matter, and that he was acting under instructions of Hilbert. So he could not just drop the matter. Given the sensitivity of the matter he turned to Einstein, who was also a managing editor.[85] Explaining the situation, he suggested a solution along the lines of Carathéodory. Painlevé, he thought, was no longer an active mathematician and the invitation would be just pro forma. But with Painlevé's moral blessing one could safely invite Hadamard. In view of Brouwer's hard-line views, he added, the managing editors, Hilbert, Carathéodory, Einstein and Blumenthal, would have to take a decision in the matter. Not wishing to upset the sensitive Brouwer, he wondered if Einstein could give his opinion on the proposed procedure. Einstein was perfectly willing to act in this matter on behalf of the *Mathematische Annalen*. 'Painlevé', he wrote, 'is a man who has already for years been active in cleaning up the spiritual atmosphere. He may have erred; yet he is of a sort of which we unfortunately have none, or not enough.'[86] In his opinion Carathéodory was quite right; one could only approach other French mathematicians through Painlevé. In the case of Hadamard such a detour was certainly advisable, 'for Hadamard is timid and something of a Germanophobe, not at all with a free view in matters that are far from his science.' He promised to approach Painlevé via Langevin.

Blumenthal had, however, been too optimistic, thinking that only Brouwer needed pacifying. In January Bieberbach (one of the associate editors) sent him

[82]The editorial board contained two kinds of editors: the proper editors, listed on the cover as '*Herausgeber*' (publishers), and associated editors, listed un the heading '*unter Mitwirkung von*' (with the cooperation of). When relevant, we will call the first kind 'managing editors' and the second kind 'associate editors'.

[83]Carathéodory to Brouwer 6.XI.1924.

[84]'*ohne Gefahr zu laufen, von der ganzen Meute der Banausen angebellt zu werden*.'

[85]Blumenthal to Einstein 15.XII.1924.

[86]Einstein to Blumenthal 16.XII.1924.

a letter with a formal motion to renounce a decision taken by the editorial board, when meeting at the occasion of the annual meeting of the German mathematicians in Innsbruck in 1924.[87] This so-called Innsbruck resolution (*Innsbrucker Beschluss*) is referred to in a few places; it probably dealt with the organization of the Riemann volume.

After reading Brouwer's note, Bieberbach had come to the conclusion that an invitation of Painlevé was out of the question. A day later Otto Hölder joined the dissidents; the matter had more or less escaped his attention, he wrote, but even without reading the statements of Painlevé, he had strong objections to the participation of French or Belgian mathematicians.[88] Somewhat later Walter von Dyck from München sent a violent protest to his fellow editors; Einstein spoke of 'a rabid letter from München, containing a protest against the participation of Frenchmen in the *Festschrift* [i.e. the Riemann volume] .... Who is the author of this letter anyway?'[89] After quoting some objectionable statements of Painlevé, von Dyck had concluded that 'I cannot imagine, although Brouwer reports to the contrary, that after learning this monstrous insult, anyone would contemplate the contribution of Painlevé even for a moment!' Not content with barring Painlevé from the Riemann volume, he went even further: foreigners could only be admitted if there 'was absolute proof of their pro-German attitude and activity.'[90] The letter was dated '*am Tage der Gründung des Deutschen Reichs*, *18 Januar 1925*' (on the day of the founding of the German empire). Von Dyck was not alone in his outrage over the anti-German feelings and actions following the war; sentiments of this sort were not all that exceptional. In a covering letter to Bieberbach, von Dyck expressed his vexation over the procedure for the Riemann volume, and over the *Annalen* in general. In particular he wondered about the role of Einstein, who 'at that memorable stay in Paris had declared that he was not a German!' He added that anyway 'one believes rather to be in Hungary, Poland, Russia and Bulgaria, reading the names of those who publish nowadays in the Annalen.' This side remark is a small illustration of the frustration of an older scholar who, quite apart from his nationalistic feelings, finds it difficult to accept the natural influx of talent from rising nations. On this point Brouwer was without prejudice; as we have seen he warmly encouraged the young topologists from Russia and Poland to publish in the *Annalen*.

Von Dyck's letter was not the only worry for Blumenthal; Carathéodory also had unpleasant news for Blumenthal: his letter had never reached Brouwer, as he found out from Brouwer a few days ago.[91] So now the whole matter had gone wrong. He added that he found 'the solution that Frenchmen will not be invited at all, quite good. I only objected to trying to invite the cooperation of French,

[87] Bieberbach to Blumenthal 12.I.1925.

[88] Hölder to Blumenthal 13.I.1925.

[89] Einstein to Blumenthal 20.I.1925.

[90] '*wenn absolute Beweise deutschfreundlicher Gesinnung und Betätigung vorliegen*.'

[91] Carathéodory to Blumenthal 20.I.1925.

before contacting Painlevé. It seems to me, however, that the resumption of normal relations should be left to a later generation.'

It was Bieberbach who suggested a compromise. Although he has gone down in history as an evil man,[92] it should not be forgotten that he was an active member of the mathematical community, with a talent for organizing and with excellent social graces. He certainly was not after a big conflict in the *Mathematische Annalen*. So his efforts in the present matter should be viewed as sincerely directed at the interest of the *Mathematische Annalen*. Since Einstein had learned through Langevin that Painlevé had reacted favourably towards a cooperation in the matter of the Riemann volume, the following approach seemed promising: Einstein could gratefully acknowledge Langevin's assistance, and ask him to see to it that suitable contributions would reach him in time. No special invitations would be sent out, and thus a special invitation to Painlevé could be avoided. Bieberbach guessed that the editors might agree to this.[93]

Blumenthal apparently was happy to adopt Bieberbach's proposal, for he wrote on January 31 to Einstein that the Bieberbach proposal could avoid a conflict in the editorial board. If Einstein agreed, he could submit the plan to Hilbert. On the 19th of February Hilbert informed the editors of the *Mathematische Annalen* of the unanimous decision of the managing editors. Basically the Bieberbach plan was adopted; to avoid misunderstanding he added that foreign mathematicians who submitted contributions, thus indicating their interest and position, should not be excluded on grounds of their nationality.

This almost settled the matter, had not Bieberbach called Hilbert's attention to a few awkward points.[94] In view of the fact that a number of editors were opposed to French participation, he asked Hilbert to have the decision approved by not just the managing editors, but by all editors—'I must fear that otherwise this matter will seriously interfere with the otherwise good cooperation.' The implicit purpose of his proposal had been to discourage French submissions, and possible French authors, as far as possible. Furthermore the blanket invitation of French mathematicians via Langevin would result in an unequal treatment of German and French mathematicians, for in Germany only a few mathematicians were selected. He had discussed the point with Einstein, and the latter was prepared to explain the predicament to Langevin, and to inform him that French contributions were not welcome. So it would probably be best to accept Einstein's offer.

It seems rather uncharacteristic for Einstein to suggest such an inelegant rebuff of his French colleagues. Whatever Einstein may have had in mind did not matter, for exclusion of the French was completely unacceptable for Hilbert. He wrote in no uncertain terms to Einstein and Bieberbach, declaring that the managing editors were after all responsible for the policy of the *Mathematische Annalen*, so it was only proper that they should take the decision. A unanimous decision of all

[92] Cf. p. 695.

[93] Bieberbach to Blumenthal 23.I.1925.

[94] Bieberbach to Hilbert 25.II.1925.

editors would have been even better, but was not to be expected. He did see the point of the invitation to the French mathematicians. There would probably not be that many papers submitted by Frenchmen. Moreover, in the end it was the board of managing editors that decided which papers to accept. For Painlevé and Hadamard, of course, an exception should be allowed. Both had 'given spontaneous and clear evidence that they respect us and want to cooperate with us.' They should not be turned down on account of their nationality. '*This decision is for me irrevocable, and I would like to ask you to act accordingly*' (underlined by Hilbert in the letter to Einstein), and to Bieberbach he added that the decision was in the interest of Germany at large and of the German mathematicians.

The decision of the managing editors had effectively ended the discussion. When the volume appeared there were no French contributors; was there still an aversion to German scientists and their institutions? The volume contained only 14 papers and there were nine nationalities involved. Four editors published in the volume: Brouwer, Carathéodory, Einstein and Hilbert. The affair did not attract much attention; it remained in the seclusion of the editorial board, where it had come up.

Brouwer had more or less triggered the crisis, and probably made certain that the problem was taken seriously. Hilbert gave in when confronted with a compromise organized by Bieberbach and Blumenthal. The whole affair could hardly have endeared Brouwer to Hilbert, indeed later developments confirmed that Hilbert could neither forget, nor forgive Brouwer's interference.[95] The Riemann volume could have served to bring a rapprochement between the French and the Germans, or at least it could have broken the unity of the anti-German front in France. It was an opportunity missed, but it probably was too early. More time was required.

13.4 International relations

Brouwer's involvement in the discussion on the Riemann volume was in the end a corollary of his struggle for the re-integration of German scientists into the international community. In chapter 9 we have seen that Brouwer went to great lengths to set a real or imagined injustice right. His crusade against Denjoy was carried out with his usual tenacity, but without tangible result. There are no documents to trace Brouwer's anti-*Conseil* activities, but we may be convinced that like a slumbering volcano, Brouwer's resistance went on under the surface. After the policy for the Riemann volume was determined, the *Conseil* again asked Brouwer's attention. He had already been in contact with Karl Kerkhof and his *Reichszentrale für naturwissenschaftliche Berichterstattung*. The year 1925 was going to be a crucial year for German science; there were signs that the wind was changing, some members of the *Conseil* had had enough of the French-Belgian supremacy and rumours of an action to invite Germany to join, started to circulate. Kerkhof was apparently mustering his forces.

[95] Cf. p. 596.

Brouwer's role in the affair of the Riemann volume is just one illustration of his implacable opposition to the *Conseil* and its boycott of German scientists. He had fought Denjoy, and when his own academy and his mathematical society had compromised themselves by joining the *Conseil* and the *Union mathématique*, he had conducted a campaign to get the *Wiskundig Genootschap* to sever its ties with the *Union*. In the meeting of February 24, 1923 Brouwer, Mannoury and Hk. de Vries tabled a motion to the effect that the *Wiskundig Genootschap* withdraw from the International Mathematical Union. Brouwer argued that by remaining in the *Union*, the *Wiskundig Genootschap* had taken sides in a political issue, namely the exclusion of Germany. Even England, he added, had remained aloof from the *Union*, so not even all Allied nations had joined. To illustrate the lack of principles and coherent policy of the French, Brouwer pointed out that in 1908, at the Rome Conference, the French had made fun of international cooperation in mathematics, claiming that mathematics was a matter of a purely private nature.

His former teacher, Korteweg, took issue with Brouwer's claims. In his opinion the *Wiskundig Genootschap* would make a political statement exactly by terminating its membership of the *Union* on the grounds of the boycott of Germany. There was no evidence, he said, that the *Union* had been founded to keep the Germans out; 'Being prepared to cooperate to the same degree with all nations, ignoring the discord that may exist between nations, he deemed the right neutrality.'[96] The chairman, Wolff, recalled the arguments of the *Wiskundig Genootschap* for joining the *Union*. These were not political, but rather financial. It was hoped at the time that the *Union* would take over the *Revues Semestrielles*, the Dutch review journal, or at least share the financial burden. Although he could see the point of Brouwer's proposal, he thought it wiser not to act hastily. And so the matter was postponed to the next meeting.

In the November meeting the matter was again postponed. Brouwer, fearing that the issue would not get proper attention, then called a special meeting of the *Wiskundig Genootschap* on January 26, 1924. At the meeting Mannoury acted as the spokesman of the *Union* critics. Since the situation had not changed, he said, the original proposal could be maintained. He advocated a 'gentle solution'; it was best to tiptoe quietly out of the bad company in which the *Wiskundig Genootschap* had fallen. Brouwer added that the policy of the *Union* with respect to the exclusion of the Germans had not changed. 'The leaders of the *Conseil*', he said, 'were fierce German-haters'. The proposal of Brouwer, Mannoury and De Vries was strongly advised against by Korteweg. He pointed out that so far no country had left the *Union*. That the English mathematicians had not joined the *Union* was correct, he said, but it was deplored by many individual mathematicians. In fact the London Mathematical Society had turned down the invitation to join. Thus the United Kingdom was represented in the *Union* by a committee of the Royal Society. It was obvious that so soon after the war direct contact between the French and the Germans was not possible, but he saw signs of change. He indeed expected that

[96] Minutes of the meeting of the *Wiskundig Genootschap* of 24.II.1923.

in the meeting of the *Conseil* in 1925 the rules would be changed in favour of the admission of the Germans. And if this were not the case, he added, it should seriously be considered to get out of the *Conseil*. Brouwer immediately countered that the admission of the Germans in 1925 was irrelevant, 'On the German side the unanimous opinion is that no German will wish to join this *Union*.' Mannoury also took part in the discussion, noting that the Dutch mathematicians should not have assisted in the immoral act of excluding the Germans; they were in principle opposed to the exclusion, but had hoped for a change. That was wrong, and invited revenge. The error, he urged, should be corrected as soon as possible. After some more discussion a vote was taken on the motion of Korteweg not to discuss the possible resignation of the *Wiskundig Genootschap* before the next meeting of the *Conseil*. The motion was carried with 7 votes to 3.

This small episode illustrates the irresoluteness of official organizations. Although there was a general feeling that the international situation in science was unsatisfactory, the tendency was not to rock the boat. It showed, by the way, that Brouwer's influence had its limits. Even in his own professional organization he could not have his way. This may have induced him to look elsewhere, mainly in Germany, for a resistance to the *Conseil* and its subsidiaries.

History does not simply side with either Brouwer or Korteweg. The latter knew what he was talking about; being close to Lorentz, he probably was reasonably well-informed about the *Conseil* and the undercurrent of discontent about the inflexible position of France. Lorentz was a true internationalist, and his many personal relations with scholars all over the world guaranteed a perfect understanding of the need for international cooperation. He was recognized by scientists and politicians alike as a man of high moral values without prejudices.

When Einstein in 1923, two months after the occupation of the Ruhr area by the French, gave up his membership of the *Commission internationale de Coopération intellectuelle*, Lorentz was appointed in his place.[97]

In 1925 the scientific world was already attempting a cautious return to a more normal state of relations in certain quarters. In particular Painlevé showed himself inclined to bridge the existing gap between the French and the Germans. On the German side Fritz Haber was paving the way for a reconciliation. These developments raised hopes that the *Conseil* might revoke the clause of the exclusion of the Germans. And so Lorentz proposed on July 8, 1925 at the meeting of the *Conseil*, on behalf of the Dutch, Danish and Norwegian delegates to drop the exclusion rule. Six years after the war, he said, it is time to make scientific research really international. He urged the meeting not to forget that even during the war there had been German scientists who showed goodwill towards the enemy. The nominally international organization should now become a real international one. If the meeting could not adopt a truly international attitude, he feared for the future of

[97]In 1924 Einstein joined the C.C.I., an organization of the League of Nations, after all. The larger part of my information on the scientific organizations after the war is drawn from Schröder-Gudehus' books [Schroeder-Gudehus 1966], [Schroeder-Gudehus 1978], which are highly recommended for its treatment and the use of sources.

the organization. Lorentz' call and the implicit warning were countered by the delegate of Belgium. He rejected the proposal as premature and dangerous for internal peace and the indispensable tranquility of the atmosphere. Lorentz' proposal was defeated, as it did not get the obligatory two-thirds of the votes. It was a fateful result; in a sense it had been the last chance to turn the *Conseil* into a beneficial organization. We now know that not many years were left before a new and far more deadly danger announced itself.

It is one of the quirks of history that the French and the Belgians tended to consider Lorentz too soft on the Germans, whereas the German nationalists viewed him as a friend of the French. The German historian Karo did not hesitate to suggest that Lorentz was rewarded for his pro-French sympathies with a *légion d'honeur*.[98] Even Brouwer, who admired Lorentz, wondered in a letter to the editor in a newspaper, how Lorentz could reconcile his internationalist principles with this decoration in view of the French inspired anti-international policy. Commenting on Lorentz' reply to the French ambassador, 'that science will after all put an end to the dissension that nowadays is manifest in mankind', Brouwer sourly remarked that 'Mr. Lorentz is waiting the further auspicious task, to point with the full authority, which he enjoys among Dutch scientists, the KNAW in the first place at the proper limits of Christian charity, which encompasses the person who has gone astray, but not his aberration, in the second place at the motto of our country.[99]

Emil Picard, the imperturbable president of the *Conseil*, failed to see the omens. 'Six years is a long time for some nations', he said in his closing speech, 'who think that the forgetting can now commence. To others this time span seems too brief, to cover so much abomination and crime with the veil of oblivion—all the more if not the least repentance is shown over it.' We note that the two leading mathematicians, Picard and Painlevé, who had started as implacable enemies of the Germans, had gone their separate ways. Picard had remained unrelenting, but Painlevé had realized that hatred and punishment were not a suitable basis for a flourishing community of scientists. Painlevé even had openly remarked in a telegramme that there had been attempts to influence an 'intransigent scholarly specimen, in particular Picard.'[100]

The outcome of the meeting of the *Conseil* had proved Korteweg wrong, and vindicated Brouwer. This had no practical consequences; the *Wiskundig Genootschap* did not break with the *Union*. Brouwer, however, must have felt that the regular means for a rehabilitation of German science had been exhausted. He turned towards the Germans and German organizations that fought the boycott with varying success. He was in fairly close contact with Karl Kerkhof and provided

[98] [Karo 1926]. Karo, by the way, was a Jew, and he emigrated to the USA during the Nazi period. The protest against the boycott of German science and scientists was supported by a large section of the German academic community.

[99] The motto is: *Je maintiendrai*.

[100] [Schroeder-Gudehus 1966], p. 248.

him with material and advice. At one occasion Kerkhof asked for references about Blumenthal, who had been recommended to him as a particularly well-informed man where international relations were concerned.[101] He had, however, his doubts whether Blumenthal had the necessary understanding for the dignity of the German scientists. For he noted that Blumenthal had in spite of a severe rebuke of Denjoy, see p. 345, tried to establish again and again friendly relations with French scholars who had harshly attacked German science. Brouwer replied that he felt that as an editor of the *Mathematische Annalen* he was not in a position to provide information on (or rather against) a fellow editor. Kerkhof would do better to ask Bieberbach, he added. In a separate confidential note, however, he agreed with Kerkhof that Blumenthal would be the worst possible authority in this matter. He added cynically that Blumenthal's behaviour seemed to justify the French when they accused the German scientists of servility.[102]

Before long, on the German side an intolerance had emerged that matched that of the Allied side at the time of the founding of the *Conseil* (cf. p. 340). Brouwer, who somehow had become more radical anti-*Conseil* than most of his German colleagues,[103] supported Kerkhof's schemes with information and documents. He did not lapse into the mistake of holding all Frenchmen responsible for the sins of the *Conseil*, but he fought the latter with determination.

In the KNAW Brouwer had not played a role in the pro- and contra-*Conseil* movements. When the Academy joined the *Conseil*, there had been a fierce opposition of various fractions and individual members. In the meeting of January 25, 1919 Brouwer had suggested a formal way out of the membership invitation of the *Conseil*, by pointing out that the old associations still existed, and that cooperation with those should be maintained. The members of the Academy from Groningen were likewise opposed to the *Conseil*. They submitted a proposal to found an independent '*Conseil*' of neutral nations, which could act as an intermediary between scholars of all nations. And somewhat later the group sent a note to the Academy, pointing out that the proposal of the board to join the *Conseil* in fact was 'a polite and humble request' to the Allied academies, to be allowed to cooperate in an organization that 'on the ground of the exclusion of Germany and Austria, is and will be of a pronounced hostile nature to the Central academies.' The Academy did nonetheless join the *Conseil* and this was for the astronomer Kapteyn a reason never again to attend Academy meetings. The philosopher Heymans went so far as to resign from the Academy. In due time peace was restored, but there were some attempts to dislodge the KNAW from the *Conseil*, or at least to admit Germany and Australia. In 1922 Cornelis van Vollenhove, the law professor, proposed in the meeting of the *Conseil* to drop the exclusion clause. This motion was easily defeated. After the rejection of Lorentz' proposal in 1925, there was another

[101] Kerkhof to Brouwer 4.V.1925.

[102] Brouwer to Kerkhof 11.V.1925.

[103] Witness the remark of Sommerfeld in a letter to Blumenthal (26.I.1925): 'I can but experience it as humiliating that Brouwer feels more German than we do.'

anti-*Conseil* action in the meeting of the KNAW of September 25; Brouwer, J.F. van Bemmelen and J. Versluys condemned the *Conseil* and advised the KNAW to withdraw from it. The attempt failed.

Brouwer's observation that no German scholar would wish to join the *Conseil* had been somewhat sweeping, but he did have a point. The more nationalistic scholars had, no doubt influenced by Kerkhof's organization, decided that they would rather not join a company which characterized them as brutal, slavish killers. The insults addressed to the collective German scholars were still ringing in the ears of the sulking scientists, who were no more familiar with forgive-and-forget, than their French colleagues. And so gradually out of the boycott a counter-boycott was born.

An influential group was actively campaigning against membership of the *Conseil*. Many pamphlets and brochures appeared that sketched in vivid notes the offensive treatment of German scientists. Already in 1923 the *Hochschulverband* had issued a list of five instructions how to cope with international contacts.[104] The instructions were a perfect counterpart of the rules and instructions of the *Conseil*. Here are some passages:

> —The German Universities (*Hochschulen*) and individual German scholars should on principle and carefully avoid everything that could in some way be interpreted as an attempted approach to the Entente.
>
> —[with respect to private contacts] Contacts via the backstairs should be turned down as unworthy with the request that first the slanderous statements[105] should be publicly withdrawn in the same solemn formulation in which they were enounced.
>
> — Any cooperation with France and Belgium should under all circumstances be out of the question.

The radically anti-French section of German science had started to organize itself, and their statements did not give much hope for a compromise. Professor Georg Karo compiled a long list of anti-German measures and statements in his 'The spiritual war against Germany'.[106] There is no denying that the Germans were sinned against, but they certainly gave as much as they received. Karo spoke for a sizable number of his colleagues when he concluded 'We are not served by a solemn retraction of the boycott, as long as the system of reglementation of the scientific world that was founded together with it, remains. Basic dignity forbids us any rapprochement with the *Conseil Internationale de recherche*, which was born out of hate and contempt for us, even when it would be reorganized.'

Brouwer did play a modest role in the anti-boycott, e.g. he once ordered (and presumably, distributed) 100 copies of Karo's monograph. He would have subscribed to some of the texts of the anti-*Conseil* movement, but certainly not all of them. He cultivated, for example, contacts with French and Belgian colleagues. But

[104] Cf. [Schroeder-Gudehus 1966], p. 207.

[105] Cf. p. 338.

[106] *Der geistige Krieg gegen Deutschland*, [Karo 1926].

he shared the radical sentiment that demanded apologies for the post-war insults. His role probably was no secret in Germany. Kerkhof made use of Brouwer's disposition and Planck, for example, asked Brouwer to find out certain disputed facts about the *Conseil*. Apart from his role in the Riemann volume, Brouwer's influence was marginal. His big moment came at the time of the Bologna conference.

13.5 The Dutch topological school

In 1925 Brouwer was back in topology, not that he prepared new publications or continued his research, but the encounter with Urysohn and Alexandrov had reawakened his interest on a more contemplative level; he realized that a new generation of topologists had come forward, and he liked what he saw. In particular Urysohn's work was after his own heart; large parts could be seen as a continuation of his innovating papers. As always, there were the dangers of priority matters, but as we have seen, Urysohn and Brouwer came to a perfect agreement; Urysohn must have been an unusually engaging and mature person. During his short sojourn in the mathematical community in Germany and France, he had succeeded in making friends everywhere. In particular, he and Alexandrov were highly regarded in Göttingen, where the tragic death of Urysohn was felt more deeply than one would generally have expected from such a meteor-like appearance in the mathematical firmament. Emmy Noether, Bernays, Felix Bernstein, Hausdorff, Fréchet, each and everyone of them felt the loss of this brilliant but modest young man.[107] Brouwer, who had a singular gift to reach the soul of another person—if he cared to do so—had held long and penetrating conversations with Urysohn during the period of their brief encounter in Blaricum. He had almost instantaneously developed an immense liking and appreciation for the young man. The sudden loss of the young genius who had so briefly entered his life brought him immeasurable desolation. Perhaps he cared all the more for the remaining young Russian, whom he subsequently invited to Amsterdam.

In 1925 Amsterdam suddenly became the centre of topology in Europe, and Brouwer was tacitly recognized as the founding father of the 'new topology'. Menger had arrived in Holland at the end of March and Alexandrov in May, in the autumn Vietoris also joined the group and Newman came over for a short visit. Furthermore Wilson, a student of Brouwer, regularly took part in the activities.

An important source of information on the period described here are the published memories of Alexandrov and Menger.[108]

As customary in those days, professors worked largely at home, unless they had their laboratories at the university, and they only turned up for lectures and faculty meetings. It was Brouwer's custom to work at home in Blaricum, and he more or less insisted that his visitors and collaborators should be within easy reach. This implied that they had to find lodgings in 't Gooi. For Alexandrov this was no problem, on the contrary, there was a remarkable affinity between his and Brouwer's

[107] As testified by letters in the Alexandrov archive.

[108] [Alexandrov 1980], p. 322 ff., [Menger 1979] p. 241, ff.

Cor Jongejan (Brouwer Archive).

life style. Both liked the outdoors, swimming, hiking, etc., although Brouwer no longer went in for excessive physical exercises. It was no problem to find lodgings for Alexandrov in Laren in the house of a friend of Cor Jongejan, for Menger something similar was envisaged, cf. p. 479.

Brouwer's way of life had not altered since he came to live in Blaricum, as a rule his wife attended the pharmacy and during the week lived mostly in Amsterdam. Brouwer's home was run by Cor Jongejan, who not only took care of the house, but also acted as his secretary and friend. She assisted Brouwer in keeping order in his files, typing letters—sometimes writing them by hand copying manuscripts that were submitted to the *Proceedings* of the Academy or to the *Mathematische Annalen*, etc. Eventually Cor got the proper recognition for her secretarial duties: she was appointed as an assistant in 1925. So far Belinfante had filled the position of assistant, and in the academic year 1925/26 Brouwer managed to get a record number of assistants: Belinfante, Menger (first assistant), Alexandrov (first assistant) , Vietoris (first assistant), C. Jongejan (adjoint assistant). Belinfante had been admitted to the university as a 'privaat docent', and in this capacity he taught his specialities: intuitionistic analysis, theory of infinite series, complex function theory. In order to make a living he also taught in high schools.

One would expect that by now Brouwer's fame would attract the brighter students in mathematics. This was only partly the case. Brouwer did not care for public relations, he taught and operated rather low key. Most students did not even know that their local mathematics professor was a topological genius, let alone a man suspected of overthrowing the mathematical world. It was more or less a fortunate coincidence that in the early twenties one of the brightest young men of the day came to study mathematics in Amsterdam: Bartel Leendert van der Waerden.

The Aljechin–Euwe match in 1935, which made Euwe world champian. (Courtesy Mrs. S.M.P. Brouwer-Euwe).

Van der Waerden enrolled in Amsterdam in 1919. The study of mathematics was for him the proverbial 'piece of cake'. Reminiscing about his studies, he said: 'I heard Brouwer's lectures, together with Max Euwe and Lucas Smid.[109] The three of us listened to the lectures, which were very difficult, he treated the integration theory of Lebesgue along intuitionistic lines, and that works. It was very curious, Brouwer never paid any attention to the audience. All the time he gazed at a point on the opposite wall. He lived in Laren, rather isolated; he came roughly twice a week to lecture in Amsterdam.'[110] Brouwer's commuting was rather time consuming, but it had an attraction for him: he could meet acquaintances, students, or colleagues on the tram (or bus, or train, as it happens to be). In winter, when darkness fell early, Brouwer as a rule carried a storm lantern, as the streets in Laren-Blaricum were not all that well lighted. His neighbour, Mr. Crèvecœur met him once on the bus. Brouwer tapped him on the shoulder and said, 'Mr. Crèvecœur I see you smoke, and I do not. You must have matches, could you light my lantern?' Arriving in Blaricum, Brouwer accompanied Crèvecœur home, but being in a good

[109] Smid became a specialist in insurance mathematics, Max Euwe won in 1935 the world title in chess. Later he became a professor in computer science.

[110] Brouwer actually lived most of the time in Blaricum, but right at the border with Laren. This border has moved back and forth in the course of time. Sometimes he rented a house in Laren or in Blaricum. A large part of his letters bears the name of Laren, later letters show Blaricum.

mood, he talked and talked, explaining how one could determine the surface of a ball by emptying it. Crèvecœur had heard quite enough, so that when they reached the house of Crèvecœur, he said at his door, 'I am tired, now I want to go to bed.' Brouwer would not hear of it, 'Come along to my house, and we can talk a bit more.' So they went from house to house a couple of times, until Brouwer suddenly said, 'I go home, good night,' leaving Crèvecœur in the dark.

Van de Waerden did not find it simple to talk to Brouwer, 'He immediately departed after the lecture, so that it was very difficult to make contact with Brouwer.' Van der Waerden meticulously took notes in class, and usually that was enough to master all of the material. Brouwer's class was an exception. Van der Waerden recalled that 'at night he actually had to think over the material for half an hour and then he had in the end understood it.'[111] It was not until he visited Göttingen that he heard about Brouwer's topological work. Students in Amsterdam were on the whole unaware that their department housed one of the great topologists of the century. Van der Waerden was an extremely bright student, and he was well aware

[111] Interview Van der Waerden

Van der Waerden and Emmy Noether in Göttingen. (Alexandrov Archive).

of this fact. He made his presence in class known through bright and sometimes irreverent remarks. Being quick and sharp (much more so than most of his professors) he could make life miserable for the poor teachers in front of the blackboard. During the, rather mediocre, lectures of Van der Waals jr. he could suddenly, with his characteristic stutter, call out: 'Professor, what kind of nonsense are you writing down now?' He did not pull such tricks during Brouwer's lectures, but he was one of the few who dared to ask questions. This was no easy matter, Brouwer lectured in a very precise, concentrated manner. He was completely wrapped up in himself and reconstructed the material as he went; he presented his mathematics in a long monologue, any interruption was apt to break the line of thought. Normally the students accepted this phenomenon as part of the unavoidable hardships of university life, but at one occasion they conquered their awe and sent a delegation to Brouwer. Admitted into Brouwer's office, the spokesman solemnly brought his message, 'Professor, we have difficulty following your lecture, you go so fast.' Brouwer thought for a moment and replied 'I see, well, I will try to lecture as slowly as the gentlemen think.'[112]

When Van der Waerden gathered all his courage and asked Brouwer a question in class, Brouwer replied politely, but later he sent his assistant to Van der Waerden with the message that further interruptions would not be appreciated.[113]

One would think that such a bright student was a man after Brouwer's heart. The truth is that Brouwer had no affinity with Van der Waerden's mathematics;

[112]Oral communication F. Kuiper.

[113]Max Euwe told a different story. When Van der Waerden asked his question, Brouwer replied sternly and then concluded: 'Mr. Van der Waerden, I advise you to study the material completely before you start a discussion.' The story went from student generation to student generation. Freudenthal's version was that 'Van der Waerden thought to have found a mistake. But Brouwer was completely right.'

furthermore, Brouwer wanted to be left alone to do his own mathematics. A clever young man who would interrupt his own contemplations with bright remarks and questions, was the last thing in the world he wished for. He certainly appreciated Van der Waerden's mathematical gifts, and he went out of his way to get him a Rockefeller stipend for Göttingen. Given Van der Waerden's algebraic interests, the person to take care of him was Emmy Noether. Once in Göttingen, under Emmy's wings, Van der Waerden became a leading algebraist. Emmy was very pleased with the young Dutchman, 'That Van der Waerden would give us much pleasure was correctly foreseen by you. The paper he submitted in August to the *Annalen* is most excellent (Zeros of polynomial ideals)...', she wrote to Brouwer.[114] Notwithstanding his popularity in Göttingen, Van der Waerden came back to Amsterdam for his doctor's degree. Perhaps he would have liked Brouwer as a Ph.D. adviser, but Brouwer systematically discouraged students from writing a dissertation under his supervision. Brouwer was not interested in the honour, pleasure and toil of the Ph.D. advisor role. Whenever a student turned to him for a Ph.D. project, he referred him to Weitzenböck or Hk. de Vries. Weitzenböck was regarded as a leading specialist in the theory of invariants, and quite a number of Ph.D. theses were written under his supervision. Max Euwe was one of his Ph.D. students, and so was Griss, the founder of negationless mathematics. Hk. de Vries had a totally different personality; he was a cultured man deeply in love with mathematics, but his admiration for the subject of his love did not express itself in deep or original contributions. He was well aware of the fact that depth and originality were not his strong points; he could jokingly refer to his colleague Brouwer, whom he admired, as a 'point collector' (*puntverzamelaar*). Even his lectures, although enthusiastically presented, were somewhat superficial. The students loved him and his courses; the love for his subject is illustrated by the following story. At the end of one of his lectures, he was so excited, that he left the room with a resounding 'long live mathematics'. He wrote quite a number of attractive textbooks, more praised for the style and presentation, than for depth or precision, including books on the history of mathematics. In spite of his modest reputation as a creative mathematician, he was the man who inspired generations of students. 'If you had heard De Vries' lectures, you would forever be under the spell of mathematics', said Euwe. It was de Vries who took the role of Ph.D. adviser of the young Bartel upon himself. The topic of Van der Waerden's dissertation was enumerative geometry, a subject that was later treated in a monograph by de Vries himself.[115] Van der Waerden's dissertation earned him instant fame in the world of algebraic geometers for its importance as a solid basis of the subject.[116] Although later in life Brouwer and Van der Waerden drifted apart, their relationship in the early days was unproblematic. Van der Waerden expressed in his dissertation his intellectual debt with

[114] Emmy Noether to Brouwer 14.XI.1925.

[115] [Vries 1936].

[116] 'The algebraic foundations of the geometry of number' [i.e. enumerative geometry] [Waerden, van der 1926].

the words, 'You, learned Brouwer, I thank you for your lectures, which went up to the ultimate limit of exactness, and for your vigorous help, amply shown to me on various occasions.'

The obligatory list of theses which were part and parcel of dissertations in Holland, dealt with a rich variety of topics; one of these treated a traditional source of confusion in intuitionistic mathematics (and logic): negation. It read, 'It is recommendable to use the word *not* in intuitionistic mathematics exclusively with the meaning *impossible*, and no longer with the meaning *there is no reason to assume that*.' This thesis demonstrated Van der Waerden's familiarity with the strong intuitionistic negation, and it embodied an attempt to do away with the confusion between, e.g. 'the Goldbach conjecture is false' and 'we have no proof of the Goldbach conjecture.' However, the weak negation is firmly entrenched in daily language, and its abolishing would make discussions most awkward. Brouwer, when questioning the candidate at the public defence of his dissertation, said as much by pointing out that 'you are not a doctor, but that does not mean that you will never be one.'

In mathematics Van der Waerden was easily recognized as an outstanding scholar, but in the 'real world' he apparently did not make such a strong impression. When Van der Waerden spent his period of military service at the naval base in Den Helder, a town at the northern tip of North-Holland, his Ph.D. adviser visited him one day. He said later that the commander was not impressed by the young man, 'he is a nice guy, but not very bright.'

After getting his doctorate in 1926, Van der Waerden became an assistant in Hamburg, and in 1927 he became a *Privatdozent* in Göttingen. A year later he accepted a chair in Groningen, where he wrote his famous *Moderne Algebra*. A few years later he moved on to Leipzig, where he held a chair from 1931 till 1944.[117]

Alexandrov saw a great deal of Brouwer; together they edited Urysohn's papers, which often came to reconstructing the text on the basis of Urysohn's notes. Brouwer conscientiously read Alexandrov's drafts and made small editorial corrections, when necessary. They often got together in Brouwer's cottage to work and talk. Alexandrov was a man with an engaging personality, a ready wit and of a cheerful disposition. Of course, the loss of his close friend and co-topologist, Urysohn, had thrown him off-balance, but his subsequent stays in Blaricum and in Göttingen did much to revive his spirits. Alexandrov, with his wit and graciousness, made quite an impression with the female inhabitants of 't Gooi, including Cor Jongejan. Brouwer even imagined a real threat to his relationship with Cor. Gerda Holdert, the wife of Van Eeden's former secretary and member of Brouwer's inner circle, reported that he once asked her to keep an eye on the two.[118] Alexandrov used the first half of July to be with his old love: the sea. He made a walking tour

[117] For biographical information on Van der Waerden see [Springer 1997] and the papers in *Nieuw Archief voor Wiskunde*, 1994(12) no.3.

[118] Oral communication Gerda Holdert.

along the seacoast and settled for a fortnight in Katwijk, at that time a small fishing town, which in summer hosted a good number of holidaymakers. In his autobiographical notes he mentioned that Princess Juliana, the future queen, stayed in Katwijk during that period.

Alexandrov spent the rest of July 1925 in Göttingen, where he presented, at Hilbert's request, a survey of topology. In August he returned to the fateful Batz where he was joined for a month by Urysohn's father. In the month of September he set out on a walking tour in the Pyrenees. At the end of October he returned again to Batz, where Brouwer joined him to spend the first two weeks of November together. They returned together to Blaricum and almost immediately the topological activities started.

Menger had also left Holland for the summer. Staying in Heidelberg, a telegram that was forwarded to him by Brouwer, brought him the sad news of his mother's death. Menger was heart-broken, in the depth of his suffering he lamented[119]

> I thank you most warmly for your kind letters of condolence with the most terrible blow of fate, which has struck me. What I lost in my dear mother, I cannot sketch in words; her goodness of heart was unbounded. And to the pain about what has been taken from me, the unspeakable thought is added, that she, who as long as I live has done and sacrificed so infinitely much for me, died precisely now, when a more quiet evening of her life, which she looked forward to with pleasure and was still capable enjoying, had begun. She followed from afar everything concerning me, grateful in particular to you, for all the support you gave me. That news was her last enjoyment.

After hurrying back to Vienna, Menger suffered a severe nervous breakdown. As a result he had to take a cure of several weeks. His thoughts wandered to the tragic fate of Urysohn, 'Again and again I thought in this period of the poor Urysohn, and wished that I would have perished instead of him. Only the thought, that I may not destroy that what my beloved mother had built with her life's effort, gives me now the will, to regain, if possible, my health and then to achieve something.'

Brouwer, who considered his topological visitors more his foster children than just a group of scientists ('postdocs' we would say nowadays) on their way to outstanding careers, sympathized with Menger's loss in its full tragic degree. He replied on July 8 with a deeply felt, caring letter; he was indeed relieved to see that Menger had survived the shock, because during Menger's stay in Holland Brouwer had surmised

> to what extent the sphere of your mother has radiated through your life. I also sensed the extent of your loss, and expected the crisis that the sudden emptiness, and the sudden necessity, to adopt another spiritual mode of breathing, would bring forth in you. But after this first crisis has been

[119] Menger to Brouwer 3.VII.1925.

> borne, I am certain that you will find the required concentration and religious devotion, to find yourself a way through, and that the certainty of the dear deceased's wish, aimed at these, as well as the thoughtful memory of her that will be with you, will help you in this.

Being uncertain how well-off Menger was financially, he offered him an assistantship in Amsterdam, as he was not certain how long it would take to wind up the details of the Rockefeller stipend.

Menger managed, nonetheless, to carry on his mathematical activities and on August 16 he could send Brouwer a copy of the manuscript of a large expository paper on dimension theory, to be submitted to the *Jahresbericht*; in the accompanying letter he asked Brouwer's permission to dedicate a separate publication as a small monograph (in fact off-prints with a special cover) to him, 'Dedicated to L.E.J. Brouwer in deep reverence.'[120] In the same letter another key figure from dimension theory is introduced: Witold Hurewicz. Hurewicz was born in 1904 in Lodz as the son of a rich industrialist; he studied in Vienna where he became informally Menger's student. Menger promised to send Brouwer a copy of Hurewicz' recent manuscript on dimension theory, which was subsequently published in the *Mathematische Annalen*.

The letter also shows that Menger was by no means a narrow specialist. There is a passage in the letter that shows Menger's occupation with graph theory; he in fact believed that he had found the key to a proof of the four-colour problem.

In Menger's autobiographical sketch 'My memories of L.E.J. Brouwer' [Menger 1979], a penetrating description of Brouwer and his *entourage* is given; Menger showed himself a keen observer—one cannot but deplore that he did not go on and write down the total story of his life. In a way his fate is typical of the middle European scientist of the pre-war era, highly successful, versatile, cultured, with many contacts, who through a forced emigration suddenly became separated from his scientific-cultural background and lived on in relative obscurity (he would never reach the heights of his European period again).

Coming to Blaricum, he met Brouwer for the first time in person. As he remembered the first impressions,

> he looked older than his 44 years. His figure was lank and youthfully agile; but his hollow-cheeked face, faintly resembling Julius Caesar's was extremely nervous with many lines that perpetually moved changing his expression from one moment to the next.
>
> His features seemed incessantly to reflect intense inner reactions to what he was hearing and seeing, just as outward intensity in speech and movement in action was the hallmark of his personality.[121]

The picture is wholly accurate, Brouwer had an unusually expressive face and his movements have been described as agile and almost fluttering. He could come

[120] *L.E.J. Brouwer in tiefer Verehrung gewidmet*.

[121] Here and in the following pages I quote from [Menger 1979].

down the aisle of the great auditorium like a gigantic bird, his large delicate hands flapping like wings. Menger was struck by Brouwer's fluency in various languages:

> Whichever the language, Brouwer talked very fast and with great precision, though often in long and involved sentences. This he did also in lectures, where he went into minute details. But he delivered them with an impressive emphasis.

If Menger had expected to find a quiet scholar, devoting his time to the creation and dissemination of knowledge, he was in for a surprise. Brouwer was largely occupied with organizational matters, non-mathematical activities and innumerable legal cases. For example, at that time Brouwer still spent a great deal of time fighting the *Conseil International de Recherche* (as we will see later).

Soon after his arrival Menger accepted Brouwer's offer of an assistant position; the duties were fairly light, varying from reading manuscripts that Brouwer had to referee for the *Proceedings* of the Academy or the *Mathematische Annalen*, lecturing, etc. Both Alexandrov and Menger shared the normal teaching duties, be it that they taught their own speciality. Alexandrov gave a course on general topology, and Menger a course on dimension theory. Brouwer conscientiously attended these courses and took part in the lively discussions that followed. Brouwer himself lectured on intuitionistic mathematics and on continuum mechanics. In the first one he covered a lot of ground, in particular he treated topology from an intuitionistic point of view, for example the theorem of Jordan. He also gave an elaborate exposition of the intuitionistic theory of order. Advanced courses were given in German for the benefit of his visitors. The second one also contained a fair amount of topology, as Alexandrov formulated it 'much homology theory (in the specifically Brouwerian manner, with pseudo-manifolds and so-called 'fragments' instead of cycles and chains').[122]

Menger was in close contact with Brouwer:

> I frequently saw Brouwer alone and came to know more about his habits and tastes. I never found him depressed, but I rarely saw him laugh or display signs of a sense of humour. He usually manifested a high-strung personality—least tense when listening to or talking about music. He loved classical music and frequently attended concerts, especially of Beethoven's works, about which he spoke beautifully. In painting he seemed to favour the style of the Renaissance and was fond of portraits of that period. He looked with obvious distaste at the expressionistic graphic with social themes by the Dutch and Flemish artists of the 1920's (which I greatly admired)—even at Masereel's marvellous novels in woodcuts. They lacked the kind of beauty he seemed to expect from art. He also looked for comeliness in handwriting and developed in his middle years an aesthetic style of writing. But even more interesting were his ways of crossing out passages in manuscripts. In order to delete an entire

[122] [Alexandrov 1969], p. 117.

> paragraph he would draw many diagonal lines in both directions forming networks of perfect regularity; some had wider meshes, others were quite narrow grills. Single words or sentences he blocked out with solid black lines or rectangles. Some corrected manuscripts of his were quite picturesque (in a remotely Mondrian-like way). The handwriting was also of interest to him in letters that he received; but there, his chief concern was the consistency of style of writing in a person's consecutive letters.

Alexandrov, too, mentioned Brouwer's love for music; he regularly accompanied Brouwer and Cor Jongejan to the Sunday afternoon concerts at the Concertgebouw where the great Mengelberg conducted. They also saw Stravinski conduct his *Sacre du printemps*. Quite often important visitors of Brouwer joined them for the concerts. Brouwer was for the greater part of his life a familiar figure at the Sunday concerts, the traditional meeting place of the Dutch music lovers. Brouwer, although not consciously drawing attention to himself, was a conspicuous figure in the Concertgebouw, one could not help noticing this extraordinary person.

Even though Brouwer was no longer actively involved in topological research, Menger warmly appreciated Brouwer's interest in his work:

> In the late spring and the fall of 1925, I reported to him my latest results almost every week and immensely enjoyed these occasions. He never formulated problems to me; but he listened with an eagerness that I found very stimulating. Through the perpetually changing expression of his face he had the rare ability to be, as it were, even silent with intensity. He also showed concern for my then still unstable health and expressed warm sympathy when I suffered a bereavement. He warned me against overworking 'A mathematician's mind is like a fruit tree,' he once said to me. 'Not all blossoms turn into fruit; and if one year they all do, then the tree bears no fruit the following year.' I revered him with a deep affection during those months.
>
> Brouwer regarded mathematical creation as a decidedly youthful activity. 'Even Gauss', he once remarked, 'spent his later years on boring numerical computations.' I observed that according to what I had heard, Gauss enjoyed numerical computations from childhood on, all his life. 'Mathematicians' old age is sad', Brouwer answered sombrely, 'Gauss' old age was sad, too.' It was the only time that I ever discovered in him something like gloom. Only much later did I realize that these remarks were the key to understanding Brouwer's personality, especially several traits of his that I was soon to discover and which (together with some of his political views) I would find incompatible with the idealized picture that in youthful enthusiasm I had formed of him.

Emmy Noether also joined the group at Blaricum for a brief period; she spent a month in Blaricum staying over for Christmas and New Year. In his paper 'Topology in and around Holland in the years 1920–1930'[123] Alexandrov reports about

[123] *Die Topologie in und um Holland in den Jahren 1920–1930*, [Alexandrov 1969].

this visit, 'Right then Emmy Noether formed the opinion, that group theory is the proper foundation for combinatorial topology, that in particular the numerical invariants—the Betti numbers and the torsion numbers—must be replaced by homology *groups*.' Alexandrov recalled 'a dinner at Brouwer's in her honour during which she explained the definition of the Betti groups of complexes, which spread around quickly and completely transformed the whole of topology.' Back in Göttingen she gave a lecture on 'the algebraization' of topology at a meeting of the Göttingen mathematics society, of which the dinner crash-course in Blaricum had been a preview.[124]

This may be a good point to dispel a widespread misconception about Brouwer's views on women and society. In his student days he had aired some radical opinions on the matter, but his actual policy in his private and professional life was in such a striking contrast, that one is inclined to attribute his early statements to a provocative student mentality and to a genuine mystical introversion. Brouwer did, however, enjoy and appreciate female company until the end of his life, and not only as the charming creatures that spread happiness and light, but equally in professional circles. In science it did not matter a bit to him what the gender of a scholar was, an interesting and active mind counted more than conventional distinctions. Emmy Noether, in particular, was highly regarded by him, and her arrival in Blaricum in a period during which the Grundlagenstreit had already dangerously soured the relations between Hilbert and Brouwer, is a telling proof that she reciprocated Brouwer's regards. As a matter of fact, their acquaintance went back to before 1914; in answer to a postcard from Carathéodory and Brouwer[125] she wrote 'I also recall with pleasure the days in Karlsruhe, and I hope that there will soon be another mathematics conference where we can meet. Won't you come soon to Göttingen and give a lecture?'

In November 1925 she had approached Brouwer about the extension of Alexandrov's Rockefeller grant for a stay in Göttingen. She had come to appreciate Alexandrov and Urysohn so much during their stay in 1923, that she had invited Alexandrov to Göttingen. Alexandrov was, understandably, gratified by the invitation, but under no circumstance did he want to antagonize Brouwer, so he discussed the matter with Brouwer, who did not object, although he remarked to Emmy Noether[126] that 'In my opinion it would be an excellent choice of the Georgia Augusta,[127] if it could get Alexandrov a temporary teaching position (*Lehrauftrag*) for Göttingen. [...]. I would even prefer to secure Alexandrov for a longer period in Holland, or even keep him here permanently, ...'

Alexandrov, who like so many young mathematicians had fallen for the Göttingen Goddess of modern algebra, apparently knew the world well enough to realize that his attachment to Brouwer would not be considered a recommendation

[124] Cf. [MacLane 1981].

[125] E. Noether to Brouwer 7.1X.1919. For the Karlsruhe meeting see p. 178 ff.

[126] Brouwer to E. Noether, 21.XI.1925.

[127] the University of Göttingen

in certain places. So when corresponding with Emmy Noether, he begged her to consider his letter as strictly personal,[128]

> I would not like that my purely moral dependence on Brouwer in these and many other matters, which I take freely upon me, and which therefore, does not clash with my personal and scientific freedom, would lead to any misunderstandings at all.

For some time the paradise-like situation lasted in the topological family but soon tensions built up. It is hard to put a finger on the exact cause, probably a number of factors played a role. In a sense Alexandrov and Menger were, temporarily at least, Brouwer's topological sons—Vietoris was also a member of the topological family but he seemed to have had a knack for keeping out of trouble, indeed for remaining unaware of frictions around him. To extend the metaphor a bit farther, Alexandrov strongly resembled the beloved brother of Urysohn, whose premature death cast such a deep shadow over his close associates. This introduced an undesirable asymmetry in the family, and one must fear that Brouwer did not obey the golden rule of all parents: not only love your children equally well, but make certain that they feel equally well-treated. The relation between Alexandrov and Menger, with Brouwer in the background is the classical story of two brothers, the charming, intelligent, sport-loving boy adored by all friends of the family, and the awkward boy who cannot tell his jokes properly and who is not applauded for the same performance as his brother.

Indeed, where Alexandrov was the accepted darling of any mathematical community, Menger confused his fellow men with his statements that may have been intended as jokes, but certainly did not enhance his popularity. The more active and outspoken scientists usually generate anecdotes that in one way or another illustrate their personality; Menger was no exception. A single one may serve as an example; on one occasion, he refused to take the thermometer[129] from a nurse with the words: 'Do you know who I am, I am the Napoleon of mathematics.'[130] Also his worries about priority (something he shared with Brouwer!)—in particular the story about the deposited manuscript—were the subject of comments in the circle of his colleagues. Alexandrov, when discussing a particular meeting of the Berlin Seminar on topology in Göttingen, jokingly remarked that Menger would certainly go with three notaries to the patent office, and get himself an 'original cover protected by law for his course notebook, where there would be 2 forms of this cover: one exclusively for national, and the other one for international use, as is customary with famous laxatives and the like.'[131] The difficulty with anecdotes is that they are not fair, they blow up a particular feature, not unlike a caricature. The anecdotes about Menger tend to highlight his tendency for an inflated feeling of importance. Those who have known Menger agree that this was only one side

[128] Alexandrov to E. Noether 11.XI.1925.

[129] rectal, as the hospital tradition was.

[130] Oral communication Mrs. J.F. Heyting-van Anrooy.

[131] Alexandrov to Hopf 10.IV.1927.

of his personality; nonetheless it made contacts with him not any easier. As to the matter of the deposited manuscript, one should bear in mind that this was the story of a 20 year old boy, seriously ill and in confusion, acting on the advice of his 80 year old father. Altogether, the combination of characters in Blaricum was one that spelled trouble—and that would have thrilled an Agatha Christie.

The atmosphere was gradually changing in the Blaricum centre. The exact cause is a matter of some uncertainty. There are two main views: those of Menger and of Brouwer. The older of the two was at the zenith of his career, respected, and occasionally feared in the scientific world. The younger one was making his way in the mathematical world; he was already basking in the gratifying admiration of the topological world. Brouwer, from his place at the mathematical Olympus, had no intention to suffer any belittling attacks or comments, and Menger, with his not inconsiderable self-esteem, was inclined to see conspiracies to do him out of his rightful place at the top. From these two perspectives conflict material was abundantly available. Brouwer had seen enough of the mathematical world to know the value of correct citations and references; he therefore insisted on appropriate reference in matters of dimension theory. Basically this came down to the references to the three founding fathers: Brouwer himself, Urysohn and Menger. He quite correctly took the position that as far as the new dimension theory was concerned, the priority lay with Urysohn, albeit that Menger's ideas were conceived at roughly the same time.[132] The independence of the work of Urysohn and Menger was never disputed.

During Menger's first year in Amsterdam the 'historical' aspects of dimension theory must have been the subject of the conversation in the group of topologists. After all, Menger was teaching a course in dimension theory, and the other members of the group would attend whenever possible. We may assume therefore that questions were asked and replies given, perhaps even debated with some fervour. Menger was not the man to stand aside, and let Alexandrov and Brouwer dominate the discussion. Having a strong interest in the proper distribution of credits, Menger in the end set out to present the historical development of his dimension theoretic ideas in their proper order. This required a complete set of manuscripts and notes that would count as objective evidence. This evidence had to be collected in Vienna. On April 10 Menger could inform Brouwer that 'the priority matter is now completely resolved'.[133] The material had been kept in a safe, and he promised to hand over the whole collection (four items) to Brouwer. The matter had put tremendous pressure on Menger's mental state, 'Had I not intended to put

[132]We recall that Urysohn discussed his dimension theory for the first time (with Alexandrov) in August 1921 (cf. p. 443 [Johnson 1981] p. 228). He lectured on the topic in Moscow in the academic year 1921/22, and submitted three printed notes on the subject to the Moscow Mathematical Society. The first internationally accessible version appeared in the *Comptes Rendus* in 1922. Menger developed his ideas on dimension theory between April 1921 and February 1922. An account was submitted to Hahn in November 1922, and was withdrawn after Hahn discovered a mistake in it. Menger's first publication followed in December 1923 [Menger 1923].

[133]Menger to Brouwer 10.IV.1926,

the documents, dear Professor, into your hands, for what you have written about the theory, and what you have done for me—I could not have suffered, what I had to live through'. A nervous collapse indeed followed. There was also corroboration of Menger's assertions by third parties; Otto Schreier, a close friend of Menger, went through his notes to ascertain the dates and events that were relevant,[134] and Menger's teacher, Hahn, gave an account of Menger's work on dimension theory.[135]

It did not take Menger long to complete the survey of his dimension theoretic work; already on 29 May Brouwer submitted a paper, entitled 'On the genesis of my papers on dimension and curve theory'.[136] Three years later, when the dimension conflict was at its height, Brouwer recalled that he had to use all his persuasive powers, 'to get him [Menger], to write down only verifiable [*beweisbar*] matters with an indication of the proof in his historiography; and to accept personally and openly the responsibility for historical views, for which only he possessed the documents, therefore neither to demand publication of these views from less expert friends, nor to convey these, instead of explicitly, only implicitly as a consequence of irrelevant marginal facts'.[137]

Menger's genesis paper later became a subject of controversy; Brouwer had insisted on a precise record of the mentioned manuscripts and papers. He therefore demanded that those documents that could be exhibited, be referred to as 'still available' (*noch vorhanden*).

> The refutation of my assertion, that the ms of February 1922 could not 'be found' remains a second difficult point (also in the new version of Menger). For when Menger during his stay in Amsterdam handed over his old documents for publication in the Amsterdam Proceedings, I initially had objections to submitting the text in which he refers to the ms of February 1922, without prior inspection. Only after my repeated requests to Menger (also in the presence of other interested parties) had failed, I have agreed to the publication of the text in such a form, that a distinction was made between 'still available' and 'not available' documents, and that the ms. of February 1922 would be classified in the latter category. Even when I subsequently had accepted a very moderate and almost veiled form of this distinction, it was nonetheless maintained with respect to content, and expressed unmistakably in the particular publication.[138]

The manuscript of Menger's February 1922 note was indeed not available at the time of the genesis paper, but a correction of the note was produced. Freudenthal, in his edition of Brouwer's topological work, commented, 'It is hard to understand why the *correction of a paper of 1921 is published in 1926 without the paper itself*, and

[134] Schreier to Menger 7.IV.1926.

[135] Hahn to Brouwer 10.IV.1926. It is not clear whether Menger or Brouwer asked for this information.

[136] [Menger 1926c].

[137] Brouwer to Hahn 22.X.1929.

[138] Brouwer to Hahn 27.VIII.1929.

only the careful reader of [Menger 1926c] will discover that the mark 'noch vorhanden' is to be associated with the correction, and not with the paper itself, which, indeed, was not available when Brouwer asked for it in 1926'.[139] Brouwer's device was so subtle indeed that hardly anybody could have observed the above effect. In particular Menger suffered no loss of face. But one may well conjecture that the discussions that preceded the publication did little to endear Menger and Brouwer to each other.

The composition of the group of topologists changed towards the end of the academic year 1925/26, Alexandrov left in the spring of 1926 for Göttingen, and Hurewicz took his place; Brouwer procured a Rockefeller stipend for Hurewicz, who stayed in Amsterdam until he left in 1936 for the Institute of Advanced Study, and subsequently Brown University and MIT.

In August 1926 there was a sudden outbreak of unpleasantness; the event that triggered Menger's displeasure was objectively speaking wholly insignificant, but it is characteristic of the changed mood. One day, Menger had as usual visited Brouwer in his home, and when parting Brouwer asked him to come round in a few days to discuss the dedication of Menger's dimension *Bericht*. And so Menger duly went to Brouwer's house twice during the next two days, but he found Brouwer out. The next four days he was confined to his bed with a flu. As soon as he recovered, he went to Brouwer's house, where he found Cor Jongejan, who told him that Brouwer had left the country for a longer period.[140] She added that no message was sent to him, as Brouwer assumed that Menger had left. Menger flew into a temper. He returned home and immediately composed an angry letter. 'I must tell you, professor, that I have for the first time in my life heard such a comment, which contains such an imputation of lack of character, education and manners.'[141] It is a fact that a well bred young scholar from Vienna would consider it unthinkable to fail to keep a social obligation towards his host, and thus Menger's dismay can be imagined. It would probably go too far to assume that Brouwer thought Menger guilty of a breach of good manners; Menger probably read too much into the event.

Having vented his feelings, Menger turned to a matter that urgently needed attention: the dedication of the Dimension Report. He suggested the formulation

	Herrn L.E.J. Brouwer
entweder:	dem grossen Förderer der Topologie
oder:	dem bahnbrechenden Bearbeiter der Topologie
	zugeeignet.
	Dedicated to L.E.J. Brouwer,
either:	the great promoter of topology,
or:	the pioneering researcher of topology.

[139] [Brouwer 1976] p.567.

[140] Brouwer was off to Batz, to meet Alexandrov.

[141] Menger to Brouwer 19.VIII.1926.

It is curious that a simple matter like the formulation of a dedication had become the subject of a serious exchange of thought. Brouwer, with his penchant for refined and precise formulations, may have stressed the importance of the right words, or Menger may have wished to make it clear that he was praising a giant of the *past*. Whatever considerations may have played a role, one does not need a great deal of imagination to have second thoughts about the dedication; true—Brouwer was promoting topology even now, and he had done pioneering work—but all the same, the formulation would equally well suit a civil servant who had promoted, say, the export of tulips, and '*Bearbeiter*' sounds like a person who is sorting out an existing field. The proposed formulations definitely had a lame ring. In view of the later developments and of Menger's report of his Amsterdam experiences, it does not seem far fetched that the dedication was a well meant, but calculated, present. Menger could not simply have dropped the dedication after first asking permission to dedicate the *Bericht* to Brouwer. When the dimension report appeared the special offprints carried a simple dedication: '*Herrn* L.E.J. BROUWER *zugeeignet*', and Brouwer's copy carried the handwritten dedication 'Dedicated to you by the author in sincere appreciation of your work and in gratitude.'[142]

Much later Brouwer mentioned this dedication to Hahn;[143] the topic was Menger's alleged disparaging view of Brouwer's activity in dimension theory. On Hahn's assurance that Menger did not in the least question Brouwer's quality in that respect, Brouwer dryly pointed out that Menger 'would thus only confirm a conviction that he continually stated in the past, which was expressed most clearly, when he wanted to dedicate his dimension-Bericht to me. I had at first to decline such a dedication—which Menger had originally had in mind as "in admiration and gratitude"—in this form, but which I was prepared to accept if it would bear on my quality as creator of the notion of dimension. Only after Menger had declared his agreement with this view (which was in accordance with the formulation of his Bericht) in the presence of others, did I accept this dedication.'

In the above mentioned letter Menger thanked Brouwer for the honour of being his assistant for a year, but he bade Brouwer at the same time not to extend the assistantship for another year. This, he added, he was certain would also 'satisfy your own wishes.' One does not have to be an accomplished psychologist to read between the lines, that the two men had become estranged. Even the last line, in which Menger bade Brouwer to visit him when in Vienna, could not take away that impression.

In contrast to the Brouwer–Hilbert conflict, where no relevant correspondence is available at all (and it is doubtful if there was any), there is quite a bit of written evidence in the Brouwer–Menger relationship. In reconstructing the development, one has to take the correspondence seriously; later reflections could easily have

[142] *In oprechte waardering van Uw werk en in dankbaarheid U opgedragen van den schrijver.* Menger had for all practical purposes mastered the Dutch language.

[143] Brouwer to Hahn 27.VIII.1929.

been coloured by rationalization after the facts, and by subsequent experiences. There is a curious discrepancy between the accounts of Menger and Brouwer; we will have to look carefully into the actual exchange of letters in the twenties.

Menger, the other party, was not blessed with social graces. The few surviving reports on him show him an introverted person, with a vastly exaggerated feeling of his own importance. Alexandrov, in a letter to Hopf of 23.XII.1926, reported that 'Menger seems, as before, to remain Brouwer's assistant. He will not live in Laren or Blaricum, but in Amsterdam. The wagging tongues of the Laren-Blaricum ladies say that Menger had not found in any house in Laren-Blaricum (according to his pretensions) a fitting admiration, and since he had fallen out with all the housewives, none of them seems to be willing to forward his mail.' The specialist who at the time was called in to treat Menger for a disorder of the lungs, had without much ado diagnosed Menger's affliction as the result of a pathological inflation of his ego, and prescribed a treatment accordingly.[144]

It would go too far to say that bringing two characters like Brouwer and Menger together was asking for problems, but those who knew both persons were aware of the volatile situation. A careless remark or a conflicting view could easily blow the delicate balance to pieces. Let there be no misunderstanding, there was no lack of appreciation between the two men. Menger revered Brouwer as a profound scholar and a keeper of the Holy Grail of mathematical wisdom, and Brouwer recognized in Menger a gifted thinker and a man with a precious geometrical spirit.

Reading Menger's memoir of his Dutch intermezzo, one wonders why, after all, he stayed on for another year. Perhaps Brouwer managed to convince him that he was an appreciated member of the mathematical group in Amsterdam, perhaps Menger took another look at the situation, and decided that one more year in Amsterdam would be alright.

During this summer of 1926 Menger set himself, among other things, the task of digesting Brouwer's courses on intuitionism which he had attended in the spring term. It suddenly struck him that Brouwer's spreads (and fans) were nothing but the analytic sets from descriptive set theory;[145] the reason that nobody had seen this before was, he guessed, that the set theoreticians found Brouwer's intuitionistic papers unreadable, whereas those who could understand Brouwer's intuitionism had no taste for descriptive set theory.[146]

Menger's discussion of the analogy between spreads and analytic sets in 1928 did not draw the attention it deserved. His paper has been overlooked by set theorists and intuitionists alike; Menger justifiably complained about this.[147] At the publication of the paper Heyting told Menger that he hoped that it would help

[144] Brouwer to Hahn 22.X.1929.

[145] [Menger 1979], p. 246.

[146] I can sympathize with Menger, although my experiences were the other way round. When I first learned about analytic sets, I thought, 'Why, these are spreads!'

[147] [Menger 1979], p. 86, 246.

people to understand the Brouwerian terminology, and indeed, Hausdorff confessed that now he understood finally what Brouwer's '*Menge*' was about.

The problem with Menger's paper[148] was that it pointed out superficial similarities, obscuring the finer points, which were at the bottom of the differences. In particular, the peculiarities of Brouwer's spreads derived from the choice sequences that they were made of, and their properties were (at least in Brouwer's approach) best seen through functions defined on spreads. It would be unfair to blame Menger for this, because it took the foundational community as a whole a long time to see Brouwer's point. Brouwer had seen Menger's manuscript while still in preparation; this is confirmed by the note in Brouwer's handwriting, which served as a draft for a letter to Menger. In the note he comments on the text, one particular remark is 'Delete remark about Hurewicz, because this observation has been made by many, namely so far by every one who got to know my 'Punktmengen' [spreads] and also the Alexandrov, or Souslin [Brouwer's spelling] sets.'[149] From this draft we learn that Hurewicz had also been shown (or perhaps had noted) the correspondence, and that Brouwer considered the observed similarity a matter of common knowledge, in other words, that it was not all that unnoticed. Since the name of Hurewicz does not occur in the paper, one may conclude that Menger saw and adopted Brouwer's advice. The note further implies that Menger's observation was not the novelty Menger considered it to be. Heyting, in his review [Heyting 1931b], noted that Menger presented a historical discussion of the external features of intuitionism, without however doing justice to the conceptual content, in particular Brouwer's contributions. Menger's analogy is mentioned with the warning that the analogy is essentially a classical phenomenon. In his monograph *Mathematische Grundlagenforschung. Intuitionismus. Beweistheorie* Heyting observed somewhat uncharitably that the interest for intuitionists was negligible, as Menger's paper made free use of the principle of the excluded third. After that, the paper seems to have dropped out of sight. Menger probably viewed the neglect of his contribution as part of an intuitionistic conspiracy, but in all fairness, it is hard to see what the analogy could contribute more than a didactical device. Thus it would be fair to say that Menger's paper was likely to promote as much misunderstanding as enlightenment. Menger was certainly interested in the aspects of constructive mathematics, but somehow he missed the point of Brouwer's intuitionism, and by the time the theory of recursive functions entered the stage, he had lost interest in the subject. It is therefore not surprising that Menger's papers on constructive mathematics play only a modest role in the literature.

The academic year 1926/27 showed, at least on the surface, no signs of conflict. Menger had decided to stay on in Amsterdam, but he no longer felt comfortable. In his 'Memory of L.E.J. Brouwer' he reported an increasing awareness of Brouwer's preoccupation with matters of reference and credit. Attempts to discuss points

[148] [Menger 1928], '*Über Verzweigungsmengen*', translated in [Menger 1979] as 'An intuitionistic-formalistic dictionary of set theory.' See also [Menger, K. 2003], p. 3–22.

[149] See [Menger, K. 2003], p. 4.

brought up by Brouwer only led to sharp altercations. So he was relieved, when Hahn offered him a position in Vienna. The actual situation in Amsterdam is hard to assess. Menger's feelings towards Brouwer had changed from admiration to aversion and distrust. Menger was not willing to take second place to anyone, and the intermezzo of the genesis paper must have been an embarrassing occasion. Even though all face saving steps were taken, the fact that one had to render an account of one's past research to the world at large, was not flattering. Being inclined to find his surroundings at fault rather than contemplate the possibility of a weakness on his side, he did not take it well to be on the defensive. Brouwer no doubt saw the strong as well as the weak points of Menger, and he felt it his responsibility to steer him in the right direction, in particular to instruct him in the subtle art of crediting. The surprising fact is that Brouwer's letters remained friendly in tone; there are no hidden reproaches 'between the lines.' Menger's ambitions had by no means escaped him, as a letter to Hahn shows.[150] Looking back at Menger's stay in Holland he remarked that 'Menger has in the past in Laren expressed himself in the same insulting way with the same injustice as now about me, about anyone whom he felt to be in his way to a lightning world-fame.' In short, Brouwer saw a gifted young man with megalomaniac ambition, and Menger perceived Brouwer as a scheming member of the old generation who wanted credit for some old, imperfect ideas.

Nonetheless, he wanted to conduct topological research in Amsterdam on a 'business as usual' basis.

The relation between Menger and Brouwer gradually worsened, and Menger reported in his 'Memories of L.E.J. Brouwer' that Brouwer insisted on certain changes in Menger's manuscripts and became incensed when Menger tried to argue with him. By themselves these interventions were not strange; Brouwer always inserted comments and corrections in papers that he was handling for the *Annalen* or the *Proceedings* of the KNAW. It is in fact a normal editorial procedure. Although Menger does not say so, his account seems to suggest that Brouwer's interference was not quite on the level. The available information does not seem to bear this out; it is more likely that Menger disliked being bossed by anybody.

If there were any concrete examples of Brouwer's intervention beyond matters of formulation or mathematical detail, Brouwer's own dimension papers or Urysohn's would most likely be involved. In fact, there is such an example in Brouwer's letter of 2 November 1924, before the two had met. Menger had referred to Brouwer's definition of dimension in a formulation that suggested that only after a subsequent correction could Brouwer's dimension be considered as such, and only then could it be compared to the definition of Menger. In other words, before the correction there was no definition at all. Brouwer's viewpoint, as we have seen, was that the definition was correct up to a slip of the pen. He had convinced Urysohn of his view, and he encouraged Menger to take the same view. Menger did indeed adopt Brouwer's formulation, but that did not prevent him from sticking to his

[150] Brouwer to Hahn 22.X.1929.

conviction that Brouwer had missed the right definition, as appears from his later publications.[151]

Menger conjectured that Brouwer was no longer satisfied with being associated with the dimension *concept*, but wanted to be considered the founder of dimension *theory*.[152]

If Menger was right—and the evidence is not quite conclusive—then this was something that emerged as a reaction to certain developments. In Brouwer's papers up to 1924, the word 'dimension theory' does not occur. Only in his historical comments on dimension theory ([Brouwer 1928d], see below) does the term appear. This paper, which basically is a critique of Menger's book *Dimensionstheorie*, contains a number of explicit statements on this issue.

The paper opens with 'I have founded dimension theory . . . ', and on the same page Brouwer explains his position with respect to the further exploitation of the notion:

> In my cited paper of 1913 I have restricted myself to the founding of dimension theory and abstained from the publication of further dimension theoretic developments, on the one hand because with the justification theorem[153] the posed epistemological goal had been attained, on the other hand because for the subsequent considerations (in the first instance those which are grouped around the sum theorem and the decomposition theorem) an intuitionistic realisation, unlike that for the justification theorem, was not plausible.

This argument may have carried a limited weight in 1928, but the modern mathematician will agree that once a proper definition of dimension is available, the immediate consequences are not all that surprising, and a superior topologist like Brouwer would certainly have seen the first steps. However, his dimension paper appeared at a time when his interest in topology was already waning, while at the same time his foundational interests had become stronger. He may very well have sensed the difficulties offered by, for example, the continuum in an intuitionistic setting. Modern developments in this field have fully vindicated Brouwer's views.[154]

It should be noted that at no time during his life did Brouwer feel inclined to exhaust the consequences of his ideas. In almost all cases he proved the fundamental

[151] For comparison, here are the two formulations: Menger—'*in einem allerdings weniger bekannten kurzen Aufsatz (Crelle Journ. 142, S.146–152) eine Definition n-dimensionaler Kontinua gegeben, die nach Korrektur (Amsterdamer Akademieber. XXVI, 1923) mit unserer Definition des n-dimensionalen Kontinuums äquivalent ist*'; Brouwer—'*in einem allerdings weniger bekannten kurzen Aufsatz (Journ.f.Math. 142, S.146–152; vgl. auch die Korrektur eines daselbst befindlichen Schreibfehlers in den Amsterdamer Proceedings 26, S.796) eine Definition n-dimensionaler Kontinua gegeben, die mit unserem Definition des n-dimensionalen Kontinuums äquivalent ist*.' The reader will appreciate the difference in suggestive force.

[152] [Menger 1979], p. 247.

[153] i.e. the theorem that R^n has natural dimension n.

[154] Cf. [Dalen, D. van 1999a], in which it is shown that, e.g., the irrationals (the complement of the rationals) are indecomposable, and hence one-dimensional.

results and left the exploitation of his ideas to others. The first topological period was in a way an exception. There it was the challenge of Lebesgue and Koebe that produced such a wealth of results.

So why did Brouwer, apparently, object to a modest role of the inventor of the notion of dimension? The most plausible answer seems to be that somehow Menger claimed the ownership of dimension theory for himself, and that he exuded, or even explicitly stated this belief. Tact was not Menger's strongest point, and a discerning man like Brouwer could easily have felt slighted, not to mention the belittling of Urysohn's role. In cases like this both sides tend to overreact and escalation sets in.

Menger was overly sensitive where his rights were concerned, and even before the Brouwer–Menger conflict flared up, he was inclined to see conspiracies to rob him of his well-deserved credit. Alexandrov was at one point his *bête noire*. In a letter of February 21, 1926 Menger wrote to Brouwer, begging for support:

> I would have liked to say a few words about my own business. You had, dear professor, dropped the last few days in conversation comments about my dimension theory papers that have deeply saddened me, because I believe not to deserve them. I can imagine, how Alexandrov during the last weeks of his stay here has worked under full pressure, to put the finishing touch to what he has been trying systematically and with all means since a year: to discredit me and my papers in your eyes. Please, do not lend him your ear. If I should feel that he succeeds in this plan, I would consider myself robbed from my total present existence.

In the absence of corroborating material there is little one can say about the matter. It is true that Alexandrov took a jocular view of Menger's preoccupation with priority rights, but he was not likely to go beyond a few quips. Another Russian author, Tumarkin, had also entered the field of dimension theory, and in his papers he had overlooked the work of Menger. Since Tumarkin was a protégé of Alexandrov, Menger blamed Alexandrov for the lapse. Alexandrov duly apologized, and accepted full responsibility for the omission and promised a printed recognition of Menger's priority.[155] In a letter to Brouwer, Menger drew his attention to Tumarkin's paper, but insisted that he should not get annoyed, 'the matter is by no means so important, that it is worth half an hour of your time.'

Roughly at that time Brouwer must have decided that the priority claims of Menger should be settled once and for all. The result was Menger's 'genesis' paper, discussed above.

With Menger safely in Vienna, the friction had disappeared, and both parties could heave a sigh of relief. Most of the information on the increasing friction between Menger and Brouwer comes from Menger's reminiscences. The curious fact is that, if one reads the letters from Brouwer to Menger, the situation does not look as bleak as Menger makes it appear. Up to 1928 the letters from Brouwer are cordial and contain no accusations or attacks. There are no signs that Brouwer

[155]Alexandrov to Menger 1925 (no precise date known).

considered Menger as an (even potential) adversary. Perhaps Brouwer considered the dimension matter settled after Menger's account of the genesis of his ideas on the dimension notion. As we will see, he was well aware of Menger's personal streaks, but he probably accepted those, as one accepts medical complications in a person—things that could be straightened out in the long run. In all, he considered Menger as a bright and promising postdoc, and treated him as a scientific collaborator. Once having accepted Menger as a student and fellow researcher, he felt it his responsibility to guide Menger both in purely scientific matters, and in the *mores* of the world of scholars. In these respects he fully recognized the strong and the weak points of his assistant. It would be absolutely incompatible with Brouwer's personality to detest Menger, and write kind letters at the same time.

In Vienna Menger became the key figure in the mathematical community, he continued his own research in topology, initiated the new field of metrical (distance) geometry, published on a wide spectrum of topics and organized the influential Mathematical Colloquium in Vienna. For the foundations of mathematics his role as scientific promoter and protector of Gödel was particularly significant.[156]

The peace between Brouwer and Menger was now and then interrupted by a small salvo, for example when Brouwer sent the proofs of Tumarkin's paper, 'On the dimension of non-closed sets'[157] for comment, Menger sharply protested against the violation of his priority; he (too) politely informed the editors (*die verehrliche Redaktion*) that publication in the present form might make a rejoinder necessary. One wonders how Menger judged the situation; Freudenthal stated that 'Correspondence up to 1928 shows Menger attached to Brouwer',[158] but letters are not so easy to judge—what is old fashioned civil politeness, and what is sincerity? Menger could with some justification think that Brouwer was after Menger's priority rights. After all, if a manuscript of Tumarkin could reach the stage of proof-sheets, then had Brouwer failed to protect his (Menger's) rights?

Another source of discontent was the choice of authors (editors) of the prestigious Encyclopaedia of Mathematical Sciences; Tietze and Vietoris had been asked to prepare a chapter on 'Relations between the various branches of topology.'[159] Brouwer informed Menger about the contribution and promised to remind the authors to send the proofs, but Menger was in no mood to lend a hand to a project that had been arranged behind his back. He was already aware of the Tietze–Vietoris project, as he had met Vietoris in the Vienna seminar. The authors had added insult to injury by categorically refusing to show the proofs to Menger; only

[156] Cf. [Menger 1994], p. 200.

[157] [Tumarkin 1928], Menger to Brouwer 8.IV.1927.

[158] CW II, p. 564.

[159] *Enzyklopädie der mathematischen Wissenschaften*, *Dritter Band*, *Geometrie*, *Ch. 13*. Enzyklopädie der mathematischen Wissenschaften (submitted 15.X.1929). In fact Klein had asked Brouwer to engage Vietoris to write the chapter. Alexandrov and Menger had opposed the idea (interview Vietoris).

Rosenthal and Kneser were allowed to see (and read) the proofs! Menger, who had definite views on the pecking order in topology, commented angrily:[160]

> If the authors expect more help from those two gentlemen than from me, let them believe it. [...]. That I must fear under the present circumstances, also in view of Vietoris' total ignorance of the fundamental theorems of dimension theory (published in 1926), to have serious objections against this chapter, is clear indeed. I suppose of course, that you will have arranged the formulation of the word of thanks to you in such a way, that it will still be possible, in view of the highest regard for you, to formulate possible objections against the trash of Tietze–Vietoris.

The Encyclopaedia chapter was, when it appeared, not the shocking piece of injustice or ignorance that Menger had feared; it was loaded (as usual) with references, and Menger got his fair share.

The above letter also contained a cryptic message: you will soon get a letter from Hahn and Ehrenhaft. The announced letter was part of an attempt to invite Brouwer to Vienna for a talk on the foundations of mathematics. Before we can discuss the 'Vienna connection' we shall have to go back and retrace our steps to the *Grundlagenstreit* and the reception of intuitionism.

The developments of topology until the middle twenties were adequately summed up by Georg Feigle in the *Jahresbericht*, a perspicuous survey of topology,[161] in which Brouwer got full recognition for his contributions to the field. Together with Poincaré and Fréchet he was listed as one of the founders of modern topology. Feigl, by the way, had no problems in giving Brouwer credit for making 'the first and most important step towards dimension theory', by providing an internal topological definition of dimension.

[160] Menger to Brouwer 17.I.1928.

[161] *Geschichtliche Entwicklung der Topologie*, [Feigl 1928].

14

FROM BERLIN TO VIENNA

14.1 More intuitionism

More or less against his will, Brouwer had to prolong his stay in the fair country of topology. The Russians, and subsequently the Austrians Menger and Vietoris, had re-awakened his interest in the area. The sudden influx of gifted young mathematicians had turned Amsterdam into a centre for topology, and it remained so until the Second World War. Being the leading authority on topology on the editorial board of the *Mathematische Annalen*, Brouwer was the obvious person to handle the steady stream of pioneering papers. Moreover, he submitted a large number of topological papers to the KNAW. But although he invested a great deal of time in his activity as editor and supervisor, his main research efforts were undeniably directed at the development of his intuitionistic programme. He was sincerely convinced that this was the new mathematics, that, if it would not drive out the traditional mathematics, at least would be recognized as a viable alternative.

Brouwer had organized his campaign along two lines; there was his massive exposition of the basic parts of intuitionistic mathematics in the *Mathematische Annalen*, '*Zur Begründung der intuitionistischen Mathematik I, II, III*[1], and there were quick, effective expeditions into the territories of the mathematical empire.

His great expositions were basically an expansion and revision of the old '*Begründungs*' papers of 1918, 1919. New insights were incorporated, old notions were refined, but the content was not unfamiliar to those who had followed Brouwer from the beginning of his second intuitionistic programme. It is no secret that few had actually read Brouwer's earlier expositions, and we can say that this new series offered in fact a first large scale international dissemination of the intuitionistic ideas.[2]

The material of the three parts of the 'Founding of intuitionistic mathematics' was not a mere update and rearrangement of the first *Begründungs* series, e.g. the notions of spread (*Menge*) and choice sequence were extended to allow higher order restrictions (cf. p. 496). In the second paper, which treated the notion of order, Brouwer shifted his attention to the so-called *virtual ordering* (cf. p. 390), which, roughly speaking, extended the natural ordering by a double negation. Brouwer

[1] [Brouwer 1925b, Brouwer 1926b, Brouwer 1927c].

[2] The reception of the papers was, judging from the review in the *Fortschritte*, rather nondescript. The reviewer, Arthur Rosenthal, one of Brouwer's topology friends, restricted himself to a few lines, referring to his earlier review of the old *Begründungs* papers. But even that review of more than two pages did little to lift the veil of mystery that covered Brouwer's writings (*Jahrbuch der Fortschritte der Mathematik* 47, p. 171, 51, p. 164).

chose to make his ordering relation as strong as possible. Indeed, his paper 'Virtual ordering and inextensible ordering'[3] established that the virtual ordering is the best one can get. Virtual order on the continuum just orders 'better' than the natural order'. History has shown Brouwer wrong in his preference for more order, in fact the order which is the traditional companion of apartness (i.e. satisfying $a\#b \Leftrightarrow a < b \vee b < a$) turns out to be the most useful notion for practical constructive mathematics. From 1925 onwards, Brouwer gave virtual order a prominent place in his expositions. In [Brouwer 1949b] he used the term 'negative order' for 'virtual order', perhaps a better, albeit less colourful, name. He also gave an exposition of his analysis of the fine structure of ordering, as introduced in his paper 'Does every real number have a decimal expansion?'[4] The third paper in the series treated the notion of well-ordering, much along the same lines as [Brouwer 1919a].

The shorter papers demonstrated convincingly that intuitionistic mathematics could indeed handle certain topics that were generally seen as test cases for a mature mathematics. Brouwer dealt successfully with the fundamental theorem of algebra[5], Heine–Borel[6], the Jordan theorem for the plane[7], intuitionistic metric spaces, and the definition and justification of the notion of dimension[8].

The last mentioned paper makes good Brouwer's claim that his pioneering work in dimension theory was basically intuitionistically correct. Of course, a number of intuitionistic refinements had to be introduced, but the results of his 1913 paper could clearly be upheld.

The paper that is most often quoted for Brouwer's proof of the bar theorem, the fan theorem and the continuity theorem, is his contribution to the Riemann volume of the *Mathematische Annalen*, 'On the domains of functions'.[9] The title of the paper referred to a serious problem that is characteristic of intuitionistic mathematics:[10] if all total functions on the continuum are continuous, how does one handle discontinuous functions? Clearly discontinuous functions can only exist on domains smaller than the continuum. For example, the classically total function

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ \frac{1}{x} & \text{else} \end{cases}$$

can be mimicked intuitionistically by

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ \frac{1}{x} & \text{if } x\#0 \end{cases}$$

[3] Virtuelle und unerweiterbare Ordnung [Brouwer 1927b].

[4] Cf. p. 326 [Brouwer 1921].

[5] Cf. p. 388 and [Brouwer 1924, Brouwer 1924f].

[6] [Brouwer 1925a].

[7] [Brouwer 1925a].

[8] [Brouwer 1926a].

[9] [Brouwer 1927a], also cf. p. 383, 384.

[10] Of course, also for later branches of constructive mathematics.

Its domain is, however, a proper subset of R; namely $\{x \in R | x = 0 \vee x \# 0\}$. The last section of the paper dealt with the following important problem: what are the proper intuitionistic analogues of the classical total discontinuous functions. After a number of case studies, Brouwer offered the suggestion that 'Only those subsets of the unit continuum should be admitted as pseudo-full domains, which are in the first place congruent to the unit continuum, and which are measurable for all measures of the unit continuum and have measure 1'.[11]

The paper is as scholarly as one could wish; there is just one footnote that betrays the tensions of the times. In footnote 8 (p. 64) Brouwer pointed out that 'canonical' proofs are well-ordered objects (in the intuitionistic sense), to be distinguished from their "finite necessarily inadequate" linguistic representation'. Summing up his objections to the formalist programme, Brouwer continued:

> The preceding remark contains my main argument against the claims of Hilbert's metamathematics; a second argument is this, that the settling of the problem of reliability of the principle of the excluded middle (taken, by the way, from intuitionism) is sought by Hilbert in a vicious circle; for if one wishes to found the correctness of this principle by means of a proof of its consistency, the principle of the reciprocity of the complementary set, thus of the principle of the excluded middle itself is implicitly presupposed.

It is worthwhile to point out that Brouwer's position on infinite (well-ordered) proofs predates that of Zermelo and Hilbert[12].

14.2 Feelings of crisis and German science

The First World War had left Germany in a crisis, not only socially and economically, but also spiritually. The newspapers and the army bulletins had given the citizen the general impression that Germany was fighting a just war, and that the military situation was not unfavourable. The sudden armistice, and the eventual peace of Versailles, left the Germans with a serious trauma. In a surprisingly short time the empire was transformed into a republic, and suddenly Germany was confronted with uprisings and revolutionary movements. Although the new republic managed to restore a measure of order, the average German citizen felt betrayed and insulted. The scientists were not excepted from the general feeling of malaise.

They were, in a sense, worse off than the general citizen; they suffered not only from the disastrous economic situation, but they were also systematically excluded from international contacts; officially (although not always in practice) they were made the pariahs of the western scientific world. The German scientific community saw itself treated in a harsh and, for them, incomprehensible way. For example

— the Germans were excluded from 129 of the 195 international conferences between 1920 and 1924.

[11] 'Congruent' means roughly 'identical up to a double negation.'

[12] [Zermelo 1929], [Hilbert 1930].

— the role of the German language in international publications was severely reduced, from 40% in 1909 to 27% in 1929.
— the German monopoly in review journals was broken up, some new review journals went so far as to refuse the review of papers in the German language.
— personal contacts with scholars from the *Conseil*-associated countries were forbidden.[13]

In this climate the German intellectual community was thrown back onto itself. It saw itself excluded from the international exchange of ideas and scholars, for which it was famous. The intolerance of the scientific boycott and the general humiliation provided a fertile soil for a rich variety of nationalistic movements, under the collective name of *Deutschnationalen*. A considerable part of the German scientific establishment was far from happy with the new political institutions that replaced the old Wilhelminian order, e.g. the physician Müller, at the Nauheim conference[14] sketched a black picture of the status of science and scientists:

> ...until now it was the custom at our congresses that at the inaugural session we greeted with veneration the Kaiser and the local prince, in whom we saw the embodiment of our country.—That we can now no longer do!—[mournful pause]—But is it not our duty, gratefully to recall the support which Germany's princes have accorded science and especially the natural sciences? ...Monarchies care for the sciences and honour their significant scholars.—Republics support instruction and leave ...concern for the sciences largely to the initiative of private individuals ...Revolutions, however, destroy, letting a Pavlov starve and beheading Lavoisier.[15]

And although mathematics has a reputation for unworldliness, it was equally well influenced by the post-war malaise. Weyl's proclamation of the revolution in mathematics was far from an abstract pun of an occupant of the ivory tower, but a symptom in the wider context of the German state of society. It was indeed a rather geographically restricted crisis, that concerned the German (and German speaking) mathematicians. The threats to mathematics were not purely abstract; the post-war authorities contemplated for example a reduction in the mathematics and science curriculum of secondary and higher education. The newly founded national union of German mathematical associations and organization[16] warned 'At the current revision of the school systems there are forces active, which aim at a reduction or elimination of the mathematical curriculum. These attempts find a well-prepared ground in the hostile mood, that certain world-wide distributed works of the *belles lettres*, in spite of the weakness of their arguments, have been

[13] Cf. the Denjoy affair, p. 344 ff.
[14] Cf. p. 325 ff.
[15] [Forman 1986], see also p. 325 ff.
[16] *Reichsverband deutscher mathematischer Gesellschaften und Vereine*.

able to generate against mathematics.'[17] It is not difficult to guess the 'world-wide distributed'-source of irritation: Spengler's book, *Die Untergang des Abendlandes*, was a best-seller; how serious it was taken may be illustrated by the fact that the mathematician Hessenberg explicitly warned in his inaugural address of 1921 for this 'danger to the young'.

It is hard for the present reader to imagine how influential Spengler's *Decline* was; the intellectual community in the western world indulged in apocalyptic visions of a fall of western civilization. The book was found in almost any bookcase, it was widely discussed and quoted, until it quietly became invisible during the Nazi period. Although most mathematicians saw through the flimsiness of the arguments of the culture-pessimists, some were left with an uneasy feeling—was something wrong after all? For example, Richard von Mises, the eminent applied mathematician, thought to discern a 'crisis in mechanics'. In general there was a certain tendency to counter the cultural pessimism by a measure of accommodation, e.g. by stressing or acknowledging the cultural roots of mathematics. The optimism of the natural sciences, the belief in progress, based on the spectacular and solid advances of the past, had become suspect.

The mathematician Timerding, also at the Nauheim conference, remarked that 'We feel in Nature a spiritual essence . . . in its core incomprehensible . . .. The transcendent . . . has thus finally appeared even in scientific research, and therewith optimism is destroyed forever'. The new passwords for science in the Weimar Republic had become 'irrationality', and 'individualism'; 'causality' and 'determinism' were out.

Under these circumstances it is hardly surprising that Brouwer's foundational programme caught the attention; it offered a scientific basis for an anti-deterministic, man-based mathematics. It remains, of course, questionable in how far the participants in the broader foundational discussions were informed about Brouwer's underlying philosophy. Before 1928 no complete exposition of Brouwer's philosophical views was available; the Dutch publications, *Life, Art and Mysticism*, the dissertation, and 'The Unreliability of Logic' could hardly be considered to be accessible to the German public.[18] The first of these publications was not even known in Holland, and the latter two would have found few readers outside Holland. In all likelihood, those who appealed to intuitionism as a basic philosophy for a 'back to intuition' (*zurück zur Anschauung*), based their beliefs, if not on Weyl's 'New crisis' paper, or maybe a lecture of Brouwer himself, on the seductive call of 'intuition' (*Anschauung*).

The friction (to put it mildly) between 'modern abstract' mathematics, including formalism, and conceptual, intuitive mathematics worried a great many mathematicians and in the discussion of the twenties many viewpoints will draw the attention of the patient spectator. The well-known mathematician Ludwig Bieberbach, who took in his inaugural address of 1913 a strikingly formalistic posi-

[17] For further information and literature cf. [Mehrtens 1984].

[18] [Brouwer 1905, Brouwer 1907, Brouwer 1908].

tion, defended in the twenties his view that the true spirit of mathematics was that of the *Anschauung* (intuition), and he praised Felix Klein for promoting the practice of mathematics along the lines of geometric tradition. He strongly condemned Hilbert's formalism for severing the ties between intuition and application, and mathematics.

In a talk in 1926 (unpublished), he said:

> ... the time will not be far off ... where its overrating [i.e. of formalism] will fade away, and the catastrophic consequences that follow from it, namely the ignoring of the problems of the concrete reality, will belong to the past.

Intuitionism appeared to him 'a breath of spring air' and 'the escape from a nightmare'. To avoid any misconceptions, it should be pointed out that at that time there were no political overtones. Like so many, Bieberbach was seriously worried about the health of mathematics.

There is no reason to associate the views of Bieberbach with his later political activities; even a man as mild as Blumenthal spoke out at the DMV meeting at Bad Kissingen (1927) against the new fashion, 'The misfortune is our cowardice. It is far easier to occupy oneself with ideals, groups and groupoids, and to cocoon in a system of abstract axioms, than to tackle the really important, concrete problems of mathematics and its applications.'[19]

Traditionally there had been two main centres for mathematics in Germany: Berlin and Göttingen. In the nineteenth century Berlin was the more prestigious of the two; although Göttingen certainly had its star scientists, the Berlin establishment had a larger mass and more famous names. The balance started to shift towards Göttingen at the end of the nineteenth century. Felix Klein moved to Göttingen and made it the basis for both pure and applied mathematics. After his exploits in group theory, geometry and automorphic functions, he dedicated himself to the organization of a department for the study and advancement of high school mathematics. Being virtually in charge of the mathematics department, his view in matters of appointments carried considerable weight. His choice of colleagues could hardly have been better: among his first appointees were Minkowski and Hilbert. Both of them were top mathematicians, and in a short time they made Göttingen the Mecca of European mathematics. Hilbert's extraordinary qualities were publicly recognized when he was invited to give a large survey talk at the 1900 mathematical congress in Paris—resulting in the famous list of Hilbert's problems. Since then a long series of present and future leading mathematicians made their name in Göttingen; some stayed on, but most accepted positions at other universities, where they set an example to students and staff.

Berlin, of course, played its part in the mathematical culture of the early twentieth century, but Göttingen was undeniably the place that set the tone for the new

[19] [van der Waerden, 1928].

mathematics. Among the mathematicians of that period a certain division of loyalties could be discerned, not in any serious way, but enough to be quietly pleased if either Göttingen or Berlin scored a point. In this atmosphere of friendly (sometimes not so friendly) competition the Berlin mathematicians more or less sided with Brouwer against Hilbert. It was therefore not surprising that efforts were made to get Brouwer to Berlin, if not as a full professor, at least then as a visitor. Erhardt Schmidt had campaigned for a chair for Brouwer and when nothing came of it, he may have opted for a visiting appointment. Already in June 1921 the faculty in Berlin had proposed Brouwer as a visiting professor for the winter term, but that time things did not work out. It is likely that Bieberbach renewed the efforts to get Brouwer to Berlin as a visitor, and indeed, in January 1927 Brouwer, after obtaining leave from the University of Amsterdam, made his triumphant entrance into the Berlin mathematical community. He had asked the curators in Amsterdam permission to teach a course on intuitionistic mathematics. In his opinion this was 'in the interest of science and the University of Amsterdam'.[20] He started his course on January 15.

14.3 The Berlin lectures

Contrary to what most people thought in the later years of the century, intuitionism had a definite appeal; serious mathematicians were inclined to agree that something was wrong in mathematics and that Brouwer and Weyl had a point, some even went so far as to use intuitionistic methods in their elementary teaching. Loewner and Study, for example, taught at one time their first course in analysis (or calculus) according to intuitionistic principles.[21] Loewner's constructive inclinations are also mentioned in Lipman Bers' introduction to [Loewner 1988]. In his Berlin period Loewner became very much impressed by the intuitionist critique of classical mathematics. He once taught a calculus course using only constructive proofs and, as he recalled many years later, concluded the first term by proving that a uniformly continuous function, defined on a bounded closed interval, is bounded. Loewner did not remain an intuitionist, but did retain a strong preference for constructive proof. It was not only among the established mathematicians that intuitionism found a certain acceptance, the more adventurous among the students, taking their cue from Hermann Weyl, fervently supported the revolution in mathematics. The actual appearance of the great revolutionary in the lecture halls in Berlin caused a furore. Somewhat to the surprise of the audience, the Dutch revolutionary was a soft spoken man with a gift for long, intricate sentences, and flowery language; a man with a quick wit and steel blue twinkling eyes. A Merlin rather than a Robespierre. The lecture hall was filled till the last seat—intuitionism and foundations became the talk of the town. Even the newspapers followed the events

[20] Brouwer to Curators of the UVA 10.I.1927.

[21] According to Freudenthal, Loewner's approach was far too complicated—'In short, a catastrophe', see [Freudenthal 1987a].

with interest. The followers of Brouwer, proudly adopting Hilbert's taunt from the 1922 talk in Hamburg, called themselves *Putschists*, and the mathematician-poet Hubert Cremer rhymed in his Carmina Mathematica ([Cremer 1927, Cremer 1965])

Und wird mir das ganze
Getu hier zu trist,
Dann kauf ich mir 'ne Kanone
Und werde Putschist.

Ja klassisch da schließen s'
Mit falschem Genie;
Sie mag net die andern,
Drum mag's also mi.

Wir Putschisten aber sagen,
Des stimmt net deswegen,
Denn sie braucht nämlich leider überhaupt
kein net z' mógen!

Ach, zwischen "Sie liebt mich"
Und "Sie liebt mich nicht"
Da gibt's noch ein Drittes,
Die alte Geschicht.

Und grade das Dritte,
Des gibt mir an Riß,
Man kann's net entscheiden,
Man weiß's halt net g'wiß!

The lectures were attended by a mixed audience, consisting of students, professional mathematicians and interested laymen.

Among the listeners there was a young student, Hans Freudenthal, a man who was going to play an important role in Brouwer's life and in Dutch mathematics. Freudenthal had privately studied Brouwer's work; in the first place, of course, his topology, but also the foundational work of Brouwer. Reichenbach, the young mathematician-philosopher, was also drawn to Brouwer's lectures. Another noteworthy member of the audience was the young André Weil, who later in life became a leading number theorist, and one of the star mathematicians of the Princeton Institute for Advanced Study. Weil had received his mathematical training at the *École Normale Supérieure* in Paris. Having a strong mind of his own, he had decided to visit a number of prominent mathematics departments in Europe, including some of the German ones. That he was breaking the boycott decreed after the First World War, did not worry him overmuch.

Weil spent some time in Göttingen, and subsequently visited Frankfurt and Berlin. In his memoirs he related his encounter with Brouwer.[22] No comments

[22] [Weil 1991].

on the subject matter of Brouwer's course are to be found, but Weil recalled how he found himself after one of the lectures next to Brouwer in the café, where the *Nachsitzung* was held. He told Brouwer that he had agreed to visit Mittag-Leffler in Stockholm, and had promised to assist him in a certain project. He confessed that he was not looking forward to discussions with the older man who had little to offer in the way of actual mathematics. He would indeed have been happy to get out of his obligation. 'Nothing easier then that', Brouwer immediately replied, 'just pick a fight with him'.[23] It must be admitted that Brouwer sometimes made use of this strategy—it saved a lot of time.

Freudenthal followed Brouwer's lectures with a keen interest. He had already become acquainted with the ideas and practice of intuitionistic mathematics through Loewner's course on differential- and integral calculus. Furthermore Freudenthal had read and studied a great deal of philosophy and history. So after Brouwer's lectures he could pose sensible questions, which betrayed a more than average intelligence and intellectual curiosity. He corresponded with Brouwer, and asked for reprints of his intuitionistic papers. Brouwer was indeed so impressed by the young man, that when Freudenthal had obtained his doctor's degree in Berlin, Brouwer offered him a position as an assistant in Amsterdam.

One should not get, by the way, the impression that Brouwer was quietly sitting in Berlin, giving his courses and discussing mathematics. In February we find him in Vienna where he had business to conduct. There is a card to Alexandrov[24] —'Tomorrow I dine with Wirtinger, Ehrenhaft, Hahn, Vietoris and Loewy. In Berlin the colleagues are very good to me and my lectures are well attended.' And Cor Jongejan, who accompanied Brouwer to Vienna, had added in a corner 'Brouwer is very much lionized. He drags me along everywhere. Now a smoking will be bought.'

At home the old fears revived, would Holland lose Brouwer to Berlin? Henri Borel wrote: 'Have you said farewell to the Amsterdam University, and accepted a chair in Berlin? Is your beautiful house now for always deserted?'[25] And Frederik van Eeden, his ally from the days of the signific circle, complained that he could not miss Brouwer for three months.[26]

Among Brouwer's correspondence there was a letter of historical interest. His old teacher and friend, Gerrit Mannoury, had composed a question for the annual Prize Contests of the Dutch Mathematical Society, which had Brouwer's intuitionism for a subject,[27]

> *Draft Prize Question*
> Although Brouwerian set theory is essentially not to be identified with the conclusions which can formally be drawn from some pasigraphy,[28]

[23] *Verkrachen Sie sich mit ihm.*

[24] Brouwer to Alexandrov 3.II.1927.

[25] Henri Borel to Brouwer, 19.I.1927.

[26] Frederik Van Eeden to Brouwer 16.I.1927.

[27] Mannoury to Brouwer 26.I.1927.

[28] A somewhat outdated term for 'symbolic (universal) language', often used by Mannoury.

> certain regularities can be observed in the language with which Brouwer accompanies his mathematical intuition, which can be collected into a formalist-mathematical system. It is asked, 1. to formulate such a system and to indicate the deviations of the formalism following from that system and Brouwer's theories, 2. to investigate if by means of a (formal) exchange of the principium tertii exclusi and the principium contradictionis an associated dual system can be derived. [...].

This is the influential prize question that was to be answered by Arend Heyting. According to the rules contestants had to submit their essays anonymously, with only a motto for identification. After the evaluation of the essays the committee would solemnly open the envelope with the winners motto, and destroy the other envelopes. Heyting had chosen the telling motto 'Stones for bread'.

The interest of Brouwer in real estate, which we have noted before, must have extended far over the borders of his native country. We have seen that Cor Jongejan

Cor Jongejan, Lize and Bertus in the Harz. (Brouwer Archive).

The house in Bad Harzburg. (Photo Dokie van Dalen).

bought a house in Bad Harzburg, no doubt on Brouwer's advice. For some reason he bought somewhere in the twenties a house in Berlin-Zehlendorf (maybe he did consider to settle in Berlin, but it could also have been an investment); from 1921 to 1928 there is a considerable amount of correspondence with real estate agencies and the city housing office. At the time of the economic slump, the house proved an awkward liability. Brouwer spent a great deal of time on his Berlin house. The house had to be let, there were repairs, the rent had to be collected, and in the end he had to find a buyer and arrange a sale. It is a saddening thought that so much time, money and ingenuity wasted on—money!

During his stay in Berlin, Brouwer renewed old friendships and made some new ones. One name has already come up in connection with Alexandrov, that of Heinz Hopf, a man thirteen years younger than Brouwer. He was born and raised in Breslau (Wroclaw) where he also had started his university studies. Called into military service, he happened, during a short leave, to hear the set theory course of Erhardt Schmidt (the man who, according to Weil, was as quick and sharp as Hadamard) just when he was treating Brouwer's proof of the invariance of dimension by means of the mapping degree. This determined the course of his mathematical career. In 1920 when Schmidt moved to Berlin, Hopf went with him. Brouwer came in contact with Hopf when he handled the latter's paper 'Vektorfelder in n-dimensionalen Mannigfaltigkeiten' for the *Mathematische Annalen*. Hopf's paper dealt with matters that were close to Brouwer's own work; as a matter of fact Brouwer informed Hopf about the connection between the work of Hadamard and Brouwer with respect to developments in 1909/10.[29]

Brouwer was so pleased with the work of the young author that he expressed in this letter the hope to meet the author in person. This was the beginning of a lifelong friendship that survived all storms. Before Brouwer came to Berlin, Hopf had already visited Brouwer in Amsterdam. The second topologist who was influenced

[29] Brouwer to Hopf 21.XII.1925. Cf. p. 139.

by Brouwer was Georg Feigl. Both Hopf and Feigl were teachers of Freudenthal. Indeed all the Berlin topologists or analysts more or less adhered to Brouwer-style topology.

There is a perspicuous survey of the topology of the twenties,[30] from Feigl's hand, published in the *Jahresbericht* in 1927, in which Brouwer got full recognition for his contributions to the field. Together with Poincaré and Fréchet he was listed as one of the founders of modern topology. Feigl, by the way, had no problems in giving Brouwer credit for making 'the first and most important step towards dimension theory', by providing an internal topological definition of dimension. At the same time he acknowledged the dimension theory of Urysohn and Menger. Feigl's survey represented the general view of the mathematical community. For a short while it looked as if the dimension discussion had reached a happy ending.

Hopf had of course attended Brouwer's lectures, and could therefore be expected to appreciate some of the finer points. After he had finished his course, Brouwer wrote him a card commenting on the topic of his final lecture; Brouwer had presented as the grand apotheosis of his lecture series a counterexample to his fixed point theorem, an act that never ceased to baffle people. How could one prove such a marvelous theorem and subsequently renounce it?[31] Of course, there is a simple explanation: the theorem is false from a constructive point of view, and it happens that there is a perfectly good substitute for the fixed point theorem. The above mentioned postcard elaborates the intuitionistic viewpoint,[32]:

> Dear Hopf,
> It occurred to me that I owe you and Feigl an additional statement as to what I said about the fixed point theorem, and I would like to pay this debt before my departure. When I said that the classical fixed point theorems could not be saved intuitionistically as fixed point theorems, I did not, by any means, want to say that these theorems would not admit an interpretation from an intuitionistic point of view, which remains valid there too. On the contrary: the classical theorem that the transformation τ of the compact space R (which we assume to be metric) $\varepsilon > 0$ exhibits a fixed point, has the intuitionistically correct meaning that for every $\varepsilon > 0$ a point P of R can be determined that is less than ε apart from its image. And the classical theorem that the transformation τ of R yields n distinct fixed points has the meaning, which remains intuitionistically correct, that there is an $a > 0$ with the property that for each $\varepsilon > 0$, n points $P_1 \ldots, P_n$ can be determined which all are less than ε apart from

[30] *Geschichtliche Entwicklung der Topologie*, [Feigl 1928].

[31] There were more rumours about Brouwer's behaviour and personality than about most mathematicians, some of them absolutely unfounded. I recall that in the sixties a leading mathematician gave a talk at a monthly meeting of the Dutch Math. Soc., who at one point remarked 'Brouwer did not believe any longer in the real numbers, didn't he?'

[32] Brouwer to Hopf 8.III.1927.

> their images, and each two of which have a distance $> a$. These theorems are, however, no fixed point theorems, for one has no means to indicate, i.e. to approximate, a fixed point.
> Please show this card also to Feigl. It is meant for both of you.

The content of this card did not find its way into the mathematical literature. It is another example of Brouwer's reticence to publish his results. His courses often contained new material; Brouwer was well aware of the novelty value of the material, and in some instances he required the students to sign a statement to the effect that they would not publish (parts of) the material of the course. It was common knowledge that a book published by Haalmeijer and Schogt (the former Brouwer's first PhD student and the latter his first assistant) used material from Brouwer's early courses on point set theory in their book on set theory,[33] a fact that rather displeased Brouwer. In a letter to his publisher Noordhoff of 10 October 1929 he mentioned the matter, 'I don't consider this matter as of great importance, but nonetheless I think that you should be informed about it, if only to prevent as much as possible repetitions of such matters.' Quite often Brouwer assigned some student the task to prepare a set of notes of his course. In the case of the Berlin lectures he could not leave the composition of a transcript of the lectures to students, but he managed to get assistance in transferring his handwritten notes into typescript. The task was carried out by Eva Wernicke, a most competent secretary. Eva went about her job with great enthusiasm, and in the process the author fell completely under her spell, and she under his. One can, looking at the remaining correspondence, easily imagine what the mutual appeal was. For her, Brouwer was the great man with startling unorthodox ideas, who had completely preserved his own boyish charm and sparkling originality, whereas she (to judge from the letters) was a lively person with a fair knowledge of philosophy, literature and with a lively sense of humour. She not only prepared his manuscript, but also—after his departure— looked after his business interests. She talked to the representative of the publisher De Gruyter and tried to convince Brouwer of the necessity to postpone no longer a book on intuitionism (to be published in the series *Göschen Lehrbücher*), which also contained Hausdorff's *Set theory*. If she had been on the pay-roll of De Gruyter, she could not have been more persuasive:

> *Please don't get a fright!* This idea, that finally a **book** of yours would come into being (which would doubtlessly be snatched from your hands), is worth not only to be 'discussed', but to be *realised*.

She even offered to come to Holland to take notes,

> Dear Brouwer, don't cry the moment you are requested to write a book. This is not a matter for you to produce new scientific material, but to collect together and present what is already there. So it is basically about

[33] [Haalmeijer–Schogt 1927].

> you, scraping your publications up till now from their dungeon in the wooden cottage, spreading them on a big table and then generating the linking text. I am at your disposition for this job.[34]

Not only did she decipher Brouwer's manuscripts and type a first version of the envisaged book, but she also negotiated the royalties with the publishing house. Eva took Brouwer's cause firmly in hand and did her utmost to coax Brouwer into producing a text for a book. She must have been an exceptionally humorous and charming lady, who indeed could twist Brouwer around her little finger. In a mixture of admiration and mockery, she could quote tongue in cheek the newspapers and the general opinion to him 'I have read in the newspaper about your "crown of fame". Great man I congratulate you and all yours. I enjoy reading something like that, in particular in the present which otherwise manifests itself so plebeian. I hope that the beautiful sentence of the "*selten geworden Menschen*" (Man who has become rare) applied to you . . . ' Eva had completely mastered the refined art of appeasing Brouwer; with a mixture of flattery, nonsense and appreciation she arrived at results that would have baffled his friends. Her letters would open with 'Verehrter und liebster Professor, *Freund und Übermensch*', or '*Teuerste Lucian*' ('Lucian' for 'Luitzen'), or just '*Lieber Professor*'. When Brouwer sent a worried telegram about the typescripts that had not arrived in time, she filled half a page with references to German craftsmen, Strindberg, class hate, and the underworld, before she came to the point. One could almost say that she had taken a leaf out of Tucholsky's book.

The reader should not get the impression of a secretive affair; Eva had met Cor and Lize in Blaricum, and she was adopted as a friend of the family. Greetings were exchanged and plans for visits to Blaricum were discussed. Eva not only assisted Brouwer with his manuscript, she also represented his interests as a Berlin landlord. As to be expected, his house in Berlin proved to be something of a disaster—taxes, rent protection, realtors, . . . A steady stream of letters on the matter of the house in Berlin passed from Berlin to Blaricum and vice versa. Real estate and land property played an important role in Brouwer's whole life; old contracts, insurance policies, contractor's bills of all sorts are to be found in Brouwer's archive. There is, for example, a reference in one of the letters to Eva to a piece of meadow in Harzburg, left over from the earlier property—as so often the taxes required attention.

Although in July 1928 Brouwer announced that 'the Berlin lectures will appear in print soon',[35] the book did not materialize after all; Brouwer kept revising the typescript until the middle thirties; one guesses that his perfectionism played a major role in the failure to deliver a book on the subject of his heart. Moreover, after 1928 Brouwer was so disillusioned that he lost interest in the project. The manuscript in the end was published posthumously together with parts of a book on real functions (style Baire–Borel) that Brouwer was working on in the twenties.[36]

[34] Eva Wernicke to Brouwer 30.III.1927.

[35] Brouwer to Heyting 17.VII.1928

[36] [Brouwer 1992].

Brouwer in the garden of his Berlin house. (ETH Archive).

The material of the Berlin lectures has a certain overlap with the series of papers in the *Mathematische Annalen*, but it touched more on everyday mathematics. It contained an extensive analysis of the continuum, not quite identical with that of the *Mathematische Annalen* presentation. A number of the results is based on the idea of the *creating subject*, which appeared in print only after the Second World War. He showed furthermore that if the continuum can be ordered at all, the ordering has to coincide with the natural ordering.[37] Since the latter is not an order in Brouwer's sense, there is no ordering whatsoever. This material was not published until 1950.[38] The final chapter of the lectures consisted of applications and counterexamples, culminating in a counterexample to the fixed point theorem, plus a formulation the intuitionistic version of a special case of the theorem: For a

[37]A partially ordered set is *ordered* under Brouwer's definition if $a \neq b \rightarrow a < b \vee b < a$; this is weaker than the trichotomy property [Brouwer 1992], p. 50.

[38][Brouwer 1950].

The house in Berlin (Zehlendorff). (Photo Dokie van Dalen).

topological transformation of a circular disk κ, for each $\varepsilon > 0$ a point P of κ can be found whose distance to its image is less than ε. A similar theorem for the sphere appeared in print in 1952.[39]

The notes of the Berlin lectures did not contain a proof of the 'virtual ordering = inextensible ordering' theorem, neither did the *Mathematische Annalen* presentation. One wonders if Brouwer thought the material too sophisticated, or did he have second thoughts? The theorem is mentioned, so he had no doubts about the fact itself. In Amsterdam he had indeed presented a proof in his 1925 course 'Institutionistic Order'. A polished proof was published in the year of the Berlin lectures in *Crelle*, the same journal that had published his dimension paper. One cannot think of a convincing reason to prefer *Crelle* over the *Mathematische Annalen*; Brouwer was probably invited to contribute to the jubilee volume of *Crelle's* centenary.[40] Whatever the reason was, the paper suffered from a definite lack of visibility.

There is a puzzling letter from Henri Borel, which quotes Gutkind[41] saying that: 'you have made an enormous discovery which influences the foundations of mathematics.'[42] Neither in Brouwer's papers, nor his private notes is such a discovery recorded, assuming that Gutkind referred to a recent event. The basic facts of

[39] [Brouwer 1952a].

[40] *Jubiläumsband I. Aus Anlass des 100 jährigen Bestehens.*

[41] See volume 1.

[42] Henri Borel to Brouwer 26.III.1927.

the 'Domains of functions' paper were already established earlier, and the 'virtual = inextensible' theorem was new and surprising, but it was not going to rock the foundational world. It is just possible that Brouwer had discussed the idea and the use of the creating subject as a tool to study the full continuum.

The outstanding event of 1927 remained the Berlin course on intuitionism. Brouwer had every reason to view the future with confidence: Berlin had received him in grand style and intuitionism was on its way to recognition. After a period of friction between formalism and intuitionism, a modest place under the sun for intuitionism, seemed a realistic option. Brouwer's lecture series was not just an internal mathematical matter, it was received in Berlin as a cultural phenomenon. So much so that the newspapers thought it worthwhile to solicit contributions, also from Brouwer to illustrate the foundational debate in mathematics. Brouwer made it clear to Eva Wernicke that he would do no such thing as long as the newspaper offered him a fee that—according to the prevailing standards—was rather that of 'the better science journalist' than of a 'foreign expert'.[43] Neither did he feel like publishing a collection of his old papers; the time did not seem ripe, and he would certainly not do so in Berlin as long as:

> they offered second class fees, and the academy in this city did not consider him worthy of its membership (but they did so for a mediocre loudmouth, and also a counterfeiter and plagiarizer, like Koebe, who is younger than I am, and moreover thanks part of his fame to a theft practiced on me, a circumstance which was known to several members of the Berlin Academy before his appointment was announced).[44]

The last part of the quotation shows that even after all those years he had not succeeded in putting the old insults of the Koebe conflict behind him, and that, although he could be magnanimous and warm hearted, there were moments when he could not govern his sharp tongue.

Alexandrov, who had created his own topological kingdom in Moscow, kept up his contact with Brouwer, and in recognition of Brouwer's role in topology he saw to it that Brouwer was made an honorary member of the Moscow Topological Society. In his letter of February 15, 1927, he informed Brouwer that both Hopf and he were appointed honorary members. Hopf was the leading Berlin topologist at the time, and Alexandrov had met him at one of his many stays in Germany. Between the two a close friendship developed. They were eventually to write a textbook of topology that dominated topology for years on end. The letters of Alexandrov to Hopf are little gems of impish humour, Alexandrov being an expert in spotting and exploiting the little absurd aspects of the world. Here, as a modest example, is a comment on fixed points: 'Thus I can with the same conviction with which Brouwer speaks of the failing of all marriages, state my point of view, which

[43] Brouwer to Eva Wernicke 22.XII.1927.

[44] For the Koebe affair see p. 181 ff.

is beyond any doubt, that the method for proving all kinds of fixed point theorems is that of yours and Brouwer (and not of Lefschetz).'[45]

Brouwer's intuitionism was attracting attention not only in Germany, the country with the oldest foundational tradition, but also in the Soviet Union. One of the brightest Russian mathematicians of that period, the young Kolmogorov, had in 1925 published a paper on intuitionistic logic, that showed a sharp insight into the impact of intuitionism on logic. His brilliant analysis predates Heyting's formalization, but since the paper was in Russian, it unfortunately went unnoticed in the West. One wonders if he had sent a reprint to Brouwer? There was another Russian mathematician, one with a philosophical turn of mind, who was attracted by the constructive aspects of Brouwer's intuitionistic mathematics, Samuel Osipovich Shatunovsky. Shatunovsky was already in his late sixties when he decided to inform Brouwer about his own ideas on constructive mathematics. In the Brouwer archive there is a translation of a text of Shatunovsky on constructive mathematics, in fact the introduction to his dissertation. Shatunovsky was a mathematician with original ideas on various subjects. He suffered from the fact that he did not have a school certificate, so that he could not enter upon a formal university career. He attended courses at a number of institutes in Petersburg, he heard for example Chebyshev's lectures. After a stay in Switzerland, he found recognition in Odessa, where he spent the rest of his life. There is a scathing reference to Shatunovsky in a letter from Alexandrov, 'You give me your opinion on the nonsensical paper of Shatunovsky. As this opinion was always privately mine too, I saw no reason to make somehow a statement about it, and I have restricted myself to pass this on to Mrs. Ehrenfest in order to inform her.'[46] Cor Jongejan sent Mrs. Ehrenfest a German translation with Brouwer's compliments, and that was the end.[47]

The name of Shatunovsky later crops up in a letter to Veblen of 2 April 1932, Alexandrov—probably in answer to a question of the former—described him as a very gifted mathematician, a precursor of Brouwer and Moore. As a constructivist Shatunovsky was closer to Kronecker then to Brouwer, who rejected him. Shatunovsky, Alexandrov wrote, was lacking in mathematical culture—a gifted solitary eccentric![48] According to Alexandrov, Shatunovsky paid Brouwer a visit, which turned out a disappointment for both.

Apart from the Berlin lectures, there is little to report about Brouwer's scientific activities in 1927. The paper that contained a substantial contribution to the intuitionistic programme, 'On the domains of functions', had already been submitted

[45] Alexandrov to Hopf 15.II.1927.

[46] Alexandrov to Brouwer, 15.III.1927. Shatunovsky may have approached Mrs. Ehrenfest for an introduction to Brouwer.

[47] Brouwer to Mrs. Ehrenfest, 3.VII.1927.

[48] A gifted '*Eigenbrötler*'.

in 1926, and the 'virtual order' paper in *Crelle* was an imaginative piece of work, but not really in the centre of the intuitionistic programme.[49]

Although the *Grundlagenstreit* was in full swing, Brouwer chose not to take part. But at the end of the year he for once spelt out his views on Hilbert's programme. Perhaps he could not resist the challenge laid down in Hilbert's 'On the infinite'. Hilbert's conviction that classical mathematicians were not going to be cast out of Cantor's paradise was one thing, but that the leading formalist judged the mathematical progress of the 'recent so-called "Intuitionism" ' to be modest, went too far. To straighten out the issue, he presented in December 1927 his paper 'Intuitionistic reflections on formalism' to the Dutch Academy.[50] We will return to this and other papers in a later section.

Brouwer was now forty-six years old, he had seen the light and the shadows of life, but so far he had observed the human tragedies of others. This year fate struck closer to him: he lost his mother. On May 3 she died of a neglected stomach ulcer.[51] Brouwer was a curious mixture of family man and a hermit guarding his privacy. Whereas we know that the relationship with his father was tense, to say the least, little is known of his relationship with his mother. She was a strict but fair woman; educating her sons came easier to her than cuddling. When she died she had saved out of the meagre salary of a schoolmaster the tidy sum of 23.000 guilders.[52] There are no documents or oral communications shedding light on the impact on Brouwer of the death of his mother. Mourning was a very private matter to him.

Busy as he was, carrying the banner of the intuitionistic revolution into the capital of Germany, Brouwer found time to indulge in his dabbling in real estate in Blaricum. His neighbour, Mr. Crone, contemplated the sale of a few lots and the villa 'De Pimpernel'. Months were spent in negotiations and in the end Brouwer became the owner of a pleasant villa, bordering on his hut and grounds. As a result he owned a considerable piece of land in Laren-Blaricum, covered with the typical trees and shrubs of the sandy soil. Brouwer occupied himself quite intensely with financial affairs. He negotiated loans, mortgages, bought land, and also tried his hand in investments. In 1927 he made a minor, and relatively insignificant investment—he bought obligations issued by the *Sodalitas Oblatorum Ordinis Sancti Benedicti*. Innocuous as this may seem, the 'Sodalitas affair' was to haunt Brouwer for the better part of the thirties, as we will see.

On Brouwer's grounds there was a number of small cabins, constructions of diverse kinds, some of them reminders of the colony of Christian Anarchists of

[49] Cf. p. 390.

[50] The paper was also published in the reports of the Berlin Academy [Brouwer 1928c].

[51] Lize to Louise, 27.XI.1949.

[52] This was indeed an incredible feat, the savings amounted to more than twice the year income of a full professor.

The villa 'De Pimpernel'. (Photo Dokie van Dalen).

professor van Rees.[53] In 1925 a modest prefab cottage, called the *Padox* after its factory, was erected on the grounds; it was usually occupied by visitors, and occasionally it was let. Although he did not have a car (and Cor Jongejan got hers only after the war), a garage was added to the grounds. It was a small building with a high roof; according to the neighbours, it was mainly built to block their view on Brouwer's property. In practice the garage was used as an extra cabin for housing visitors (girl friends as the neighbour said[54]).

By now Brouwer was one of the icons of Amsterdam University. His fame was well established, the offers from Berlin and Göttingen had not gone unnoticed. His authority in the senate of the university had probably something to do with his status in science, but it was also to no small degree based on his verbal virtuosity. He could convince the Senate of almost anything by his convoluted but fascinating arguments. Heyting, who had occasion to observe Brouwer's behaviour in the faculty and senate later in life, recalled that 'even in meetings, Brouwer spoke in complete sentences; also at meetings he often uttered nonsense, but impeccably formulated and motivated, so that one had to think hard to find the error.' In the minutes of the faculty or the Senate Brouwer does not figure prominently, but one should not conclude that therefore his role was negligible.

[53]Cf. p. 61

[54]Interview H.J. Looman.

The Padox, a prefabricated cottage. (Photo Dokie van Dalen).

In 1927 family and faculty matters got mixed up; a vacancy had occurred when the geologist-palaeontologist Dubois retired, and the faculty was looking for a geologist. In May the Curators recorded in the minutes that 'H.A. Brouwer had not yet put his desiderata on paper'. The matter dragged on and in the minutes of September 10, we read that L.E.J. Brouwer had pointed out that all the geologists in The Dutch Indies were foreigners 'because there is no training here.' He urged the University to take the matter in hand. The faculty and the board of the University must have been satisfied with the qualities of Aldert Brouwer, because his appointment followed in the beginning of 1928.[55]

Intuitionism had gradually become an accepted, be it controversial part of mathematical life. In the twenties an interest arose in its logical aspects. As we have seen, Kolmogorov analysed intuitionistic logic in a masterly paper.[56] By means of a syntactical translation he also established the embedding of classical logic in intuitionistic logic. Here Kolmogorov not only anticipated Heyting's full system of intuitionistic logic, but also Gödel and Gentzen by providing the first example of a

[55]Aldert had been very successful in his profession. After his PhD in 1910 (Delft) he worked as a geologist in the Dutch Indies (now Indonesia); in 1918 he got a chair in Delft, and in 1925 he was an exchange professor in Michigan. Utrecht made him an extraordinary professor in 1925. The Royal Academy in Amsterdam elected him a member in 1922. His chair in Amsterdam was in 'general and practical petrology'.

[56]He actually formalized a fragment of full intuitionistic predicate logic, cf. [Heijenoort 1967] p. 414, [van Dalen 2004].

translation. This so-called Kolmogorov translation basically adds double negations to every step of the construction of a proposition.[57]

Already in 1926 Heinrich Scholz, the German theologician who had turned logician and philosopher, followed Fraenkel's example and visited Brouwer to discuss foundational matters. Although this did not result in any significant progress in intuitionistic logic, it showed that logicians of an independent mind started to visit the founder of the subject. Intuitionistic logic gave rise to an impressive body of technical research, but it also tended to confuse the lesser minds. An example is the long cooperation of two Belgian mathematicians Barzin and Errera. Already in 1925 they published a paper on Brouwerian logic which contained a not uncommon misconception. In the early thirties Heyting engaged in a discussion with the two; we will return to the matter, see p. 675. In 1927 Alonzo Church conducted a correspondence with Barzin and Errera, without convincing them.[58]

The greatest blow in 1927 for Brouwer was, in a sense, the plan of the City of Amsterdam to expropriate the pharmacy of Brouwer's wife at the Overtoom.[59] The pharmacy was left over from the medical practice of Lize's father. After his death in 1880, Lize's mother carried on the pharmacy with the help of a so-called *provisor*, that is certified manager. Lize later studied pharmacy and took charge of the pharmacy in 1908 after she got her degree. The building and the practice had been bought by Brouwer from his mother-in-law in 1905 for an annuity of 1100 guilders. Brouwer had developed an almost emotional relation with the pharmacy, it had become an important part of his life. He often handled the administration, he personally dealt with the tax office, helped to select staff, etc.

And now the City demanded the building and the grounds for development. In Brouwer's eyes this was not only sacrilege, but also a base attack at his wife's livelihood. He, of course, rejected the City's offer of 40,432 Dutch guilders for the property.[60] He contested the expropriation procedure, and in due course found himself in the district court at the Prinsengracht where he pleaded his own case on November 22. When his objections were turned down, he tried to get a better deal. He argued that the pharmacy had at that time 2142 clients, and resettling the pharmacy meant a serious loss of clients plus stiff competition at another location. So he requested a higher compensation, or a location at the Overtoom not too far from the existing shop. The negotiations lasted until the end of 1928, but Brouwer lost on both counts, and so the laborious process of relocating the pharmacy had to be started. The pharmacy at the Overtoom was demolished in 1929.

[57] Cf. [Plisko 1988b, Plisko 1988a], [Troelstra, A.S. and D. van Dalen 1988], p.59; [Troelstra 1978]; [Heijenoort 1967]. Another famous Russian, Molotov, also proposed a translation, albeit in a completely different setting. According to Leonard Mosley in *Dulles*, 1978, Molotov once during a negotiation said 'I propose that we insert a "not" before every verb in the text'.

[58] Cf. [Church 1928].

[59] Lize ran the pharmacy, but Brouwer was formally the owner. See p. 53 ff., 197 ff.

[60] There were extensive grounds behind the building, stretching all the way into what is now the *Vondelpark*.

Politically speaking, Holland was a quiet island in the turmoil of the twenties. Brouwer's active role in those years was mostly in the world of scholars, where the fight between the old adversaries was fought around the *Conseil International de Recherche*. By 1927 the diehards on the allied side were loosing ground. One decisive battle was still looming; the abolition or the reshaping of the Mathematical Union, a subsidiary of the *Conseil*. That battle was going to be fought in all fierceness in 1928.

Brouwer's close friend, Gerrit Mannoury, was active in national and international politics. He was a staunch Marxist and member of the Communist Party of Holland, K.P.H. He had joined the SDAP, Social Democratic Labour Party, in 1909, and followed it when it changed its name to *Kommunistische Partij Holland*. In the party organs and pamphlets he published innumerable expositions of the Marxist doctrine for the workers. Mannoury was a rather exceptional person; he maintained the highest possible standard of moral and intellectual integrity in the middle of the infighting of his political friends. Far from sticking devotedly to the party line, he had already in 1925 spoken out against the persecution of Trotski. Mannoury had the curious mixture of brotherly love for mankind and Marxist class war that one could find in socialists and communists in the early part of the twentieth century. In his many publications in *De Tribune*, the communist newspaper, one recognizes both the spirit of the pietist preacher and the marxist reformer. In fact, Mannoury was one of the kindest men on earth and his rhetoric must be viewed in the light of the vocabulary of the time. Nonetheless it is surprising to find him defending the old 'you cannot make an omelette without breaking eggs' doctrine, as he did when one of Brouwer's old fellow students Bonger attacked the communists for using the Sacco-Vanzetti case for political advantage.[61] He had written in the *Handelsblad*, 'Even worse than the anarchists are the communists in this case. If anybody in this matter should keep silent with a blush on his cheek, they should, in view of all the injustice that the Russian henchmen bring over their political opponents—when does a worldwide protest begin against that?—the significance of the American affair becomes negligible. He who commits himself injustice or explains it away, and complains about what somebody else may have done, only makes himself ridiculous and despicable.' Mannoury felt that he had to defend the Soviet Union; he replied in *De Tribune*: 'Soviet-Russia is a bastion that communism has raised against the violence of its opponents, and that therefore needs itself violence for its defence. In the first period of the revolution this struggle of violence took the form of an overt war, of late the enemy has taken recourse to hidden violence of murder attempts and conspiracies; and in the face of that violence, in my firm conviction, terror, that is summary justice in all its horrors, is necessary and just. But I am also convinced that it is possible, in the middle of those horrors and violence, never to lose sight

[61] Bonger was a seasoned socialist.

of the lofty standard of humaneness and justice …' And yet, Mannoury was a man who would and could not 'harm a fly', as the saying goes. It is greatly to Mannoury's credit that he did not hesitate to expose crash cases of party injustice.[62]

In 1928 Mannoury found himself in the centre of a party conflict. Stalin had banned Trotski to Alma Mata; this was in Mannoury's eyes an act that no true communist should condone. He thus sent on January 17 a protest telegram to Stalin (with a copy to Trotski). The Dutch comrades could, as to be expected, not appreciate such a gesture, as Mannoury was to find out in due time. He was indeed expelled from the party on May 20, 1929. Not wishing to accept this injustice, he carried a determined defence to all corners of the party, including the Moscow organs—to no avail.

In December 1927 Brouwer fell once more victim of the flu; in his letter to Eva Wernicke of January 24, 1928 he spoke of 'a bad case of flu, at which roughly 12 days ago my right eye has gone blind'. The ophthalmologist[63] mentioned a haemorrhage in the optic nerve. So far, he wrote, there had been no recovery, but the specialist was optimistic. In the meantime Brouwer trained himself to see with one eye only, and he hoped to be able to travel to Vienna in a few weeks' time, albeit with a dark patch over one eye. The correspondence with Eva, of which no letters later than April 30, 1928 have been preserved, contain little glimpses of Brouwer's affection—'Again I am not writing anything personal. You know however, also without words, how my relationship with you is' (22.XII.1927), or 'I admire you, I thank you and I embrace you' (24.I.1928). The main part of the letters, however, deals with Brouwer's house in Berlin. There are all kinds of instructions for Eva concerning the real-estate agents, legal advisers, the tax collector, tenants, prospective buyers, etc.

In February the topology school bore another fruit, the young Englishman, Wilfrid Wilson, who had come to study with Brouwer defended on the tenth his dissertation in the Dutch tradition. His thesis *Mappings of spaces* was a generalization and extension of Brouwer's work.[64]

14.4 The Vienna lectures

The big event of the early months of 1928 was undoubtedly Brouwer's visit to Vienna. He had been invited by the 'Committee for the organization of guest lectures of foreign scholars in the exact sciences'[65] to give an exposition of his intuitionistic mathematics and philosophy. The members, Ernst Egger, Felix Ehrenhaft, Hans Hahn, Anton Lampa and Rudolph Wegscheider, envisaged a promotion of the exact sciences through the invitation of prominent scientists. The choice of Brouwer was not all that surprising; in Vienna there was considerable interest in

[62] Cf. [Mannoury 1930], e.g. the executions following the assassination of Wojkof.

[63] Prof. Van der Hoeve, the same person who met Urysohn and Alexandrov in Norway.

[64] *Afbeelding van Ruimten* [Wilson 1928], [Brouwer 1924g].

[65] *Komitee zur Veranstaltung von Gastvorträge ausländischer Gelehrter der exacten Wissenschaften*.

the sciences in general, but in particular in the philosophy of the sciences. There was the Schlick circle, and later the Wiener Kreis. Furthermore at least two of the members, Hahn and Ehrenhaft, were close friends of Brouwer. Brouwer, by the way, had the privilege of opening the series of lectures. He gave two lectures, 'Mathematics, Science and Language' and 'The Structure of the Continuum'.[66] The first one on March 10, and the second one on March 14. Both lectures contained, among other things, material that roughly speaking was conceived at the time of Brouwer's dissertation (1907), which had never been presented to the public at large. A first version appeared in Brouwer's dissertation, where, however, a substantial part was suppressed by his PhD adviser, Korteweg (cf. p. 91 ff). This time Brouwer presented a concise survey of his views on the role of the individual. The first lecture presented Brouwer's views on the genesis of the basic entities of the subject's inner and outer world. This exposition was most helpful indeed, as in the dissertation the central intuition more or less seemed to come out of the blue.

I will quote a few parts from the lectures.

> Mathematics, science and language constitute the main functions of the activity of mankind, by means of which it dominates nature and maintains order in its midst. These functions find their origin in three forms of action of the will to live of the individual:[67] 1. mathematical attention, 2. mathematical abstraction, and 3. the imposing of will by means of sounds.
>
> The *mathematical attention* comes about as an act of will serving the instinct of self-preservation of the individual in two phases: the *temporal disposition* and the *causal disposition*.[68] The first one is nothing but the intellectual principal phenomenon of the falling apart of a moment of life into two qualitatively distinct things, of which the one is experienced as giving way to the other, and nonetheless is experienced as preserved by the act of memory.
>
> At the same time the split moment of life is separated from the Ego and shifted to a world of its own, called the world of perception [*Anschauungswelt*].

Once this sequence of the manifestation of two phenomena is recognized, one can easily create longer finite sequences. These sequences are then subjected to the *causal attention*:

> Now the causal disposition consists of the act of the will to 'identify' certain of these temporal sequences of phenomena which extend over the past as well as the future. Thereby comes into being a common

[66] *Mathematik, Wissenschaft und Sprache*, [Brouwer 1929b] and *Die Struktur des Kontinuums*, [Brouwer 1930].

[67] In German Brouwer uses 'einzelner Mensch'. We will use 'individual'. It is important to note that in this paper Brouwer does not take a solipsist position.

[68] In [Brouwer 1933a] Brouwer uses '*tijdsgewaarwording*' and '*causale aandacht*', translated as 'perception of time' and 'causal attention'.

> substratum of these identified sequences, which is denoted by as a 'causal sequence'.

These causal sequences form the building material of the universe. In particular the stable, permanent causal sequences are called 'objects'.

The mathematical disposition finds its sole justification, according to Brouwer, in what in *Life, Art and Mysticism* was called 'the jump from end to means'.[69] In the Vienna lectures Brouwer spoke of the *mathematical act*. This act consists of the following: if we know that a particular desirable phenomenon occurs in a causal sequence, and the phenomenon cannot directly be attained, then one may try to realize some intermediate event in the sequence, in the expectation that the causal sequence will (more or less) automatically take us to the desired phenomenon. Here the means is substituted for the end.

As Brouwer points out, the efficacy of the mathematical act[70] is greatly enhanced if 'external' interference can be blocked. We may add that if all interference is eliminated, so that the end follows with absolute certainty, algorithmically so to speak, from the means, a prototype of the constructive interpretation implication is obtained.

The paper furthermore contains a fairly detailed treatment of the role of language and communication, including the moral techniques and principles necessary for an organization of human society.

Brouwer's views on language and communication had not changed much since the early days:

> Now there is, however, for the transmission of will, in particular for the transmission of willl accomplished by language, neither exactness nor certainty. And this state of affairs unrestrictedly remains the case, if the transmission of will concerns the construction of purely mathematical systems [...]. This situation cannot be fixed by the procedure, that one, as the formalist school does, objects the mathematical language itself (i.e. the symbol system that serves to evocate purely mathematical constructions in other people) to mathematical contemplation, and lends it by means of an overhaul the precision and stability of a material instrument or a phenomenon of the exact science, and communicates about it in a language of the second order or a metalanguage.

After pointing out that none of the above can exclude the possibility of misunderstanding, Brouwer passed on to a discussion of the misplaced trust in logical principles.

The final section dealt with intuitionistic mathematics and its consequences. The novelty here, which probably escaped the audience, was a presentation of his method of the creating subject *avant la lettre*. That is to say, Brouwer introduced a method for counterexamples, which went beyond the Brouwerian counterexam-

[69] See p. 69, 91 ff. and [Brouwer 1905] p. 19.

[70] Which much later is called the 'cunning act' [Brouwer 1948b].

ples. One could, however, easily miss this point, as Brouwer did not stress the new methodology. The Berlin lectures were clearer in this respect. The two terms that are remembered from this exposition are 'fleeing property' [*fliehender Eigenschaft*] and 'pendulum number' [*Pendelzahl*]. The section closes with the remark that the extended principle of the excluded third has contradictory instances, the example being 'all real numbers are either rational or irrational'.[71]

There is a report of Brouwer's lecture by Karl Menger in his memoirs [Menger 1994], ch. X.

> While none of the other members of the Circle knew Brouwer as a lecturer, I had often heard him speak in Amsterdam. So, thinking of the conversation with Schlick a few weeks before, I said to Waismann "Why don't you invite Wittgenstein to these lectures? Brouwer is a stimulating speaker and his topics may arouse Wittgenstein's interest." Waismann reflected for a split second and said "That is a very good idea. I will speak to Feigl. Perhaps we can induce Wittgenstein to attend." Two days before Brouwer's lecture, Waismann told me that Wittgenstein would be present.
>
> Hahn, who was to introduce Brouwer, was notified by Waismann or Feigl when Wittgenstein entered the auditorium, (Like myself, he had until then seen Wittgenstein only in a photograph, which incidentally proves that the latter had never attended a meeting of the Circle in its first years.) From a distance I watched Hahn walking down the aisle to introduce himself and to welcome the guest. Wittgenstein thanked him with an abstract smile and eyes focused at infinity, and took a seat in the fifth row or so.
>
> I always tried to avoid making the acquaintance of someone who appeared to be not interested in making mine and thus stayed far away. But I was curious to see how the guest would behave during the lecture which he attended, probably unbeknownst to him, at my suggestion. So I took a seat two rows behind him and well to his left. Motionless from beginning to end, Wittgenstein looked at the speaker with a slightly startled expression, at first, which later gave way to a faint smile of enjoyment.

Feigl described Wittgenstein's reaction after Brouwer's lecture as follows: 'He became extremely voluble and began sketching ideas that were the beginnings of his later writings. That evening marked the return of Wittgenstein to strong philosophical interests and activities.' The opinions on Brouwer's influence

[71] The terminology may seem confusing. Brouwer had observed that PEM itself is not contradictory. For consider some open problem, e.g. the Riemann hypothesis R. Then if 'R or not-R' were contradictory, both R and not-R would yield contradictions, So both not-R and not-not-R would be true, which is impossible. Hence the negation of PEM for a single statement is consistent.. The same argument works for finitely many statements, $R_1 \wedge R_2 \wedge \ldots \wedge R_n$. However for an infinite set of statements (the extended case of PEM) this is no longer the case. Consider 'for all real numbers x, x is rational or x is irrational.' This cannot be true, because it would yield a decomposition of the continuum, which is impossible, cf. p. 385.

on Wittgenstein diverge. Brouwer's views on communication must have appealed to Wittgenstein, and his later use of language games seems an adequate consequence of these views. In mathematics there are certain traces of Brouwer's influence, e.g. Wittgenstein's use of the decimal expansion of π to illustrate some points concerning real numbers. It should be noted, however, that in the printed version of the Vienna talks the decimal expansion of π does not occur. Brouwer may of course have used the example in the talk, or Wittgenstein could have read Brouwer's earlier papers, or—for that matter—even Borel.[72]

Four days later Brouwer gave his talk 'The structure of the continuum', which was intended for a mathematical audience. The paper has a historical introduction which is not quite identical with the introduction to the Berlin lectures, nor with the historical sketch Brouwer wrote in connection with Fraenkel's '*Ten Lectures on Set Theory*'.[73] The old-intuitionistic school of, in particular, Poincaré and Borel, is criticized quite properly for freely using classical logic, only a certain caution is practiced in the presence of impredicative definitions.

The paper concentrates on a refined analysis of the continuum with respect to the standard notions, such as order, separability, compactness and the like. A special feature which the paper shares with the Berlin lectures, is the separate treatment of the full continuum (i.e. as based on choice sequences) and the reduced continuum (as based on lawlike sequences). This corresponds perfectly to the techniques of the creating subject and that of the Brouwerian counterexamples.[74] Whereas the Berlin lectures anticipate the presentation of the basis of intuitionistic mathematics in the form of the first and second act of intuitionism (creation of natural numbers and creation of choice sequences), the choice sequences as such are played down in 'The structure of the continuum'. The term *Wahlfolge* (choice sequence) occurs only once, and in one of Brouwer's private copies it is replaced by *Folge von solchen Wahlen* (sequence of such choices), but though the word hardly occurs, the notion plays its role alright. The representation of the (unit) continuum by a fan demonstrates the basic role of choice sequences. One might wonder what happened to the 'ur-intuition' of the dissertation of 1907. Under the influence of Brouwer's later expositions the impression became prevalent that the ur-intuition was just a beginner's device, a thing of the past. In the paper 'The structure of the continuum' the intuitive continuum, including the ur-intuition, is however once more given its prominent place. A private note, acting as a reminder, reads, 'In the continuum lecture, add at the end of section I, that the continuum is thus given by means of the *ur-intuition* just as with Kant and Schopenhauer.' Section I indeed pays at the end generously attention to this topic.

Wittgenstein, in his *Philosophical Remarks* (1930), discusses a large number of issues pertinent to, or derived from Brouwer's lecture. He energetically rejects the legitimacy of choice sequences as objects of mathematics; e.g. when discussing

[72] [Brouwer 1908, Brouwer 1923e], [Borel 1908].

[73] [Brouwer 1992], [Brouwer 1923e].

[74] Cf. [Dalen, D. van 1999a].

the unending choice process of 0 and 1 in the generation of a decimal expansion, he concludes 'That, what in the process of throwing a die is arithmetic, is not the factual result, but the infinite non-decidedness. But that indeed does not *determine* a number.'[75]

In the same volume, no. 173, Wittgenstein comes very close to Hilbert's 'there is no ignorabimus'.[76] After conceding that Brouwer was right in stating that the properties of the pendulum number are not compatible with the principle of the excluded third, he continued 'For if the question whether a statement is true or false is a priori undecidable, then the statement loses its meaning and exactly for that reason the laws of logic lose their validity.' So the pendulum number and choice sequences in general are for Wittgenstein, on the basis of meaning considerations, not a part of mathematics.

On the other hand there are points with a certain Brouwerian ring, e.g. no. 159 says: 'One cannot describe mathematics, but only practice it'. Compare this to Weyl's rendering of Brouwer's viewpoint as 'Mathematics is an activity, not a theory'.[77] The use Wittgenstein makes of the decimal expansion of π is closer to Borel's examples. In 'Les paradoxes de la théorie des ensembles,' Borel draws attention to phenomena, by mathematicians usually viewed as 'ill-considered'; here is an example: consider the decimal expansion of π and replace all digits 7 by 8, and vice versa. Question: is the resulting real number algebraic? In general there is little one can say about these problems, they excellently illustrate the fact that at a very elementary level hard questions can be posed. Brouwer uses the decimals of π also for the illustration of unsolved problems, but his trade-mark is the occurrence of certain sequences. Wittgenstein, in his *Philosophical Remarks* adopts the 'switch of digits trick' (no. 182 ff.) Since Wittgenstein is parsimonious with references; it is not obvious what his source was. Given the date and place, it is however likely that Brouwer was his inspiration (his mentioning of the pendulum number also points in the direction of Brouwer).

It would be interesting to know with whom Brouwer met in Vienna. He certainly met Hahn and Ehrenhaft, Wirtinger and Menger. But did he meet the Viennese sphinx Ludwig Wittgenstein or the young genius Kurt Gödel, or the other members of the Vienna Circle?

As to Wittgenstein, we have some definite, albeit scarce, information from Roy Finch. Finch met Brouwer in 1953 when Brouwer was doing the grand tour of the United States.[78] In a conversation Brouwer mentioned 'that he had had a private all-day meeting with Wittgenstein on an island, during which they discussed Brouwer's lecture which had made such a strong impression on Wittgenstein'. Finch assumed that this island was off the coast of Holland, but apparently no

[75] [Wittgenstein 1984], no. 179.

[76] Wittgenstein's arguments, however, should be distinguished from those of Hilbert, Brouwer, Heyting, Martin-Löf.

[77] P. 105, p. 323.

[78] Cf. p. 863. Finch to Van Dalen 10.X.1990.

records of the meeting have survived. A small Danube island would fit the bill as well.

Gödel and Brouwer probably did not meet; at that time Gödel would not so easily have approached a famous visitor, moreover his name had still to be made. There is, however, a letter from Gödel to Menger of April 21, 1972, in which he wrote 'I was never introduced to Wittgenstein and have never spoken a word with him. I only saw him once in my life when he attended a lecture, I think it was Brouwer's'. With a little goodwill one might infer that Gödel attended Brouwer's lecture.[79]

Gödel was quick to understand Brouwer; there are some notes in Carnap's diary which shed some light: 'Gödel says: when I want to follow strictly the constructivist point of view, then I have either to reject the principle of the excluded third (because it is not the case that either p or *not* p is provable), or on the other hand to presuppose a complete (decidable) logic. That seems correct.' 'Gödel on the inexhaustibility of mathematics. It was Brouwer's Vienna Lecture that gave the first impulse to these thoughts.'[80] Gödel's subsequent research beautifully illustrates how to use the new tools of logic to obtain conceptually significant insights.

In the early part of 1928 Holland was honoured by a visit of one of the most prominent European philosopher, Edmund Husserl. Husserl visited Holland in April at the invitation of the Dutch philosopher Pos.[81] Husserl was on the whole rather pleased with the occasion; in a letter to Heidegger he wrote 'The most interesting events were the long conversations in Amsterdam with Brouwer, who made a most significant impression on me, that of a completely original, radically honest, real, completely modern man'.[82] There is also a reference to this visit in the letter of Brouwer to Eva Wernicke of April 30:

'Here, at the moment, Husserl is darting around, which strongly draws me in. He thinks Miss Gawehn the most intelligent person he has come across in Holland'.[83] The brief passage makes clear that Brouwer was as strongly impressed by Husserl as Husserl by him.

Apparently Husserl had met in Amsterdam a gifted young lady, who impressed him by her intelligence. Since she did not make a reputation in philosophy one may well wonder who she was and what became of her. Irmgard Gawehn was born in 1900 in Memel, where she went to school. She studied in Heidelberg between 1920 and 1924, interrupted by a summer semester in Göttingen. She specialized in topology under Rosenthal, who became her PhD adviser. She obtained her doctorate in

[79] See [Köhler 1991]. There is however a letter from Gödel to the American Philosophical Society of 19.I.1967 in which he says 'I have seen Brouwer only on one occasion, in 1953, when he came to Princeton for a brief visit'. If 'seen' means 'met' there is no contradiction.

[80] Gödel über Unerschöpflichkeit der Mathematik. Erst danach Brouwer Wiener Vortrag zu diesen Gedanken angeregt worden'. Carnap 14.II.1928 and 23.XII.1929. See also [Wang 1987], p. 50.

[81] The *Amsterdamer Vorträge* were published in [?].

[82] Husserl to Heidegger 5.V.1928, []

[83] Hier tummelt sich jetzt Husserl herum, wobei ich stark mit herangezogen werde. Er findet Frä ulein Gawehn den intelligensten Kopf, der er in Holland angetroffen hat.

1931.[84] The dissertation *Über unberandete 2-dimensionale Mannigfaltigkeiten*, was accepted by the *Mathematische Annalen*, where is was published in 1927. Brouwer handled the paper; the correspondence concerning her paper was put in Menger's hands.

Irmgard was already a legend in Heidelberg when she finished her studies. She had the reputation of a first-class topologist. With this reputation she went to Berlin to pursue her career further. Freudenthal, who studied in Berlin at that time, recalled that everybody was excited to get such a wonderful topologist in Berlin. However, when she gave on June, 25, 1926 her first colloquium talk, *Set theoretic definition of 2-dimensional manifolds without bounds*, it turned out to be bordering on the incomprehensible. Whatever knowledge she might have had she could not get across.

Soon she discovered, however, that her real interest was philosophy, and she built herself a reputation as a philosopher. But again, she could not really make a career in philosophy. In a letter to Hopf of April 9, 1927 Brouwer asked Hopf to keep an eye on Irmgard, and to see that her philosophical paper got published quickly. So there was, or there seemed to be, some publishable philosophical activity.

In order to provide Irmgard (who was always referred to in a formal manner as 'Fräulein Gawehn') with a basis for a secure income, Brouwer had already suggested that she should take a '*Staatsexamen*', that would give her a licence to teach at gymnasia or high schools. The exam out of the way, Brouwer would apply for a Rockefeller stipend for her to study with him in Amsterdam. When she took the advice, she was in for an unpleasant disappointment. The stumbling block was the oral examination; the committee would ask questions on all kinds of topics, and she simply could not convince the examiners of her mathematical knowledge.

So she came to Amsterdam without her *Staatsdiploma*. There she became Brouwer's assistant from 1927 to 1930. And once in Amsterdam she surprised everybody by showing a striking lack of ability both as a topologist and a philosopher. Brouwer apparently kept her on for light work. The veil over the enigma Gawehn was lifted by Rosenthal, who later told Freudenthal that Irmgard was for whatever reason the object of admiration of the male student population. In Heidelberg she found a topologist as a lover, and the man wrote her dissertation. In Berlin a philosopher proved his love for her in a similar way. When left to her own resources, she could not cope. Hence the disaster of the state examination. Brouwer, who only knew her dissertation, was not aware of her history. In Amsterdam she had also a number of lovers, whom she invariably dumped after some time. But finally one of her suitors dumped her, and according to the local gossip, that was more than she could bear. She mentally caved in and spent the rest of her life in a mental institution in Holland. Cor Jongejan and Lize visited her regularly until she died, after the Second World War.

[84]The oral exam was in 1925. Rosenthal and Liebmann examined her in mathematics, Lenard in physics and Jaspers in philosophy.

The story, tragic as it is, illustrates that even in mathematics, the inner sanctum of reason, not all is what it appears to be, and that famous philosophers may be fooled.

It never became clear who wrote the thesis in topology.

The University of Oslo commemorated in 1929 the centennial of the death of Niels Henrik Abel and at the occasion the mathematics faculty awarded a large number of honorary doctorates to the most prominent mathematicians of the day. Brouwer received an invitation for the festivities with the announcement that the university proposed to appoint him as an honorary doctor.[85] Brouwer gladly accepted the honour, but fate decidenst him: he wrote on April 4 that he could not attend the ceremony because of an illness. The list of honorary doctors was impressive; it contained the names of Engel, Fueter, Hadamard, Hardy, Hensel, Juel, Landau, Lindelöf, Painlevé, Phragmèn, Pincherle, Takagi, de la Vallée-Poussin, Veblen and Weyl. On the day itself, however, only four of the them showed up.

14.5 Other activities

With the founding of his new intuitionism, the topological school, the *Grundlagenstreit*, the fight against the *Conseil*, and his normal teaching Brouwer had enough on his hands to be fully occupied. Nonetheless he found time for activities outside the academic sphere. The signific enterprise, in a sense, was one of those side activities. The signific circle has sometimes been compared to the Vienna Circle, and certainly it had for some time the potential of growing into an influential centre. There was, however, no driving power behind significs. The leaders, Brouwer and Mannoury, were not ambitious enough to get significs on the map. That is to say, the members of the circle indulged in grand schemes, such as the encyclopaedia project and the attracting of influential foreign experts, but somehow it never came to anything. Moreover, the only one who was willing to devote his scientific career to significs was Mannoury. Neither Brouwer, nor Borel, Van Ginneken or Van Eeden, were willing to give up their own practice, be it scientific, social or artistic. But Mannoury, successful as he might be as a teacher, never managed to get influential philosophers to join his enterprise.

When in 1924 the circle was running into problems (cf. p. 372), Mannoury seriously considered its dissolution. After some discussion, it was decided to keep the circle going as long as a glimmer of hope was left. The members of the circle set themselves the goal of publishing the *Signific Dialogues*, a report of some of the discussions of the circle. Even this proved to be problematic. The circle met two more times, on 12 November 1925 and 2 December 1926.[86] After that the circle just faded away. The *Signific Dialogues* eventually appeared in 1937.[87] In the late

[85] Rector Oslo University to Brouwer 19.II.1929.

[86] For details see [Schmitz 1990], 6.3.

[87] [Brouwer, L.E.J. et al. 1937,1939].

thirties a new organization for significs was founded; Brouwer only took part in this as an interested party.

By the time Brouwer got involved in his other schemes, significs was a thing of the past. What was left, was his friendship with Mannoury and Borel. The friendship with Mannoury lasted; it was not at the same level as the friendship with Scheltema, but there was an immense appreciation and trust between the two. Brouwer was well aware of the destructive element in his relations with fellow human beings; he once remarked to Mannoury, 'Gerrit, you bring the good out of people. I bring out the bad'.[88] Indeed, Mannoury had the makings of a saint; he was highly valued by his colleagues, and revered by his students. And his role in politics had shown him to be a man of principle.

The other friend that the significs episode had left him was Henri Borel. After the foundering of the circle the two remained in contact; there are a number of letters from Borel to Brouwer preserved. They show Borel as a tragic figure, unable to find a proper place in life. When he died he left a large number of books and brochures behind, and curiously enough, in spite of his vehement criticism of Van Eeden for turning to Catholicism, he embraced the faith of the Mother Church in the end.

A considerable part of Brouwer's time was taken up by matters concerning his wife's pharmacy. The original building and grounds at the Overtoom had been expropriated by the City of Amsterdam. In spite of his protests, Brouwer had to relocated the pharmacy. He found a suitable location at the Amstelveenseweg 21-23 in a street which is a continuation of the Overtoom. After some effective bargaining he bought the house for 44,500 guilders, which was a substantial sum at the time.[89] Even if the city compensated Brouwer for a comparable sum (the original offer was 40,342 guilders), Brouwer had to furnish quite a bit of capital. In fact the cost of converting the building came to an extra 1500 guilders. It may be said that Brouwer remodelled the building in style. The shop was panelled in 'Slavonic oak', the central part was designed in the style of an old library, complete with a gallery and fitting staircases. In matters of furnishing the pharmacies and providing the necessary equipment Brouwer kept rather high standards; in the case of the Amstelveenseweg pharmacy, for example, he engaged one of the better architects.

The second and third floor of the building were occupied by tenants. On the first floor lived Louise, the daughter of Lize, and Brouwer had his study on the same floor. His study contained a grand piano, and Mr. Guasco, the son of a tenant, remembered that usually when Brouwer returned from his lectures, 'the heavy tones of the overture Egmont resounded through the house'. The Guasco's lived on the second floor, and the third floor was let to a music teacher. Louise, in spite of her preoccupation with religion and the eternal, had conceived a personal

[88] Oral communication C. Vuysje-Mannoury.

[89] He got a mortgage of 29,000 guilders

Silver wedding of Bertus and Lize. (Brouwer Archive).

interest in the man. She could hear all the traffic on the stairs, and when the teacher passed her door, she happened to be at her door, and invited him in for a neighbourly chat. Eventually he tried to sneak out on stockinged feet, but even that could not always mislead Louise's sharp ears.[90] At roughly the same time Brouwer acquired two more premises, not far from the shop at the Amstelveenseweg. He bought a shop at the Surinameplein[91] and rented a wooden temporary building at Surinamestraat number 12.[92] The premises at the Amstelveenseweg and those at the Surinamestraat and Surinameplein were separated by a canal. It seems that Brouwer had counted on a future bridge in front of his Amstelveenseweg pharmacy, which would make it more attractive for clients. When the city decided to build the bridge some distance to the North he was rather angry, but though he contemplated legal action, there was little he could do.

There is a tendency to equate 'owning a pharmacy' and 'being rich', but that did not work for Brouwer's pharmacy; it was on the whole rather poorly managed. Somehow Brouwer could not operate in a businesslike manner where the pharmacy was concerned. It was overstaffed, and, since Brouwer and his wife took a somewhat

[90] Interview R.A.F. Guasco.

[91] Suriname place or square.

[92] The transacting was concluded on 2 April 1930.

paternalistic view of the assistants, they were almost appointed for life In the end the shop started to look like a home for elderly pharmacy assistants. Brouwer took an active part in the running of the pharmacy, he carried out administrative duties, prepared the tax declarations, dealt with the municipal bureaus, and represented the pharmacy at the national organization of pharmacists. At the meetings of the latter he could act in an overbearing manner, until finally he was barred from the meetings, on the ground that he was not a pharmacist.

Family life remained as hectic as before. Lize usually spent the weekdays in Amsterdam, where she slept over the pharmacy. This had the advantage that she could keep an eye on Louise. In spite of the fact that Louise was not accepted in the convent as a novice, she remained a devoutly Catholic, and—unknown to Brouwer—Lize was baptized as a Catholic at roughly the same time of Van Eeden's conversion (cf. p 373). The ceremony took place on 24 December 1924 in the St. Dominicus Church in Amsterdam. Neither her husband nor her friends were aware of her conversion, it was a secret between her and Louise.

In 1927 Lize spent some time in Antwerp in a paramedical institute of some sort, probably halfway between a boarding house and a health centre. From the scarce information seems to indicate that she suffered from pernicious anaemia.[93]

[93]Lize to Louise 17.V.1935.

15

THE THREE BATTLES

> If mathematics would seriously retire to this status of pure game for the sake of its safety, it would no longer be a determining factor in the history of mind.
>
> H. Weyl

15.1 The *Grundlagenstreit*

We left the foundational debate in a fairly peaceful state. Hilbert's talk at the Weierstrass conference in 1925 was rather moderate in tone; there were a few references to dissident views, but more admonishing than belligerent in tone. Apart from a repeat of the Münster lecture in Copenhagen no further major activities took place that year. In November Hilbert was diagnosed with pernicious anaemia; he had not been well for some time, but the actual cause was not discovered earlier. At that time the disease was more often than not fatal. The immediate consequence was a serious decline in working power, and the disease took its toll on Hilbert's intellectual activities. Hilbert, nonetheless, tried to carry on his research as usual.

The years 1925 and 1926 were for Brouwer mainly filled with duties that had little to do with research. He spent time editing the Urysohn memoir for *Fundamenta*, and the necessary correspondence with Sierpinski and Alexandrov.

As an editor of the *Annalen* he was also very much occupied with the new influx of topological papers of, among others, Alexandrov, Urysohn, Menger, and Tumarkin. In between he found time to work on his contribution for the Riemann volume.

The foundational debate was watched with interest in mathematical circles, but only a handful of mathematicians actually took a position. If they did, it was mostly on the formalist side. The choice was, mostly, determined by feelings of loyalty—any student or alumnus of the Göttingen university fostered 'right-or-wrong my teacher' feelings.[1] Another argument for siding with the formalists was of a pragmatic nature—life with PEM is easier than without it. It is, however, fair to say that the majority of the mathematical community deplored the hostilities of the last few years. A perfect occasion for a peaceful exchange of ideas arose, when in the summer of 1926 Brouwer once more visited Göttingen.

Alexandrov, who was already in Göttingen, told later the charming story how the two opponents came to lay down their arms.[2] Brouwer, he wrote, was

[1] Cf. [Lietzmann 1942].

[2] [Alexandrov 1969].

immediately at home, as in the old days, in the circle of mathematicians. He was warmly welcomed in the intimate circle around Courant and Emmy Noether. The ill-feelings and suspicions having melted like snow for the sun, plans were made to get Brouwer and Hilbert together again. One evening a group of mathematicians, including Hilbert and Brouwer, was invited to the apartment of Emmy Noether. And so Brouwer, Hilbert, Courant, Landau, Alexandrov, Hopf and a few other young mathematicians found themselves at the table under the roof of Emmy's friendly quarters (Landau used to question the validity of Euler's polyeder theorem for this room). It fell to Alexandrov to start a conversation that would break the ice between Hilbert and Brouwer. Alexandrov's solution was as ingenious as simple: what better expedient was there to bring two persons together, than the mentioning of a third person both enjoyed criticizing. Alexandrov cleverly introduced the acknowledged paragon of vanity Paul Koebe (in his paper he discretely referred to the well-known function theorist of Luckenwalde). The trick worked better than could be expected, it did not take long before Brouwer and Hilbert were outbidding each other in criticism of poor Koebe, 'at the same time they were nodding more and more friendly at each other, until finally they completely agreed in a mutual toasting.' The thus established peace lasted all during Brouwer's visit. Even his lecture in the Göttingen mathematical society on 22 July 1926, 'On everywhere- and seemingly everywhere defined functions',[3] could not disrupt the peace. There are no notes of the lecture, but one may safely assume that it contained material from Brouwer's paper for the Riemann volume.

After his stay in Göttingen, Brouwer pilgrimaged in the company of Alexandrov to Batz to commemorate the lamented Urysohn. The topological fire in Brouwer had apparently not been extinguished completely, for Alexandrov wrote excitedly to Hopf that 'Brouwer has proved a topological theorem.'[4]

Less than a week later Brouwer was to be found at one of those meetings where he could indulge in the spiritual side of life. He attended a Sufi meeting in France with a lady named 'Mies'. 'I am undergoing in humility the first impacts of the Sufi-order' he wrote to Bertha Adama van Scheltema, the sister of his deceased friend.[5] Brouwer was no stranger to spiritual intermezzo's of this kind. It is not easy to decide what interested him more, the spiritual experience and reflection offered, or the attraction of the colourful participants.

Not much later Brouwer turned up at the Düsseldorf meeting of the DMV, where he delivered on 23 September his talk 'On domains of functions'.[6]

Hilbert, in the meantime, was very poorly. He had his ups and downs, but generally speaking there was not much improvement.

In Amsterdam the faculty did something that certainly must have pleased Brouwer, and one is tempted to see his hand in it. In the meeting of October 24

[3] *Überall und scheinbar überall definierte Funktionen.*

[4] Alexandrov to Hopf 12.VIII.1926.

[5] Brouwer to B. Adama van Scheltema, 22.VIII.1926.

[6] *Über Definitionsbereiche von Funktionen.*

the faculty decided no longer to admit philosophy as a major in the curriculum. Four years ago his colleague Kohnstamm had managed by clever manoeuvring to get philosophy accepted as a major, and Brouwer, in spite of his sharp intelligence had lost that battle.[7]

The next significant event in the *Grundlagenstreit* was Hilbert's second talk in Hamburg in July 1927. His health had somewhat improved, but it was a tired old man who appeared at the rostrum. Nonetheless, the event was a tremendous success. Courant, who accompanied Hilbert to Hamburg reported to Springer, 'The days in Hamburg with Hilbert were in every respect most satisfying. Hilbert's condition was so good, that he could speak for an audience of over 100 with a soft and tired voice, it is true, but with intense temperamental outbursts and very impressive, and that he even took part in a big festive dinner in the evening of the second day. The whole occasion was a great triumph for him and did him psychologically exceptionally good, without harm to the body. Objectively, his blood picture has strongly improved and now our greatest hope is a liver cure. It will be, however, hardly possible to get Hilbert to eat liver in the required large quantities, but we have cabled to Harvard medical school, and we probably soon get the liver preparation produced there, from which we expect much.'[8] The liver cure started not long after the lecture, followed indeed by the American preparation. The influence of American colleagues and students had certainly been helpful in procuring Hilbert this newly discovered treatment.[9] This was by no means the end of the story; the right dose had to be found, and annoying and painful symptoms reared their heads. It was a surprising proof of the success of the treatment—and the determination of Hilbert—that he was again one of the main speakers at the annual meeting of the DMV in Leipzig. His topic was 'The axiom of choice in mathematics'. The title is somewhat misleading, as 'choice' referred here to the ε-operator.

The Hamburg lecture of 1927, was like its predecessor of 1921, published in the *Abhandlungen des Hamburgischen Seminar*, this time with the simple title *The foundations of mathematics.*[10]

The paper was in content a leisurely exposition of the Infinity paper, a didactical introduction to proof theory of arithmetic and analysis, laced with some popular foundational motivation, and the usual pot-shots at dissenters. Again, we must bear in mind that the paper should be excused for its heavy handed propaganda, since it was doubtlessly the sort of language that could and would rouse an audience. One cannot read the paper without being touched by the belief of the author in his philosophical credo, and by his fighting spirit. Here was a man who had discovered the final answer to the foundational problems of mathematics. The wisdom of publishing all these belligerent remarks may be doubted. As a rule they reflect on the author, and moreover, they stand in the way of a proper scientific discussion.

[7] Cf. p. 361, ff.

[8] Courant to Springer 2.VIII.1927.

[9] See [Reid 1970], Ch. 21.

[10] *Die Grundlagen der Mathematik* [Hilbert 1928].

In particular, the formulations used by Hilbert tended to demonize the opponent. One might almost conclude that Hilbert did not want a *discussion*, he wanted to *win*.

As in earlier publications, Hilbert was determined not to give up the *Tertium non datur*. After pointing out that from a finitist point of view the negation of statements like $\forall x A(x)$, where each instance $A(n)$ can be finitistically checked, is problematic, he continues:

> But we cannot give up the use of the applications of the Tertium non datur or any of the other laws of Aristotelian logic which are expressed in our axioms, as the construction of analysis is impossible without them.

It may be remarked here that Hilbert was too pessimistic about a *Tertium non datur*-free mathematics. Work in the intuitionistic school and above all the results of the school of Erret Bishop gave a powerful impetus to constructive mathematics by actually rebuilding large parts of analysis in a constructive manner. Bishop made it clear that 'constructive' does not stand for 'ugly and cumbersome'. He convincingly demonstrated that constructive mathematics possesses the same elegance as classical mathematics. [11] We now know that (i) a substantial part of analysis can be developed in the intuitionistic frame work, (ii) the proofs are not as cumbersome as people in Hilbert's day expected, and (iii) that the mathematical problems arising in the Brouwer–Bishop tradition are interesting and rewarding. Of course, it would be anachronistic to expect Hilbert to see this point. Hilbert's solution was exemplary for the methods of the working mathematician. If you cannot solve the equation $x^2 + 1 = 0$, extend your system with $i(= \sqrt{-1})$. So if you cannot negate and quantify at the same time, add the ideal statements (which allow negation and quantification), and add *Tertium non datur*. It is like the extension of an algebraic structure. And then, of course, show the consistency.

After a quick survey of some of the tools and methods of proof theory, Hilbert returned to answer the various objections that had been raised, and which 'he found one and all as unjustified as possible.' He started by reconsidering Poincaré's objections against his consistency proofs that intended to safeguard induction by applying induction. This time Hilbert elaborated his defence, as promised in 1922, cf. p. 486; Poincaré, he said, had not understood the distinction between conceptual induction and formalized induction. On this point Hilbert's position was weak, to say the least. Unless he managed to pull off the finitist consistency proof that he had announced, he could not uphold his own view. Brouwer and Poincaré had a better intuition on this point,[12] Hilbert saw Poincaré's negative comments as a consequence of his pronounced prejudice against Cantor's theory. The argument does not convince; why should a mistrust in set theory lead to a rejection of a combinatorial practice as proof theory?

On the whole Hilbert was not impressed by what he read in the foundational literature. Most of it he considered as backward, as if belonging to the pre-Cantor period. Since Hilbert's yardstick was calibrated by the continuum hypothesis,

[11] See [Bishop, E. 1967] and [Bishop, E. 1985].

[12] Cf. [Brouwer 1913a], p. 88.

Hilbert's dogma, 'consistency $\Leftrightarrow$ existence', and the like, he was by definition right. But if one is willing to allow other yardsticks, no less significant, but based on alternative principles, then Brouwer's work could not be written off as obsolete nineteenth century stuff.

However, Hilbert clearly saw Brouwer as his prominent opponent, and thus he spent the rest of the paper to give him a thorough roasting. As he put it: 'By far the largest amount of space in the present literature on the foundations of mathematics is taken up by the doctrine that Brouwer has formulated and called intuitionism. Not out of an inclination for polemics, but to express my views clearly, and to prevent misconceptions about my own theory, I have to go further into certain statements of Brouwer.'[13]

There are actually two main points that Hilbert singled out for comment: the meaning of existence and the *Tertium non datur*. 'Brouwer declares existential statements by themselves to be one and all meaningless, insofar they do not at the same time contain the construction of object that is asserted to exist, for worthless scrip: it is through them that mathematics degenerates into a game.'[14] Hilbert's main defence against this was based on the ε-operator. What Hilbert exactly wished to achieve is not clear; in the following argument the ε-operator plays no role. A possible option might be the use of the ε-operator to obtain a term that can be seen as the solution promised by $\exists x A(x)$, namely $\varepsilon_{x+2=7}$ is the solution of the equation $x + 2 = 7$.[15] What is left is a passionate plea for pure existence proofs on general grounds; they do away with lengthy computations, they provide real insight, ... No mathematician will disagree, but the fact remains that Hilbert evades the issue: can the promise $\exists x A(x)$ be fulfilled? Instead Hilbert stuck to his lofty views: 'Pure existence theorems have thus in fact been the most important landmarks in the historical development of our science.' 'But' he added, 'such considerations do not trouble the devout intuitionist.' As to the game, so deprecated by Brouwer, it is Hilbert's opinion that the formula game reflects exactly the *technique of our thinking*. And indeed, 'the basic idea of my proof theory is nothing else but to describe the activity of our intellect, to make a protocol about the rules according to which our thinking actually proceeds.'[16]

The defence of the *Tertium non datur* is, so to speak, left to the ε-axiom, from which it follows; the informal argument for the *Tertium non datur* comes down to the bottom-line: 'it has never yet caused the slightest error.' In the end Hilbert's arguments are pragmatic, and emotional, as the following quote shows:

> Taking this Tertium non datur from the mathematician would be the same as, say, denying the astronomer his telescope, and the boxer the use of

[13] Ibid, p. 77.

[14] The reference to 'worthless scrip' suggests that Hilbert has Weyl's new crisis paper in mind. But it covers Brouwer's views equally well.

[15] it is simple to prove $\exists x(x + 2 = 7)$.

[16] A most interesting claim. Given a number of extra specifications, there are good arguments for Hilbert's claim when made for the system of Gentzen.

> his fists. The ban of existence theorems and the Tertium non datur boils roughly down to renounce the science of mathematics as a whole.

The last part reflects a prevailing pessimism with respect to intuitionistic mathematics. One simply could not see how the essential parts of mathematics could be salvaged. Even those who were sympathetic to Brouwer's cause, felt discouraged by the complications that had to be faced. Abraham Fraenkel saw the intuitionistic landscape as a bleak place; he feared that after the intuitionistic revolution, only ruins would remind the passing traveller of the former splendour of the architecture of mathematics. Perhaps some corners of the old buildings, e.g. arithmetic, might be saved, cf. p.489.

Hermann Weyl was no less pessimistic:[17]

> Mathematics attains with Brouwer the highest intuitive clarity. He succeeds in developing the beginnings of analysis in a natural way, retaining contact with intuition much closer than before. But one can not deny, that progressing to higher and more general theories, the fact that the simple principles of classical logic are not applicable finally results in a hardly bearable awkwardness. And with pain the mathematician sees that the larger part of his tower, which he thought to be joined from strong blocks, dissolves in smoke.

Understandably, Hilbert, who did not particularly love the competition, shared these bleak views:

> For what can the wretched remains, the few incomplete and unrelated isolated results, that the intuitionists have obtained without the use of logical ε-axiom mean, compared to the immense expanse of modern mathematics!

There is little doubt that Hilbert's assessment was right, but what was his point? After all, a new discipline, say group theory, had to start from scratch. Around the turn of the last century abstract group theory was not everywhere received with open arms. How could this new subject of group theory ever hope to catch up with the enormous mass of techniques and results of, say, projective geometry? If one takes into account that intuitionism aimed at rebuilding mathematics, then it is evident that Hilbert was premature. Moreover, his own programme would, under the above criteria be disqualified as well.

Hilbert's ultimate disproval of Brouwer's foundational tenets is expressed in an observation following his own credo, 'The fact is that thinking proceeds parallel to speaking and writing, by forming and concatenating sentences.' From Brouwer's early writings, and from the Vienna lectures, we know that this belief is diametrical opposed to Brouwer's view on language and communication. Quite consistently, Hilbert continues, 'after all, it is the task of science to liberate us from arbitrariness, sentiment and habit, and to protect us from the subjectivism, which has made itself noticeable in Kronecker's views, and which, as it seems to me, finds it apex in

[17] [Weyl 1928].

intuitionism.' This is again one of those populist phrases that go well with almost any audience. One may be almost certain that Hilbert's statement was interpreted as 'Kronecker and Brouwer act without objective justification, they ban what they dislike.' It is more than likely that even Hilbert intended this interpretation, because otherwise he would not lump Kronecker and Brouwer together. Kronecker, so to speak, was the super-objectivist, his rejection of such things as arbitrary sets and functions, which are ultimately a matter of belief, makes him a stricter rationalist–objectivist than even Hilbert. Brouwer, on the other hand, accepted such subjective objects as choice sequences, that by their nature escaped description and communication.[18] This could rightly be seen as subjectivism in mathematics, and Brouwer would insist on doing so. But here Hilbert's attitude seems somewhat unreasonable, for would it not be a matter of progress if such notions could be handled mathematically?

The critique of Brouwer and his intuitionism found an apotheosis in the following, undeniably subjective, expression of exasperation:

> Under the circumstances, I am baffled that a mathematician doubts the strict validity of the Tertium non datur. I am stunned even more about the fact that, as it seems a whole community of mathematicians has nowadays constituted itself, that is doing the same. I am most astonished about the fact that also even in the circle of mathematicians, the power of suggestion of a single, high spirited and imaginative man, can exert the most improbable and eccentric influences.

Strong language, understandable in the euphoria of an exciting lecture, but out of place in a paper in a respectable scientific journal.

After this emotional intermezzo Hilbert returned to business; he sketched some progress made by Ackermann, and closed with the optimistic statement that the success of his programme was just round the corner, 'the remaining problem is just to carry out a purely mathematic finiteness proof.' His conclusion was therefore, 'mathematics is a science without presuppositions. For its foundation I need neither God, as Kronecker does, nor a special faculty of our intellect attuned to the principle of complete induction, nor Brouwer's ur-intuition, nor, finally like Russell and Whitehead, axioms of infinity, reducibility, and completeness, which are in fact actual, contentual assumptions that cannot be compensated by consistency proofs.'

It is interesting to read these proclamations, in view of the lessons Gödel taught us about Hilbert's programme. Nonetheless, the certainty and enthusiasm of Hilbert must have been infectious, as Courant's report indicates.

One can only speculate what made Hilbert so angry; after all in August 1926 he and Brouwer had made their peace, and since then not that much had happened. Or had Brouwer's spectacular success in Berlin upset Hilbert? Had he become convinced that Berlin was a hotbed of intuitionists; it is possible, where else would he have conjured up 'a whole community of mathematicians' abjuring the *Tertium non datur?* In fact, one may doubt if the number of practising intuitionists would

[18] Cf. [Dalen, D. van 1999b].

surpass five. If anything, the cause of Hilbert's anger and anxiety must have been psychological. In fact here was a younger man, with an impeccable scientific past, who with great tenacity kept pointing out the Achilles heel of the formalist programme, and who was not daunted by the displeasure of the reigning sovereign of mathematics. And worse, a man who had even inspired the defection of Hilbert's star student, Hermann Weyl. Hilbert was reputed to take a common sense position in political matters, to steer away from his conservative colleagues, and to accept the republic for what it was. He was, however, not able to practice the same common sense with respect to himself. He could not forget that he was the famous professor who was always right; indeed his students usually went out of their way to spare the great man embarrassments or inconvenience. So when this Dutchman offered him admiration for his mathematical achievements and for his role as a promoter of mathematics and mathematicians, but declined to drop his foundational programme, he committed something comparable to high treason. Finally, Hilbert's mental-physical situation was put under pressure by the persistence of his pernicious anaemia. In all, there were enough factors present to worry even an experienced diplomat. A diplomat can, however, consult advisers, but Hilbert had isolated himself from potential advisors on foundational matters (and not only those). Advice was to be given, not to be taken. Under the circumstances, there was probably no one to question the wisdom of such a crude course. One might almost think that Hilbert was provoking an open conflict, which would allow him to sever all ties with Brouwer. Whether he was enough of a Bismarck to hatch such a plot is of course questionable. From Courant's report to Springer, we may believe that the Hamburg audience applauded the address of the Grand Master; the provocative and aggressive character could, however, not have escaped anyone in the audience. Whereas some saw Hilbert's talk as a triumph for mathematics, Hermann Weyl did not share that view. Following Hilbert's lecture he made a number of remarks, published in the same volume of the *Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität*.[19]

Weyl took his place at the lectern as if he were the counsel for the defence, who saw that his client had just been sentenced in absentia. 'Allow me to say a few words in defence of intuitionism', he opened. Mathematics, he said, was considered a system of contentual, meaningful and evident truths. This was the generally excepted platform before the advent of Hilbert's proof theory. Poincaré took that position, and Brouwer seconded him. 'But he was the first to see exactly and in its full extent how it had in fact transgressed the limits of contentual thought. I think that we are all indebted to him for this recognition of the limits of contentual thought.' Hilbert too recognized these limits in his metamathematical considerations, and he accepted that there was nothing artificial about these limits. 'Thus', he went on, 'it does not seem strange to me that Brouwer's ideas found a resonance (*Gefolgschaft*); his point of view followed necessarily from a thesis, that was shared by all mathematicians before the formulation of Hilbert's formal approach,

[19] [Weyl 1928, Heijenoort 1967].

and from a fundamental new, indubitable logical insight, that was recognised by Hilbert too.' The consequences of these insights led, however, to widely differing reactions. Brouwer rebuilt mathematics from the inside, putting up with the limitations; Hilbert opted, however, in Weyl's words, for a wholesale reinterpretation of the meaning of classical mathematics, that is the formalization. At this point Weyl made a gesture that caused some sensation, 'Also in the epistemological appraisal of the thus created new situation, nothing separates me from Hilbert, as I am happy to acknowledge.' Did this mean the definite farewell to intuitionism? That seems too hasty a conclusion, for we have to keep in mind that it was the same Weyl who wrote in that year,

> The ice field was broken into floes, and now the element of the flowing will soon completely be master over the rigid. L.E.J. Brouwer designs—and this is an accomplishment of the greatest epistemological importance—an exact mathematical theory of the continuum, which conceives it not as a rigid being, but as a medium of free becoming.[20]

The recognition of the epistemological basis of system does not automatically lead to the rejection of another system, which is compatible with it. In fact, one can very well be an intuitionist, and practise proof theory, and one could even, as Weyl seems to suggest, share the epistemology of the proof theorist. Let us consider one of the major points of difference, the interpretation of existential statements. Even the success of Hilbert's programme did not provide an existential truth, like $\exists x A(x)$, with a construction of an a such that $A(a)$. The commitment of proof theory *à la* Hilbert is to finitary statements, not to all real statements. The latter would only be consistent.

It looks from our point of view more as if Weyl was granting Hilbert his project. For a discerning man like Weyl must had seen that the goal of the programme was still not reached, and a few private calculations would have convinced him that this was a treacherous swamp to enter. So he might have, quite reasonably, accepted the philosophical part of Hilbert's programme, without committing himself to either proof theory or intuitionism. Weyl's career, his work and personality show, I think, that he in the first place was a 'working mathematician' with an interest that could not be pinned down to one project or doctrine. There are enough statements to be found in his later writings, that make it clear beyond doubt that basically Brouwer was right. For example, in his review of *The philosophy of Bertrand Russell*:[21]

> Brouwer made it clear beyond a doubt, that there is no evidence supporting the belief in the existential character of the totality of all natural numbers, and hence the principle of the excluded middle ...

[20] '*Die Eisdecke war in Schollen zerborsten, und jetzt ward das Element des Flieszenden bald vollendes Herr über das Feste. L.E.J. Brouwer entwirft—und das ist eine Leistung von der grószten Erkenntnistheoretischen Tragweite—eine strenge mathematische Theorie des Kontinuums, die es nicht als starres Sein, sondern als Medium freien Werdens faszt* [Weyl 1928].

[21] [Weyl 1946], p. 275.

And in *The Open World*:[22]

> If mathematics would seriously retire to this status of pure game for the sake of its safety, it would no longer be a determining factor in the history of mind. (p. 77)
>
> If mathematics is taken by itself, one should restrict oneself with Brouwer to the intuitively recognizable thruths and consider the infinite only as an open field of possibilities; nothing compels us to go farther. But in the natural sciences we are in contact with a sphere which is impervious to intuitive evidence; here cognition necessarily becomes symbolic construction.(p. 82)

In 'David Hilbert and his mathematical work' he returned to the issue:[23]

> L.E.J. Brouwer by his intuitionism had opened our eyes and made us see how far generally accepted mathematics goes beyond such statements as can claim real meaning and truth founded on evidence. I regret that in his opposition to Brouwer, Hilbert never openly acknowledged the profound debts which he, as well as all other mathematicians, owe Brouwer for this revelation.

Although Weyl was reluctantly prepared to live in a world where Hilbert's formalism prevailed, one in which a doctrinaire meaninglessness should be the ultimate scientific moral, he notes with sincere regret that 'If Hilbert's view prevails over intuitionism, as appears to be the case, *then I see in this a decisive defeat of the philosophical attitude of pure phenomenology*, which thus proves to be insufficient for the understanding of creative science even in the area of cognition that is most primal and most readily open to evidence—mathematics.

Why was Weyl so defeatist? To be honest, there was no reason to give up hope so soon. After all, Hilbert's programme had no palpable results to show. Most of the steps on the road to success had yet to come. There is a well-known quote of Weyl that may provide a hint: 'My work has always tried to unite the true with the beautiful and when I had to choose one or the other, I usually chose the beautiful.'

Hilbert was a master of sweeping presentations, but his technical evidence was not always solid. As Von Neumann remarked in a letter to Carnap, 'There are many programmematic publications of Hilbert, in which Hilbert states that something is being proven or almost proven, for which this is not even approximately the case.'[24] It could not have escaped Weyl that the grand old man of Göttingen had enough faith in himself to confuse wish and fact. And even if Hilbert were to succeed, why give up 'meaning' for the evidence of the table of multiplication (Kreisel)? It is more plausible that Weyl's motivations were of an altogether different sort. Let us not forget that here was his revered teacher, who was in a state which could very well be terminal. And circumstances like that tend to inspire a

[22] [Weyl 1932].

[23] [Weyl 1944], p. 157.

[24] Von Neumann to Carnap 7.VI.1931.

certain compliance. Moreover, if one shares the intuitionistic conviction that there is a real, sound mathematics, created by ourselves, then it is a matter of magnanimity to grant the pleasure of 'playing the game,' which formalism professes. In short, Hilbert could argue, if Brouwer is right, he can prove the consistency I am looking for, so why worry? For Hermann Weyl this would have been too cheap a solution of a deep problem; but who knows, he may have had his reasons to prefer a compromising policy. We will probably never know. Whatever was the meaning of Weyl's epistemological move, it was a serious blow for intuitionism. There are no comments known of Brouwer on the above. He never lost his esteem for Weyl, and kept his new crisis paper on the reading lists until the fifties.

Brouwer's reaction to the Hamburg lecture is not known, although it can easily be guessed. Apparently, Hilbert's infinity-paper was already as much as he could take, for he decided to write down his version of the foundational conflict. On December 17, 1927 he submitted his paper of 'Intuitionistic reflections on formalism', together with a paper on an embedding of spreads[25] to the KNAW. Two months later the same paper was submitted at the Prussian Academy in Berlin.

The paper lists all the relevant articles that had appeared so far. The last paper of Hilbert on this list is the infinity paper, so one may reasonably conclude that Brouwer had not yet seen the Hamburg paper when he submitted his manuscript. Compared to the rather excited paper of his adversary, Brouwer's reflections paper is moderate in tone. It opens with what could be considered a peace proposal:

> The disagreement of which is correct, the formalistic way of founding mathematics anew or the intuitionistic way of reconstructing it, will vanish, and the choice between the two activities be reduced to a matter of taste, as soon as the following insights, which pertain primarily to formalism but were first formulated in the intuitionistic literature, are generally accepted. The acceptance of these insights is only a question of time, since they are the results of pure reflection and hence contain no disputable element, so that anyone who has once understood them must accept them. Two of the four insights have so far been understood and accepted in the formalistic literature. When the same state of affairs has been reached with respect to the other two, it will mean the end of the controversy concerning the foundations of mathematics.

Brouwer shows himself optimistic, not to say certain. The acceptance of his points was just a matter of time, he said, as it is only a matter of reflection; the points contain no disputable elements, and everybody who has understood them must accept them. It is striking indeed, that Brouwer had given up the radical condemnation of formalism of his early years. It is not unlikely that in the changing world of mathematics, he started to see that logic and formalization at best offered certain benefits after all, and at worst, were harmless. After all, his own student had designed a logical codification for intuitionistic mathematics, and he, Brouwer, had approved the project.

[25] [Brouwer 1928b, Brouwer 1928a].

After the brief introduction, which ended with a prediction of the end of the *Grundlagenstreit*, Brouwer formulated his four insights:

> First insight. *The differentiation, among the formalistic endeavours, between a construction of the 'inventory of mathematical formulas' (formalistic view of mathematics) and an intuitive (contentual) theory of the laws of this construction, as well as the recognition of the fact that for the latter theory the intuitionistic mathematics of the set of natural numbers is indispensable.*
>
> Second insight. *The rejection of the thoughtless use of the logical principle of excluded middle, ass well as the recognition, first, of the fact that the investigation of the question why the principle mentioned is justified and to what extent is valid constitutes an essential object of research in the foundations of mathematics, and, second, of the fact that in intuitive (contentual) mathematics this principle is valid only for finite systems.*
>
> Third insight. *The identification of the principle of excluded middle with the principle of the solvability of every mathematical problem.*
>
> Fourth insight. *The recognition of the fact that the (contentual) justification of formalistic mathematics by means of the proof of its consistency contains a vicious circle, since this justification rests upon the (contentual) correctness of the proposition that from the consistency of a proposition the correctness of the proposition follows, that is, upon the (contentual) correctness of the principle of excluded middle.*

The list is followed by a critical discussion, in which Brouwer precisely checks the provenance of these insights.

> The first insight is still lacking in [Hilbert 1905] (see in particular section V, pp. 184-185, which is in contradiction with it). After it had been strongly prepared by Poincaré, it first appears in the literature in [Brouwer 1907], where on pp. 173- 174 the parts of formalist mathematics mentioned above distinguished by the terms *mathematical language* and *mathematics of the second order*, and where the intuitive character of the latter part is emphasized[2)] In [Hilbert 1922] (...), under the name *metamathematics*, mathematics of the second order broke through in the formalist literature. The claim of the formalist school to have reduced intuitionism to absurdity by means of this insight, borrowed from intuitionism, cannot very well be taken seriously.

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2) [Brouwer's note] An oral discussion of the first insight took place in several conversations I had with Hilbert in the autumn of 1909.

We have seen that in the fall of 1909 Brouwer and Hilbert met in Scheveningen, (cf. p.128.). At that occasion Brouwer discussed the various levels of language, logic and mathematics from his dissertation with Hilbert.

The precise distinction *formal, meaningless mathematics–contentual mathematics (metamathematics)* is only much later introduced in Hilbert's first Hamburg paper (1922). There is no reference in any of Hilbert's papers to Brouwer's role in clearing up the confusion about the two levels. Perhaps Hilbert had completely forgotten

about the exchange in 1909. Courant confirmed what was common knowledge at the time, that Hilbert had great difficulty in keeping track of the information that he picked up. Courant mentioned a few instances, which concern substantial research, by no means small, neglected corners of mathematics—'so indeed, Hilbert's theory of integral equations, one of his greatest achievements, was triggered by a bad memory.'[26] Whatever was behind Hilbert's unreliable memory, one should not forget that at the time of the *Grundlagenstreit*, Hilbert had passed his sixtieth year, and he was at the height of his career. The success and admiration had left its trace in the form of a '*roi soleil*' mentality; it would not do to remind the king of such matters of credit. As usual, there is hardly any reason for a scientist to suppress reference to his sources, and certainly not for a successful mathematician like Hilbert. It is well known that some scholars only need the slightest hint, and they will build a beautiful theory. Others excel in originality, but have no urge to carry out the follow up. It is the cooperation of the two kinds that often produces striking results. We have seen that in the case of dimension theory, Brouwer was satisfied to work out the underlying idea so far that its significance was beyond doubt. It took a man like Menger (and had he lived, Urysohn) to work out all the interesting consequences. In the present case of the mathematics and language levels, one is hard pressed to find a reason for Hilbert to deny Brouwer his credit. Hilbert's grand design for proof theory was there for all the world to see; the most plausible explanation is that Hilbert simply forgot about Brouwer and his levels. That still leaves the question, why Hilbert did not acknowledge Brouwer's contribution after the subtle hint in the reflections paper. At worst, he could have said, 'Brouwer independently arrived at the idea of the mathematics levels.'

Brouwer definitely had not forgotten, and this time he decided to mention the fact; the above mentioned footnote serves the purpose. In the main text there is a hidden allusion to this matter, the two level distinction occurs in the dissertation where 'the mentioned parts of formalistic mathematics are distinguished as *mathematical language* and second-order mathematics, and where the intuitive character of the latter part is emphasized. It broke through in the formalist literature under the name *metamathematics* for second-order mathematics in the first Hamburg lecture.' In Brouwer's archive there is a small note to the effect that the 'Hilbert of 1900 did not feel the contradictory consequence of the lack of the intuiting of metamathematics, of which I freed him only in 1909 on a walk in the dunes, whereupon he changed my description "mathematical language of the second order" in "metamathematics".'[27] Clearly, in the absence of written evidence or witnesses, it is hard to prove that, on this walk in the dunes, the topic of formalization and levels was discussed. However, Brouwer was a scrupulous man when it came to historical facts; there is no reason to doubt the 'discussion in the dunes', and it is most plausible that, apart from Lie groups and topology, also foundational matters came up. And

[26] [Courant 1981], p. 161.

[27] The note is not dated, but it is almost certainly written in his later years. It also mentions Russell, Kant and Mannoury.

what is more plausible than that Brouwer turned to the criticism formulated in his dissertation. Hilbert never reacted to Brouwer's statement. This may or may not be significant. One would guess, however, that a public reference to a poor credit policy would not have amused him. Hilbert had a reputation for a certain cavalier attitude with respect to credits. Courant, who belonged to Hilbert's inner circle, lists in his *Reminiscences* a number of instances where Hilbert completely forgot where he picked up what.[28] Being a man of undisputed technical capacities, he could easily work out a topic for himself on information that reached him in conversation. Courant cites F. Schur and Fredholm as two fellow mathematicians who fell victim to Hilbert's absent-mindedness; there were undoubtedly many more, but apparently nobody went as far as Brouwer to mention the matter of intellectual debt in public. Courant was acutely aware of Hilbert's practice, and he observed the influence on his circle of students, 'it did create in Hilbert's students and assistants a feeling of neglect. A certain duty exists, after all, for a scientist to pay attention to others and to give them credit. The Göttingen group was famous for a lack of feeling of responsibility in this respect. We used to call this process—learning something, forgetting where you learned it, then perhaps doing it better yourself, and publishing without quoting correctly—the process of "nostrification". This was a very important concept in Göttingen.'

For the remaining three insights Brouwer also analysed Hilbert's papers, and showed that in each case an acceptance of the insight by the formalists came *after* the intuitionist allegation. It is interesting to see that usually Hilbert, after some rearguard skirmishes, adopted Brouwer's views. It is certainly strange that Hilbert never bothered to take Brouwer's arguments seriously in public.

Number three on the list is a bit surprising, as Hilbert's dogma is a rather marginal matter; one of those slogans that belong rather to popular lectures than to science proper. That Hilbert consistently missed the point is not all that surprising, as one must view the dogma, just as 'PEM' with an intuitionistic eye, something that Hilbert could not, or would not, do.

Number four was, of course, the heart of the matter. Accepting it would mean for Hilbert to declare his programme bankrupt. It took the soft spoken Gödel, and an equally withdrawn Gentzen, to present Hilbert with a *fait accompli*.

After analysing his four insights, Brouwer devoted the second and last section to the consistency of various forms of PEM, but not before answering politely, but firmly, Hilbert's snubs:

> According to what precedes, formalism has received nothing but benefactions from intuitionism and may expect further benefactions. The formalistic school should therefore accord some recognition to intuitionism, instead of polemicizing against it in sneering tones while not even observing proper mention of authorship. Moreover, the formalistic school should ponder the fact that in the framework of formalism *nothing* of mathematics proper has been secured up to now (since, after all, the

[28] [Courant 1981] p. 160.

> meta-mathematical proof of the consistency of the axiom system is lacking, now as before), where intuitionism, on the basis of its constructive definition of spread[29] and the fundamental property it has exhibited for finitary sets[30] has already erected anew several of the theories of mathematics proper in unshakable certainty. If, therefore, the formalistic school, according to its utterance in [Hilbert 1926], p.180, has detected modesty on the part of intuitionism, it should seize the occasion not to lag behind intuitionism with respect to this virtue.

One can see that Hilbert's qualification of the intuitionistic efforts as 'modest', or worse, 'wretched remnants', rankled.

We know that Brouwer was no stranger to emotional outbursts, but he always kept his public reactions under control. The above quotation is an illustration: there is no name-calling, just a summing up of some facts, and a mild admonition.

The paper contained a novelty, namely the strong refutation of the principle of the excluded middle. In fact this was an immediate corollary of his theorem that the continuum is indecomposable. Hence the statement 'every real is either rational or irrational' is contradictory. The indecomposability was proved directly in the Berlin lectures, but it is obviously an immediate corollary of the continuity theorem.

The *Grundlagenstreit* was warming up, yet nobody could foresee that it would soon be over. Brouwer used his Vienna lectures to lay a coherent philosophical picture before the world; two talks without antagonistic elements. Hilbert was preparing his grand lecture for the Bologna conference. A storm was brewing, and when it broke, its rage threatened to sweep away substantial parts of the castle of the German mathematical community.

15.2 The Bologna conference

After the war the mathematicians had twice been called to an international meeting: one in 1920 in Strasbourg, and the next in 1924 in Toronto. The atmosphere in Strasbourg had been one of exhilaration, of victory. The vanquished, not allowed to attend, had met in Bad Nauheim licking their wounds. The Toronto meeting was still out of bounds for the Germans; certainly there had been an attempt to put an end to the boycott, but the motion for readmitting the Germans was not even tabled by the *Union internationale de Mathématique*, it was simply passed on to the *Conseil*.

After the defeat of the Dutch–Danish–Norwegian proposal to drop the exclusion clause from the statutes of the *Conseil*,[31] a substantial part of the mathematical community wished to see that the next international mathematics conference truly deserved that name. The Union had already decided on the place: the international mathematics conference was to be held in 1928 in the venerable University

[29] Cf. p. 313 ff.

[30] Cf. p. 381 ff.

[31] Cf. section 9.1.

of Bologna. And so the various mathematics societies duly advertised the conference. The German mathematics society, DMV, had inserted the flyer announcing the meeting in its journal, the *Jahresbericht der Deutschen Mathematischen Vereinigung*. The DMV traditionally informed their members of relevant events, and it clearly saw no objection to honour the old policy of free information. The more radical adversaries in Germany, however, viewed an announcement of the organization that did not recognize them, in their own periodical, as a case of adding insult to injury. Brouwer, the implacable foe of the *Conseil* and its satellite, the Union, made himself the spokesman of the *Deutschnationale* opposition. Already in January, Brouwer wrote to Bieberbach about possible actions against the Union. Apparently, he had approached Bieberbach, as the editor in chief of the *Jahresbericht*. Bieberbach, in his reply of January 20, informed him that as the managing editor of the *Jahresbericht*, he could not possibly publish a comment of Brouwer on the invitation of the Union. 'It would create a novum if we wished to accept political arguments in the *Jahresbericht*, thus we have avoided, because of the political side of the matter, to speak of the planned conference in Bologna.' Brouwer, never to be caught out, agreed that his comments on the *Conseil* were only in form, but not in content, more political than the invitation for the Bologna conference.[32] His account, he said, would expose the hidden content of this invitation. Anyway, if the *Jahresbericht* would not publish his note, an enclosed leaflet would do. And indeed, a leaflet was produced, in which Brouwer reminded the readers of the motivation and formulations of the advocates of the boycott of German scientists.[33] He quoted the statements made by Painlevé in the *Académie des Sciences*.[34] These statements belonged to the emotional atmosphere of the war and its aftermath, and as such they were understandable, but they were not well-suited to promote peace and cooperation in the scientific world. Indeed, they were nothing less than a wholesale insult and condemnation of all German scholars. Painlevé's conciliatory moves in the affair of the Riemann volume of the *Annalen*, and his general role in the scientific community, made it clear that he no longer was the rabid anti-German of 1918.[35] But a public retraction of everything said and done in the early years after the war, was probably asked too much. Nonetheless this was what the German nationalists among the mathematicians insisted upon.

Brouwer, in his leaflet, left it at a—none too subtle—suggestion: '... the readers of the *Jahresbericht* may contemplate, in how participation in the planned congress is possible, without mocking the memory of Gausz and Riemann, the humanitarian character of the science of mathematics, and the independence of the human spirit.'

Not content with a passive resignation, Brouwer, possibly after consultation with like-minded Germans, decided to visit the Italian organizers in person, in the hope that the role of the Union could be reduced to naught. The physicist

[32] Brouwer to Bieberbach 23.I.1928.

[33] Dated March 1923, and addressed to the members of the DMV.

[34] Cf. p. 338.

[35] Cf. [Schroeder-Gudehus 1966] p. 248,250.

Sommerfeld wrote Brouwer in March that the new invitation for the Bologna congress did not mention the name of the *Conseil* or similar organizations. He expected that Brouwer would find an open ear in Italy.[36] Sometime late March or in April Brouwer visited Pincherle, the president of the organizing committee, and Levi-Civita. The discussions were friendly, and for both sides most satisfactory. From the side of the organizers, it was agreed that the conference would not be a Union conference, but an international congress under the aegis of the University of Bologna. Brouwer could thus return home with good news: the Union was sidetracked.

In the German mathematical community the neutral mathematicians and the anti-*Conseil* group had strong feelings about the conference. The neutrals advocated participation in Bologna, and the anti's were, in spite of the Italian initiative, still wary. At the annual meeting in Bad Kissingen the DMV had decided not to send representatives to the Bologna conference, but it had at the same time sent out the parole to its members, to attend in as large numbers as possible.[37] The Göttingers largely supported the Bologna conference, and the Berlin group contained some outspoken anti's. It was no secret that a certain amount of friction existed between the Berlin group and the Göttingen group. Although there was no lack of cooperation between the mathematicians of both groups (and it should be kept in mind that Berlin and Göttingen were icons for many mathematicians all over Germany), there was a certain tendency to accuse the Göttingen group of smugness. Vice versa there was little feeling of antagonism; a Göttingen mathematician was usually well aware of the true or imagined superiority over sister universities.

Pincherle, in his role of president of the congress, had to face one unpleasant duty: to inform Picard, the president of the *Conseil* and the Union of the decisions of the Bologna committee. And he proved himself up to the occasion. On June 8 he sent a long letter to Picard—'Monsieur et illustre Maître!' He respectfully, but firmly, explained that one could no longer stick to the format of the Strasbourg and Toronto conferences. If one wishes, he wrote, to re-establish between the scholars who practice the most pure of all sciences, the good relations which are so necessary for their progress, one should leave formal considerations behind. The organizers felt that they had to shape the congress, in such a way that this meeting of all mathematicians would be possible. And so he informed Picard of the decision to make the congress an Italian matter, independent of the Union. Of course, he added, the Union will still have its business meeting during the conference, where they could fix their policy for the future. He hoped, nonetheless, that the great Emil Picard would be there to open the lecture series of the conference.

Picard, confronted with a *fait accompli*, replied that his quality of president of the *Conseil* did not permit him to attend the congress.

[36] Sommerfeld to Brouwer 24.III.1928.

[37] Perron to Bieberbach 6.V.1928. Perron inquired if Bieberbach would go to Bologna and give a talk.

In view of the pleasing initiative of Pincherle, one would guess that the problem, at least in Germany, had been solved to everyone's satisfaction. Perhaps a number of French mathematicians would feel hurt, but the international character of the congress was at least saved. Yet, in Germany new problems and discussions arose. Bieberbach, in reply to a circular letter of Geheimrat Ziehen, the rector of the University at Halle, summed up the touchy issues that still rankled.[38] He pointed out that the information on the Bologna congress was far from consistent. Indeed, there must have been some mix-up affecting the invitations. Some mentioned the involvement of the Union, others did not; by way of illustration—Brouwer obtained a personal, albeit printed, invitation from Peano on July 8, in the usual flattering wording, full of misprints, and mentioning explicitly on the third line '*Sous les auspices de l'Union Mathématique Internationale*.'

Bieberbach's list of grievances contained more: the French invitation listed the Bologna congress as the eighth international mathematics congress, thus elevating the Strasbourg and Toronto congresses, from which the Germans were excluded, to a status of internationality they did, in his eyes, not deserve. As far as he knew the effects from a certain neutral side, had resulted in the fobbing off in the form that in the latest circular the Union was no longer explicitly mentioned. 'A diplomatic feat, by which the real state of things was even more underlined.'

Furthermore, he took exception to the tactless plans for an excursion to the power station at the Ledrosse in South Tirol, which had changed hands after Versailles.

The call of the rector of the University at Halle to the universities and academies, to send representatives to Bologna, amazed Bieberbach. He could not see a justification in customs of the past. The Prussian Academy had left the decision to attend congresses of the *Conseil* to the individual scientists, but advised a large measure of restraint. 'Only if this congress, compromised beyond rescue, essentially suffers damage through the lack of German participation, we can expect with certainty that the next international mathematics congress will be really international, without involvement of the Union, either that this totally superfluous and ineffective organization cleanses itself by extricating itself from the Conseil, or by a complete reorganization satisfying the German wishes.' Here was a concrete goal; whereas Brouwer vented his moral objections against the *Conseil* and the Union, Bieberbach saw a solution to the present discontent in mathematical circles, and not only in Germany. The breaking up of the Union and the *Conseil* could start in Bologna, he thought. Ziehen must have informed the leading mathematicians of Bieberbach's letter, an unfortunate and rash action. For the letter was a personal reaction, and Bieberbach had not foreseen that Ziehen would circulate it.[39] Had he known the use Ziehen was going to make of his letter, he would probably have been more guarded in formulation and content. Bieberbach had not expected any such

[38] Bieberbach to Ziehen 18.VI.1928.

[39] Cf. Bieberbach to Courant 15.VII.1928.

action from Ziehen, as the *Rektorenkonferenz* and the *Hochschulverband* had agreed that all foreign affairs would be handled by the latter.

The Bieberbach–Ziehen exchange was not entirely ignored: there was at least one sharp and aggressive reaction, it came from the grand-master of German mathematics. Hilbert wrote an angry letter to the German rectors and directors of mathematical seminars, resolutely repudiating Bieberbach's views.[40] 'An die Herren Rektoren der deutschen Hochschulen und die Leiter der mathematischen Seminare. *Betrifft den Internationalen Mathematiker-Kongress in Bologna*.'

Bieberbach, he said, was ill informed, and citing rumours. The congress had officially severed its ties with the Union; agitating against the Italian organization and boycotting the congress would alienate friendly colleagues and organizations, and could only be seen as a slap in the face of the Italians.

The letter ended with an explicit call to the German colleagues, 'It would be in the interest of German science and German prestige to wish most urgently, that no university and no academy will desist from accepting the official invitation to participate in a friendly manner.'

The letter clearly outlined Hilbert's political position; he did not speak about the past insults, but stuck to the possibility offered by the Italian organization. At the same time he put Bieberbach in his place, and this caused—as to be expected—some hard feelings.

For many German mathematicians Hilbert's action was the ardently awaited signal. Thus Perron wrote to Landau 'Finally the circular of Hilbert, for which I had been waiting passionately, has arrived.'[41]

Hilbert, in a manner of speaking, had viewed the discussion and the arguments concerning the Bologna congress from the height of his Olympus, blurring the finer details. He was probably quite right to do so, because an overly discerning gaze is no asset of one wants to reach a compromise. But it should be noted that in this way he did not do justice to Bieberbach's observations. Small wonder that Bieberbach replied publicly to Hilbert's attack. Since his '*hochverehrter Lehrer*, *Herr Geheimrat Hilbert*' had severely reprimanded him, he felt obliged to defend himself. The letter is a clever balancing act, between not insulting Hilbert and sticking to his guns. On the whole, one must say that, viewed as an academic exercise Bieberbach performed most credibly. E.g. in countering Hilbert's argument, that Picard had felt offended after Pincherle's letter, he remarked that this was beside the point, 'for our aim is only the satisfaction of our own value, not the degree of irritation of Mr. Picard.' He deplored that Hilbert, by using the word 'denunciated', had introduced a false note into the discussion.[42] His veneration for his teacher and the level of courtesy in a discussion would forbid him to follow suit.

[40] Hilbert to rectors of German universities 29.VI.1928.

[41] Perron to Landau 2.VII.1928.

[42] ... *Dieses Schreiben nimmt in schroffster Weise Stellung gegen den diesen Herbst in Bologna stattfindenden Internationalen Mathematiker-Kongress, denunziert den Kongress als eine Veranstaltung des conseil internationale de recherches und rät daher dringend an zu einer Ablehnung der an die Universitäten usw. ergangener Einladung.*

There must have been an enormous amount of correspondence concerning the Bologna congress; most of it will have found its way to the wastepaper basket, but some of the letters of the influential mathematicians have been preserved. There are basically two kinds of views, one, like Hilbert's, shows itself satisfied with the Italian results, another one, like that of Bieberbach and Brouwer, considers the concessions promising but not conclusive. Von Mises, for example, in his letter to Courant,[43] agrees with the latter that 'we should have nothing to do with the Union.' But he disagreed with Courant (and the Göttingen mathematicians)[44] about the real content of the Italian communications. The role of the Union is left completely vague; there is no evidence that the Union is put out of action. For one thing, Pincherle, who became president of the organizing committee in his function of chairman of the Union, should have abdicated in the latter function.

From von Mises' letter we learn that Brouwer had not aimed in his discussion with Pincherle, at a complete separation between congress and Union, but rather at a more modest goal: the appointment of an independent international committee that would after the conference decide on the future conferences, so that there would be a guarantee that these would no longer be congresses of the Union. That much was promised to him. The main thing was, he argued, to prevent that the Union, 'which at this moment is fighting its final battle', could present the Bologna conference as 'sufficiently international', for then 'the Union, this pronounced *political organisation*, would still exist in four years time, and all the present conflicts would have to be repeated.' He concluded that the only definite solution could be reached through a wholesale ignoring of the conference by the German mathematicians.

By now, in the early days of July all positions had been taken up. It was unlikely that any German mathematician would change his views. The remaining correspondence can be viewed as damage control. Courant, who had become Hilbert's main lieutenant, tried to pacify Bieberbach in a letter of July 10. Hilbert had acted in such a radical manner, as he had promised to give a talk in Bologna, and believed that the political issues had been straightened out. To his surprise, the letter from Halle, followed by Bieberbach's letter, had given substance to the idea that going to Bologna was un-German. Hence his sharp reaction, which was in no way intended personally. As Courant put it, 'Hilbert did not want to react once more to your reply, he had, however, expressly assured me how little from his side a personal insult was intended.' One might wonder why Hilbert did not send a short note to that effect to Bieberbach, but one has to keep in mind that he still suffered from his pernicious anaemia. He probably tried to avoid all action that was not strictly necessary. Of course, there is also the possibility that the king of mathematics does not apologize to a former student, who happened to be a Berlin professor.

[43] Von Mises to Courant 7.VII.1928.

[44] He referred tongue in cheek to the Hilbert–Bieberbach discussion, noting that 'it is a good illustration of your remark that inside the mathematics society no controversies exist.'

Courant went on to explain that Hilbert had been seriously annoyed by the Riemann affair: 'Hilbert had felt deeply insulted, when as a consequence of Brouwer's exhortation and, as Hilbert thinks, with your cooperation the collaboration, desired by Hilbert, of French mathematicians to the Riemann volume, was blocked.' Courant's, no doubt kindly intended, attempt at reconciliation, met with doubt and scorn. Bieberbach replied five days later that he assumed that Courant did not agree with the use of the term 'denunciate' to describe his action. 'I presume that, this is solely on Hilbert's conto.'[45] Courant was completely wrong, Bieberbach added, in his view of Brouwer's role in the Riemann affair. Bieberbach's letter shows some of the discontent one would find in Germany over the high-handed actions of Hilbert.

> As far as this matter of the Riemann-volume is concerned, there was, as you know, a discussion about it in Innsbruck.[46] And I have made use of my right, to ventilate my opinion. I have never committed myself to voice only opinions that pleased Hilbert; and I would find it foolish if such a demand would seriously be made to me. Also in this matter I have not unfolded any agitation, as I have better things to do. I have only when my opinion was asked, stuck to my point of view.

Bieberbach also felt that Hilbert had not done his homework, but nonetheless accused Bieberbach of 'using secondary information.' Here Bieberbach had a point: right from the beginning he had been involved in the discussions concerning the Bologna congress. He was well-informed about all the negotiations. Thus it is not surprising that he was upset, if not offended by the amateurish diplomacy of the Göttingers, 'Rather one could reproach the Hilbert clique, that they attach decisive importance (*Bedeutung*) to ad hoc formulations, e.g. like the letter of Bohr, without taking into account the previous history and the laborious correspondence over many years.'

Bieberbach's letter also provides some useful information on Brouwer's role. From the point of view of Göttingen it seemed as if there was a huge conspiracy with Bieberbach, Brouwer and Von Mises as the main characters. But actually, Brouwer's actions in Italy were his own; Bieberbach only learned about them from Von Mises. The only significant contact about the Bologna matter concerned Brouwer's letter to the editor of the *Jahresbericht* (see p. 588). As to Brouwer's involvement in the Bologna affair, Courant had confirmed that Hilbert's thoughts went beyond just suspicions: 'The idea that Brouwer's disposition was actually normative for the colleagues in Berlin, was moreover confirmed by reports of Von Mises; and I can only say that Hilbert rejected most passionately this interference and this playing the judge.'

In his reply Bieberbach soberly asked 'Why, by the way, is Brouwer's interest in the matter an "outside interference", but in contrast, that of Bohr and Hardy not?'

[45] Bieberbach to Courant 15.VII.1928.

[46] Apparently a meeting of the editors of the M.A. editors at the time of the annual meeting of the DMV.

Ludwig Bieberbach (Courtesy U. Bieberbach)

As Bieberbach wrote, Hilbert had already at the time of the Bad Kissingen meeting, thus before the conflict arose, put himself forward as a lecturer at the conference. This certainly was an extremely visible sign from the German side of the wish for appeasement, in particular since it came from the greatest mathematician, at a time when the congress was still a full-blooded Union congress. He, Bieberbach, had acquiesced, as one might assume that few Germans would attend. Hilbert's role as a symbol of German good will was part of Bieberbach's master plan. He had not dreamed that 'Hilbert was essentially so little independent, that he demanded a general participation, and considered the alternative as an insult for his person.' In short, Bieberbach was not subdued by the carrot-and-stick letter of Courant.

In another letter, to the applied mathematician Walther, Bieberbach again analysed the key statements concerning the separation of Union and congress, pointing out the various readings and glaring inconsistencies (e.g. in June Bortolotti had told Bieberbach that Picard's objection to the procedure was that the invitations for the conference were not offered for inspection to the *Conseil*, and that the rector of the University of Bologna had sent them out). He had seen too many diplomatic formulations, not to be sceptical. 'But', he wrote, 'mundus vult decipi and who wants to go to Bologna, and thinks that he has to let his glory shine in the beams of the international sun, to him such formulations are *Butter aufs Brot*.' The correspondence between Courant and Bieberbach went on for another

round. Since only Bieberbach's letter has been preserved, one has to guess the content of Courant's letter. Bieberbach in his vehement protest against Courant's claims, maintained that Brouwer's disposition had not been of decisive influence on him. It had not exercised any influence at all, he wrote. Hilbert's conciliatory move towards Bieberbach must again have been brought up by Courant. But Bieberbach remained adamant. Hilbert had publicly insulted Bieberbach—even after Bieberbach's reaction of July 3, Hilbert's circular had been distributed further; for example to the Prussian Academy. Instead of authorizing Courant to make friendly noises, he should have stated his apology before the same Forum that had witnessed the insult. But, he went on, 'in view of the circumstance, that Hilbert is substantially older than I am, and that he is my teacher, I will be satisfied if you will get yourself an authorization from Hilbert, to state to me that he regrets and retracts the expression 'denunciate.'

Since the negotiations concerning the Bologna congress were now all water under the bridge, he advocated a composed conduct of the German delegation in Bologna: 'It is more important to prevent that in Bologna the Germans present there, sneer altogether too much at those who stayed away to paint a picture of their own excellence at the expense of German prestige and unity. Perhaps you should just think about it if you could not contemplate this idea. So far, everything that comes from Göttingen has a ring of such disdain for dissenting Germans, that I am really worried how it will be in Bologna, if this mood comes to an eruption, just as it is said to have burst out in Göttingen before foreigners.'

Brouwer's role had become marginal; after his negotiations in Rome and Bologna, he carried on his correspondence, but no longer played a role. He once more sent out his leaflet with the unfortunate statements of Painlevé (August).

The unity of the foreign advocates of ending the boycott of Germans had begun to crumble. Harald Bohr had, after an exchange of letters with Pincherle, come to the conclusion that the Italians were offering a really international meeting where all nationalities participated with equal rights. He saw the Union as definitely defeated—'we, the internationally minded, (i.e. people like you, Hardy, etc.) have in fact won so completely, that from my point of view it would be neither natural, nor advantageous, if now the congress would be blown up by the opponents of the Union. The relatively minor questions and formalities should not create a rift between us.'[47]

Hardy also wrote to Brouwer about the conference boycott; he had, like Brouwer, been a staunch enemy of the *Conseil*—'I detest it, and have never had anything to do with it.' But—contrary to Brouwer—he hoped 'that the Germans would on the whole, adopt the policy of being more magnanimous than they could reasonably be expected to be.' In short, he shared Bohr's view. Having fought, with Brouwer, Bohr and others, the *Conseil* tooth and nail, he could not share Brouwer's adamant views, as appears from the closing hints of his letter: 'Finally, if you are to demand that everybody should formally retract all the imbecilities which has

[47] Bohr to Brouwer 3.VII.1928.

uttered during the war, then assuredly there will never be any Congress of any kind until everybody born before 1914 is dead. And I should hope that, whatever happens at Bologna, it will at any rate be enough to make everybody realize that the era of imbecility is passed.'

So this was the end of the road; a small group of diehards refused to go to Bologna, so close to Canossa, as Bieberbach had remarked. A sizeable contingent of the German mathematical community did accept the assurances of the organizers, and participated. There was also a number of mathematicians that did not go, but for other than political reasons (e.g. Hardy and Menger).

The conference became a huge success. The mathematician Härlen reported to Brouwer that Pincherle had opened the congress by declaring that the Italians were the only organizers of the congress, and on many occasions it was proclaimed 'that this was the first truly international congress after the war.'[48]

Three weeks later Härlen sent a more complete report to Brouwer. The German aspects were quite satisfactorily reckoned with, he wrote, there were for instance a large number of German posters instructing the participants, more than English or French ones. The congress organization clearly had done its best not to offend the Germans. In particular there were quite a number of apologetic remarks about the recent past. Pincherle was elected with general approval as president of the meeting. The choice of the vice presidents was judicious: Hadamard, Hilbert ('exceptionally strongly applauded, very striking'), Fehr (Switzerland), Young (UK), Birkhoff (USA), Bohr (Scandinavia), Rey Pastor (Spain & South America), Sierpinski (Poland), and Lusin (USSR).

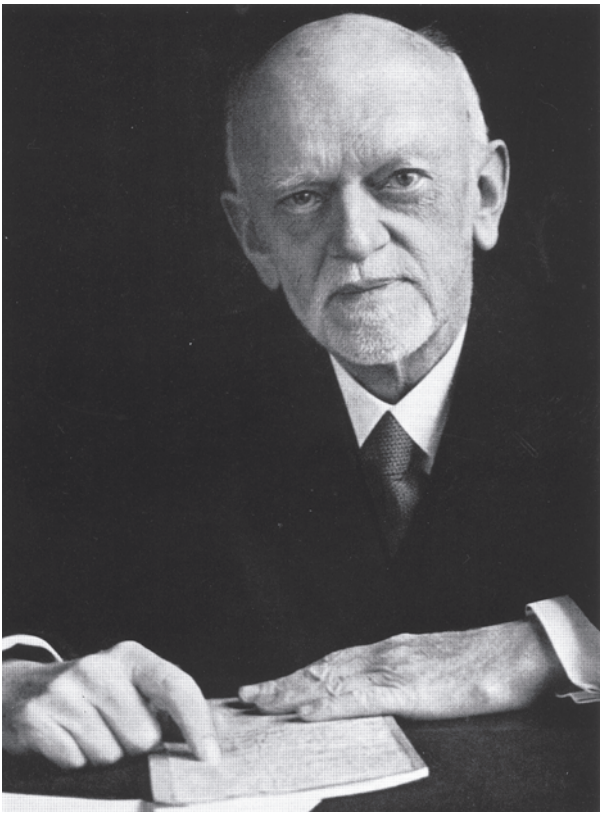
The series of lectures was opened by Hilbert. Constance Reid described the occasion in her biography of Hilbert: 'At the opening session, as the Germans came into an international meeting, for the first time since the war, the delegates saw a familiar figure, more frail than they remembered, marching at their head. For a few minutes there was not a sound in the hall. Then spontaneously, every person present rose and applauded.'[49] Härlen also described this event:

> ... the first lecture of Hilbert, who was greeted with a storm of applause. Numerous repetitions; power of concentration clearly much hampered by physical suffering. Content familiar by latest publications. Strong applause—Also a strong applause greeted Hadamard, whose lecture was also very good in presentation much more effective than Hilbert's. At Hadamard's lecture the applause much stronger than before. In Hilbert's case the applause was meant for the person, in Hadamard's case also for the lecture.

It should probably not surprise us that the Italian government made the most of this meeting. At the opening the Podesta greeted the participants in the name of 'the fascist city, that was happy to show the foreign visitors the accomplishment of the

[48] Härlen to Brouwer 6.IX.1928.

[49] [Reid 1970], p. 188.



David Hilbert (Courtesy Springer Verlag)

fascists.' The participants were given tags with bands in the Italian colours and at breakfast there were little Italian flags, for the participants to wear. This unnecessary and somewhat childish nationalism did not exactly please all participants.

The Union was active after all, it distributed flyers with invitations to join its business meeting. At this meeting Pincherle announced that Switzerland had offered to host the next congress. In his closing address the Union did not figure at all. In preceding private sessions invitations from Prague and the Netherlands had reached the meeting. Switzerland had declined, in spite of pressure from all sides. The Dutch professor Schouten had telegraphed after the meeting to the Dutch government, and he had obtained a positive reaction. The next day in informal discussions objections against the Netherlands were raised on the grounds of Brouwer's actions. After some more pressure Switzerland came back on its refusal, and the Netherlands and Prague withdrew their offers.

After the congress there were some retrospective discussions, but on the whole the hopes were high that from now on the boycott would be a thing of the past.

Brouwer may not have been successful in discouraging the Germans to participate, but he could well be satisfied with the final outcome. If one tries to give a reason for the uncompromising attitude of the anti-Unionists, one should look at the past history of the *Conseil* and the Union. All attempts to drop the exclusion clauses had been rudely blocked by Picard and his followers. In certain quarters there was absolutely no faith in the will or capability to accept the enemy of the past as the colleague of today, hence the uncompromising stance of people like Bieberbach, Brouwer and Von Mises. On the other side there was a more optimistically minded group, led by the Göttingers, who believed that a compromise at the Bologna meeting would pave the way for a restoration of the old free cooperation. History has proved the Göttingers right, but in 1928 the future still offered two options.

Looking back at Brouwer's role in the Bologna affair, one is almost surprised by his innocence. For him it was a fight between good and evil, between a closed shop of *Conseil*-connected scholars and institutions, and the free world of science. His attacks were conspicuous for the quotations of his adversaries, not for his own strong language. The most effective action he undertook was his personal visit to Rome and Bologna, and the negotiations with his Italian colleagues. It probably was the turning point in the action for detaching the congress from the Union. His pamphlet might be considered a clever attempt to keep the German mathematicians away from Bologna, but it is doubtful if it changed the view of those who had not made up their minds to stay home. For a successful action of that sort, one needs an organization and strongly motivated activist-supporters. None of this was within Brouwer's province. He had always been a loner, and even the sympathy of a number of German colleagues was not more than the support of kind well-wishers. Against a network like that of the Göttingen mathematicians, an unworldly Amsterdam professor stood no chance. Brouwer, like the legendary Roland, was famous for his lost battles.

Brouwer's actions and pamphlets were personal in the sense that he referred to certain mathematicians in the higher ranks of the *Conseil*, but no attacks at German colleagues were launched. Hilbert on the other hand personally attacked Bieberbach, and there is a private note of his, probably of this period, in which he poured his wrath over Brouwer:

> *Erpressertum.*
>
> In Germany a political blackmail of the worst kind has come up. You are not a German, not worthy a German birth, if you don't speak and act as I tell you now. It is very easy to get rid of these blackmailers. One has but to ask, how long they have been in the German trenches. Unfortunately German mathematicians have fallen victim of this blackmail, for example Bieberbach. Brouwer has known to make use of this state of the Germans, and without having been active himself in the German trenches, all the more to work towards inciting and to cause discord among the Germans, in order to pose as the master of German mathematics. With complete success. He will not succeed in this for the second time.

One should of course keep in mind that Hilbert's health was very poor at the time, and this may well have influenced his judgement and emotional balance. A far more fateful manifestation of Hilbert's illness would in the aftermath of the Bologna congress bring German mathematics to the brink of 'civil war'.

Who will rid me of this meddlesome priest
Henry II

15.3 The war of the frogs and the mice

The dreaded and perilous pernicious anaemia had not left Hilbert alone. After a modest improvement in January 1928, Hilbert was plagued again by its various symptoms. In this condition he had a heart attack in the weeks before he set off for Bologna.[50] So when he appeared in Bologna, he was physically in a poor state. His triumphant reception, and the success of his lecture, must have given him the strength to make it through the congress, but after the congress he completely broke down. He was taken to a sanatorium in Luzern where he spent a full five weeks. His situation was for a longer period so desperate, that he saw himself at death's doorstep. It was this unfortunate and sad combination of events that lay at the root of the following tragic history of a conflict in the German mathematical world.

On 27 October 1928, a curious telegram was delivered to Brouwer, a telegram that was to plunge him into a conflict that for some months threatened to split the German mathematical community. This telegram set into motion a train of events that was to lead to the end of Brouwer's involvement in the affairs of German mathematicians and indirectly to the conclusion of the *Grundlagenstreit*. The telegram was dispatched in Berlin, and it read:

> Professor Brouwer, Laren N.H. Please do not undertake anything before you have talked to Carathéodory who must inform you of an unknown fact of the greatest consequence. The matter is totally different from what you might believe on the grounds of the letters received. Carathéodory is coming to Amsterdam on Monday.
>
> Erhard Schmidt

Following Schmidt's instructions, Brouwer, puzzled as he was, went about his daily routines. Two registered letters that had arrived at the post office added to the mystery. Brouwer collected these letters from Göttingen and waited for the arrival of Constantin Carathéodory. The letters were still unopened when Carathéodory arrived in Laren on 13 October. Carathéodory's visit figures prominently in the history that is to follow.

In order to appreciate the full dramatic magnitude of the following history, one must keep in mind that Brouwer was on friendly terms with all the actors in this

[50] Courant to Springer, 3.X.1928.

small drama, with the possible exception of David Hilbert. Some of them were even intimate friends of his, for example Carathéodory and Otto Blumenthal.[51]

Carathéodory found himself in the embarrassing position of being the messenger of offensive news, in which he was involved against his will. The first letter, he explained to Brouwer, should have carried more signatures, or at least Blumenthal's signature. Carathéodory's name was used in a manner not in accordance with the facts, although he would not disown the letter should Brouwer open it. Finally, the sender of the letter would probably seriously regret his action within a couple of weeks. The second letter was written by Carathéodory himself, although Blumenthal's name was on the envelope. He, Carathéodory, regretted the contents of the letter.

Thereupon Brouwer handed the second letter over to Carathéodory, who proceeded to relate the theme of the letters. The contents of the second can only be guessed, but the first letter can be quoted verbatim. It was written by Hilbert, and copies were sent to the other dramatis personae in the tragedy that was to follow.

Constantin Carathéodory (Courtesy M. Geogiadou)

[51] A wealth of information on Carathéodory can be found in the recent biography [Georgiadou 2004].

Hilbert's letter was brief:

> Dear Colleague,
> Because it is not possible for me to cooperate with you, given the incompatibility of our views on fundamental matters, I have asked the members of the board of managing editors of the Mathematische Annalen for the authorization, which was given to me by Blumenthal and Carathéodory, to inform you that henceforth we will forgo your cooperation in the editing of the Annalen and thus delete your name from the title page. And at the same time I thank you in the name of the editors of the Annalen for your past activities in the interest of our journal.
>
> Respectfully yours,
> D. Hilbert.

The meeting of the two old friends was painful and stormy; it broke up in confusion. Carathéodory left in despondency and Brouwer was dealt one of the roughest blows of his career.

Although Brouwer had kept himself completely under control during the visit, he suffered under a strong reaction. After the visit he was ill for a few days and ran a temperature.[52]

The *Mathematische Annalen* was the most prestigious mathematics journal at that time. It was founded in 1868 by A. Clebsch and C. Neumann. In 1920 it was taken over from the first publisher, Teubner, by Springer.

For a long period the names of Felix Klein and the *Mathematische Annalen* were inseparable. The authority of the journal was largely, if not exclusively, based on the mathematical fame and the management abilities of Klein. The success of Klein in building up the reputation of the *Annalen* was to no small degree the result of his choice of editors. The journal was run, on Klein's instigation, on a rather unusual basis; the editors formed a small exclusive society with a remarkably democratic practice. The board of editors met regularly to discuss the affairs of the journal and to talk mathematics. Klein did not use his immense status to give orders, but the editors implicitly recognized his authority.

Being an editor of the *Mathematische Annalen* was considered a token of recognition and an honour. Through the close connection of Klein—and after his resignation, of Hilbert—with the *Annalen*, the journal was considered, sometimes fondly, sometimes less than fondly, to be 'owned' by the Göttingen mathematicians.

Brouwer's association with the *Annalen* went back to 1915 and before, and was based on his expertise in geometry and topology. In 1915 his name appeared under the heading 'With the cooperation of'.(*Under Mitwirkung der Herren*) Brouwer was an active editor indeed; he spent a great deal of time refereeing papers in a most meticulous way.

The status of the editorial board, in the sense of by-laws, was vague. The front page of the *Annalen* listed two groups of editors, one under the head *Unter Mitwirkung von* (with the cooperation of) and one under the head *Gegenwärtig*

[52]Gawehn to Von Mises 9.XI.1928.

herausgegeben von (at present published by). I will refer to the members of those groups as associate editors and chief editors. The contract (25 February 1920) that was concluded between the publisher, Springer, and the Herausgeber Felix Klein, David Hilbert, Albert Einstein, and Otto Blumenthal speaks of *Redakteure*, but does not specify any details except that Blumenthal is designated as managing editor.

The loose formulation of the contract would prove to be a stumbling block in settling the conflict that was triggered by Hilbert's letter. At the time of Hilbert's letter the journal was published by David Hilbert, Albert Einstein, Otto Blumenthal and Constantin Carathéodory, with the cooperation of L. Bieberbach, H. Bohr, L.E.J. Brouwer, R. Courant, W.v. Dyck, O. Hölder, T. von Kármán, and A. Sommerfeld. The daily affairs of the *Annalen* were managed by Blumenthal, but the chief authority, undeniably, was Hilbert.

Given the status of the *Annalen*, any mathematician would be more than happy to join the editorial board, and once on the board, expected to remain there until a ripe old age. Two of the most prominent editors, Klein and Hilbert served respectivly. from 1876 till 1924, and from 1901 till 1939. So Klein retired at 75, one year before his death, and Hilbert at 77, four years before he died. Under the circumstances, Brouwer, who was an active associate editor, could expect eventually to become an editor, and to remain so for years to come. Hilbert dashed these expectations with one stroke of the pen. Even if the procedure remained a secret, which was to be doubted, Brouwer would be the laughing stock of the mathematical world. The readers of the *Mathematische Annalen* would note that Brouwer's name had disappeared from the list, and they would draw their own conclusion. Had Hilbert contemplated these consequences, when he signed the dismissal in an emotional impulse? And did he realize the enormity of the insult he was committing?

It is hard to imagine what Hilbert had expected; he could not have counted on a calm resigned acquiescence from the high-strung, emotional Brouwer. In Brouwer's eyes (and quite a few colleagues would have taken the same view) a dismissal from the *Annalen* board was a gross insult.

Carathéodory, in trying to win Brouwer's acquiescence, had apparently revealed only part of the underlying motive, as appears from Brouwer's letter of November 2 to Blumenthal,

> Furthermore Carathéodory informed me that the *Hauptredaktion* of the *Mathematische Annalen* intended (and felt legally in the position) to remove me from the editorial board of the Annalen. And only for the reason that Hilbert wished to remove me, and that the state of his health required to give in to him. Carathéodory begged me, out of compassion with Hilbert, who was in such a state that one could not hold him responsible for his behaviour, to accept this shocking injury in resignation and without resistance.

Hilbert himself made no secret of his motivation; in a letter of 15 October he asked Einstein for his permission (as a *Mitherausgeber*) to send a letter of dismissal (the draft to the chief editors did not contain any explanation) and added

> Just to forestall misunderstandings and further ado, which are totally superfluous under the present circumstances, I would like to point out that my decision–to belong under no circumstances to the same board of editors as Brouwer–is firm and unalterable. To explain my request I would like to put forward, briefly, the following:
>
> 1. Brouwer has, in particular by means of his final circular letter to German mathematicians before Bologna, insulted me and, as I believe, the majority of German mathematicians.
>
> 2. In particular because of his strikingly hostile position vis-a-vis sympathetic foreign mathematicians, he is, in particular in the present time, unsuitable to participate in the editing of the Mathematische Annalen.
>
> 3. I would like to keep, in the spirit of the founders of the Mathematische Annalen, Göttingen as the chief base of the Mathematische Annalen—Klein, who earlier than any of us realized the overall detrimental activity of Brouwer, would also agree with me.[53]

In a postscript he added: 'I myself have for three years been afflicted by a grave illness (pernicious anaemia); even though this disease has been taken its deadly sting by an American invention, I have been suffering badly from its symptoms.'

Clearly, Hilbert's position was that the chief editors could appoint or dismiss the associate editors. As such he needed the approval of Blumenthal, Carathéodory and Einstein. Blumenthal had complied with Hilbert's wishes, but for Carathéodory, the consent was problematic; apparently he did not wish to upset Hilbert by contradicting him, but neither did he want to authorize him to dismiss Brouwer. Hilbert may easily have mistaken Carathéodory's evasive attitude for an implicit approval. Carathéodory had landed in a awkward conflict between loyalty and fairness. He obviously tried hard to reach a compromise. In view of Hilbert's firmly fixed conviction, he accepted the unavoidable conclusion that Brouwer had to go; but at least Brouwer should go with honour.

Being caught in the middle, Carathéodory sought Einstein's advice. In a letter of 16 October he wrote 'It is my opinion that a letter, as conceived by Hilbert, cannot possibly be sent off.' He proposed, instead, to send a letter to Brouwer, explaining the situation and suggesting that Brouwer should voluntarily hand in his resignation. Thus a conflict would be avoided and one could do Brouwer's work justice: 'Brouwer is one of the foremost mathematicians of our time and of all the editors he has done most for the *Mathematische Annalen*.' The second letter we mentioned above, must have been the realization of Carathéodory's plan. Einstein answered, 'It would be best to ignore this Brouwer affair. I would not have thought that Hilbert was capable of such emotional outbursts.'[54] The managing editor, Blumenthal, must have experienced an even greater conflict of loyalties, being a close personal friend of Brouwer and the first PhD student (1898) of Hilbert, whom

[53] Carathéodory had also received a copy, and Blumenthal probably as well.

[54] Einstein to Carathéodory 19.X.1928.

he revered. Einstein did not give in to Hilbert's request. In his answer to Hilbert (19 October 1928) he wrote:[55]

> I consider him [Brouwer], with all due respect for his mind, a psychopath and it is my opinion that it is neither objectively justified nor appropriate to undertake anything against him. I would say: 'Sire, give him the liberty of a jester (*Narrenfreiheit*)!' If you cannot bring yourself to this, because his behaviour gets too much on your nerves, for God's sake do what you have to do. I, myself, cannot sign, for the above reasons such a letter.

Carathéodory, however did not possess Einstein's strength to cut the knot, once the moral issue was decided. He was seriously troubled and could not let the matter rest. He again turned to Einstein:[56]

> … your opinion would be the most sensible, if the situation would not be so hopelessly muddled. The fight over Bologna … seems to me a pretext for Hilbert's action. The true grounds are deeper—in part they go back for almost ten years.[57] Hilbert is of the opinion that after his death Brouwer will constitute a danger for the continued existence of the *Mathematische Annalen*. The worst thing is that while Hilbert imagines that he does not have much longer to live (…) he concentrates all his energy on this one matter, (…). This stubbornness, which is connected with his illness, is confronted by Brouwer's unpredictability … If Hilbert were in good health, one could find ways and means, but what should one do if one knows that every excitement is harmful and dangerous? Until now I got along very well with Brouwer; the picture you sketch of him seems me a bit distorted, but it would lead too far to discuss this here.

This letter made Einstein, who in all public matters practised a high standard of moral behaviour, realize that these were deep waters indeed:[58]

> I thought it was a matter of mutual quirk, not a planned action. Now I fear to become an accomplice to a proceeding that I cannot approve of, nor justify, because my name—by the way, totally unjustifiably—has found its way to the title page of the *Annalen* … My opinion, that Brouwer has a weakness, which is wholly reminiscent of the *Prozessbauern*,[59] is based on many isolated incidents. For the rest I not only respect him as a man with an extra-ordinarily sharp eye, but also as an honest man, and a man of character.

[55] Einstein to Hilbert 19.X.1928.

[56] Carathéodory to Einstein 20.X.1928

[57] An enigmatic remark. It is far from clear what Cara had in mind. The only possible point of friction around 1918 could have been the offer of a chair in Göttingen to Brouwer, who subsequently turned it down. But is hard to see how that could have bothered Hilbert.

[58] Einstein to Carathéodory 23.X.1928

[59] This probably refers to the troubles in Schleswig-Holstein during roughly the same period when farmers resisted the tax policies of the government. Hans Fallada has sketched the episode in his *Bauern, Bonzen und Bomben*.

> While I beg you not to blame me for my stubbornness and while I assure you that I will never make use of the fact that it was *you* who informed me about this, I remain with warmest greetings

From these letters, even before the real fight had started, it clearly appears that Einstein was firmly resolved to reserve his neutrality. Einstein had called Brouwer 'an involuntary champion of Lombroso's theory of the close relation between genius and insanity', but Einstein was well aware of Brouwer's greatness, and did not wish him to be victimized. It is not clear whether Einstein's opinion was based on personal observation or on hearsay; the two knew each other well enough, they had met in 1920 when Einstein visited the Academy in Amsterdam, and Einstein had stayed with Brouwer in Blaricum.[60] Furthermore they had probably met during some of the visits of Einstein to Leiden, and at meetings of the editorial board of the *Annalen*.

Einstein's somewhat crude characterization of Brouwer in the letter of 19 October may also have been prompted by a wish to pacify the unstable Hilbert. There is no better remedy to calm a person down, than by outdoing him.

It did not take Brouwer long to react. Brouwer was a man of great sensitivity, and when emotionally excited he was frequently subject to nervous fits. The days following Carathédory's visit, Brouwer was actually physically incapacitated (see p. 593).

On 2 November Brouwer sent letters to Blumenthal and Carathéodory, from which only the copy of the first one is in the Brouwer archive—it contained a report of Carathéodory's visit. The letter stated that 'in calm deliberation a decision on Carathéodory's request was reached.'

The answer to Carathéodory, as reproduced in the letter to Blumenthal was brief:

> Dear Colleague,
> After close consideration and extensive consultation I have to take the position that the request from you to me, to behave with respect to Hilbert as to one of unsound mind, qualifies for compliance only if it should reach me in writing from Mrs. Hilbert and Hilbert's physician.
>
> Yours
> L.E.J.Brouwer

This solution, although perhaps a clever move in a political game of chess or in a court of law, was of course totally unacceptable—even worse, it was a misjudgement of the situation. The prevailing view is that a gentleman rather suffers the accusations of an unaccountable person, than to mention this unaccountability. Cara was thoroughly upset; he reported the outcome of his trip to Blaricum in a letter to Courant;[61] the idea had been that by explaining that Hilbert had been

[60] Oral communication Mrs. F. Heyting-van Anrooy.

[61] Carathéodory to Courant 3.XI.1928.

acting under the pressure of his illness, the pill could be sugared, thus making it possible for Brouwer to withdraw voluntarily. Erhard Schmidt had thought that this would satisfy Brouwer. At first Brouwer was quite sensible, Cara wrote, and he promised to do nothing before he had talked to Schmidt. 'Unfortunately I have today received a totally absurd letter from Brouwer, so that my whole action seems to have fallen through.' If there was any bright spot at all, he said, it was that now the Berlin mathematicians would no longer unconditionally support Brouwer, and that would reduce the risk that the whole matter got into the open. Being a man of honour, Carathéodory added, 'However, after my mediation has so sadly failed, I must resign as soon as possible from the board of the *Annalen*.'

Blumenthal's first reaction was guarded, 'With Brouwer it's complete chaos, you will hear soon enough. Hilbert must not hear about Cara's trip.'[62]

Hilbert's brief note had triggered a development that would have greatly surprised him. He had dismissed an associate editor, and that was it. The idea that he had to justify his decision would not have crossed his mind. For Brouwer, on the other hand, it was unthinkable that one could fire an editor just because of an 'incompatibility of fundamental views.' In particular, since the formulation allowed only one interpretation: 'no intuitionist on my board,' Brouwer had every right to be upset. And so would every well meaning editor. For Brouwer, giving in meant swallowing a grave insult; for the Hilbert side, Brouwer's demand as formulated above, was equally unacceptable.

Hilbert, like one of his ideal statements, had in the mean time been eliminated from the discussion. For his protection, his friends and students had decided to avoid any excitement that could harm his precarious health. And so the defence of the old master was taken up by the younger Hilbertians. There must have been quite a bit of hurried consultation, which could not have been all that easy, as Hilbert's extremely negative view of Brouwer was certainly not universally shared. Most editors were of course aware of the skirmishes in the *Grundlagenstreit*, but they would not dream of considering Brouwer a poor editor or a political risk for the *Annalen*. In fact, Brouwer was on friendly terms with most German mathematicians, be it from Göttingen, or elsewhere. There were of course obvious exceptions, such as Koebe, but on the whole he was a welcome guest at any university.

The conflict had presented itself so suddenly and so totally unexpectedly to Brouwer that he failed to realize to what extent Hilbert saw him as a deadly danger for mathematics, and as the bane of the *Mathematische Annalen*. His belief that the announced dismissal was the whim of a sick and temporarily deranged man emerges from a letter he dispatched to Mrs. Hilbert three days later:

> I beg you, use your influence on your husband, so that he does not pursue what he has undertaken against me. Not because it is going to hurt him and me, but in the first place because it is wrong, and because in his heart he is too good for this. For the time being I have, of course, to defend myself, but I hope that it will be restricted to an incident within

[62] Blumenthal to Courant 4.XI.1928.

the board of editors of the *Annalen*, and that the outer world will not notice anything.

A copy of this letter went to Courant with a friendly note, asking him (among other things) to keep an eye on the matter: 'As a matter of course, I count especially on you to bring Hilbert to reason, and to make sure that a scandal will be avoided.'[63] Courant, after visiting Mrs. Hilbert, replied to Brouwer that Hilbert was in this matter under nobody's influence, and that it was impossible to exert any influence on him.[64] The reader should realize that there was no animosity between Courant and Brouwer. As the *Annalen* affair left in the end deep scars and lasting aversions, it is well to keep in mind that there were no hidden or open personal conflicts between Brouwer and the other members of the editorial board. It was not as if a bone had to be picked. In particular there was no bad blood between Brouwer and Courant, the favoured assistant of Hilbert. Indeed sometime in the past Brouwer had warmly recommendeed Courant for a mathematics chair in Münster.[65]

Apart from Einstein, who kept a strict neutrality, all the editors (mostly reluctantly) did take sides—the majority with Hilbert. Hilbert himself, however, no longer took part in the conflict. His position was fixed once and for all, and in view of his illness the developments were as far as possible kept from him (e.g. Blumenthal stressed in a letter to Courant that Cara's attempted intervention with Brouwer should be kept secret from Hilbert cf. p. 606). One might wonder whether Brouwer, as a relative outsider (one of the three non-Germans among the editors), stood a chance from the beginning; his letter of November 2 to Carathéodory, doubtlessly lost him a good deal of sympathy and proved a weapon to his opponents.

In a circular letter of 5 November 1928 Brouwer appealed directly to the publisher and the editors, thus widening the circle of persons involved:

To the publisher and the editors of the Mathematische Annalen.

From information communicated to me by one of the chief editors of the Mathematische Annalen at the occasion of a visit on 30-10-1928 I gather the following:

1. That during the last years, as a consequence of differences of opinion between me and Hilbert, which had nothing to do with the editing of the Mathematische Annalen (my turning down of the offer of a chair in Göttingen, conflict between formalism and intuitionism, difference in opinion concerning the moral position of the Bologna congress), Hilbert had developed a continuously increasing anger against me.

2. That lately Hilbert had repeatedly announced his intention to remove me from the board of editors of the Mathematische Annalen, and

[63] Brouwer to Courant 6.XI.1928.

[64] Courant to Brouwer 10.XI.1928.

[65] Cf. Freudenthal to Hopf 22.I.1932. Freudenthal wrote that in Brouwer's opinion Hilbert was not pleased at all, feeling that Brouwer was trying to do him out of a valuable collaborator.

this with the argument that he could no longer 'cooperate' (*zusammenarbeiten*) with me.

3. That this argument was only a pretext, because in the editorial board of the Mathematische Annalen there has never been a cooperation between Hilbert and me (just as there has been no cooperation between me and various other editors). I have not even exchanged any letters with Hilbert since many years and that I have only superficially talked to him (the last time in July 1926).[66]

4. That the real grounds lie in the wish, dictated by Hilbert's anger, to harm and damage me in some way.

5. That the equal rights among the editors (repeatedly stressed by the editorial board within and outside the board*)) allow a fulfilment of Hilbert's will only in so far that from the total board a majority should vote for my expulsion. That such a majority is scarcely to be thought of, since I belong to the most active members of the editorial board of the Mathematische Annalen, since no editor ever had the slightest objection against the manner in which I fulfil my editorial activities, and since my departure from the board, both for the future contents and for the future status of the Annalen, would mean a definite loss.

6. That, however, the often proclaimed equal rights, from the point of view of the chief editors, was only a mask, now to be thrown off. That as a matter of fact the chief editors wanted (and considered themselves legally competent) to take it upon themselves to remove me from the editorial board.

7. That Carathéodory and Blumenthal explain their cooperation in this undertaking by the fact that they estimate the advantages of it for Hilbert's state of health higher than my rights and honour and freedom of action (*Wirkungsmöglichkeiten*), and than the moral prestige and scientific contents of the Mathematische Annalen that are to be sacrificed.

I now appeal to your sense of chivalry and most of all to your respect for Felix Klein's memory, and I beg you to act in such a way, that either the chief editors abandon this undertaking, or that the remaining editors split off and carry on the tradition of Klein in the management of the journal by themselves.

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*) [Brouwer's note] (From the editorial obituary of Felix Klein, written by arathéodory) 'He (Klein) has taken care that the various schools of mathematics were represented in the editorial board and that the editors operated with equal rights alongside of himself—He has (...) never heeded his own person, always had kept in view the goal to be achieved.' (From a letter from Blumenthal to me, 13-9-1927). 'I believe, that you overestimate the meaning of the distinction between editors in large and small print. It seems to me that we all have equal rights. In

[66] Cf. p. 574. The last preserved written communication was dated 20.VIII.1919.

particular we can speak for the *Annalenredaktion* if and only if we have made sure of the approval of the editors interested in the matter under consideration. — Although I too take the distinction between the two kinds of editors more to be typographical than factual (I make an exception for myself as managing editor), I understand your wish for a better typographical make up very well. You know that I personally warmly support it. However, we can for the time being, as long as Hilbert's health is in such a shaky state as it is now, change nothing in the editorial board. I thus cordially beg you, to leave your wish for later. At the right moment I will certainly and gladly bring it out.

L.E.J. Brouwer Laren, 5. November 1928

The letter clearly was addressed to all editors, both chief and associate. This, of course, widened the circle of the informed, and it would make it more difficult for the Hilbert side to sweep the matter under the carpet.

The above circular letter was dispatched at the same day as Brouwer's plea to Mrs. Hilbert; the two letters are in striking contrast. One letter is written on a conciliatory note, the other is a determined defence and closes with an unmistakable incitement to mutiny.

Blumenthal immediately took the matter in hand, he wrote to the publisher and the editors[67] to ignore the letter until he had prepared a rejoinder. The draft of the rejoinder was sent off to Courant on November 12, with instructions to wait for Carathéodory's approval and to send copies to Bieberbach, Hölder, von Dyck, Einstein and Springer. It appears from the accompanying letter that Carathéodory had already handed in his resignation, although he had given Blumenthal permission to postpone its announcement, so that it would not give food to the rumour that Carathéodory had turned against Hilbert.

Blumenthal, being the acting managing editor, had more or less assumed responsibility for the defence of Hilbert. It is hard to understand that he, who had been a close friend of Brouwer's, could from one day to the next turn against his former friend, and organize a campaign against him. It is not unlikely, however, that the Riemann affair, and the subsequent Bologna affair, had already introduced a measure of estrangement. It was of course well-known that Hilbert ran a tight ship, but it still comes as a bit of a shock to see that Blumenthal actually feared that Carathéodory would be banned from Hilbert's circle: 'Poor Cara has, in spite of his best intentions, got himself into a tight corner, and I don't know yet if Hilbert will break off relations with him.'[68]

In the meantime Brouwer had travelled to Berlin to talk the matter over with Erhard Schmidt, and to explain his position to the publisher Ferdinand Springer. Brouwer, accompanied by Bieberbach, called at the Berlin office of Springer. The

[67] Blumenthal to the editors 16.XI.1928

[68] Blumenthal to Courant 28.XI.1928.

visit is described in a memorandum *Aktennotiz* 'Unannounced and unexpected visit of Professor Bieberbach and Professor Brouwer' (13 November 1928). As Springer wrote, his first idea was to refuse to receive the gentlemen, but he then realized that a refusal would provide propaganda material for the opposition. Springer opened the discussion with the remark that he was firmly resolved not to get involved in the skirmishes and that he did not consider the *Annalen* the sole property of the Company (like other journals), but that the proper *Herausgeber*, Klein and Hilbert, had in a sense entrusted it to the publisher. Moreover he would choose Hilbert's side, out of friendship and admiration, if he would be forced to choose sides.

The unwelcome visitors then proceeded to inquire into the legal position of Hilbert, a topic that Springer was not prepared to discuss without the advice of his friends and which he could not enter into without consulting the contract.[69] Thence the two gentlemen proceeded to 'threaten to damage the *Annalen* and my business interests. Attacks on the publishing house, which could get the reputation of lack of national feelings among German mathematicians, could be expected.'

Springer took this stoically, and assured that he would know how to react to such statements, but that he would accept any negative effects without complaint.

The implicit threat was definitely in bad taste, not in the last place because the Springer family had Jewish ancestry. Bieberbach's later political views have gained a good measure of notoriety;[70] it certainly is true that already before the take-over of the Third Reich his views had grown more and more nationalistic. Brouwer's position on these matters was neither political, nor nationalistic, it was dictated by his extreme aversion of the scientific boycott of Germany. Nonetheless, the above lines show that, wittingly or not, he ran the risk to be associated with right wing Germans.

Thus rejected Bieberbach and Brouwer asked if Springer could suggest a mediator, to which Springer answered that he was not sufficiently familiar with the personal features involved, but that two *deutschfreundliche* foreigners like Harald Bohr and G.H. Hardy might do.[71] Before leaving, Brouwer threatened to found a new journal with De Gruyter, and Bieberbach declared that he would resign from the board of editors if it definitively came to the exclusion of Brouwer. In a letter to Courant (13 November 1928) Springer dryly commented 'On the whole the founding of a new journal, wholly under Brouwer's supervision, would be the best solution to all difficulties.'[72] He also conveyed his impression of the visit: 'I would like to add that Brouwer, as a matter of fact, does make a scarcely pleasant

[69]The contract had indeed a clause to the effect that changes in the editorial board required the assent of the publisher. The problem was of course how to interpret 'editor'.

[70]Cf. [Mehrtens 1987]

[71]This suggestion of the publisher encouraged the impression that the conflict had a political origin. Blumenthal complained to Courant (letter of 18 November 1928) '… the bad thing is, that Brouwer managed to move everything on to the political plane, just what Carathéodory thought he had prevented.' The idea of mediation was not pursued.

[72]Brouwer indeed founded a new journal, the *Compositio Mathematica*, with the Dutch publisher Noordhoff.

(*unerfreulich*) impression. It seems, moreover, that he will carry the fight to the bitter end (*der Kampf bis aufs Messer führen wird*).'

In Aachen, Blumenthal was preparing his defence of the announced dismissal of Brouwer and, following an old strategic tradition, he took to the attack. After consulting Courant, Carathéodory and Bohr he drew up a kind of indictment. From a letter from Bohr, and Courant to Blumenthal,[73] one may infer that the draft of 12 November was harder in tone and more comprehensive than the final version. There is mention of a detailed criticism of Brouwer's editorial activities and of matters of formulation ('... leave out capriciousness (*Schrullenhaftigkeit*) ...').

Carathéodory remained an uncertain factor in the coming power play; Bohr and Courant realized this, and they therefore would prefer him to do his bit: 'We feel that Cara should make a stand himself because of the misuse Brouwer made of his kindness, respectively that he should explicitly authorize you to use defensive words.' A clever suggestion, but not exactly considerate. Bohr and Courant explicitly warned Blumenthal:

> To what extent Brouwer exploits without consideration every tactical advantage that is offered to him, and how dangerous his personal influence is (Bieberbach), can be seen from the enclosed notice which Springer has just sent us.[74]

The correspondence of Blumenthal, Bohr, and Courant shows an unlimited loyalty to Hilbert, which it would be unjust to ascribe to Hilbert's state of health alone. There is no doubt that Hilbert as a man and a scientist inspired a great deal of loyalty in others, let alone in his students. Sentences like 'We don't particularly have to stress that we are, like you, wholly on Hilbert's side, and also, when necessary, prepared for action,'[75] illustrate the feeling among Hilbert's students.

A revised version of Blumenthal's letter is dated November 16, and it is this version that was in Brouwer's possession. It incorporated remarks of Bohr and Courant, but not yet those (at least not all of them) of Carathéodory. It contained a concise *resumé* of the affair so far, and proceeds to answer Brouwer's points (from the letter of 5 November 1928).

Blumenthal partly based his handling of the matter on correspondence, partly on conversations with Hilbert in Bologna. The contents of the latter conversation remain a matter of conjecture, but it may be guessed that in August at the conference Hilbert had made clear his objections to Brouwer—in particular after Brouwer's opposition to the German participation in the conference.

From Blumenthal's circular letter, the editors—and also Brouwer—learned the contents of Hilbert's letter of October 25.

In view of the importance of the letter, it is worthwhile to reproduce Blumenthal's letter here.

[73] Bohr and Courant to Blumenthal 14.XI.1928.

[74] The above mentioned *Aktennotiz*.

[75] Ibid.

To the publisher and the editors of the *Mathematische Annalen*.

As manager of the board of the *Annalen*, I feel obliged to reply to Brouwer's circular letter to the publisher and the editors of the *Mathematische Annalen*. My exposition relies in part on letters of Hilbert, Carathéodory, and Brouwer, in part on an extensive discussion I had with Hilbert in Bologna.

I would like to point out in advance that the formulation of Brouwer's letter is misleading: one can get from it the impression that the editor who visited Brouwer on October 30 (Carathéodory) formulated the statements 1—7. This is of course not the case for any of them, these are rather opinions that Brouwer formed himself.

In the following I give a brief report of the developments, and react to the relevant points of Brouwer's letter.

Blumenthal proceeded to quote Hilbert's letter of 25 October to Brouwer in full (cf. p. 602) and continued

Brouwer has not opened this letter, as I should note already here, and as I explain later. He is, however, informed by Carathéodory about its content, in particular also of the motivation for Hilbert's action given in the first sentence. Brouwer's points 2 and 3 refer to this. About this I have to say the following:

On points 2 and 3. Brouwer interprets the notion of cooperation in a literal sense (point 3). This is complete misapprehension of the true meaning. It is rather the case that Hilbert had become convinced, that Brouwer's activity was detrimental for the *Annalen*, and that he could therefore no longer take the responsibility to act as a chief editor in an editorial board, to which Brouwer belonged. By no means does this concern a pretext.

On points 1 and 4. The motivations for Hilbert's way of operating, indicated by Brouwer, in these points is not correct. The motivation in point 4 is spiteful and therefore requires no refutation. Also the scientific differences with respect to the foundations, of which one could think, plays no role. In particular it is not true what Brouwer seems to suggest in point 5, that the mathematical direction, represented by him, will in future get less opportunity to speak. Also Brouwer's circular letter before the Bologna congress, the expression of which Hilbert found insulting, has only in conjunction with other, perhaps more important factors acted as a catalyst on his decision. The causes lie much deeper. I will give them in my formulation, but I am certain to get Hilbert's meaning precisely.

Felix Klein had, until his resignation from the editorial board, formed among us a kind of supervising body, that in difficult cases could be called in, or that acted on its own initiative, to support important decisions (for example the transfer of the *Annalen* to Springer Verlag), or to smooth disagreements inside the editorial board. It is good and necessary that in

a numerous board like ours, there is such a supreme body available, that, relieved from the details of the management, keeps an eye on the general relations and feels responsible for these. After Klein's death, Hilbert had felt obliged to fulfil this position, and has already acted in this sense, and I for one have also personally always instinctively recognized him as such.

Hilbert saw in Brouwer a headstrong, unpredictable and domineering character. He had feared that, when he at some time should have left the board, Brouwer would bend it to his will, and he has judged this such a great danger for the *Annalen*, that he wanted to stand in his way as long as he still could do so. Probably under the influence of his renewed illness, he felt obliged in the interest of the *Annalen*, to order Brouwer's exit from the board, and to tackle this measure immediately and with all energy.

Cara and I, who were associated with Brouwer in a long-standing friendship, had objectively to recognize Hilbert's objections to Brouwer's editorial activity.

True, Brouwer was a very conscientious and active editor, but he was quite difficult in his dealings with the managing editor and he subjected the authors to hardships that were hard to bear.

For example, manuscripts that were submitted for refereeing to him, lay around for months, while in principle he had prepared a copy of each submitted paper (I recently had an example of this practice). Above all there is no doubt that Klein's premature resignation from the editorial board is to be traced back to Brouwer's rude behaviour (in a matter in which Brouwer was formally right). The further course of events has shown that Hilbert was even far more right than we thought at the time.

Since we could not reject the objective justification of Hilbert's point of view, and were confronted by his immutable will, we have given our permission for the removal of Brouwer from the editorial board.[76] We only wished—unjustifiedly, as I now realize—a milder form, in the sense that Brouwer should be prevailed upon to resign. Hilbert could not be induced to this procedure, so we finally, though reluctantly, have decided to give in to him (*den Weg freigeben*). Einstein did not comply, with the argument that one should not take Brouwer's peculiarities seriously.

Point 5 and 6 In how far it was justified that the other editors were not first informed of Hilbert's plan, I don't want to go into here. Formally speaking the justification seems to be given by the distinction between '*Mitwirkenden*' and '*Herausgeber*' on the cover.

The events after the dispatch of Hilbert's letter.

[76] Blumenthal is here less than truthful. Cara had not authorized Hilbert. He had probably tried to pacify Hilbert, who took this for the desired approval. Cara wrote, referring to Blumenthal's draft, 'In fact I have not given an authorization in the legal sense in my letter to Hilbert.' (Carathéodory to Courant 14.XI.1928).

> On October 26 and 27 Cara and I were in Göttingen, to discuss the situation. Cara travelled subsequently for the matter to Berlin. Although he saw objectively that Brouwer's eviction was unavoidable, he decided in Berlin, to make a last attempt, to settle the matter in an amicable sense, by weakening the categorical form of the expulsion. For that reason he came on the thirtieth of October to Laren, after Brouwer had been asked in advance, by telegram, not to take any steps until Cara's arrival. Since Brouwer had not opened Hilbert's letter, Cara informed him of the content (but not the formulation), and proposed him, to resign of his own free will from the editorial board and to leave the letter unopened. He wanted to prevent that Brouwer would feel insulted by the form, and he felt justified to so, as the rudeness seemed partially determined by Hilbert's ailing health.
>
> He did not make it clear to Brouwer that in our opinion he had to leave the board, and bade him, out of consideration with Hilbert and his illness of that moment, to withdraw by himself. Brouwer reserved a decision until further calm deliberation. He had left Hilbert's letter unopened, and written to Cara on November 2 the following letter:

At this place Blumenthal inserted the text of letter of Brouwer of 2 November, see p. 605.

> For this frightful and repulsive letter, which Brouwer also sent to me in copy, I can only offer the explanation, that Brouwer (on purpose or inadvertently) had put together from Cara's statements and entreats precisely the ugliest part. I must confess, and Cara has written me likewise, that I have been thoroughly deceived in Brouwer's character, and that Hilbert has known and judged him better than we did. I too am no longer in a position to cooperate further with the writer of this letter in the board, and I now side actively with Hilbert. I cannot understand, that Brouwer after this letter can appeal to the chivalry of the editors and to the memory of Felix Klein.
>
> I beg the gentlemen either for a speedy reaction, or for their tacit consent, that from the next issue Brouwer's name is no longer on the cover, and that he does no longer get my Annalen-information.

So far the 'case for the prosecution.' As an indictment the above letter did not make a convincing impression. It was written to refute Brouwer's points, and to justify Hilbert's decision. In neither was Blumenthal very successful. Admittedly he was in a difficult position; he had read Hilbert's letter to his fellow '*Herausgeber*', and that letter listed some concrete complaints. But he could not very well use these; as the letter was for the chief editors only, he could not ask Hilbert's permission to use the letter, because Hilbert was not supposed to know what was going on, and finally, Hilbert's complaints were very subjective, another editor would probably have seen the mentioned facts in a completely different light. Worse, these facts would have given ammunition to Brouwer.

Blumenthal indeed acted as if he had direct access to Hilbert's thoughts. For example, in the case of the refutation of Brouwer's points 1 and 4, he gave no evidence, he just denied Brouwer's statements. Looking at the evidence, cf. p. 598, one cannot but conclude that the *Grundlagenstreit*, the Bologna affair, and the Riemann affair brought out some uncontrolled emotions on Hilbert's side. Hilbert's references to intuitionism and Brouwer go beyond scholarly comment. The matter of the Göttingen chair is not as clear. Position bargaining was a normal thing in Germany, and that Hilbert's offer was used by Brouwer to improve his situation in Amsterdam would not have surprised anybody. And yet—might not Hilbert have taken Brouwer's decision ill? After all, he was planning to get the leading topologist for Göttingen, and perhaps, in a corner of his heart, he saw Brouwer as a useful addition to his foundational team—the discussion in 1909 might have left some memories. And there is Carathéodory's statement that Hilbert's objections go back ten years.

Considering all evidence, one would be inclined to side with Brouwer rather than Blumenthal. Point 4 is a debatable one. When not under the influence of a fatal disease, would Hilbert have had no wish to hurt Brouwer? From the height of his Olympus he might have acted objectively and impersonally: 'I cannot get along with this man, so he'd better go.' But perhaps Hilbert, when not on his Olympus, was enough of a man of flesh and blood to be susceptible to the 'I'll get you' mood. Whatever was the real state of affairs, the point is subjective, but precisely for that reason Blumenthal should not have dismissed the point so perfunctorily.

Hilbert, in the letter to his fellow chief editors, mentioned three points. (1) the insult implicit in Brouwer's Bologna circular letter, (2) Brouwer's anti-*Conseil* and anti-Union feelings as standing in the way of his editorial work, (3) the *Annalen* should remain in Göttingen. The heart of Hilbert's argument lies in (2). For, one could hardly accept (1) as a ground for dismissal.[77] If Hilbert was so sensitive about insults, had he forgotten his words in the second Hamburg lecture? And (3) was rather irrelevant. Journals have no fixed abodes (not counting the proceedings of academies, and the like). Of course, it is a pleasure and a honour for a department to be almost a synonym for a prestigious journal, but all these things would pass one day. Whatever Hilbert's assessment may have been, there is not the slightest indication that Brouwer would try to move the *Annalen* elsewhere. Knowing Brouwer's attachment to Göttingen, he would have insisted on keeping the journal where it was.

Yet, even a man like Carathéodory, to whom the term 'the milk of human kindness' could justifiably be applied, had his reservations on this point. It tells something about the internal frictions in German mathematics, that this man, who obtained a doctorate and habilitation in Göttingen, and who was for some time Felix Klein's successor, was critical of the Göttingen imperialism. It should be added that he knew the mathematical world better than most, having studied in Berlin, where he briefly held a chair, and at the time of the *Annalen* conflict he was professor in Munich. Nonetheless, it might have come as a surprise to Courant,

[77]If anything, Brouwer's circular letter could have been condemned rather as divisive, than as insulting.

when Carathéodory told him that Hilbert's claim was for him one of the grounds to resign from the *Mathematische Annalen*.

No, Hilbert saw problems with, in particular, French authors. Brouwer, if he were a chief editor, might, in his opinion, block a paper of Painlevé, or of Picard. Here he was mistaken; Brouwer could very well distinguish between an individual and an organization. We may recall that Brouwer lectured Denjoy at length for not observing this distinction, cf. p. 344. His objections against an invitation of Painlevé to participate in the memorial volume of Riemann was based on the invitation and on the special occasion. As it was, Brouwer was only an associate editor with no influence on papers that were not refereed by him. No matter how one looks at this point, one cannot but conclude that Hilbert's fears were mostly the product of his unfortunate health situation.

Anybody with a cool mind could see that the Bologna conference had restored the international character of mathematics, and that the nationalistic differences would no longer play a role. This clearly had escaped Hilbert, when he was in Bologna. Either he was fixed on the past, or he had an axe to grind.

A few editors responded to Blumenthal's letter in writing, but the majority remained silent. Only von Dyck, Hölder, and Bieberbach[78] sent their comments. Von Dyck could 'neither justify Brouwer's views nor Hilbert's action' and he hoped that a peaceful solution could be found. Hölder was of the opinion that he could not approve of a removal of Brouwer by force (November 27).

Bieberbach's letter showed a thorough appreciation of the situation. And he at least was willing to take up the case of the underdog. In view of his later political extremism, one might be inclined to question the purity of his motives; however in the present letter there is no reason not to take his arguments at their face value. Like Brouwer, and probably the majority if not the whole of the editorial board, he contested the right of the *Herausgeber* to decide matters without the support of the majority of all the editors, let alone without consultation. Indeed this seems to be a shaky point in the whole procedure. Bieberbach referred to the 'Innsbruck resolution', which laid down that the chief editor had no claim to the dismissal of editors. 'And now', he said, 'the right to dismiss editors should follow from typographical characteristics. That would reduce the members of the editorial board to subalterns, who can be dismissed any day by the chief editors.'

As a matter of fact, the contract between Springer and the *Herausgeber*[79] is not very concrete in this particular point. It states: 'Changes in the membership of the editorial board require the approval of the publisher.' The correspondence makes it clear that Hilbert did not observe this rule. Bieberbach observed that a delay in handling papers cannot be taken seriously as grounds for dismissal: such things ought to be discussed in the annual meeting of the board.

He devoted a few lines to the procedure proposed by Blumenthal. Blumenthal had not asked for a vote on the expulsion of Brouwer, his plan was to decide the

[78] Bieberbach to Blumenthal 24.XI.1928.

[79] 25.II.1920

matter on the basis of individual reactions plus the 'silence lends consent' principle, without even mentioning a time limit. Such a procedure contradicted the most elementary principles of justice, Bieberbach observed. The exclusion of an editor should be handled with extreme care. 'I would consider it correct', he continued, 'that in our circle the question of expulsion should only be made a point of discussion, if such a sensational failure of one editor is the case, that one count on unanimous agreement, not however, if beforehand a prominent member of the board of chief editors such as Einstein, opposes the exclusion,…'

As one of the main dissidents in the matter of the Bologna affair, Bieberbach added some 'scholastic' comments to Blumenthal's report of Hilbert's feelings. Brouwer's pamphlet was directed at those visiting the Union congress in Bologna. As Hilbert himself claimed, the Bologna conference was *not* a Union congress, so how could he be insulted?

Bieberbach found no difficulty in dissecting Blumenthal's case against Brouwer. He concluded that 'A dismissal without any notice of an editor, who is moreover a scientist of world fame, after thirteen years of diligent activity could only be justified by defamatory actions or so, not by incidents that only hold inconveniences for the editor in chief (*geschäftsführender Redakteur*).

Blumenthal had been making the most of Brouwer's, admittedly tactless, letter to Mrs. Hilbert. Bieberbach correctly spotted a serious flaw in Blumenthal's charge involving Brouwer's 'terrifying and repulsive' letter.

> Finally I hold it totally unjustified to concoct material against Brouwer from letters that he wrote after learning about the action that was mounted against himself. For it is morally impossible to use actions, to which a person is driven in a fully understandable emotion over an injustice that is inflicted on him, afterwards as a justification of this injustice itself.

The point is well taken. It does not exonerate Brouwer, but it at least makes clear that to use it against Brouwer is in poor taste. Bieberbach explicitly stated that he would not support Brouwer's dismissal; on the contrary, he strongly sided with Brouwer, without, however, attacking Hilbert.

There is a certain tendency to dismiss Bieberbach's statements and opinions on the basis of his later political views and actions. Needless to say this does not go well with rational reflection. Be that as it may, Bieberbach's letter painfully exposed the weaknesses in the case against Brouwer.

The publisher reacted in a cautious way. Springer thought that Brouwer was 'an embittered and malicious adversary', and that he should not receive a copy of the circular letter without the permission of the lawyer of the firm. Springer also concluded that the publisher should not state in writing that he officially agreed to Brouwer's dismissal, because it would imply a recognition of Brouwer's membership of the board of editors in the sense of the contract. In short, Springer abstained from voting on Blumenthal's proposal.

At this point the whole action against Brouwer seemed to have reached an anticlimax. One may surmise a good deal of activity in the camp of the Göttingers. The matter now began to take on national proportions. In view of the barely veiled

animosity between Berlin and Göttingen, as the ultimate bastions of mathematics, and Brouwer's close connections with the Berlin mathematicians, the temporary leaders of the Göttingen group started to worry that the Brouwer dismissal might lead to an open rift between the two groups. This might even lead to further unpleasantness, as the main loyalties among German mathematicians were with Berlin or Göttingen; once a Göttinger, always a Göttinger, and the same for Berlin. It thus became a matter of some urgency to settle the Brouwer matter. Had not Hilbert, much like the ghost of Hamlet's father, continually hovered in the background, the editor's would have been able to find a compromise, so that Brouwer could stay on. Unfortunately the outcome was fixed in advance. Nonetheless the Hilbert party had its worries.

If only Einstein could be persuaded to give his assent . . . ! Although it seemed doubtful that anyone could succeed where even Hilbert had failed, Max Born tried to convince Einstein to remain at least neutral. In a lengthy letter he summed up the arguments, sketched the mood, outlined the consequences. Hilbert, he said, was so ill that he wouldn't live much longer. Every emotion could do serious harm, and would shorten the time left to finish his work. 'What is more, he is full of a strong will to live, and he sees it as his task, to carry out his new founding of mathematics, to which he has to devote himself with his last strength. His mind is clearer than ever, and the rumour, spread by Brouwer that Hilbert were not completely *compos mentis* is an extraordinary heartlessness.' Born had spoken with Hilbert on the topic 'Brouwer', and Hilbert had declared that he saw Brouwer as an eccentric and unbalanced man. According to Born, Hilbert considered Brouwer's ultra-German behaviour in the Bologna matter a folly, 'but the dreadful thing was that the Berlin mathematicians fell for Brouwer's nonsense.' Born mentioned Bieberbach, Von Mises and Schmidt as dupes of Brouwer, a claim that was not borne out by evidence; at least Bieberbach and Von Mises were anti-Bologna-Union on their own accord. Born had travelled with Von Mises in the USSR, and on one occasion Von Mises opened the conversation with, 'the Göttinger simply run after Hilbert, who was no longer quite responsible for his actions' (*unzurechnungsfähig*). Born immediately broke off the conversation, since in his opinion Von Mises was too insignificant to have an opinion on Hilbert. As he pointed out, this was before the Bologna conference, thus before Hilbert's collapse. In every community there is invariably an amount of gossip floating around, in mathematics no less than in other subjects. It is more than likely that the rumours of Hilbert's disease, combined with his strange emotional behaviour at the Hamburg and Münster lectures, had encouraged speculation about his mental health. Thus von Mises' view may have displeased, but not surprised, Born.

In view of a meeting to be held at Springer's in Berlin, Born begged Einstein not to do anything that might harm Hilbert's interest. It was important that the chief editors should speak with one voice.

The pressure on Einstein was from a strategic point of view, understandable. His immense scientific and moral prestige made him a key figure in any debate. If he could be persuaded to side with Hilbert the battle would be half won. In spite

of personal pressure from Born (20 November 1928) on behalf of Hilbert, Einstein remained stubbornly neutral. In his letters to Born and to Brouwer and Blumenthal one may sense a measure of disgust behind a facade of raillery. In the letter to Born (November 27) the apt characterization of '*Frosch-Mäusekrieg*' (war of the frogs and the mice) was introduced.[80] After declaring his strict neutrality he went on:

> If Hilbert's illness did not lend a tragic aspect, this ink war would for me be one of the most funny and successful farces performed by that sort of people who take themselves deadly seriously.
>
> Objectively, I might briefly point out that in my opinion there would have been more painless remedies against an overly large influence on the managing of the *Annalen* by the somewhat mad (*verrückt*) Brouwer, than eviction from the editorial board.
>
> This, however, I only say to you in private, and I do not intend to plunge as a champion into this frog-mice battle with another paper lance.

Einstein's letter to Brouwer and Blumenthal of November 25 is even more cutting and reproving.

> I am sorry that I got into this mathematical wolf-pack (*Wolfsherde*) like an innocent lamb. The sight of the scientific deeds of the men under consideration here impresses me with such cunning of the mind, that I cannot hope, also in this extra-scientific matter to reach a somewhat correct judgement of them. Please, allow me therefore, to persist in my 'boohnor-bah' (*Muh-noch-Mäh*) position and allow me to stick to my role of astounded contemporary. With best wishes for an ample continuation of this equally noble and important battle, I remain
>
> Yours truly,
> A. Einstein

The whole affair now rapidly reached a deadlock. A week before, Springer had, after seeking legal advice at Blumenthal's urging, written optimistically to Courant[81] that the legal adviser of the firm, E. Kalisher, was of the opinion that it would suffice that those of the four chief editors who did not want to advocate Brouwer's dismissal actively would abstain from voting, thus giving the remaining chief editors a free hand. Apparently Springer did not realize that since two editors with a high reputation had already decided not to support Hilbert, the solution, even if it was legally valid, would lack moral support. If this solution should turn out to raise difficulties within the editorial board, the publisher could still fire the whole editorial board and reappoint Hilbert and his supporters, so the advice ran. In the opinion of the legal adviser the publishing house was contractually bound to the chief editors (*Herausgeber*) only; there was no contract with the remaining editors.

[80] War of the frogs and the mice—a Greek play of unknown author; a late medieval German version, *Froschmeuseler*, is from the hand of Rollenhagen.

[81] Springer to Courant 17.1.I.1928

Bieberbach's letter, mentioned above, apparently worried Carathéodory to the extent that he decided to ask a Munich colleague from the law faculty for advice. This advice from Müller-Erzbach plainly contradicted the advice from the Springer lawyer. It made clear that

1. Brouwer and Springer-Verlag were contractually bound since Brouwer had obtained a fee.
2. Hilbert's letter was not legally binding.

Müller-Erzbach sketched three solutions to the problem:

(a) Springer dismisses Brouwer. A letter of dismissal should, however, contain appropriate grounds.
(b) The four chief editors and the publisher form a company (*Gesellschaft*) and dismiss Brouwer.
(c) A court of law could count the *Mitarbeiter* as editors. In that case the only way out would be to dissolve the total editorial board and to form a new one.

Carathéodory considered the first two suggestions inappropriate because it would not be fair to saddle Springer with the internal problems of the editors. Hence he recommended the third solution.[82] Here, for the first time, appeared the suggestion that was to be the basis of the eventual outcome of the dispute.

Hilbert, the main contestant in the *Annalen* affair, had quite sensibly withdrawn from the stage. The developments, had he known them, would certainly have done his health no good. He had authorized Harald Bohr and Richard Courant to represent him legally in matters concerning the *Mathematische Annalen*. Thus the whole matter became more and more a shadow fight between Brouwer and an absentee.

At this point the dispute had reached an impasse. Although Springer upheld in a letter to Bieberbach the principle that the chief editors could dismiss any of the other editors, the impetus of the attack on Brouwer seemed to ebb. A meeting between Carathéodory, Courant, Blumenthal and Springer had repeatedly to be postponed and finally had been cancelled.

Courant agreed with Carathéodory that the dissolution of the complete board would be a convenient way out;[83] however, it would require a voluntary action from the editors and the ultimate organization of the editorial board should not have the character of a legal trick with the sole purpose of rendering Brouwer's opposition illusory.

Carathéodory, who, on the basis of Müller-Erzbach's information, had come to the conclusion that the original plan of Hilbert, even in a modified form, would not stand up in a court of law, expressed his willingness to assist 'out of devotion to Hilbert' in the liquidation of the affair, but quite firmly refused to be involved in the future organization of the *Annalen*.

[82] Cara to Blumenthal 27.XI.1928.

[83] Courant to Carathéodory 30.XI.1928

The reluctance of Carathéodory to be involved in the matter beyond the bare minimal efforts to satisfy Hilbert and spare Brouwer (his friend) is throughout understandable. As far as we can judge from the correspondence, only Blumenthal exhibited an unbroken fighting spirit. He realized, however, that his circular had not furthered an acceptable solution,[84] and he leaned towards alternative solutions. In particular, Blumenthal wrote, the time was favourable to Carathéodory's plan. The *Annalen* were completing their hundredth volume, and it would present a perfect occasion to open with volume 101 a 'new series' or 'second series' with a different organization of the editorial board. But at the present time he was facing a dilemma. Because Hilbert's letter clearly had no legal status, Brouwer was still a *Mitarbeiter* and his name should appear on the cover of the issue that was to appear—this, however, conflicted with Hilbert's wishes. Could Bohr and Courant as proxies of Hilbert, authorize him to print Brouwer's name on the cover? Otherwise the publication would have to be postponed. The authorization probably was given.

It seems that Bohr had also put forward a solution to the affair. From the correspondence of Carathéodory and Bohr with Blumenthal, one gets the impression that Bohr's proposal was a slight variant of Carathéodory's suggestion. The main difference was that Bohr advocated a total reorganization of the editorial board. In his proposal there would only remain *Herausgeber*, and no *Mitarbeiter*. So the solution would look like a fundamental change of policy, and hence it would no longer be recognizable as an act levelled against Brouwer.

Apparently Bohr envisaged Hilbert, Blumenthal, Hecke and Weyl as the members of a new board. And should Weyl decline, one might invite Toeplitz. Blumenthal questioned the wisdom of reinstating himself as an editor; it could easily be viewed as the old board of chief editors in disguise.[85] In his letter to Courant, the next day, he considered the dissolution of the editorial board at large as necessary, and he fully agreed that Hilbert should choose the new editors.

From then on things moved smoothly; Springer accepted the dissolution of the editorial board and agreed to enter into a contract with Hilbert on the subject of the reorganized *Annalen*. By and large only matters of formulation and legal points remained to be solved.

One might wonder where Brouwer was in all this—he was completely ignored. In a letter of November 30 to the editors and the publisher he confirmed the receipt of Blumenthal's indictment which had only just reached him. In a surprisingly mild reaction he merely asked the editors to reserve their judgment—blissfully unaware that nobody was going to cast a vote—for the composition of a defence would take some days.

Because the dissolution of the editorial board had to be a voluntary act, it was a matter of importance to get Einstein's concurrence. The contract of 1920

[84] Blumenthal to Courant and Bohr 4.XII.1928.

[85] Blumenthal to Bohr 5.XII.1928.

presented an elegant loophole that would allow both parties to settle the matter without breaking the rules. In §5 the clauses for termination of the contract were listed, and one of them stipulated that if the editors (*Redaktion*) renounced the contract, without a violation from the side of the publisher, the latter could continue the *Mathematische Annalen* at will.

Possibly Einstein's agreement could be dispensed with, but it is likely that a decision to ignore Einstein's vote would influence general opinion adversely; moreover, it would be wise to opt for a watertight procedure, as Brouwer would not hesitate to test the outcome in court.

So pressure was brought to bear on Einstein. James Franck, a physicist and a friend of Born, begged him to listen to the new plan. He stressed the political side of the issue, 'At this time, ..., whether the mathematicians split into factions or whether the affair is arranged smoothly, depends on your decision. It would almost be an unapropriate joke (*ein nicht all zu guter Witz*) if in this case you would be claimed for the nationalistic side' (undated). Franck was not the only person to discover a (real or imaginary) political aspect in the controversy at hand. Blumenthal had already complained to Courant (November 18) that Brouwer had managed to introduce the political element into the matter. Born also, in his letter to Einstein of November 11, tied the conflict to the political issue of the German nationalists and the animosity of Berlin vs. Göttingen.

The successful conclusion of the undertaking was conveyed to Springer by Courant. In his letter of December 15 he announced the cooperation of Einstein, Carathéodory, Blumenthal and Hilbert in the transition. At the same time he proposed that a new contract be made between Hilbert and the publisher, and that Hilbert get *carte blanche* for organizing the editorial board. Blumenthal should be invited to continue his activity as managing editor and, according to Courant, he would probably accept. Also—and this is a surprising misjudgement of Einstein's mood—Courant thought that there was a 50% chance that Einstein would join the new board. As far as he himself was concerned, Courant thought it wiser to postpone his own introduction as an editor until the dust had settled (the matter apparently had been discussed earlier).

Courant had to work hard to prepare the various documents, to solicit comments, make changes, etc. The new style *Annalen-* would have one *Herausgeber*, Hilbert, and a variable (but small) number of *Mitarbeiter*. Hecke und Blumenthal were eventually chosen for the latter function.

The new arrangement promised a satisfactory end to the Annalen affair, but not everyone was happy. Blumenthal, for example, cautiously pointed out that Hilbert would become the only chief editor; if he intended any criticism, he was careful to leave it to Courant to read it between the lines.[86] Carathéodory on the other hand openly expressed his disappointment. He, too, deplored the end of the old regime. When confronted with Hecke's comments on the practice of the past (letter from Courant to Carathéodory, December 17): '... that Hecke, when he learned about

[86] Blumenthal to Courant 16.XII.1928.

the organization of the editorial board and the competence of the *Beirat* [the advisory editors] grasped his head and judged a revision and a more strict organization absolutely necessary.' Carathéodory heartily disagreed 'For, Klein had organised the board of editors of the *Mathematische Annalen* in such a way that it formed really a kind of Academy, in which each member had the same rights as the others. That was in my opinion the main reason why the *Annalen* could claim to be the first mathematics journal in the world. Now it will become a journal like all other ones.'[87] It did not take Blumenthal long to recognize the negative sides of the new set-up. Already on 2 February, 1929 he sent out a note, 'On the future organization of the Annalen', in which he drew the attention to the decline of the journal compared to other journals. Since the associated editors (*Nebenredaktion*) had been eliminated one simply needed a larger staff: 'the increasing necessity of scientific advisers follows inevitably from the increasing specialisation.' In short Blumenthal proposed to reinstate something like the old associate editors under a different name. In the same letter he broached the question of the successor of Hilbert, should he step down. One finds it difficult to reconcile this letter with the arguments that were put forward in favour of the solution to the conflict.

Finally Courant suggested that the publisher alone should inform all present editors of the collective resignation. With respect to Brouwer, he advised Springer to write a personal letter explaining the solution to the conflict, and to stress that he [Springer] would regret it if Brouwer were left with the impression that the whole affair would restrict his freedom of action, and that the publishing house would be at his disposal should he wish to report on his foundational views. It is not known whether this letter was ever written, but Courant's attitude certainly was statesmanlike and conciliatory.

The fact that the whole board was going to be dismissed, and that only Hilbert and Blumenthal were going to be reappointed was an unpleasant message for most, if not all of the sitting editors, but in particular for the senior members Von Dyck and Hölder. Van Dyck had even at one time been a chief editor. It is greatly to Blumenthal's credit that he asked Courant to intervene with Hilbert, so that the latter would write a few nice words to the two; the pill needed a strong dose of sugaring, he said.

Like a good statesman, Courant realized that the past events carried the potential for a long period of friction. Of course he was aware of the reputation of arrogance of the Göttingen group, but he was sensible enough to see that there was life outside Göttingen. As he wrote to Carathéodory,[88] 'We should also think of the future relations between the German mathematicians. If a part of the colleagues does not learn to understand, what really motivated Hilbert, then the vexation will not yield and can burst out here and there. If such a latent tension—that will not come from Hilbert's circle—is to be avoided for the future, then one must make

[87] Carathéodory to Courant, 19.XII.1928.

[88] Courant to Carathéodory 23.XII.1928.

use of the present moment to rob the matter of any unjustified ugly appearance and enter into a basis of mutual understanding and trust. It would be gratifying and comforting, if you would help us to make all persons involved, in particular our Berlin colleagues, to adopt this position.'

Once the decision was taken, no time was wasted; after the routine legal consultations the publisher carried out the reorganization and the editors were informed of the outcome (December 27). In spite of Courant's considerations mentioned above, the letter was signed by Hilbert and Springer. Brouwer, like everybody else, was thanked for his work and was given the right to a free copy of the future *Annalen* issues. The matter would have been over, were it not for some rumblings among the former editors and for a desperate but hopeless rearguard action of Brouwer.

Carathéodory had been considerably distressed during the whole affair; from the beginning he had been torn between his loyalty to Hilbert and his abhorrence of the injustice of Brouwer's dismissal. His efforts to mediate had only worsened the matter and the final solution was an immense relief to him. In a fit of despondency he wrote to Courant:[89] 'You cannot imagine how deeply worried I was during the last weeks. I envisioned the possibility that, after I had parted with Brouwer, the same thing would happen with all my other friends.' He had even considered accepting a chair at Stanford that was offered to him. In his answer Courant tried to set Carathéodory's mind at ease:[90] he believed that he had succeeded in convincing Hilbert that Carathéodory, in his position, could not have acted differently; the matter was settled now 'without fears of a residue of resentment on Hilbert's part.' Two days later he wrote that the night before he had discussed the whole matter with Hilbert, who had asked Courant to tell Carathéodory that 'he thinks that you would have done everything for him, as far as possible.' Hilbert was completely satisfied with the result of the undertaking, and in his opinion the *Annalen* were even better protected now than through his original dismissal of Brouwer '… and by and by it has become completely clear to me that in fact no personal motives have inspired Hilbert's first step, …' Carathéodory expressed his pleasure with Hilbert's views but he was not satisfied with Courant's evaluation of the motives behind Hilbert's move.[91] 'Now, he himself has given as the exclusive motive for his decision that he felt insulted by Brouwer; I would find it unworthy of him, to construe after the fact, that only impersonal motives had guided him.' This last remark could hardly be left unanswered by Courant. He had worked hard to pacify the participants in the affair, and here one of the former chief editors was lending support to the rumour that Hilbert was not completely devoid of some personal feelings of revenge. In an attempt to quench this source of dissent he and Bohr admonished Carathéodory. Courant calmly repeated his view and referred to Hilbert's personal statements that he 'fostered no personal feelings of hate, anger or insult against

[89] Carathéodory to Courant 12.XII.1928.

[90] Courant to Carathéodory 15.XII.1928.

[91] Carathéodory to Courant 19.XII.1928.

Brouwer.'[92] Even a bit of subtle pressure was brought to bear on Carathéodory: 'Our responsibility to Hilbert at this point is even greater, as he is not yet filled in on the development of the conflict; in particular he does not surmise your visit to Laren and the disconcerting report of it by Brouwer.'

Bohr was less subtle in his approach (same letter); if Carathéodory were not convinced of Hilbert's impersonal motives, he should ask Hilbert himself. 'For, that Hilbert—without being aware of it and without being able to defend himself—should first be considered "of unsound mind" and then "not to the point" (*unzurechnungsfähig* . . . *unsachlich*), that is a situation, that I, as a representative of Hilbert, cannot in the long run witness without action.' In spite of Bohr's sabre rattling Carathéodory stuck to his guns: 'To judge Hilbert's motives is a very complicated matter; I believe that I see through his motives because I have known his way of thinking for more than 25 years. It is true that the motivations that you indicate, and which H. also expounded in Bologna in discussion with Blumenthal, were there. The total complex of thoughts, that caused the explosion of feeling of October 15,[93] was much more complicated.'

Who was right, Courant and Bohr, or Carathéodory? The matter will probably never be completely settled. There is no doubt that the question of 'how to safeguard the Annalen from Brouwer's negative influence (real or imagined)' was uppermost in Hilbert's mind. But who is to say that no personal motives were involved? There are Hilbert's own statements to the effect that no personal grudge led to his action, e.g. to Blumenthal and Courant, but how much weight can be attached to them? In any case they contradict the letter of October 15.

Finally, there was the 'blackmail' note (see p.598), which indicates a strong emotion and vexation, if not more. And if Hilbert had personal motives, so what? Would we think less of a person if he were not the cardboard saint that some would prefer him to be?

Were Courant and Bohr themselves all that certain about Hilbert's motives? This question will probably never be answered. The available correspondence is not really informative, the lack of personal motives is systematically given credence by quoting Hilbert. They may have realized that, when they wrote 'Thus it is nothing less than a reconstruction after the fact, if one stresses now at the liquidation these objective motives, although the first step of Hilbert, made under such singular circumstances, could perhaps create another impression.'

The whole problem seemed to have been settled satisfactorily. Hilbert, who was only partially informed of the goings on, wrote to Blumenthal 'a triumphant letter, that everything was glorious.'[94] Courant had written a conciliatory letter to Brouwer in which he expressed the hope that the solution to the matter satisfied Brouwer. He also wished to convince Brouwer that no personal motives had played a role in Hilbert's action, and definitely no motives 'whose existence were in

[92] Courant to Carathéodory 23.XII.1928.

[93] Cf. letter to Einstein, October 15

[94] Blumenthal to Courant 31.XII.1928.

conflict with the respect for your scientific or moral personality.'[95] Little did he know Brouwer!

As a matter of fact Brouwer launched another appeal to the publisher and the editors the same day Courant was offering Brouwer the 'forgive-and-forget' advice. Brouwer insisted that in the interest of mathematics the total editorial board of the *Mathematische Annalen* should remain in function. As he realized that a written defence from his hand would inevitably wreck the unity of the editors, he was willing to postpone such a letter; moreover, Carathéodory, in a letter of December 3, had promised him to do his utmost to find an acceptable solution, and had begged him to be patient for a couple of more weeks. Sommerfeld had also pressed Brouwer to wait for Carathéodory's intervention. The final solution, as formulated in the Hilbert–Springer letter, did *not* satisfy Brouwer. He recognized that the reorganization of the *Annalen* was mostly, if not wholly, designed to get rid of him. Also, Brouwer had explicit views on the ideal organization of the *Annalen*. In a circular letter (23 January 1929) to the editors, Blumenthal and Hilbert excluded, Brouwer rejected the final solution. According to him, the *Mathematische Annalen* were a spiritual heritage, a collective property of the total editorial board. The chief editors were, so to speak, appointed by free election and they were merely representatives *vis a vis* the mathematical world. Thus, Brouwer argued, the contractual rights of the chief editors were not a personal but an endowed good. Hilbert and Blumenthal, in his view, had abstracted this good from their principals, and hence were guilty of embezzlement, even if this could by sheer accident not be dealt with by law (the reader may hear a faint echo of Brouwer's objections to the consistency programme, see p. 488, [Brouwer 1923b].). Brouwer then proceeded to attack Blumenthal's role in the *Annalen*. He repeated Blumenthal's earlier views on the equal rights of all editors and referred to certain irregularities in the management of the *Annalen* in 1925, when Blumenthal had committed an exceptionally strong infringement.[96] Brouwer had only given up his plan to call a meeting of the collective board to discuss Blumenthal's lapse, when Blumenthal made it clear that he planned to give up the position of managing editor no later than at the publication of volume 100. There is no information on this alleged intervention. Since 1925 was the year of the Riemann affair, it might have played a role.

There is concrete evidence that Blumenthal wanted to resign from the *Annalen* board. A stern letter from Hilbert to Blumenthal of November 18, 1925, opens with words that quelled all opposition: '*NEIN*, I do not agree at all with your plan.' Since the letter provides useful background information for the Annalen conflict, it is worthwhile, to reproduce it here in part.

> Already by itself, I would at the moment indeed follow the tendency to reduce the number of members of the editorial board, instead of increase

[95] Courant to Brouwer 23.XII.1928.

[96] The German term is '*Übergriff*', which could also be translated as 'impertinence'. In view of Blumenthal's reputation, that version seems implausible.

> it—including the chief editors, where, by the way, Springer should also have his say. When Cara once wanted to resign, I have—but only for the case that you would insist on an immediate replacement—mentioned who is by far the most important and most generous mathematician of his generation, Hecke as his successor; you turned him down with the argument that you, as a substitute for me, needed a real worker and manager for yourself, and at that point you were right. And now Weyl should be such a person? Weyl, who has a purely academic and luxury professorship, and for the rest lives for his scientific and literary activity, and for his health! And such a man should from Zürich conduct the business of the Annalen! His name on the title page were nothing but an honour from us for his person. [...] Thus I come to my main motive for rejecting your plan: if you lay down the managing of the Annalen, I would like to get it to Göttingen.

Hilbert wanted in fact to strengthen in Göttingen the connections with physics. But, he continued:

> Fortunately you are for the time being prepared to conduct the management, and it is my sincerest and warmest wish that you will do this for a long time to come. But I recommend you—in particular in your own interest not to let yourself be distracted from the trusted practice of your management by external influences, and I recommend a smooth development *without* editorial meetings, *without* changing editors, and *without* formal innovations.

And that was the end, Blumenthal had to stay on, whether he liked it or not. So, maybe Brouwer should not blame him for hanging on. The letter sheds light on a few details that bear on the previous pages. Much of what went on in Göttingen and Berlin was of course circulating in mathematicians' gossip. The problem is that this is hard to trace, and that it usually is coloured one way or another.

The *Annalen* were settling down under the new regime and due to a tactful handling of all publicity, the excitement in Germany was dying out, even, as Courant wrote to Hecke, among the colleagues in Berlin—and Brouwer was completely ignored. After waiting for months—and probably realizing that the battle was over and that everybody had gone home—he fired his parting shot, the letter of defence against Blumenthal's indictment of 16 November 1928. The letter is three and a half folio sheets long and contains a report of the events mentioned above, as experienced by Brouwer. The tone of the letter is bitter, Brouwer felt that he was let down by his supporters, 'To my astonishment and disappointment up to now no correction has followed from the other side, in spite of my challenge, of the false representations in Blumenthal's circular letter of 16.11.1928.' This applied most of all to Carathéodory, who had failed to straighten out Blumenthal's distorted interpretation of the facts about Carathéodory's visit. So there was no choice but to speak himself.

In the first place he denied Blumenthal's claim that Brouwer had substituted his own interpretation for Carathéodory's version of the developments leading to, and including, Hilbert's action. The views, he wrote, were not mine, but 'views that during the afore-mentioned visit, came up between Carathéodory and me in mutual agreement, i.e. that were successively uttered by one of us and accepted by the other.' He also elaborated the grounds for not acquiescing in the dismissal. He had told Carathéodory that

> he would consider a possible dismissal from the editorial board not only a revolting injustice, but also a serious damage to my possibility to function and, in the face of public opinion as an offending insult; that, if it really came to this unbelievable event, my honour and freedom of action could only be restored by the most extensive flight into public opinion.

At the end of the otherwise friendly visit the shadow returned, and the two parted shattered and in grief.

Reading Brouwer's report, one gets a gloomy impression of a meeting of two friends, confronted with a human tragedy they know they cannot prevent. It was an almost paradoxical situation; both knew that the request was an injustice, yet both knew that a decision had to be taken. In view of the role of Carathéodory's visit in the history of the conflict, Brouwer's report follows here:

> At his visit on 30.10.1928 Carathéodory informed me first of all, while the two letters lay unopened, that the 'fact of the greatest consequence, which was unknown to me' consisted of the following: recently the taking of a wrong medicine had brought out in Hilbert a situation of such a serious nature, that on the one hand 'he could no longer be taken serious at all' (Carathéodory's words)[97] and that on the other hand the slightest opposition to his will could be fatal for him. In this situation Hilbert got the idea, to remove me from the board of the Annalen, and wished to realize this idea by all means. It should be evident that the realization of Hilbert's plans would mean a scandalous injustice. In order not to jeopardize Hilbert's life, he (Carathéodory) begged me to take no action against it, for the time being. Hopefully Hilbert would soon return to the right medicine, and as a consequence of the improvement of his condition, regain better views, before anything definite could happen.
>
> One of the two unopened letters was Hilbert's. The message it contained, that Hilbert dismissed me as an editor, 'authorized by Blumenthal and Carathéodory', were not justified; for when he (Carathéodory) after his return from America, received a letter from Hilbert, asking for his

[97] [Brouwer's footnote] One might think for a moment, that the communication of such utterances carried something incorrect, because one assumes with respect to these naturally an atmosphere of confidentiality. This assumption of confidence, however, and the solidarity which it presumes, in as far as it is not invalidated by the end of the conversation, is certainly not compatible with Carathéodory's later silence in the face of Blumenthal's later false representations. Moreover, even justifiable scruples must take second place in the present case, where a discussion of wicked defamation and isolation of status is concerned—just as in the case of the question of a witness in a court of law.

authorization, he had answered that he would on principle not oppose Hilbert, but that he would come to Göttingen to discuss the matter. Arriving in Göttingen he had learned from Blumenthal, that Hilbert had dispatched the letter of dismissal, mentioning the above authorization.[98] In the following conversation of half an hour with Hilbert, the matter was mentioned between them, as little as it was until today. As far as the second letter was concerned (with Blumenthal's name on the cover as sender), this was written by him (Carathéodory), in which he begged me to withdraw of my free will from the editorial board, in consideration of the situation of Hilbert's health. He now regretted, however, to have written this letter.

Subsequently I have returned the second letter unopened to Carathéodory, and declared, that I would consider my possible removal from the board not only a scandalous injustice, but also a serious harming of my professional prospects, and in its public aspect, a despicable insult; that should it really come to this unheard of event, my honour and professional prospects could only be regained by means of the most far-reaching flight into publicity; as a consequence the crime practised against me would cause a public scandal—Carathéodory replied, that he had been prepared for such a position of mine, that in his expectation the Annalen would meet its doom over the realization of the plan hatched against me. And that he had already made the decision to resign from the editorial board, in which decision could for the time being—again out of consideration for Hilbert's health—not be carried out.

The further course of our conversation then came to the seven points in my circular letter of 5.11.1928.

Concerning the consideration with Hilbert's state of health, demanded by Carathéodory, I gave as my opinion, that if there were an immediate mortal danger for Hilbert, it would be a crime to assist him in ending his life with a crime; on the other hand that unreasonable tolerance would possibly only increase his sensibility and lust for power in a manner that would endanger his happiness in life. I promised, however, to consult on these latter psychological questions a suitable acquaintance. In case after further reflection my position would not change, the acceptability of Carathéodory's request, to undertake for the time being nothing against Hilbert's plans, would for me be equivalent to the probability of a cancellation of these plans even without my active interference. The conversation closed with Carathéodory's repeated reference to Hilbert's terrible state, and the words that he (Carathéodory), under these circumstances 'appealed to my mercy.'

During this conversation of two hours in the morning of October 30 Carathéodory's position was indeed that of a mediator, friend and ally,

[98] Only a thoroughly confused person would thus run the risk of a court case for falsification. If true, this supports Carathéodory's and Brouwer's view.

> who counselled me on the possibilities and means to prevent a calamity. The discussion seemed concluded in full agreement, in spite of the temporary differences in our evaluation of details of the situation. Carathéodory stayed accordingly some more hours at my house with some guests that I had invited at the occasion of his visit; all guests had the impression of a perfect mood. Only at the parting, when I was again alone with Carathéodory I mentioned a thought that came up at that moment, that, as Hilbert had survived Einstein's objections against his plans, he could also suffer without danger a repudiation of the authorization, mentioned without justification in his letter to me. Only when I did not get from Carathéodory a reply to question, but only exclamations such as 'What can one do' and 'I don't want to kill a person' (perhaps to be ascribed to the uneasiness of parting), amazement, uncertainty and irritation came up in me, which expressed themselves, under a complete change of mood at my side, in phrases like 'I cannot follow you any more', 'I consider this visit a farewell', and 'I am sorry for you.'

A fortnight later Brouwer visited Schmidt in Berlin. Schmidt's account roughly agreed with Carathéodory's, but he added one piece of information: Hilbert's anger was to a great extent caused by Brouwer's actions in the Riemann affair.

The second part of Brouwer's circular letter concerns Blumenthal and his indictment. He saw Blumenthal's hand in the action against him, for only Blumenthal had insight in the record of the individual editors. The objections listed by Blumenthal, 'could only degenerate into anecdotes, if one would ascribe them to Hilbert. He counts already for years so little as editor, that for a regular handling of the business, it had even been proved to be dangerous to submit manuscripts to him. Accordingly Hilbert does not try to mention these grounds in his well-known letter of dismissal.' As a reason for Blumenthal's alleged action, Brouwer mentioned the promise to resign at the completion of Volume 100, and the admonishments he repeatedly directed to Blumenthal in relation with arbitrariness and damaging actions in general.

In defence of Blumenthal, it must be acknowledged that in his unshakable loyalty to Hilbert he would probably go very far, but he would never seek to protect himself. Indeed, there is convincing evidence that Blumenthal almost immediately regretted his actions against Brouwer. Blumenthal was a man of high integrity, but in this case his *Doctorvater* overruled his conscience. That Blumenthal mismanaged the editorial procedure from time to time was a recognized fact; the Lebesgue note of 1911 is an example. But that would not have been a reason for ousting Brouwer.

Brouwer's refutation of Blumenthal's four points is more interesting, as it provides relevant information:

> Ad 1. There could very well be a reality corresponding to the word 'rude' (*schroff*), if the meaning of the word is fixed as follows: will for integrity (the duty of every human being), with in addition the will for clarity (destiny of the mathematicians).—These wills have manifested themselves with me if the honour and prestige of the Annalen was at stake (There

were, by the way, cases, where Blumenthal himself had called upon me). Then neither the vanity of the authors, nor Blumenthal's wish to please, could be spared—When I have occasionally carried my will through against that of the managing editor [Blumenthal], the latter has indeed found no support with his colleagues in the board, or has had reasons, not to look for it.

There is not much evidence in this case. There is hardly any correspondence between Brouwer and the authors left, but there are notes that show that Blumenthal was quite happy to use Brouwer as a trouble-shooter. Furthermore, a certain natural wish to ingratiate in Blumenthal cannot be denied. This may not be a recommendation for a managing editor.

Ad 2. The occasion Blumenthal hints at in his report on Klein's resignation, can hardly be anything but the following: I had a discussion with Klein about a paper that I had already handled, the author of which had appealed to Klein, as chief editor, concerning the changes I had demanded, and he had made him his views plausible in a personal discussion. In the conversation with me, Klein then saw that the author was wrong (not formally, as Blumenthal suggests, but contentual), and that he could as a consequence, not stick to the promise given to the author.[99] In the further course of his discussion Klein offered his view that the manner in which the chief editors were mentioned at the cover, apparently gave the public a misleading impression, and he for himself as far as it concerned his person, could not very well bear the responsibility for this impression.—Some time later he resigned as a chief editor.—Such a conduct does as much credit to Klein, as little as it does for Hilbert that, with on his part a much smaller contribution to the editorial activity, than Klein made at the time of his resignation, the opportunity existed, to exploit the internal weakness of his position for the external strengthening of it.

Ad.3. As I devoted yearly some thousand hours, it is almost obvious that manuscripts that had come in were usually for months in my possession. Only the word 'lay around' (*Lagern*) is misleading, for never papers were temporarily forgotten, or even missing (as happened with Hilbert), but they were always the subject of the most intensive editorial activity, by which their content was as a rule considerably influenced. As I moreover kept only in extremely exceptional cases, in which great deficiencies were found, manuscripts beyond the normal printing time limit, the papers were much better stored with me, than that they would have 'laid around' at Blumenthal's.—Blumenthal was, by the way, until shortly of the opinion that my procedure was normal and scrupulous, otherwise he would not so often appealed to me for refereeing, even of papers, for which the content of I could not in the least be considered an expert.

[99] The available evidence points at Mohrmann.

Brouwer's reply is, at least to us, completely convincing. A modern editor would be happy indeed if his referees were as punctual as Brouwer. Brouwer in fact had a reputation of being a most scrupulous editor, he would—in cases where it made sense—rethink the content of the paper, which resulted in considerable, and sometimes essential improvements. He mentioned the copying procedure below; this may have been the expression of a certain caution, adopted over the years. This meant a considerable extra work load, borne by Cor Jongejan, who was paid an assistant's salary for these and similar jobs.

> Ad 4. Although Blumenthal knows to give an example of my 'principle' to obtain a copy of every manuscript coming in, and although I think that this act is by itself the elementary right of the refereeing editor, I have since many years done so only then, when a paper seemed indeed acceptable, but fit for publication only after rewriting or considerable emendations. Then I considered it a duty in the face of mathematical history, and indeed because one must take into account the possibility of an unjustified reference to the date of submission.

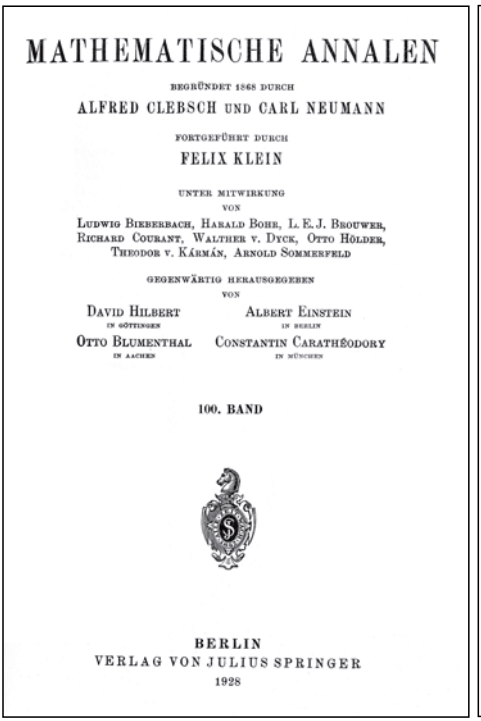
The circular ended with:

> I challenge Blumenthal to make public the Annalen correspondence, in particular with the complete correspondence between him and me. I claim that these documents will really refute those accusations of him against me.

In an open discussion in a free meeting of editors, Brouwer would undoubtedly have scored, but the days of Klein's 'academy-concept' were over. The *Annalen* had become a one-man enterprise; this one man happened to be a man of great prestige, and an outstanding mathematician (although already then some of his colleagues would prefer the past tense), but a man with absolute power is not so easily corrected, as history has taught.

Whatever one may think of the crisis management of Hilbert's lieutenants, Bohr and Courant, they had managed to outwit Brouwer. The whole matter was decided behind closed doors and the books were closed on Brouwer.

And so the curtain fell over one of the most tragic and completely unnecessary conflicts of twentieth century mathematics. It seems desirable to sum up the affair, but the motives of the invisible main actor were so confused that it is hard to come to a satisfactory conclusion. The available evidence tends to show that Hilbert's primary motivation was Brouwer's political activity and the Riemann affair. Although he nowhere explicitly says so, he felt it as a loss for the *Annalen* that French authors would keep their distance. The fear that was formulated in correspondence, that Brouwer would take over the *Annalen* after Hilbert's death, was rather a figment of Hilbert's imagination. It is not unlikely that Brouwer would have been pleased to move up to the chief editors, but that his influence would go any further than what a chief editor normally does, i.e. accept or reject papers, is not plausible. Even in his own faculty, his own mathematical association, his own academy, he had no excessive influence. So why would the *Annalen* be different? Some people have


MATHEMATISCHE ANNALEN

BEGRÜNDET 1868 DURCH
ALFRED CLEBSCH UND CARL NEUMANN

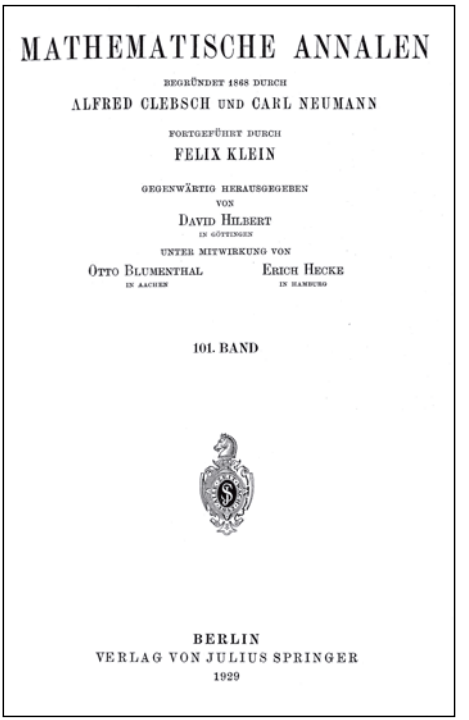
FORTGEFÜHRT DURCH
FELIX KLEIN

UNTER MITWIRKUNG
VON
LUDWIG BIEBERBACH, HARALD BOHR, L. E. J. BROUWER,
RICHARD COURANT, WALTHER v. DYCK, OTTO HÖLDER,
THEODOR v. KÁRMÁN, ARNOLD SOMMERFELD

GEGENWÄRTIG HERAUSGEGEBEN
VON
DAVID HILBERT — IN GÖTTINGEN
ALBERT EINSTEIN — IN BERLIN
OTTO BLUMENTHAL — IN AACHEN
CONSTANTIN CARATHÉODORY — IN MÜNCHEN

100. BAND

BERLIN
VERLAG VON JULIUS SPRINGER
1928


MATHEMATISCHE ANNALEN

BEGRÜNDET 1868 DURCH
ALFRED CLEBSCH UND CARL NEUMANN

FORTGEFÜHRT DURCH
FELIX KLEIN

GEGENWÄRTIG HERAUSGEGEBEN
VON
DAVID HILBERT
IN GÖTTINGEN
UNTER MITWIRKUNG VON
OTTO BLUMENTHAL — IN AACHEN
ERICH HECKE — IN HAMBURG

101. BAND

BERLIN
VERLAG VON JULIUS SPRINGER
1929

mentioned the danger that Brouwer would turn the *Annalen* into an intuitionistic bastion. The record shows that even his own journal, *Compositio Mathematica*, did not function as such.

There remains the matter of the *Grundlagenstreit*. Here we have to admit that Hilbert's emotions ran high. His aversion to this odious doctrine, with its by far too clever proponent—as witnessed, for example, by his refusal to quote Brouwer for any foundational contribution—certainly went beyond rational evaluation.

There is one more point that might have added to Hilbert's emotional outburst. David Rowe has pointed out that Brouwer's reminder of a forgotten credit, see p. 584, was the first public manifestation of this sort. There had been grumbling, before, but no one had the courage to put it in print. Here we may very well have an instance of the old-time German professor, who could not forget such a slur.

Reactions on the *Annalen* affair are scarce; of course the German mathematicians must have known that something was going on. A healthy dose of gossip is part of academic life. But it may be doubted that the facts were known, also across the borders of Germany. There is for example a letter from Ehrenfest to Van der Waerden from that period:[100]

[100] Ehrenfest to Van der Waerden 8.X.1928.

> It was by itself a great pleasure to be again in Göttingen, together with the wonderful Harald Bohr! In spite of all the sadness I actually had to laugh about this scandal bomb that exploded around the editors of the Mathematische Annalen. It looks almost as if the physicists are a more tolerant and humorous people then the mathematics-only people. They have such a terribly refined brain that it can tip over at any moment.

Alexandrov, when informed about the *Annalen* affair by Cor Jongejan, thought that 'such a dismissal is a terrible insult, and also an act that is hard to justify to the international mathematics.' But knowing both Brouwer and the Göttingers, he hesitated to take sides. 'Be that as it may, the whole matter is and remains most unpleasant, and it will hardly contribute to the moral prestige of the mathematicians in central Europe. But what I find most distressing, is that the painful, and for the collective mathematics in Germany, damaging, antagonism Göttingen – Berlin will be further intensified.'[101]

A later letter of Alexandrov to Hopf suggests that there were rumours about a boycott of the *Annalen* in protest against Brouwer's dismissal. That seems, however, never to have been a serious plan.

Looking back at the *Mathematische Annalen* affair and the surviving correspondence, and keeping in mind that before the final blow-up all the persons concerned, with the possible exception of Hilbert, had been living in the general atmosphere of friendship that was so characteristic for the international brotherhood of mathematicians, one gets a strong impression of regret and reluctance at the side of the managing and associated editors. Leaving aside Hilbert and Blumenthal, nobody was out to hurt Brouwer. The majority of the editors had at one time or another more or less close connections with Brouwer, and all of them were aware that he was a strict, perhaps difficult, but not unfair, colleague. Bohr and Courant, Hilbert's two lieutenants, for example, were doing their utmost to reach a clean, objective solution; they did not muster the zeal of Blumenthal, but looked for a minimal damage compromise, in the hope that Brouwer would play along and be so magnanimous as to swallow the insult that no cosmetic procedure could obscure for him.

Blumenthal was an exception; there had apparently been frictions between him and Brouwer before 1928, but the main source of his vehement denouncement of Brouwer—so uncharacteristic for the man of peace he was—was his unerring loyalty to Hilbert. Later in life, according to his daughter, he sincerely regretted the consequences of his actions.

Hilbert, the leading actor in this almost Shakespearean drama of the 'frogs and the mice', only played a role at the beginning and the end of the conflict. After his initial appearance in the role of defender of the realm, he was whisked backstage, only to reappear in the finale for the ceremony of signing the letters that sealed the conclusion of the conflict. There are no known reports of comments or reminiscences of Hilbert that shed more light on the episode or on his motivations. It

[101] Alexandrov to Hopf 6.XII.1928.

was as if for him the book on Brouwer was closed once and for all. As we have seen above, Hilbert told his closer associates at the time of the conflict that there was no personal element in his actions and statements, and most of his followers were happy to take his word for it. Nonetheless, one would probably do well to entertain a certain measure of doubt where motives and self-knowledge are concerned. A person who is the subject of unlimited admiration and trust is mostly not in a good position to do serious soul searching.

Brouwer, himself refused to publish further in the *Mathematische Annalen* and he convinced Heyting to follow his example. Heyting's paper was the outcome of a prize problem set by the *Wiskundig Genootschap* in 1927; it was formulated by Mannoury, cf. p. 545, and obviously endorsed by Brouwer. It may come as a surprise, but Brouwer was highly pleased with Heyting's formalization of intuitionistic logic, so much indeed that when Heyting had rewritten the material as a full blown paper, Brouwer expressed his appreciation in no uncertain terms, 'Your manuscript has interested me extraordinarily, and I am sorry that I have now to hasten to return it. In future I would appreciate it if you made a copy of your manuscripts before sending them to me, if at least you value a more than superficial reading. In the meantime I have already got such a high appreciation for your work, that I ask you to edit it in German for the Mathematische Annalen.'[102] At the time of writing Brouwer could not have foreseen what Hilbert had in store for him, but after the battle of the frogs and mice had run its course, he asked Heyting to withdraw his manuscript from the *Mathematische Annalen*. Two months later he wrote to Heyting that he had hoped that Blumenthal and Hilbert would show repentance and mend their ways before the summer was out, but that he had now given up hope.[103] Anticipating Heyting's understanding, he had decided 'not to leave the manuscript in the hands of those who call themselves falsely editors of the Mathematische Annalen.' He proposed to submit the paper to the Prussian Academy, where Bieberbach could handle it. As a consequence Heyting's historic contribution to logic appeared in the reports of the academy, which did not have the distribution and visibility of the *Annalen* by a long chalk.

That Brouwer was seriously impressed by the advances made by his student may further be illustrated by his letter to Weyl. Brouwer was proposing Heyting for a chair in Utrecht, and he asked Weyl to support his proposal.

> In Utrecht there is an important vacancy list of the mathematics faculty: Barrau, H.J.E. Beth, Schaake (all three insignificant). My (alphabetical) list: Heyting, Hurewicz, Van der Waerden (respectively intuitionist, topologist, algebraist). Heyting and Van der Waerden are Dutch, Hurewicz (my assistant) though a Polish citizen, and educated in Moscow and Vienna, is already for a long time in Holland. In order to document my list with the minister, I need foreign testimonials. For Heyting (so far my only really gifted intuitionistic student) you are the only eligible author of

[102] Brouwer to Heyting 17.VII.1928.
[103] Brouwer to Heyting 28.IX.1929.

> such a testimonial. Such a testimonial should on the one hand stress the general importance of intuitionistic research in the present stage of development of mathematics (in Holland nobody outside Amsterdam believes that), on the other hand it should qualify Heyting's papers (which are enclosed) as epoch-making.[104]

The letter also shows that Brouwer had few illusions about the acceptance of his programme in his own country; it must be said that he did little to convert his colleagues to his views. Brouwer was quite prepared to explain his views in lectures and papers, but he would not go round to practice empire building.

In 1929 there was another publication in the intuitionistic tradition: an intuitionisatic analysis of the game of chess by Max Euwe.[105] It was a paper in which the game was viewed as a spread (i.e. a tree with the various positions as nodes). Euwe carried out precise constructive estimates of various classes of games, and considered the influence of the rules for draws. When he wrote his paper he was not aware of the earlier literature of Zermelo and Dénès König. Von Neumann called his attention to these papers, and in a letter to Brouwer Von Neumann sketched a classical approach to the mathematics of chess, pointing out that it could easily be constructivized.

15.4 The endings of the *Grundlagenstreit*

The issues of the *Grundlagenstreit* were far from settled. In military terms, Brouwer had built an impregnable citadel, and Hilbert produced blueprints for an immense fortress.

Brouwer's intuitionism could only be defeated on philosophical grounds; his mathematical intuitionism consisted of honest mathematics, and there was no grand claim for which an account could be asked. Strictly speaking there was a master plan: the rebuilding of mathematics along intuitionistic lines. But that was an open-ended claim. One could not reject intuitionism, because it had, say, not yet proved the fixed point theorem for the unit square. In the first place, because the list of such challenges is endless, and none of them would be crucial. In the second place, the reply is flexible. In the case of the fixed point theorem, the intuitionist would say, I cannot give you the fixed-point theorem, because it is false in my mathematics, but I can give you our version, which is an ε-theorem, cf. p. 547.

In Hilbert's approach there was a crucial test: the consistency proof for, say, arithmetic by finitary means. In 1928 at the Bologna congress Hilbert 'saw the light at the end of the tunnel', that is to say, he had set up his machinery, and considered it just a matter of time before the remaining technicalities were carried out.

The Annalen affair rudely interrupted the conflict; Brouwer felt deeply insulted, and retired from the field. He did not give up his mathematics, but he simply became invisible. He no longer appeared at meetings of the DMV to report on intuitionistic mathematics. Even worse, he gave up publishing for a decade, there

[104] Brouwer to Weyl 16.II.1928.

[105] [Euwe 1929].

is evidence that he carried on his more reflective research, but the results remained restricted to such things as classroom notes, communications to Heyting and Freudenthal, etc.. His withdrawal from the debate did not mean a capitulation; on the contrary, he was firmly convinced of the soundness and correctness of his approach.

Looking back, one has to come to the conclusion that there never had been a discussion between Brouwer and Hilbert on the essential points. One doubts if the two ever had a private conversation on the foundations, with the exception of the Scheveningen walks. It is very likely that Hilbert never read Brouwer's basic papers, such as the 'domains-of-functions'-paper (although he probably attended his lecture in Göttingen in 1926, and the paper was a contribution to the Riemann volume). Indeed, Bernays asserted in an interview, 'he has not read these things at all.'[106] All of Hilbert's attacks at Brouwer consisted of rather superficial comments on hearsay bits of Brouwer's repertoire. Brouwer, on the other hand, repeatedly put his finger on the crucial spots of Hilbert's programme; (1) consistency of induction requires induction (siding with Poincaré), (2) consistency does not prove existence. Hilbert was in fact encouraging work on (1), and he did not see the point of (2).

The historic facts were in fact all speaking for Hilbert; he had every right to be optimistic about the success of his programme. There was considerable activity in his proof theory, and there was significant work of Ackermann (1925, 1928), Herbrand (1928–1931), Von Neumann (1927).

Yet Hilbert's programme foundered when in 1930 a young man with his soft voice announced that formalized classical mathematics was incomplete. This stunning event took place at the *Tagung für Erkenntnislehre der exakten Wissenschaften* in Königsberg. At this meeting (which was embedded in the Meeting of the Natural Sciences and Medicine)[107] an exchange of ideas between the 'big three' of the foundations of mathematics was arranged. Rudolph Carnap spoke for the logicists, Arend Heyting for the intuitionists, and Johann von Neumann for the formalists.

Reidemeister, who was in charge of this meeting, wrote to Heyting that neither Brouwer, Hilbert, nor Russell were invited as speakers. This was a meeting where the younger researchers presented their work, so it seemed more appropriate not to invite the big names. Apparently Heyting was worried that Brouwer might feel passed over. Reidemeister also begged Heyting not to be upset that Hilbert was going to be present; he was invited by the *Naturforscher*. In view of Hilbert's unusual lecture in Bologna, von Neumann was definitely a better choice. It was a conference of the second generation, and as a consequence the tone was friendly and objective.

The surprise of the meeting was Gödel's contribution to the discussion at the end. He had already presented a talk, which dealt with his completeness theorem; this solved one of Hilbert's 1928 problems. His contribution to the discussion at the closing session, however, spelled disaster for Hilbert's programme. It said, roughly,

[106] *Er hat die Sachen überhaupt nicht gelesen*. Interview Bernays 18.VII.1977.

[107] To be precise, the 6th *Deutsche Physiker- und Mathematikertagung*, and the 91th *Versammlung der Gesellschaft Deutscher Naturforscher und Ärzte*. September 5–7, 1930 Königsberg. Proceedings published in *Erkenntnis* (23), 1931.

that there were simple statements, comparable to Goldbach's conjecture, that were neither provable in classical mathematics, nor refutable. In symbols, statements A such that $T \nvdash A$ and $T \nvdash \neg A$, where T is a suitable mathematical theory, for example arithmetic.[108]

This was a dramatic moment—the end of a programme, but what a magnificent end! Hilbert, however, missed it, as he was whisked off in a taxi to the radio studio, where he was expected for a broadcast of his '*Logik und Naturerkenntnis*' (Logic and the understanding of nature). According to contemporary sources, Hilbert learned about this stroke of fate only months later; no one had the courage to disappoint the old master. When he learnt about Gödel's work, he was angry.

Bernays had heard about Gödel's result through the grapevine; he wrote to Gödel, asking for advance information in the form of proof sheets.[109] One may thus take it that Hilbert was not aware of the results either. This would be consistent with the paper that Hilbert published after the Königsberg meeting, namely 'The Founding of elementary number theory' (submitted 12.12.1930).[110] In this paper Hilbert upheld his programme as if nothing had happened. He did, however, introduce in his paper a novelty the ω-rule:

$$\frac{A(0), A(1), A(2), \ldots, A(n) \ldots}{\forall x A(x)}$$

In words: if you have derived $A(0), A(1), A(2), \ldots, A(n) \ldots$, then you have also derived $\forall x A(x)$. This could have been reaction to Gödel's theorem, but as he considered the ω-rule to be a finitary rule (p. 491), the system would be finitary and fall under Gödel's theorem. There is no reference to Gödel in this paper, but that does not say much in view of Hilbert's reference practice. The paper was the report of a talk before the philosophical society in Hamburg, in December 1930.

Hilbert's next, and last, paper, 'Proof of the Tertium non datur' (submitted 17.7.1934), does not mention Gödel either, but as it is more modest in scope, one may guess that Hilbert revised his perspective.

Gödel published his epochal paper in the Austrian mathematic journal, the *Monatshefte für Mathematik und Physik* in 1931. In the same year the *Erkenntnis* issue with the lectures of the Königsberg meeting appeared with a brief synopsis of his incompleteness results. This time it included the second incompleteness theorem, 'For a system in which all finitary forms of proof are formalised, a finitary consistency proof is not possible.'

Immediately after Gödel's talk John von Neumann had collared Gödel. Von Neumann, who was reputedly the fastest thinker of his generation, had right away seen what was going on, and he had realized that Gödel's argument could be

[108] *Erkenntniss*, 1931, p. 147, 148.

[109] Bernays to Gödel 10.XII.1930, cf. [Dawson 1997], p. 282.

[110] [Hilbert 1930].

'internalized.' This then would yield the unprovability of the consistency of (say) arithmetic in arithmetic.[111]

Gödel's incompleteness theorems brought the second ending of the *Grundlagenstreit*. Where Hilbert had won the conflict in the social sense, he had lost it in the scientific sense.

What could be learned from the history of the *Grundlagenstreit*? In fact little, the whole matter was in the real sense a struggle of Titans. Both Brouwer and Hilbert were exceptionally gifted, be it that their characters were almost complete opposite. The older man was totally engrossed in mathematics—the paradigm of the German professor as the icon of traditional *Bürgertum*, the younger man equally infatuated with mathematics, but without the compulsion, who was always glad to change the three piece suit of the professor for the bohemian outfit that belonged to his free and artistic way of life, complete with numerous love affairs. The fact that their philosophies of mathematics were so different, undoubtedly played a role, but should not be overestimated. Those who had followed Hilbert's evolution from the first steps in 1904 to the middle twenties must have seen that he sought safety for mathematics in the realm of the finitary, and that whatever intuitionism might be, it certainly encompassed Hilbert's finitary mathematics. Abraham Fraenkel who had observed the developments closely, was quoted in the *Vossische Zeitung*, 'With charming wit Fraenkel called Hilbert the second intuitionist','[112] and in his 'Ten Lectures on Set Theory' Fraenkel noted that 'one could even call him an intuitionist.'[113]

Brouwer had come to the same conclusion in his 'Intuitionistic Reflections on Formalism'. If anything, Hilbert could rather accuse Brouwer of surpassing the limits of finitary mathematics. It is doubtful whether he was aware of this, although Bernays wrote him in 1925 that 'he had discovered a certain difference between the finitary position and that of Brouwer.[114] In 1977 Bernays said that he had grasped the difference between 'finitistic' and 'intuitionistic' through the Gödel translation. Taking into account the cited letter, the plausible reading seems to be 'the Gödel translation confirmed my earler view.'

As Brouwer had put it, after acknowledging a few foundational insights, the whole 'intuitionism–formalism' matter became a matter of taste. So why the incessant harping on Brouwer? Even in his last paper but one, Hilbert repeated his complaints: 'Nonetheless there are even today followers of Kronecker, who do not believe in the Tertium non datur: it is well by far one of the crassest disbeliefs that we find in the history of mankind.'[115] Since the paper has the character (and

[111] Gödel called this the Von Neumann conjecture: 'If there is a finitary consistency proof at all, then it can be formalized. Then Gödel's theorem yields the impossibility of a consistency proof at all' (15.1.1931). See also [Dawson 1997], p.68 ff.

[112] 8.X.1929.

[113] [Fraenkel 1927], p. 154.

[114] Bernays to Hilbert 25.X.1925.

[115] [Hilbert 1930].

hence the precision) of a popular lecture, one should not worry too much about the details, but here are a few noteworthy statements:

– The a priori is nothing more nor less than a basic disposition, which I would like to call the finitary disposition.
– Kronecker has clearly enunciated the view, and illustrated it by means of numerous examples, which nowadays coincides essentially with our finitary disposition.
– The problem of the foundations of mathematics is, as I believe, definitely dispelled by proof theory.

So we note that Hilbert's a priori is even more restricted than Brouwer's of 1907. Furthermore Hilbert made after all these years his peace with Kronecker; in Hilbert's hindsight Kronecker was, foundationally speaking, a forerunner of himself, who did not go all the way. One guesses that for a similar rehabilitation of Brouwer it was too early. Moreover, Brouwer was still alive, so, in contrast to Kronecker, he could possibly protest.

All great men, with great ideas, attract criticism; one may think of Cantor, Einstein, Gödel. Sometimes (and perhaps often) the criticism is offered by persons halfway between crank and amateur, sometimes by colleagues, who lack the imagination or intuition to appreciate novel circumstances or principles.[116] Reading Hilbert's foundational papers, one gets a strong impression that he did not suffer the criticism (of fools?) gladly. In the majority of these papers he complained about all those people who held obscure views and could not see the point of his programme. His final reaction was, 'The critics of my theory should point to me exactly the place where my alleged error is to be found. Otherwise I decline to check his line of thought.'[117] A definite exasperation speaks from these lines, but was it reasonable? Assume someone were to claim to have proved that after $10^{10^{10}}$ there are no more primes. Would he not rather say, go and study your Euclid, than check the long and intricate proof. The ultimate consequence of the above viewpoint would be that Hilbert had to reject Gödel's criticism of his programme, for Gödel did not carry out a proof check of Hilbert's (non-existent!) proof. Apparently, in his emotions, he allowed himself to say things that he could not possibly mean.

The reader may have got a somewhat negative impression of Hilbert's proof theory. This should be corrected at once, it is a fact that Hilbert gave the world a new discipline of great finesse; after Gentzen added beauty and structure to it, which it was so sadly lacking, it became an elegant and powerful mathematical tool. In modern logic it occupies an important place, being the tool *par excellence* to bring out the finer details of complexity and structure. It also happens to be the part of logic that is of considerable interest to computer science. That Hilbert saw it primarily as a prop for settling all fundamental questions about the foundations of

[116] For an instructive discussion of a particular instance of perpetual criticism, see [Hodges 1998].
[117] [Hilbert 1931].

mathematics, was simply a matter of a lack of experience in topics of this complexity and intricacy. It was a gift from Hilbert to mathematics, comparable to the rich gifts he had already made. From our modern point of view it is surprising that a man like Hilbert, with a gift for sweeping methods, should not have been the first model theorist instead of the first proof theorist. In the light of the exacting demands that model theory made where intuition was concerned, proof theory was indeed more accessible to traditional mathematical techniques.

So far it seems as if the *Grundlagenstreit* was some sort of personal quarrel between Hilbert and Brouwer. What were the reactions from the mathematical community? From the isolated comments in correspondence or in print, one might easily get the impression that the foundational issues, including the surrounding gossip, went largely unnoticed. It would be a mistake, however, to conclude that the conflict was a private affair between a few prominent mathematicians. Hilbert's lectures and publications always attracted a good deal of attention, and although most readers would not be in a position to judge the merits of Hilbert's programme, they would certainly read Hilbert's diatribes. And on the other side Brouwer in his soft-voiced lectures could capture large audiences. His Berlin lectures had exerted a strong fascination on the audience. In contrast to Hilbert's more aggressive approach, Brouwer's lectures made their impression through a mixture of persuasive argument, wit, vigorous proof and cynical metaphor. The reports of these lectures leave no doubt that Brouwer could recruit and inspire a not inconsiderable following. Nonetheless there are few explicit references to the foundational crisis. The man who did most to enlighten his contemporaries in the confused matter of the foundations was Fraenkel; his books and papers covered the basic issues of the period. His expertise in foundational matters was widely recognized by his colleagues. Hausdorff, after reading Fraenkel's *Ten lectures on set theory*,[118] spoke his mind without reservation, 'I still nurse hopes that you, as the best expert in this literature, will at some time launch a vigorous and witty attack at intuitionism—although it might be more advisable to let this castrates' mathematics suffocate in its own complicate obtuseness. It is indeed stupid to stick into every mathematical theorem, like the egg of an ichneumon wasp, an unknown number fabricated out of the decimal expansion of π ....'[119] Study's reaction to Fraenkel's book was one of relief, 'I cannot find the outcry of Brouwer and Weyl, and also that of Hilbert, as remarkable as you and others do.'[120] The function theorist Konrad Knopp shared this sentiment; after the publication of Fraenkel's *Introduction to set theory*[121] of 1924, he confided, 'I am glad to draw from it the confirmation of my feeling, that the "shake-up of the foundations" by Brouwer was by no means as disastrous as it seemed to be.'

[118] *Zehn Vorlesungen über die Grundlegung der Mengenlehre*, 1927.

[119] Hausdorff to Fraenkel 20.II.1927.

[120] Study to Fraenkel 5.III.1927.

[121] *Einleitung in die Mengenlehre*.

Hausdorff was a implacable enemy of intuitionism; he was not prepared to relent, as appears from another letter to Fraenkel, in reply to the receipt of a copy of Fraenkel's 1928 edition of the *Introduction to set theory*, 'I hope that it will leave my heartfelt aversion to intuitionism untouched.'[122]

The geometer Finsler had in his inaugural address in 1922, *Are there contradictions in mathematics?*, noted that Brouwer and Weyl rejected in principle the excluded middle. He generously granted that 'such assumptions may in themselves lead to very interesting research', but, he continued, 'an exact science cannot be based on these, not mentioning the great complications that would thus arise; also many of the most certain results must be given up.'[123] A positive appreciation of intuitionism is to be found with Ludwig Bieberbach. He had earlier deplored the decline of the intuitive approach to mathematics as advocated and practiced by Felix Klein, but the propagation of formalism by Hilbert and his school had made him aware that there was a real danger that formalist tendencies could seriously harm mathematics, in the sense that intuitive and applied mathematics could become a neglected, if not extinct, part of mathematics. He sensed in Brouwer's intuitionism a healthy antidote against the formalist epidemic. In an address for an audience of mathematics teachers he stressed the importance of Brouwer's intuitionism—including choice sequences.[124] 'There is a fresh breath of air thanks to intuitionism', he called out. In how far Bieberbach was right, is debatable. After all in Hilbert's Göttingen applied mathematics flourished, e.g. in the hands of Courant. And Hilbert occupied himself intensely with theoretical physics (albeit with formalization in mind). But undeniably, there certainly was strong influence of Hilbert's formalist doctrines on the theoretical and methodological level.

Spectators, who observed the foundational conflict with more detachment from some distance, often saw clearer that this middle European muddle of loyalty, common sense and foundational ingenuity, was a serious matter, and not just a clash of personalities. As the American mathematician Pierpont observed

> And yet when one hears one of the greatest living mathematicians calmly telling the world that a considerable part of our analysis is devoid of proof, if not nonsense, and when one beholds the mighty efforts which the champions of Weierstrass are making to repel these attacks, it is reasonable, in view of such facts, to ask ourselves, 'Is all well?'[125]

The German philosopher Grelling described the *Grundlagenstreit* pointedly:

> If intuitionism has been characterized with a certain propriety as revolutionists who overturned the ancien régime, Hilbert might be compared with Napoleon who, without regard for considerations of legitimacy,

[122] Hausdorff to Fraenkel 19.XI.1928.

[123] [Finsler 1925].

[124] About the scientific ordeal of mathematicians *(Vom Wissenschafts ideal der Mathematiker.)*1926.

[125] [Pierpont 1928], p. 37.

> established, through a brilliant political stroke, a new order whose success is the substitute for legitimacy.'[126]

The working mathematician, more concerned with practicing mathematics than with reflecting on its soundness, could not stop wondering why Brouwer, the wizard of topology, had given up the riches of traditional mathematics for a life in the arid desert of the foundations. André Weil, who attended some of Brouwer's Lectures in Berlin,[127] wrote to Fréchet:

> people here are very excited at the moment, because Brouwer has just arrived and he has started a series of lectures, not on topology, but on intuitionistic mathematics; this is very particular, as you know. I have no pretensions to understand it, but Brouwer is a very interesting man. He has declared in his first lecture that the principle of the excluded third is a superstition which is about to disappear. It is a pity that such a remarkable man devotes himself exclusively to such bizarre things.[128]

Weil's view was shared, and is still shared, by many mathematicians all over the world. The often somewhat esoteric issues of the foundations of mathematics have traditionally met with tolerance at best, and distrust at worst.

15.5 The Menger conflict

The upheaval of the Bologna conference had only just become history when Brouwer had to take up arms again. This time he was confronted with a conflict that had been simmering slowly and systematically—the priority over the definition of dimension.

We have seen that Brouwer guarded the rights of Urysohn and himself as a true Cerberus. Thanks to the harmonious deliberations with Urysohn a possible conflict between them had been resolved before it could erupt. Urysohn was such a superb topologist that he could indeed imagine, and hence believe, that Brouwer had made a slip of the pen. One would, so to speak, be oneself convinced that there could only be one sensible interpretation of 'connected' in Brouwer's separation definition, and one would have to make the right psychological assessment of Brouwer's mental make-up.

On both counts Urysohn was more than satisfied, and so in Brouwer's book the situation was simple:

1. the first correct definition of dimension had been given by him and it had been put to the test by showing that it gave the right answer for Euclidean spaces.

[126] [Grelling 1928].

[127] Cf. p. 545.

[128] Weil to Fréchet, 31.I.1927.

2. Urysohn and Menger had subsequently discovered independently of each other and of Brouwer their notion of dimension.
3. Urysohn published his definition and results before Menger.[129]

In March 1924 Urysohn had not yet seen Menger's publication, cf. p. 472, so he definitely was not a party (not even posthumously) to the priority conflict, and Alexandrov had been a permanent witness to everything that went on during the last weeks of Urysohn's life. He would have observed any irregularities, had Urysohn changed his mind on Brouwer's role.

The slowly mounting pressure of Brouwer's excessive care for his and Urysohn's rights and the feeling of helplessness in the face of the cooperation between Brouwer and Alexandrov must have taken its toll on Menger, who could not be said to lack ambition. Slowly he must have reached the decision to put Brouwer in his proper place. Since he was writing the first overall account of dimension theory, the opportunity was there: simply describe matters according to the facts as he saw them, and not according to the 'Brouwer–Alexandrov fiction.'

In the summer of 1928 Menger added the final touches to his book *Dimensionstheorie*, and in September it appeared. The flyer, which announced the publication,[130] did nothing to allay Brouwer's worst expectations. After extolling the importance of the subject—'In Menger's book one of the oldest and most difficult questions of mathematics, the dimension problem (also significant for philosophy) is treated'—. The leaflet ended with:

> These and numerous other questions are answered by the dimension theory, founded by Menger and Urysohn ....[131]

A curious statement, and—from the publishers point of view—an unwise statement. Brouwer's name would certainly have increased the advertisement value. Usually these flyers are written, or at least designed, by authors; if Menger wrote this text, he was like the brave villager taunting the dragon. If he did not write it, the anonymous author at the publishing house could hardly be blamed, one could indeed not read from the text what Brouwer had to do with the theory. Brouwer must have obtained a copy soon after the publication for already on September 29 he presented a rebuttal of Menger's historical views on the subject to the Academy in Amsterdam.

The reader who wanted to find out what Menger actually did say about the credits in dimension theory was confronted with a novelty. Menger initiated the tradition of collecting the historical remarks in a paragraph at the end of each

[129] The reading of this section presupposes parts of earlier chapters. In particular chapter 12 and section 13.5 See the pages 451, 454, 454, 457, 458.

[130] There is a copy in the Brouwer Archive.

[131] In this section we will repeatedly refer to Menger's publications on topology and dimension theory. The relevant ones are [Menger 1926d, Menger 1923, Menger 1924b, Menger 1924c, Menger 1924a, Menger 1925a, Menger 1925b, Menger 1926a, Menger 1926b, Menger 1926c, Menger 1927, Menger 1927a, Menger 1927b, Menger 1927c, Menger 1927d, Menger 1928, Menger 1929, Menger 1930, Menger 1932a, Menger 1932, Menger 1932c, Menger 1933, Menger 1979, Menger 1994].

section. It definitely added to the readability; the example has found followers in more recent times. He had intentionally avoided all footnotes, to break with Brouwer's footnote obsession.[132]

The particular historical paragraph that concerns us here is at the end of II.2, *The notion of dimension*. It does not contain incorrect details, but, as Freudenthal pointed out later,[133] Menger arranged the material in such a way that the uninitiated could hardly avoid the impression that Menger came in first, followed at a short distance by Urysohn. The note opens with

> In a note submitted by me in February 1922 to the *Monatsheften für Math. und Phys.* (printed in *Proc. Acad. Amsterdam* 29, S. 1124),. . .'

to be followed on the next page by:

> In September 1922 Urysohn published (*Comptes Rendus* 175, S. 440). . .

A reader who has no reason to question the intentions of the author (and why should he?), cannot see that Menger's note was not published until 1926, nor can he guess why a paper submitted in Vienna is printed in Amsterdam, but he can see that February 1922 precedes September 1922.

After commenting on the definitions given by himself and Urysohn, he went on to list the predecessors in the quest for a correct dimension notion: Euclid, Cantor, Poincaré and Brouwer. On page 85 Brouwer's definition of 1913 is reproduced verbatim and it is pointed out that 'if one replaces in the first place Brouwer's separation definition by the following, which has emerged in the meantime, . . .,' and moreover redefines the notion of 0-dimensional, one gets the right 'dimension concept —'This notion of general degree of dimensionality already comes quite close to the notion of dimension that underlies the theory of dimension.'

If Menger wanted to insult Brouwer, then this passage would do very well; the patronizing tone, usually reserved to pat a harmless colleague, who did not make it, on the shoulder, would be enough to drive anybody into a state of extreme indignation. On the next page Menger characterized Brouwer's paper 'Remarks on the notion of natural dimension'[134] as a second attempt to provide a definition of dimension—this time with the right notion of separation, and with the right result.

Hurewicz, who had remained in Amsterdam, was an involuntary witness to Brouwer's emotional outbursts, following the publication of Menger's book. From his letters to his friend Karl, it appears that the situation in Amsterdam had definitely become unpleasant. Brouwer would of course not blame Hurewicz for the actions of Menger, but he would fly into violent harangues on Menger and his views on dimension theory. Although Hurewicz was a close friend and associate of Menger, he could not approve the manner in which Brouwer was treated in the historical section.

[132] [Menger 1979] p. 248.

[133] Brouwer, CW II, p. 565.

[134] [Brouwer 1924b].

> What you write about Brouwer, about his character and his 'morals', is nothing new for me. Nonetheless I find that your conduct with respect to him is wrong and inappropriate. As to the matter itself, in my opinion the historical survey in your book is not completely free from a subjective representation.

Why didn't you send the proofs to Brouwer, so that he could have reacted, Hurewicz continued. The matter would now only cost time and energy, and the satisfaction of the beautiful book would be spoilt. 'A "life and death" struggle because of these foolishnesses! But that you have not for a minute given a thought to the impossible situation into which you have brought me, I very much hold against you.'[135] In another letter he wrote, 'As before, it is still my opinion that Brouwer, even if he were not Brouwer but a just and sensible man, must have felt your action as an insult.'

There were more points where Menger could be found guilty of a lack of tact. On page 243, for example, he took a novel look at the invariance of dimension; he viewed this particular theorem of Brouwer as simply a theorem in, say, analysis, or topology of Euclidean space—*not* as a theorem of dimension theory. If the canonical notion of dimension were not a topological invariant, so much the worse for topological mappings. Of course, Menger did not set out to belittle Brouwer's invariance of dimension proof, but he certainly knew how to sail close to the wind when it came to innuendo. His statement had a flavour of 'invariance of dimension is a very nice property, but if it had not been the case, mathematics would not have been the worse for it.' This would not reflect on the ingenuity of Brouwer's proof, but it would suggest that Brouwer derived his fame from a rather accidental feature of geometry.

It was, however, in particular Menger's historical comments on the emergence of the notion of dimension that hurt; to be bundled with Euclid, Cantor and Poincaré into the group of forerunners was no compliment for the man who presented the first tenable definition of dimension. Was Menger putting Brouwer in his place as a repayment for the real or imagined mistrust at the end of his stay in Amsterdam? His comments in 'My memories of L.E.J. Brouwer' strongly suggest that by the time Menger left for Vienna, he felt uncomfortable, if not humiliated, by Brouwer's policy on credits, and his fixation on precise formulations. Whatever he had in mind, he could not expect to get away with it unscathed.

To make it clear beyond doubt that Brouwer was not part of the history of dimension theory proper, Menger went on, 'From the prehistory of dimension theory, finally [sic], Sierpinski should be mentioned . . . '

The above brief history of dimension theory is in curious contrast to Menger's earlier reports on the topic. In 1926, he wrote 'The second recursive definition came first. It was given in 1913 by Brouwer (where it only differed from the above definition by basing it on the discontinuous sets as 0-dimensional) and justified by

[135] Hurewicz to Menger 10.X.1928.

Karl Menger. (Courtesy Mrs. Eve Menger)

the proof that every set of R_n, which contains an open subset, is n-dimensional,'[136] and 'Also in the direction indicated here, Brouwer, following Poincaré, has given a definition of n-dimensional continua, which is equivalent to the above definition of n-dimensional continuum (*Crelles Journ. f. Math.* 142, S. 146 bis 152; cf. also the correction of a clerical mistake (*Schreibfehler*) in the Proc. Ac. Amsterdam, XXVII, S. 796).'[137]

In his 'Memories' of 1979, Menger returned to this discrepancy; he explained that at first he had just repeated Brouwer's statements concerning the slip of the pen, but that when he started to read Brouwer's 1913 paper thoroughly, he became convinced that this was not correct—'Actually Brouwer had not made a slip at all,

[136] [Menger 1926a], p. 121.

[137] [Menger 1924c], p. 139.

but had formulated a perfectly valid definition—though not quite the one on which Urysohn and I had built our theories.'[138] Indeed, in 1979 Menger had mellowed to the extent that he wrote 'As to Brouwer, I should have more strongly emphasized … that in correcting Poincaré's definition he formulated the first tenable general dimension definition.'[139] Menger was partly right, and Brouwer had explicitly said as much in his *Remarks on the natural dimension* and in his 'historiography' paper. One can build two distinct dimension notions on the two distinct notions of 'connected', called weak and strong dimension by Brouwer, cf. p. 461. But Brouwer had not intended the strong (i.e. wrong) dimension; we have seen that there is convincing evidence for the 'slip of the pen'.

One wonders why Brouwer had not shown his evidence, or sent a certified copy of it to Menger. There were two reasons: in the first place, Menger could have refused to accept the evidence—leaving Brouwer feeling rather foolish; in the second place, Brouwer had laid down strict norms for himself with respect to acceptable evidence and he could not very well violate them. He explained this to Hahn, who was attempting to mediate between Brouwer and Menger, that:

> …with respect to scientific historiography only such documents can be considered pure, thus can be printed by an author, which either have been written and dated by, if possible uninvolved, scholars, or if they come from the author himself, have been in the custody of other, still living, if possible uninvolved, scholars, who declare that they have been uninterruptedly from the time of their originating to the time of publication in their possession. One's own papers, that have been, if only for a short while, in one's own possession, can never, not even a little bit, be counted as a proof, as long as there is not at the same time an official report, in which on the basis of an analysis of the ink, the age of every page and every change has been established (assuming that this is possible).[140]

According to this criterion of purity (*Reinheits Kriterium*), he explained, he could not enter as evidence the correction inserted in March 1913 in his private copy (*Handexemplar*) of the dimension paper, nor the correction made in the proofs of Schoenflies' book.

Thus Brouwer was giving up the only written evidence available in order to fall back on the argument that 'any informed reader could immediately see that the adjective "closed" had inadvertently slipped in.' He must have had unlimited trust in the intelligence of the informed reader of 1913!

Brouwer's first reaction to Menger's book appeared in a paper in the proceedings of the Amsterdam Academy, 'On the historiography of dimension theory'.[141] In §1 he discussed his own work, in §2 that of Urysohn, Menger and Alexandrov;

[138] [Menger 1924c], p. 252. See also p. 443.

[139] [Menger 1994], p. 248.

[140] Brouwer to Hahn, 4.VIII.1929.

[141] [Brouwer 1928d].

the remaining sections were devoted to criticism of Menger's book. The paper opened boldly:

> I have founded the dimension theory in the paper 'On the notion of natural dimension' (...) for those species,[142] for which the dimension theory has a natural sense, namely for the condensed metric species, by giving a *dimensional justification theorem*, that is, I introduced a notion of dimension, meaningful in the class of condensed metric species, for which I could show that the n-dimensional manifolds had dimension n.[143]

From this, he went on to point out that there were two notions of connectedness—the (usual) notion formulated by Lennes and the old notion from Schoenflies (called resp. 'weak' and 'strong' by Brouwer), yielding two notions of separation and hence two notions of dimension. The weak notion was used in the 1913 paper (as would be clear to any reader who followed the argument),[144] but the strong one had slipped in via a clerical error.[145]

In Menger's eyes Brouwer could only be considered a precursor of dimension theory, as he had not gone beyond the definition of dimension and the justification theorem: 'It only came to the construction of an extensive dimension theory, when the fertile first definition was formulated by me,[146] and in an equivalent form by Urysohn, allowed this.'[147] Brouwer answered this claim by pointing out that he had intentionally abandoned further investigations since the plausible topics (one may think, for example, of the sum- and decomposition theorems) ran into serious intuitionistic complications. Brouwer's argument is more convincing than it seems now. One should keep in mind that in 1913 virtually no intuitionistic mathematics existed—there was no systematic exploration, only isolated results.

Menger, on the other hand had a somewhat doubtful point to score; the distinction between a precursor and a founder is tricky, to say the least. Roughly speaking, a precursor introduces a notion or a topic that has some of the characteristics of the final notion (the idea requires further investigation, it may be even partly wrong). Brouwer does not fit this description, he had the right notion and even showed that, in the traditional setting, it was the right one. It seems a bit finicky to split notions so finely, but one would be justified to say that Brouwer was the founder of dimension theory, although not the creator. Menger's definition of 'precursor' may have been just an academic point, but it certainly lacked taste to apply the term to Brouwer, in particular to award him a shared second place with Euclid.

[142] i.e. intuitionistic sets.

[143] The intuitionistic terminology did not occur in the original 1913 paper; Brouwer used it in his writings from 1924 onwards. 'Condensed metric space' means in traditional terms 'locally compact metric space'. Brouwer's intuitionistic dimension paper dealt with compact metric spaces.

[144] *jedem mitdenkenden Leser*

[145] Cf. p. 454.

[146] Menger's definition was based on boundaries of neighbourhoods, rather than separation.

[147] [Menger 1926a].

Menger's chronology of the development of dimension theory diametrically opposed that of Brouwer, as Brouwer pointed out;[148] furthermore he protested against Menger's suppression of the correction of the slip of the pen and his claim that the notion of 'connected' that Brouwer purported to use (marred by the *Schreibfehler*) did not exist at the time. The first complaint can be understood in view of Menger's earlier recognition of the slip of the pen the second one is a serious one since it can hardly be viewed as anything but an accusation of fraudulent claims of Brouwer.

In §5 Brouwer scrutinized the evidence, supporting Menger's priority. The first item,[149] the material deposited at the Academy in Vienna of April 1921, was judged by Brouwer as too vague to base claims on. The reader who checks the material will agree that it is rather an outline for research than academically recognizable evidence. A nineteen-year-old student should not be judged too sternly for writing down his ideas, expecting an eventual recognition, but the matter changes its perspective when a twenty six year old scholar bases priority claims on it.[150] The paper of 1922,[151] cited in Menger's historical survey, could not be located ('In reality such a note has not appeared and no manuscript could be found'), only the corrections to it, in a letter to Hahn, were available. This letter with corrections contained the 'justification theorem' (in Brouwer's terminology) as a conjecture. All this, concluded Brouwer, could hardly be seen as evidence for a priority claim for the year 1922. Menger only entered the field of dimension theory with Menger 1923 (*Über Dimensionalität von Punktmengen I)*, which actually appeared in 1924 (Brouwer had reached great finesses with respect to correct dating of papers![152]), and this paper did not go beyond Urysohn's *Comptes Rendus* notes. So Menger could only claim priority for his results from later publications, i.e. from 1924 onwards.[153]

With respect to Urysohn, Brouwer accused Menger of a false representation of the facts by his reference to Urysohn 1926 and Menger 1924 in his treatment of the sum- and decomposition theorem. His argument was that Urysohn, obviously had submitted his paper before 17.8.1924, and Menger his paper on 6.10.1924. Urysohn's paper was accepted without changes or corrections, so if submission decides priority, Urysohn got it.

Brouwer, who in 1926 had watched closely over the formulation of Menger's 'On the genesis of my papers on dimension- and curve-theory', see p. 527 had insisted that it should be made quite clear which documents were actually avail-

[148] [Brouwer 1928d], p. 955

[149] [Menger 1926a], [Menger 1929], 1.

[150] There is admittedly something unfair about priority and recognition. When a Gauss or a Gödel announces facts without proofs, they is not treated on the same footing as a similar statement by John Doe from the State University of X. Something like 'reputation' is involved—but even great men are not infallible.

[151] Menger 1929-2.

[152] It should be pointed out that the correct dating of contributions is not all that simple. Quite often the issues and the volume carry different dates, and sometimes reprints predate the issue itself. And then there is the date of submission, of the revised version, etc.

[153] [Menger 1924c, Menger 1924a].

able. Since apparently the manuscript of [Menger 1926d] could not be produced, Brouwer wanted this fact to be recorded. But at the same time he did not wish to cause Menger any loss of face, so he made Menger insert a footnote to the effect that the correction was still extant, without mentioning the manuscript of the paper to which it belonged. One may be certain that the point escaped most readers. To dispel any misunderstanding, Brouwer inserted the above remark in his historiography paper. A year later the missing manuscript had after all turned up.[154]

In the lull following the publication of the historiography paper, Brouwer did not wholly lose sight of the dimension conflict; with the *Annalen* conflict at its height, one would imagine him to be already fully occupied. But correspondence with Hahn shows that he considered it desirable, if not necessary, that Menger give an account of himself, now that Brouwer's historical sketch of dimension theory had appeared. The word he used was 'rehabilitation':[155]

> The main thing is that Menger rehabilitates himself by defending in a chivalrous way (*ritterlicherweise*), personally and in the same journal where he was attacked, his criticised views, and to justify them further for the public in an acceptable manner, even if they should retain, in the conflict with me, a one-sided character.

Hahn, on the other hand, could not muster any enthusiasm for a further altercation. He begged Brouwer to let the matter rest. Hahn, a kindhearted man with an unbounded trust in human nature, could probably not imagine what possessed Brouwer and Menger to fight over futilities. He seemed to think that with Menger's assurance that he did not wish to rob Urysohn of whatever priority he might have, the matter was closed.[156] But he overlooked the real thrust of Menger's attack, which was aimed at Brouwer.

Hahn had tried to persuade Brouwer to give up the ill-fated priority crusade (and his exertions for the rehabilitation of German science as well), but Brouwer did not show any inclination to do so:[157]

> As far as your advice to leave the priority matter in the dimension theory alone, it is my opinion that the tiniest moral matter is more important than the all of science, and that one can only maintain the moral quality of the world, by standing up to any immoral project. This was so for the Bologna conference, and now for Menger's falsifications.

While Menger was working on his rejoinder, Brouwer had work to do for the refutation of Menger's historical claims. In the first place there was the matter of the notion of connectedness. Menger had, roughly speaking, argued that Brouwer simply could not have given the right definition, because he did not have the right notion of 'connected.' There had been various attempts in the past to define

[154] [Menger 1929], 2.

[155] Brouwer to Hahn 8.IX.1929.

[156] Hahn to Brouwer 6.XI.1928.

[157] Brouwer to Hahn, 24.II.1929.

'connected', e.g. by Jordan, Cantor, Schoenflies, most of them fairly satisfactory. A modern, correct definition occurs in a paper of Lennes, '*A is connected if at least one of any two complementary subsets contains a limit point of points in the other set.*'[158] This is equivalent to our modern formulation, which occurs in [Hausdorff 1914]. Actually Brouwer had already, independently, given definitions of connectedness, one in a letter to Engel,[159] another in his paper on the invariance of domain.[160] The second one is perfectly modern, complete with relative topology. Curiously enough, he seemed to have forgotten this. And so he set out to show that he was aware of Lennes' definition. Here he had a strong case; it happened that Blumenthal had at one time asked him to referee a paper of Lennes for which he had to check the author's definition of connectedness.[161] Blumenthal was kind enough to provide Brouwer with copies of the relevant correspondence. In his fervour to get all details straight, Brouwer also wrote to Hasse, Hahn, and the Amsterdam University library to ascertain the date of appearance of some of the papers involved.

Menger's reactions were conspicuously absent during this interim period. Contacts with Brouwer had been dropped, so the only channels of communication were via Hahn and Hurewicz. This does not mean that Menger had put the dimension discussion out of his mind for good. As a matter of fact he was preparing the publication of all the documents that had played a role in his road to dimension theory.

In the file in the Brouwer archive, Menger's name turns up again in June 1929. At the time Brouwer was visiting Ehrenhaft in Vienna, and on the sixteenth Hahn dropped in; the conversation after some time reached the topic of Brouwer's historiography note. Hahn happened to know that Menger did not intend to answer Brouwer's note. He announced that he was himself going to write a contribution for a journal in which the topic of dimension would be mentioned, and he planned to take care to do justice to all parties.[162] At that point Brouwer remarked that a complete silence could permanently harm Menger's reputation; a suitable rejoinder would enable Menger to rehabilitate himself, and that opportunity should be offered to him. Hahn immediately agreed, and thought it a good idea if Brouwer would be willing to go over such a rejoinder.[163]

Brouwer may have been extremely angry with Menger, and he probably still was, but under all circumstances his sense of justice and decorum prevailed. So the idea to give Menger an opportunity to defend his point of view was not a tactical move to trip up Menger, but a sincere wish to straighten things out. Whatever Brouwer might have thought about Menger's personality, he was convinced that Menger was a gifted topologist, perhaps not the genius that Menger considered himself to be, but basically a capable man.

[158] [Lennes 1911], p.303.

[159] Cf. p. 132.

[160] [Brouwer 1911], p. 308.

[161] Blumenthal to Brouwer 3.II.1912, Blumenthal to Brouwer 12.II.1912.

[162] [Hahn 1929].

[163] A signed account of Brouwer, 20.XI.1929.

Through the mediation of Hahn, Menger agreed to write a reply. The only problem was that Brouwer and Menger meant different things by 'rehabilitation'; for Menger it meant refuting Brouwer and for Brouwer it meant acknowledging a few errors and revising the historic account in accordance with reality.

More than a year after this conversation, Ehrenhaft recalled the occasion; he had noticed the friendship between Brouwer and Menger at the time of the *Gastvorträge*, and therefore, when in June Brouwer stayed with him, he had arranged the visit of Hahn for the specific purpose of reducing the barriers between Menger and Brouwer. He confirmed Brouwer's account and complained that for inexplicable reasons Menger was now also mad at him—'You are completely right, that when someone has really good intentions, and defends truths and objectivity, he always gets conflicts and unpleasantness.'[164]

Hahn's suggestion worked miraculously; already on June 21 Hahn wrote Brouwer that Menger had finished his text, Brouwer would soon get his copy. On July 1 Hahn indeed dispatched the manuscript.

In the covering letter he said that he was convinced that now the difference between Brouwer and Menger would soon be eliminated, as 'Menger states clearly to have no priority claims over Urysohn, but only maintains independence and a rough simultaneity. He also acknowledges—as could not possibly otherwise—the importance of your paper of 1913, declares not to have attached any importance to the matter of the separation notion, and admits his oversight concerning the notion of "connectedness".' Furthermore he informed Brouwer that the *Monatshefte* had accepted Menger's collected documents on the dimension conflict for publication.[165] Brouwer's initial reaction to the manuscript was favourable,[166]

> I find the manuscript really successful; both the general structure and the presentation of the majority of the details seem to me well suited to bring the matter to an end in such a way that neither from my side, nor from the side of Moscow further replies will be necessary. I hope therefore to be able to restrict myself to [...] a brief epilogue, in which I can express the hope that the discussion that has taken place, will be a useful contribution to clarify the development of dimension theory, and to note that the attentive reader may conclude from both notes that there are hardly any points of difference left to be cleared up, and that there remain hardly any points of difference between Menger and me.

He promised to send a few suggestions for corrections. Two days later Brouwer again wrote to Hahn, this time to acknowledge the receipt of the proofs of Menger's collection of original documents; the letter contained one suggestion for a change in the editorial note, which ('as I ascertained in consulting impartial local mathematicians') made a threatening impression. Apparently Brouwer was optimistic about the '*Antwort*' note of Menger, he wanted to have the note typeset, so that it

[164] Ehrenhaft to Brouwer, 28.X.1930.

[165] [Menger 1929].

[166] Brouwer to Hahn 11.VII.1929.

could appear immediately after it had been formally communicated to the Academy. Hahn showed himself pleased with this reaction of Brouwer. He agreed with the editing of the editorial note and told Brouwer that Menger wanted to avoid a public row over Brouwer's 'historiography' paper and therefore would not cite it in the announced paper reproducing his original documents.[167]

When Menger's collection of documents appeared in the *Monatshefte*, the editors referred in a footnote to a forthcoming note of Menger, in which he would give his views on the genesis of dimension theory. This shows how optimistic Hahn was that the whole matter would be resolved soon, and forever. The paper contained all the early manuscripts from Menger's dimension- and curve theoretic work. One of the items was the disputed manuscript 'On the dimensionality of point sets' ([Menger 1929], 2, 2a). In Brouwer's historiography paper the manuscript was still classified as 'missing' (see p. 527, 650).

In view of the topics of Menger's draft reply, the above paper is of importance. Menger commented in his collection of documents: 'This paper (announced in the preceding note) was submitted to the *Monatshefte für Mathematik und Physik* around the turn of the year 1921/22.' The evidence for this—in 1926 missing—crucial, manuscript is confusing. Hahn reported on April 10, 1926 to Brouwer, that the definition of dimension was given in a letter from Menger of 15.2.1922,[168] in which there is reference to an earlier '*kleine Arbeit*', 'However, I cannot remember this early version.' The manuscript was also not among the material that Menger had collected for Brouwer in Vienna in April 1926.[169] The summary of Menger's work by Otto Schreier did not mention this paper either. In 1929 the manuscript must have been found in the files of the *Monatshefte*, for it was published in [Menger 1929], and certainly Hahn's reputation would guarantee that it was the genuine article. Back in 1928 Brouwer still had not seen it and he could justifiably say that no such note had been published, and that the manuscript had not been located.[170]

Indeed, when Brouwer and Hahn discussed the Menger affair on 22 July 1929 in Vienna, the former asked permission to see the text of this elusive manuscript of Menger. Hahn answered (literally[171]) 'The transfer of the documents of Menger has not yet taken place.' Hence even at that moment only Menger could be certain of its existence.

One can imagine why Menger insisted on the recognition of this paper; it moved the priority back from November to February. In the '*Antwort*' note he

[167] Hahn to Brouwer 15.VII.1929. The proposed editorial note explained that a recent publication on the history of dimension theory, to which in due time Menger would respond, induced the editorial board to reproduce a number of hitherto unpublished documents relating to the matter.

[168] Cf. [Menger 1929], 2a.

[169] Menger to Brouwer, 10.IV.1926.

[170] [Brouwer 1928d], p. 956.

[171] Brouwer's note 30.XI.1929.

had denied Brouwer's claims by referring to this paper 'of which Brouwer thinks, that it could not be found.'[172]

Why worry about such details, one might ask. It is quite likely that Brouwer was not out to deny the existence of the manuscript, or to suggest foul play, he just wanted to put on record that at the time of the preparation of [Menger 1926b], Menger could not show it. It is hard to understand why a man who took the precaution to deposit his first ideas with the Vienna Academy and who had his documents authenticated by Hahn, could have been so careless to present irreproducible and unverifiable evidence to Brouwer. In all, one is unavoidably led to the conclusion that Menger could not produce the manuscript in 1926. This, of course, does not say that it did not exist but it justified Brouwer's saying in 1926 that so far it had not been found.

In his drafted reply Menger had formulated the issue of the missing manuscript as follows: 'of which Brouwer means that it could not be found.' This clearly evaded the issue, namely that Menger in the presence of others[173] had not been able to meet the challenge to produce the manuscript. Brouwer must have written to Hahn, that this formulation was not acceptable, and after deliberation Menger conceded the point; he was willing to replace this formulation by 'Brouwer's remark that this manuscript could not be found may be caused by a misunderstanding based on some conversation. As a matter of fact I cannot recall the content of a discussion on the matter with Brouwer in 1926, but I certainly have not said that the manuscript could not be produced.'[174]

It will be clear that this did not satisfy the demands of Brouwer, but there was little he could do. Menger's memory, so to speak, set the limits to the available evidence.

With Menger's 'Reply to Brouwer' note a short period of almost scholastic, refined argument began. Menger wrote a first draft, gave it to Hahn, who passed it on to Brouwer, who, in turn, would suggest changes, send them to Hahn, …

Brouwer clearly attached great importance to the final solution of the dimension conflict; this is demonstrated by the fact that he was willing to travel to Vienna in order to discuss the matter with Hahn in person. On July 22 he visited Hahn and showed him a proposal for a postscript

> in case Menger would accept all of my proposals for change.
> *The above explanations of Mr. Menger yield, in my opinion, a welcome completion of his book. After the discussions that have now taken place I consider the controversial points sufficiently cleared up.*

After some discussion, the following version emerged:

[172] Actually, Menger did not seem to be all that certain about the data of the note; its submission to the Monatshefte is dated respectively in January, around the turn of the year, and in February.

[173] Presumably Alexandrov and one or more of the other topologists.

[174] Hahn to Brouwer 30.VII.1929.

> *I gladly accept the above explanations of Mr. Menger, and consider the points of difference, that have to my regret, existed between Menger and me, as cleared up.*

Hahn, who had already moved to his vacation resort in Bellagio, continued his correspondence with both parties in an effort to get a formulation of the '*Antwort*' that would satisfy both parties.

Menger had duly read Brouwer's typescript, incorporated changes and informed Hahn of possible further concessions, which Hahn passed on to Brouwer.[175]

A curious discussion followed that ran over several months; the central person was Hans Hahn, who went to great length to appease both angry opponents. There was no direct communication between Brouwer and Menger, every bit of information, every proposed formulation, had to pass through Hahn. The latter was in an awkward asymmetrical position, on the one hand there was Brouwer, the charismatic, visionary but difficult mathematician, who had become his friend, and on the other hand Menger, his brilliant student, a brooding, vain man, with a strong tendency to feel offended—but loved as a son in spite of his not always easy personality. The correspondence between Brouwer and Hahn has largely been preserved, but there are no letters or documents that shed light on the contacts between Hahn and Menger. Since both were in Vienna, it is possible that most communications were oral; there have been, of course, documents and drafts containing proposals and reactions.

In the middle of this correspondence there is a letter that tells of a piece of misfortune that befell Brouwer and that would be enough to bring anyone to a state of irreversible despondency. Brouwer's letter of August 9 to Hahn on his vacation address in Bellagio opened with a dramatic statement written in a remarkable mood of resignation:

> Four days ago my briefcase (*Brieftasche*),[176] which also contained my scientific diary was stolen from me on the front platform of a Brussels' tram, by a pickpocket, and both the police and the detectives consider the case as hopeless. Since in this diary my collective scientific thoughts and ideas of the last three years, which have largely disappeared from my memory, and of which only a few have already found a registration elsewhere, had been recorded, this event means for my scientific personality a serious personal mutilation (*Verstümmelung*), in a way that is like the 'decapitation' (elimination of the central process) for a pine tree. To my amazement, I remain so far, fairly calm under this blow of fate; I believe, however, from certain phenomena, that I have nonetheless suffered a nervous

[175] Hahn to Brouwer 30.VII.1929. Brouwer's document is not extant, but there is a draft in his archive, listing a number of points.

[176] Brouwer uses *Brieftasche*; it is more likely that he was carrying a small type of briefcase that was very common at the time, than a wallet.

> collapse, the consequences of which will perhaps only later become visible, together with a disorganization of my scientific thoughts.[177]

After the exchange of some more letters and cards, Brouwer reported that there was no progress in the matter of the stolen notebook.

> For the recovery of the lost papers I mobilize everything possible—advertisements in the newspapers, police, detectives, clairvoyants—but so far no success.[178]

The loss of this notebook may shed more light on Brouwer's unexpected withdrawal from research for a considerable period. A young man would go ahead, reconstructing and remembering his ideas, but Brouwer was approaching 50, and the mental scars of his battles were visible to the discerning eye.

The discussions, and no doubt Hahn's wise mediation, had reduced the discord to a few bones of contention. One was the 'slip of the pen' in the separation definition of Brouwer, cf. p. 454, 455. Both Urysohn and Menger had found the mistake, and Brouwer had acknowledged the, what he called, clerical error, excusing the late correction as partly caused by a permissive (*fahrlässig*) attitude towards slips of the pen and as something that any reader could immediately correct, as well as partly by Lebesgue's dragging out his promised proof of the paving principle. Menger, in his book, acted as if there was no mistake in Brouwer's definition, but that it was simply defining another separation notion; whereas Brouwer in his *Berichtigung* in *Crelle* (1924) had pointed out that from the arguments in his dimension paper of 1913, it could clearly be seen that the intended separation was the one based on modern connectedness. In the historiography paper he pointed out that, clerical error or not, the intended and the published connectedness led to two notions of separation and hence to two notions of dimension. In both cases R^n had dimension n. Freudenthal, in his comments on the dimension papers, carefully analysed the various notions of connectedness that were around at the time of Brouwer's first paper. He came to the conclusion that Brouwer's account has a strong ring of truth. As to the various (weak and strong) notions of dimension he remarked: 'The fine distinction between weak and strong connexion may appear as an *a posteriori* implantation—such subtleties in the stone age of topology! Yet, Brouwer was subtle.'

It is instructive to follow the discussion about connectedness. In his *Dimension Theory*, Menger reproduced Brouwer's 1913 definition of dimension verbatim, and went on to tell the reader that the definition made sense, if one replaced the separation by the notion that 'had emerged since then.' Thus suggesting that Brouwer was not up-to-date at the time, and that he made a wrong choice of notion. The correction of 1924 was not mentioned at all. So, to all appearances from a superior height, Menger had informed Brouwer and the reader of 'nice try, but not good enough.'

[177] Brouwer to Hahn 9.VIII.1929.

[178] Brouwer to Hahn 17.VIII.1929.

Brouwer replied in his historiography paper with an exposition of almost a page that (i) he did know the right definition (given by Lennes), (ii) he had made a simple slip of the pen, which anybody knowing his topology could easily correct, and (iii) the erroneous definition, like the correct one, yielded its own notion of dimension.

Then, in the first draft of his reply, which is unfortunately not extant, Menger must have reacted to this statement of Brouwer. From a later letter of Hahn it appears that Menger proposed the formulation: 'To my lapse of p. 86 Brouwer is, by the way, also guilty, as in his paper of 1913 he did not cite the statement (definition) of Lennes, which certainly was known to only a few mathematicians at the time.'

When Brouwer objected to this formulation, which left open the possibility that *he* did not belong to those 'few mathematicians', Menger modified it: 'of this lapse Brouwer is indeed also guilty, as in his paper in the year 1913 he did not cite the paper of Lennes, which, although known to him, was, as yet, actually little known.'[179]

Even this formulation did not satisfy Brouwer; he wanted Menger to insert some evidence about Brouwer's familiarity with Lennes' definition, as otherwise the reader could possibly think that the definition was known to him at the time *according to Brouwer*. Hahn correctly waved this objection aside. It was not Menger's duty to provide evidence, he said.[180]

The next week Hahn spent conferring with Menger about Brouwer's proposed changes. A number of suggestions resulted, and were conveyed in Hahn's letter of September 7. With respect to the paper of Lennes, Hahn stuck to Menger's suggestion mentioned above.

The correspondence shows how the matter of sharpening formulations assumed (on both sides) proportions between pedantic and paranoid.

Hahn took his role as mediator extremely seriously, he did his utmost to find acceptable formulations, to suggest suitable arrangements, etc. He had committed himself to his student to the extent that that the crown on the successful negotiations was to be a reconciliatory epilogue from Brouwer's hand; Brouwer probably had made his doubts known on this point, for on August 28 Hahn let him know that he could not possibly drop this condition. Giving up the planned epilogue, could not be justified to Menger, he would feel led down the garden path. Brouwer had indeed further perfected a closing text (cf. p.655):

> I gladly accept the above welcome explications of Menger, and I hereby consider the differences, that to my regret have existed between Menger and myself after the publication of Menger's book as resolved.

After the letter of September the negotiations were suspended for a time, both Brouwer and Hahn were on vacation, moreover Brouwer fell ill. But on October 15 Hahn resumed his mediations. In the meantime Menger had started his own preparations for a further battle, should that prove necessary. He wrote to

[179] Hahn to Brouwer 21.VIII.1929.

[180] Hahn to Brouwer 27.VIII.1929.

Sierpinski, the editor in chief of the *Fundamenta Mathematicae*, asking for copies of documents concerning the editing of Urysohn's memoir. On October 12, Knaster supplied him, in the name of Sierpinski, with handwritten copies and translations of letters of Brouwer and Alexandrov; begging him to ask permission before making any public use of the documents, as they were in part very private correspondence.

If anything, it shows that Menger was hedging against possible actions of Brouwer, but at its worst it could show that he was planning an action himself. Hahn's letter of October 15 was optimistic, albeit that he thought Brouwer was dragging his feet. Menger had accepted the proposals of September 7, and nothing seemed to stand in the way of a peaceful conclusion of the conflict. 'Prominent colleagues', he wrote, 'when told about the close prospect of a peace in this matter, were most satisfied, and wished for it as much as I do.'

He gave Brouwer time until the twenty-fifth; if by then there was no agreement, he would consider his mission as failed. In view of the progress Hahn had made, the grand finale seemed at hand. But with Brouwer one was never certain.

Indeed, Brouwer dispatched a long letter on October 22, in which he questioned the progress that had been made so far. But before he entered upon the state of Menger's note, he vented his feelings on a seemingly innocuous remark of Hahn. This ill-fated remark slipped in when Hahn was trying to convince Brouwer that the pacifying epilogue could not be given up.[181]

'I have exerted myself, since I got to know your note, to convince Menger that a calm, objective settlement were possible and the only thing to be desired.' Harmless as this line may seem, before Brouwer's eyes it conjured up the experiences of the past years. He immediately interpreted Hahn's words as a result of an 'evidently preceding *accusation of Menger* that a calm and objective discussion were not possible with him.' This was enough for him to burst into a description of Menger's conduct and reputation during his Dutch period. He had taken the trouble to keep Menger to the straight and narrow, where his scientific responsibilities were concerned, and in fact it was a continuation of this practice that had brought Brouwer to assist in the attempt to get Menger to rehabilitate himself in the matter of his historical inaccuracy or tactlessness. He did not see Hahn's arguments, he said. It was not clear to him why Menger's note would necessarily deserve the predicate 'calm and objective' if the note was followed by a peace-declaration of Brouwer. Nor could he be accused of a lack of calm and objectivity, when he considered to go back on his promise to write an epilogue, if all his arguments concerning the allegedly missing manuscript of Menger were met by a continuing appeal to a shortness of memory, which excluded a permanent understanding. It cannot be denied that Hahn, when pressing Menger for detail about the temporarily missing manuscript, met with a 'that has slipped from my memory.' And it is not unthinkable that after a mutual understanding was reached, Menger's memory would produce undesirable items. So Brouwer feared that, unless written confirmation was

[181] Hahn to Brouwer 29.VIII.1929.

obtained, an agreement had only a temporary validity. In Brouwer's view Menger's note should rehabilitate its author, not Brouwer.

After realizing that Menger's short memory showed more gaps than so far expected, Brouwer could not agree with Hahn's view, that nothing had changed in the meantime—but why not accept the text? He had studied Menger's manuscript closely, and discovered a few unacceptable consequences. For example, that Menger's text left the impression that the error in the separation definition had gradually dawned on Brouwer, and that he had formulated the correction only in [Brouwer 1924b], that is *after* Menger's first publication. Although nothing was explicitly asserted, the reader could easily get the impression that Brouwer only saw the right formulation after Menger had shown him the way. Furthermore, Menger had not mentioned the 'clerical error', but he had cited a paper that cited the correction. 'According to the principle that Menger adheres to, an author would be justified to cite falsely at will, if he mentions in between also some paper, whose reading would reveal the incorrectness of the "quotes".' The long and the short of this letter was that sharper formulations were necessary.

Hahn's letter of October 15 had not reached Brouwer in time for the latter to comply with the request for a decision before October 25. Brouwer had been travelling and he discovered the letter after he had sent off his letter of October 22. He refused, however, to accept Hahn's conclusion. The point was not, wrote Brouwer, a personal conflict between him and Menger, but rather the ultimate rehabilitation of Menger under the joint supervision of Hahn and Brouwer.

The end of the negotiations had now been reached. Hahn replied on November 10 that he regretted that his efforts had failed, in particular as he felt that at all points of an agreement had essentially been achieved. He could only wish that 'the continuation of the polemics, which had now become unavoidable, would neither for you nor for Menger diminish the predestined strength for scientific research.'

As far as the note of Menger was concerned, Menger had submitted it to the *Monatshefte* where it would appear. The journal would not accept a reply of Brouwer, if necessary he could publish that in the proceedings of the KNAW. No protracted polemics in the *Monatshefte*.

For some reason Brouwer had not included this possibility into his campaign plan, he was thunderstruck (*ganzlich verblüft)*; this was not what was agreed upon in our talks on June 16 in Vienna, he said. Menger had to publish in the Proceedings of the KNAW! And he could not possibly accept the decision to open the columns of the *Monatshefte* to a person close to the editors, and barring them to the other party.[182]

After being involved in a conflict with Menger, he could not bear to think of fighting a conflict with the *Monatshefte*, he wrote. If such should be the case, he added in a footnote, he could not possibly allow the *Monatshefte* to publish his second Vienna lecture, *The structure of the continuum*.

[182] Brouwer to Hahn, 26.XI.1929.

Brouwer's consternation can be easily imagined, for he had assumed, on the basis of Menger's agreement with Brouwer's text proposals, that the note could now simply be published in the Proceedings of the Academy without the peace-epilogue. He was too late, however; Menger had already submitted his '*Antwort*' to the *Monatshefte*.[183]

Menger's *Antwort* had a confusing history. There is a proof sheet in the Brouwer archive with the heading 'A note of K. Menger submitted on July 1 1929 by H. Hahn to L.E.J. Brouwer, intended for the Amsterdam proceedings.' We may assume that this was the outcome of Hahn's mediation; no letter of Menger, nor of Hahn, authorizing this submission has been found. The strange thing about the proof sheets was that they were not in the usual format of the Proceedings. Did Brouwer arrange for typesetting ahead of the actual presentation, in order to handle the publication efficiently? If so, then he paid for it out of his own pocket. In the absence of further information, the presentation to the Academy by Brouwer had probably been in good faith, but rather careless. Menger had of course reserved his freedom, as he could always withdraw the manuscript for the *Monatshefte*. Without his explicit approval the text that was in Brouwer's possession could not be published anyway. The continuing negotiations undoubtedly held up the publication of this note. Finally Brouwer must have decided to go ahead and present the paper, unaware that Menger had taken steps for the publication in the *Monatshefte*.

Indeed Menger had sent a copy of the '*Antwort*' to the Academy in Amsterdam with a covering letter and an extra copy of the note for Brouwer, which informed it of the insults suffered by the correspondent and the journal *Die Monatshefte* at the hands of Brouwer, the author of a polemic pamphlet in the pages of the Proceedings of the Academy, and of the fact that he, Menger, did not demand space in the proceedings for a retort, but that he felt entitled to do so, should another polemic appear.[184]

Unaware of Menger's letter, Brouwer planned to communicate the, let us say, Amsterdam version of Menger note to the proceedings at the meeting of November 30; he warned the secretary that the communication could be problematic, because Menger 'was said' to have published a polemic article in the *Monatshefte*:

> according to information received from Vienna, probably written in a state of mental aberration, such as I have witnessed a couple of times during his years of apprenticeship with me. If this indeed is the case, then the rather moderate and objective paper that I will submit on Saturday, will be such a clear implicit refutation of the Viennese paper, that I feel I have to take into account the possibility, that Menger, after the receipt of the proof sheets, notices the refutation and will wish to make drastic

[183]The paper is dated 15 May, 25 October 1929; this would normally mean that it had been submitted on May 15, and that a second, a revised version was sent in on October 25.

[184]Menger to KNAW, 14.XI.1929 (erroneously dated 14.XI.1928).

> changes. I think (and so does Weitzenböck) that we should in that case refuse to allow these changes.[185]

The secretary in turn informed Brouwer that a letter from Menger, of which a copy (or the original) was enclosed, had already been circulating in the board of the academy, and he invited Brouwer for a discussion of the matter. The copy of Menger's *Antwort* was not mentioned in this letter, so it is possible that Brouwer saw the note only later. The result of the discussion is unknown, but Brouwer went ahead and communicated at the academy meeting of November 30, the '*Antwort*' in the form that was along the lines of the last agreement.[186]

In fact, this submission remains curious. The least Brouwer should have done under the circumstances was to ask Menger for an authorization to submit the paper to the Academy. As was to be expected Menger, when he became aware of the submission of the note, immediately saw the trap laid by Brouwer, he protested vehemently, summing up the history of the conflict and finally, with a noble gesture, declaring that he would concur with the publication, considering it a sincere sign of Brouwer's wish for a '*Wiedergutmachung*' (making amends), provided, it would contain the peace-making epilogue. In that case Menger promised to do his utmost to stop the publication of his '*Antwort*' in the *Monatshefte* (the issue was to appear after January 15, 1930) the cost would of course have to be borne by Brouwer.[187] The letter to the Academy was accompanied by a copy of the *Antwort* in the Monatshefte, with changes in the formulation in Menger's handwriting and with the text of the required reconcilliating epilogue. Menger was no longer taking any risks; the reprint carried a signed stern clause: 'Any publication with changes, additions, or deletions will be prosecuted under copyright! Publication without the indicated epilogue will not be allowed by the author.' Furthermore Menger would send all the recipients of his reprints (which had already been sent out) a note informing them that in view of Brouwer's cooperative attitude a similar note would appear in the Proceedings of the KNAW, and the sharp foundations of the *Monatshefte* note had become superfluous. It is certainly understandable that under this shotgun diplomacy Brouwer could only withdraw from the affair.

The board of the KNAW closed the exchange in a letter of March 17, 1930, with the firm announcement that the Academy did not want any part in polemics, that it hence had informed Brouwer that it regretted the publication of his paper in the proceedings. This position of the board was rather disappointing for Brouwer. In the March meeting of the Academy he expressed his view that an Academy in matters of defending the honour of a deceased scientist should be supportive of its members, rather than address them in a way that could be interpreted as a repudiation, in upholding the truth. The minutes recorded dryly 'Mr. Brouwer finally pointed out that the polemic element plays such an important role in all his work,

[185] Brouwer to KNAW 26.XI.1929.

[186] Thereby saving Menger's face in the matter of the notion of connectedness and the missing manuscript.

[187] Menger to KNAW 14.XII.1929.

that a ban of polemics would mean for him a crippling of his activity as a scientific author.'

There was one more act to come in the Menger conflict. It was precipitated by a passage in Menger's '*Antwort*', concerning the editing of Urysohn's big *Mémoire* in the *Fundamenta Mathematicae*:

> The idea that *Urysohn* was a 'successor' (*Fortsetzer*), could look for support in the references to Brouwer's note of 1913, which are to be found in the, under cooperation of *Brouwer*, posthumously published *Mémoire sur les multiplicités Cantoriennes* of Urysohn. These references have, however, as I want to mention in passing, been *supplemented* and *changed*, in comparison with *Urysohn's* manuscript. In how far these supplements and changes should be provably identical to the formulation of later statements of the deceased, has not been indicated, indeed these supplements to and changes from the original text have not been indicated at all.[188]

This allegation, in the context of the *Antwort*, can only be classified as 'hitting below the belt.' Of course, Menger was right, there was no editorial account of the handling of the text, and in that sense one can definitely speak of sloppy editing. But this passage, embedded as it was in an attack on Brouwer, can only be read as an accusation of 'unproved fraud'. Particularly so, as Menger's claim is that the changes in the text were designed to show that Urysohn was a successor of Brouwer. If one checks the references to papers of Brouwer in Urysohn's memoir, it turns out that all of them are of an innocuous nature. Knowing Brouwer, one may be certain that during the joint discussion in Blaricum between Urysohn, Alexandrov and Brouwer, the latter would have pointed out where he should be cited. Looking at the resulting quotes, it is almost obvious that Urysohn would have agreed. Even if Brouwer had suggested footnotes referring to himself, nothing would have been wrong. After all, footnotes are the traditional places for credits. There is no obvious reason not to accept the fact that Brouwer and Alexandrov acted in good faith in inserting references, when useful. The only crucial reference is in footnote 3, page 37–38, where Urysohn mentioned Brouwer's dimension paper of 1913. The available evidence is such that Brouwer and Alexandrov cannot be accused of fraudulent editing, The correspondence between Brouwer and Urysohn, cf. p. 461, (to which Menger never had access) leaves no doubt that the published version was faithful to Urysohn's views. Alexandrov, in his edition of Urysohn's collected works, explicitly mentioned the difference between Brouwer's 1913 definition (with the slip of the pen) and Urysohn's definition.[189] He added that Urysohn had accepted Brouwer's explanation, thus endorsing the corrected version of 1923 as the intended one.

Brouwer felt that Menger failed to do justice to the memoir of Urysohn, and the editorial procedures excercised by Alexandrov and Brouwer. Stronger, he con-

[188] [Menger 1930].
[189] Cf p. 448,459.

sidered Menger's attack a clear case of foul play. In his letter of January 10 to Hahn, he vented his disconcert and indignation about the allegations.

> The passage insinuates moreover without a shred of proof, that there is no certainty concerning the exact correspondence of the proofs of Urysohn's proofs of the dimension theoretical fundamental theorems with the manuscript that was submitted to the editors of the Fundamenta in March 1925. This accusation too, which in essence is made by Menger, is absolutely libellous and unfounded: the manuscript in Urysohn's own handwriting is still *available*, and was, by the way, given for inspection to Menger for an unspecified period by Alexandrov in the Spring of 1925.

It is clear that Menger was not trying to cast doubt on Urysohn's work, he was rather suggesting that Brouwer used the occasion to bolster his claims on his place in the history of dimension theory. But strictly speaking Brouwer had a point; one might just as well conjecture that the two editing topologists had quietly been upgrading Urysohn's paper. However, nobody would in earnest claim such a thing. Nonetheless the above mentioned quotation shows that something jarred; if Menger had access to the Urysohn manuscript in 1925, why did he not speak out at an earlier moment? There does not seem to be an easy answer to that question. Perhaps Menger did speak out, and was ignored, or perhaps he did not foresee the conflict of 1928/29.

Brouwer, anyway, did not accept the implicit accusation, he pointed out to Hahn in unmistakable terms that no editorial board could accept statements of the above sort without insisting on appropriate proofs

We have in fact seen how Brouwer and Alexandrov exerted themselves in preparing the scientific estate of Paul Urysohn, including the proofreading and correcting of Urysohn's memoir. There is no doubt whatsoever that they made changes in the text (including the notorious footnote), and given the fact that both Brouwer and Alexandrov had a deep veneration for their deceased friend and colleague, it is unthinkable that they would doctor the text for selfish purposes. Urysohn, who could, during his short life, not write as fast as new ideas flooded his mind, hardly had sufficient time for reflecting on the manuscripts he sent to the publishers, and so it is plausible that during their final stay in Blaricum, Paris and Batz, Alexandrov, Urysohn and Brouwer went over the material that had already been submitted, and found improvements in the formulation.

The unfortunate Hahn, who as a mediator, had to bear the discontent of both parties, was disconsolate; he was about to lose a friend because of a conflict that was not of his making. Although the editors had declared the Brouwer–Menger discussion closed, it seemes that under Brouwer's insistence, they apparently had permitted Brouwer to publish a reaction.[190] No such reaction had been submitted, however.

Quite understandably, Brouwer was upset about the content of Menger's *Antwort auf eine Note von Brouwer*. The formulations had become less tolerant, and

[190] [Menger 1979], p. 254.

a totally new aspect was added, one that had little to do with the original issue. This was Menger's *sotto voce* attack at Brouwer's editing of Urysohn's memoir. Brouwer was furious. He immediately wrote to Hahn,[191] that this accusation was absolutely libellous and unfounded—as much as the rumour spread by Menger that Brouwer should have done something similar in a paper of Vietoris in the *Annalen*. He wondered how the editors could have accepted such a serious allegation without a shred of proof.[192]

A day later Brouwer supplemented the information on Urysohn's memoir.

> The part that appeared in Fundamenta Mathematicae 7 was typeset and corrected when Urysohn was still alive. At that time *Urysohn* himself has inserted various changes, among others, in a letter to *Kuratowski*; the only reference in the memoir to my note in Crelle, namely the one to be found in Fundamenta Mathematicae (7). p.37, footnote 3), was re-edited and brought into the present form. Urysohn had written down, but not yet sent off further corrections, when he died. When Alexandrov and I contacted the editors of Fundamenta, in order to carry through the correction of this part of the memoir taking into account Urysohn's jottings, it turned out that the paper was almost completely typeset. Only in two galleys (p.33–48 and p. 129–137) could we make correction in the text (e.g. in the above mentioned footnote, the planned references to my correction in Crelle 153 and to Menger's note in the Monatshefte 33 were inserted); for the remaining galleys only the most urgent corrections were incorporated in an extensive list of errors at the end of the volume.
>
> The part that appeared in Fundamenta Mathematicae 8, on the other hand was typeset after Urysohn's death and corrected by Alexandrov and me, and thus has a more or less posthumous character. The original handwritten manuscript of Urysohn is in Alexandrov's care, and for the authorship of the part in Fundamenta Mathematicae 8 Alexandrov and I accept full responsibility to the public.

That same day Brouwer wrote to Wirtinger, trying to convince him that certain parts of Menger's *Antwort* could not be published in the absence of evidence. He closed his letter with a sigh: 'I cannot bless the moment, in which five years ago I accepted Menger as a student.'

Again a day later Brouwer wrote a relieved letter to Hahn.[193] In a miraculous way the original proof-sheets of the pages 33–48 had been found. One in the old form with the handwritten corrections of Urysohn, and the next one with the

[191] Brouwer to Hahn 14.XII.1929, 10.1.1930.

[192] The matter of the disputed dating of Menger's paper was no longer an issue in the eyes of Brouwer as the matter had been put right in the proceedings of the KNAW.

[193] Brouwer to Hahn 12.I.1930.

corrections incorporated—but without the posthumous reference to the corrected version of 1923 of Brouwer's paper.[194]

> By a fortunate chance, I almost would say as by a miracle. Those in which the changes, sent by Urysohn to Kuratowski, of his reference to my Crelle-paper were inserted by hand by the editors of the Fundamenta Mathematicae, and those in which the printer subsequently carried out those changes, both of course without the posthumous reference to my Crelle-correction and to Menger.

He closed the letter by expressing his hope that this message would be decisive for the editorial board.

It is easy to misread the last sentence of the above quote. What Brouwer said was, that in spite of the handwritten correction, the typesetters just ignored the instructions. So the first proofs showed the old text.

He could not know that Menger, in the meantime, had done his homework, and collected directly from the *Fundamenta Mathematicae* the relevant information on the editing of Urysohn's memoir,[195] so that Hahn probably had already seen part of the evidence. It was too late to stop the publication of Menger's '*Antwort*' anyway.

Hahn, in his answer,[196] assured Brouwer that Menger's remark concerning the memoir was neither an insult nor a reproach, 'and that his claim with respect to footnote 3 on page 37 is correct, is on the whole confirmed by your letter.' Literally speaking, Hahn was right, but he charitably overlooked the innuendo in Menger's note. This letter of Hahn is the last one in the Brouwer archive. Apparently the contact was broken off at this point—one of the regrettable consequences of the Menger affair.

The last act in the Menger conflict consisted of a privately published note of Menger, containing the evidence on the editing of Urysohn's memoir. Because Brouwer declined to continue the discussion in the *Monatshefte*, or anywhere else, Menger's *Antwort* was the final note in the *Monatshefte*. The editors must have felt that one *Antwort* was enough. However, Hahn probably told Menger about Brouwer's angry reaction to Menger's accusation; this was for Menger an ideal opportunity to show his cards, that is to say the evidence provided by the *Fundamenta Mathematicae*. He published a letter to the editor (Hahn) in the *Monatshefte*, announcing his intention to publish privately a note 'On the reference to Brouwer in Urysohn's memoir'.[197] In reaction to Brouwer's continuing complaints that Menger had raised unsubstantiated suspicions about his integrity as an editor

[194]These proof sheets have not been found in Brouwer's estate. The documentation of the Menger affair is, although extensive, not complete.

[195]Knaster to Menger, 12.X.1929.

[196]Hahn to Brouwer 21.I.1930.

[197]*Eine Zuschrift von K. Menger an H. Hahn*, [Menger 1933]; *Über die Hinweise auf Brouwer in Urysohn's memoir* [Menger 1932], reprinted in [Menger 2002].

of Urysohn's work, Menger published his available evidence in a pamphlet 'On the references to Brouwer in Urysohn's memoir' (1932). He argued that Brouwer had changed the memoir, and pointed out specific places where those changes were visible.

His final conclusion was that he had proved that there had been changes in the text which were not recognizable as such. We would all agree that this was not up to the standards of good editing, but there is an unmistakable suggestion that Brouwer used the occasion to suggest more recognition by Urysohn for his own dimension paper than could be justified by this original manuscript. Although there is no explicit statement to the effect, there is clearly an implied doubt of Brouwer's integrity. As the only explicit information on part of the text concerned the by now notorious footnote, one can neither assert nor deny that Brouwer and Alexandrov inserted wholesale changes in the text. As long as one does not possess the original handwritten manuscript the extent of the changes is hard to assess.

This was the definite finale to a conflict that lasted six years. The pamphlet was not widely known, e.g. Freudenthal, when editing Brouwer's collected works discovered its existence only then. The history of Urysohn's memoir and the conflict has been related above. From our viewpoint there was no objective reason for a conflict, the matter was completely transparent: Brouwer had given the first correct inductive definitions of dimension (marred by a clerical mistake) and Urysohn and Menger had subsequently, independent of each other and of Brouwer, given a more general definition, the main difference being (a) their definition was local, (b) the zero-dimensional case was given a more elegant and general form. So far everybody would agree, even Menger. So why the fight? There were two ingredients that yielded the devastating effect: Brouwer's excessive protective feelings towards Urysohn's and his own priorities, and the ambition and aggression of the young Menger.

Menger's historical exposition in his book was at best a reckless act, at worst a slick attempt to rewrite history. It is not the facts that are wrong in Menger's story, it is the way they are arranged and the way in which information is suppressed. None but an insider could recognize the true course of events. The general impression left by Menger is that he created dimension theory first (February 1922), Urysohn then published his version (September 1922). The impression of Menger's priority is fortified by the mentioning of Menger's 1921 deposit of his first manuscript. Urysohn and Menger are mentioned as founders of dimension theory in the flyer, but not in the historical outline. Menger's claim that he never denied the fact that Urysohn's paper appeared first is correct, but it is not affirmed either (in the short history). This list can be extended *ad libitum*. Of course, by scrutinizing the book and comparing data, a reader of Menger's book could get at the facts, but the historical section would then still puzzle him.

To mention Brouwer among the forerunners of the (final) notion of dimension is curious or even denigrating. The formulation of the paragraph on Brouwer's contributions to dimension theory will strike the reader as patronizing and somewhat embarrassing. A general impression 'an old dodderer who had a nice idea but could

not work it out' is conveyed. Menger could not possibly have been so insensitive as not to have noticed this effect. His tactics of misinterpreting Brouwer on purpose were also cleverly carried out, e.g. by *literally* quoting Brouwer's papers he could truthfully state that Brouwer got the wrong notion and published the right notion later, instead of accepting (like Urysohn) that Brouwer had made a clerical error and had recognized this almost immediately. Such an act would require an amount of charitable collegiality that may have been present in the years 1925,1926, but which was sadly lacking in the later years of the twenties.

In the matter of the editing of Urysohn's memoir Menger was of course right, although he introduced the topic for the wrong reason. Urysohn's being or not being a successor of Brouwer had nothing to do with Urysohn's memoir. One could at best look for signs if Urysohn *felt* himself a successor. But the decision on these matters, as a rule, is reserved for later generations, who have a less passionate view of historical matters. The argument was most likely introduced in order to allow an oblique reference to the editorial activities of Brouwer. It is a mystery why Brouwer, who usually was a miracle of precision, did not indicate the 'changes made in print' or did not at least mention the proof- reading and correcting in the paper. There is nothing basically wrong with editing posthumous work, including the elaboration of the last ideas and wishes of the deceased—as long as they are visible. In the case of Urysohn's memoir, after all, two respected scholars could testify to its correctness. As it was, Brouwer laid himself open to suspicions, ranging from sloppy editing to fiddling.

Long after Menger's final shot, both Menger and Brouwer remained touchy about the subject. In 1930 Brouwer spoke still of the 'Menger war' (*Menger Krieg*). In the *Monatshefte*'s review section there is a curious item by Menger, that shows how the recent events in dimension theory were becoming something of an obsession. The *Jahrbuch über Fortschritte der Mathematik* had sent the issues of 1925, 1928 and 1929, for review to the *Monatshefte*. The *Monatshefte* passed the copies on to Menger, who tested the editorial procedures on (1) the choice of expert reviewers, (2) how to choose a reviewer of a controversial paper, who is in any way party to the controversy, (3) the classification of papers, (4) the control of the observance of these criteria. After discussing the embarrassing confusion with respect to classification he came to the point. There was only one recent controversy, he said, and that concerned his book on dimension theory and the priority debate with Brouwer that followed its publication. And what would have been more appropriate than assigning this book for reviewing to 'an authoritative expert in Germany or Austria. But instead I found my book reviewed by a Mister Freudenthal, whose name is not known to me for any scientific result to speak of, but about whom I learned that he had recently habilitated in Amsterdam.'[198] Freudenthal lacked the required objectivity, he claimed, as was apparent from the opening sentence of his review in

[198]Menger may not have been aware that there was (and is) no such thing as a habilitation in Holland. He may have adopted Brouwer's view that the admission as a 'privaatdocent' was the equivalent of the German 'Habilitation'.

which he confirmed Brouwer's view: 'The present work is the first textbook collecting dimension theory, of the kind that was promoted by Poincaré, founded by Brouwer and extended by Urysohn, the author and others.' Compare this, he said, to the objective reports by Tietze and Vietoris as published in the Encyclopaedia of Mathematics[199] (these were, by the way, the same authors that Menger thought incapable of the job, see p. 535). The editors had, in Menger's opinion, made a mess of the reviewing by assigning both the *Grundrisz der Mengerschen Dimensionstheorie* and *Grundrisz der Urysohnsen Dimentionstheorie* to Alexandrov. Apparently Menger had no high opinion of the integrity of his fellow topologists. He was angry indeed at the editors of the *'Fortschritte'* who were, in his eyes, 'implicated in statements that falsified historical facts.'

The whole complex of priority questions around dimension theory remains a mire of conflicting opinions. There are basically two conflicts: Brouwer–Menger and Menger–Urysohn. The Brouwer–Menger priority conflict is partly a matter of terminology, namely insofar as it concerns the term 'dimension theory'. If we reserve the name 'theory' for a coherent body of theorems, notions, methods, then Urysohn and Menger developed dimension theory. Brouwer would probably be called the founder (or father) of dimension theory: he gave the first correct definition of dimension and tested it for correctness in the case of Euclidian spaces. So he showed that the notion was correct and viable. The issue of the 'clerical error' was viewed differently by Urysohn and Menger (the two immediately interested parties); Menger was not convinced by Brouwer's arguments, he was inclined to view the 'slip of the pen' as a real conceptual error; Urysohn, on the other hand, saw that under the given circumstances Brouwer must have intended the right notion of separation. Menger, strictly speaking, is right, After all, a proof with an error is no proof at all. It is a quirk of history that the champion of precision, Brouwer, had to plead 'an oversight', where others could condemn him for 'an error'. Taking into account the available evidence, the charitable reading seems quite in order. Freudenthal's thorough historical analysis convincingly tips the scales in favour of Brouwer.[200]. The fact that Brouwer's 1913 paper remained unnoticed, cannot be used against Brouwer's priority claim.

The Urysohn–Menger priority conflict was a far more difficult matter. Here three kinds of criteria were used in the discussion: (i) the date of discovery, (ii) the date of submission, (iii) the date of publication. And these criteria were applied all at the same time, with rather confusing outcomes. Menger repeatedly stated that he was not after a priority contest with Urysohn, nonetheless he felt he had to defend himself when Freudenthal in his review in the *Fortschritte* spoke of Urysohn's curve theory. Formally Menger had a point, Urysohn's paper was published posthumously in 1928. Nonetheless, something jars.

[199] [Tietze, H. 1914].

[200] [Brouwer 1976], p. 548–552, 564–567

In 1951 Alexandrov edited Urysohn's collected works.[201] For our purpose two features of this edition are significant: (i) a survey and comparison of the contributions of Menger and Urysohn; (ii) indeed, some of the footnotes mentioned by Menger in his note of 1932 were deleted. There is no editorial comment on the adoption or deletion of references. One is inclined to believe that Alexandrov dropped the notes added in the editing process. This, by the way, may reflect on the editing, but certainly not on Brouwer's integrity. In the collected works Alexandrov did indicate the places where comments were added. The disputed footnote, [Urysohn1951], p. 37, was literally reproduced.[202]

In a detailed analysis of the documents of Menger and their history (an autopsy almost) Alexandrov comes to a rather negative conclusion with respect to Menger's priority. The first non-trivial theorem in dimension theory proved by Menger was in December 1923, 'one and half years after Urysohn completed the creation of dimension theory, and 10 months after Urysohn forwarded to the editors of F.M. a complete exposition of his theory in the form of Part I of his *Mémoire sur les multiplicités Cantoriennes*'. 'All the theorems found in the papers considered,[203]were established by Urysohn in the fall of 1921 and winter 1921/22 and presented in his main memoir.' After discussing the relevant research of Menger and Urysohn, Alexandrov reaches the following conclusion:

> Let us summarize the above brief review of Menger's work. The only piece by him which, on the one hand, was written before Urysohn's death (August 17, 1924) and, on the other hand cannot in the least degree, in view of its content, support claims to development of dimension theory, is the second part of his paper '*Über die Dimensionalität von Punktmengen*'. This article was not only much poorer in content than Urysohn's '*Mémoire sur les multiplicités cantoriennes*' thus not having had (nor even having been in a position to have) any influence on the subsequent development of topology, comparable to the influence of the Urysohn's classic study; but it was, according to the author himself, submitted for publication in the fall of 1923, i.e. a year after the publication of Urysohn's notes in the *Comptes Rendus* and half a year after Urysohn submitted his memoir,[204] which contained a truly complete and exhaustive presentation of his theory. Under these circumstances Urysohn's priority over Menger is an objective historical fact which precludes any possibility for considering Menger an equal co-creator of this theory.

[201] [Urysohn1951].

[202] The following changes are made in the text of the memoir: p. 328 lines 6–9 including footnote 1, replaced by 'Put differently, our initial *local* definition of dimension is equivalent, *for the case of closed sets*, to the global definition just given; in the latter one the dimension of *points* plays no role whatsoever.' p. 345. 'une autre … 148)' deleted. p. 351. lines 1–4. Replaced by 'A question poses itself as to the equivalence, in more general settings, of the various definitions of dimension given in the present memoir.'

[203] I.e. up to [Menger 1923] issued in 1924.

[204] 20.III.1923, cf. [Johnson 1981], p. 228.

The dimension conflict was re-enacted once more when Freudenthal analysed the facts again as an editor of the collected topological works of Brouwer, and Menger subsequently disputed his conclusions.[205]

The social impact of the dimension conflict should not be overrated. Most mathematicians deplored the fight, recognizing the scientific status of both Brouwer and Menger. It was no secret among the mathematical community that both men were difficult, and that the conflict was blown out of proportion. There are not many contemporary comments on the issue; in a letter of 14 January 1929 to his coauthor, Hopf, Alexandrov commented on the matter, 'In my view Brouwer is objectively right, and if I were asked for an opinion, I can only—in accordance with my information, and my conscience—do as I have done, that is, agree with Brouwer and Urysohn, and not with Menger.' Hopf, in his reply of 3 March cautiously agreed, 'Brouwer seems to be right.' The smoothing effect of time has taken the sharp edge from the matter. In the book of Hurewicz and Wallman, the standard text for dimension theory for a long time, the issue is not mentioned at all, and Brouwer is simply given credit for his definition of dimension.

[205] [Brouwer 1976], p. 564; [Menger 1979], p. 237.

16

THE THIRTIES

After the emotional controversies of the *Mathematische Annalen* and the dimension theory, Brouwer carried on in his characteristic way, be it that he occupied himself more with secondary activities than research. After the breakdown of the negotiations with Hahn concerning Menger's 'Reply', Brouwer had decided to withdraw the paper 'The structure of the continuum' from the *Monatshefte*. He thought it impossible to publish a paper in a journal where the editors allowed Menger to attack him without allowing him the right of rejoinder. Even when Wirtinger, Hahn's co-editor, offered to publish the rejoinder, Brouwer despaired of the possibility to give a good presentation: 'without an unabridged publication of a series of authentic letters and documents from Urysohn's estate we would not succeed.'[1] Ehrenhaft, who had also incurred Menger's wrath, still hoped to get Brouwer's 'Continuum' paper published before Menger's 'Reply', so that at least the reader would not be prejudiced by Menger's note, but even that proved impossible. So we find Brouwer deploring in a letter to Ehrenhaft the necessity to withdraw his paper, referring to the editors of the *Monatshefte* 'who had openly and actively supported the robbery of the person Menger.'[2] Having no quarrel with the committee that had invited him to lecture, he had no objections to the publication of the manuscript as a pamphlet.

When Brouwer finally saw that Menger's 'Reply' had appeared, he angrily blamed Hahn,[3] 'Hahn has finally thrown off his mask.' In his anger Brouwer completely overlooked Hahn's efforts for a solution to the Menger conflict, and Hahn's undisputed integrity. After a long exchange of letters between Brouwer and Ehrenhaft, the Continuum paper finally appeared at the end of 1930. This really and definitely put an end to Brouwer's involvement in the Menger affair. He had with meticulous care collected an immense file containing copies of letters, originals, drafts, proof sheets, … The file contains no documents later than 1930; in particular Menger's note on the references in Urysohn's memoir is not mentioned.[4]

[1] Brouwer to Wirtinger 11.I.1929.

[2] Brouwer to Ehrenhaft 1.IV.1930.

[3] Brouwer to Ehrenhaft 19.IV.1930.

[4] Part of the file was taken to Moscow after Brouwer's death by Alexandrov, and the remains that survived two fires and several transfers are in the Brouwer Archive.

16.1 Freudenthal arrives

Freudenthal, the bright student from Berlin, had come to Amsterdam in 1930 after writing his dissertation under supervision of Heinz Hopf and Bieberbach.[5] Brouwer was immediately taken with the erudite young man, who was generally well-informed about mathematics, including foundational matters, and who was equally well at home in philosophy, philology and related topics. When he arrived in Amsterdam, Brouwer more or less ordered him to live in Laren. It was a matter of principle for Brouwer that his visitors and assistants lived in his direct neighbourhood. For Freudenthal this was not attractive at all; after Berlin he longed to live in a city with all the comforts of shops, culture, etc. So he asked Brouwer permission to move to Amsterdam; Brouwer grudgingly agreed, but insisted that Freudenthal should get himself a telephone connection, for 'I must be able to reach you day and night.' So Freudenthal did acquire a telephone and ... Brouwer never did call him! In Freudenthal's correspondence descriptions of academic life in Amsterdam, or Holland, are to be found which are instructive because as a keen foreign observer, he saw clearer than the local mathematicians how the curriculum and the research compared to the international centres.

The University of Amsterdam, he said, was comparable to a small German university, say, Erlangen. The mathematics professors were decent mathematicians, but, with the exception of Brouwer, not of a level that could match one of the better German universities. So much, one could say, for Brouwer's Göttingen at the river Amstel. The other professor with an international reputation, the local expert in invariant theory, Weitzenböck, for example, was not in touch with modern mathematics. Freudenthal recalled that at one occasion Weitzenböck asked him to explain what Galois theory was.[6] Mannoury and De Vries were popular teachers, but certainly not innovators of the mathematics curriculum. Mannoury's fame was mostly based on his role in significs, but in the international philosophical community he was certainly not seen as one of the leaders of the period. Freudenthal, anyway, was not impressed by the significs aims and achievements. Brouwer gave a variety of courses, among other things a regular course in mechanics (mathematical physics)—in fact a rather old-fashioned course. Brouwer was good at the traditional mechanics, he enjoyed the topic, but his active knowledge of modern physics was limited, he had not kept up with the developments. In general Brouwer appreciated physics and physicists, he would not have approved of a mathematics curriculum without physics.

Freudenthal did not find life at the university demanding; Brouwer left him free to pursue his research, although he was originally taken on to assist Brouwer in his intuitionistic programme. He did, as a matter of fact, occupy himself with

[5] For more on Freudenthal's Berlin years see [Freudenthal 1987a].

[6] Freudenthal may have been a bit prejudiced; Rutherford mentions another experience: 'In the winter of 1929 Professor Weitzenböck pointed out to me that there was no complete account of the theory of modular invariants embodying the work of Dickson, Glenn and Hazlett. ... The substance of Part II is largely taken from a course of lectures entitled "Algebraische theorie der lichamen " which Professor Weitzenböck delivered in Amsterdam University during the session 1929–30.' [Rutherford 1932].

intuitionistic mathematics, gave alternative proofs of a number of Brouwer's results, and in 1936 published a beautiful paper on intuitionistic topology. Brouwer had been flattered that a young man from a German elite university was interested in intuitionism, and had asked sensible mathematical questions—not the routine silly ones.

When Freudenthal took up his post in Amsterdam, Withold Hurewicz had been Brouwer's assistant already for four years. Hurewicz had come to Amsterdam in the wake of Menger and had stayed on when Menger returned to Vienna. In the year following the publication of Menger's *Dimension Theory*, Hurewicz felt so oppressed by the atmosphere of hostility that he made up his mind to leave Amsterdam and look for a job elsewhere. This was, however, easier said than done; moreover with Menger safely in Vienna, life gradually returned to normal. The cooperation between Freudenthal and Hurewicz was exemplary; between the two of them they developed the fundamentals of homotopy theory and they definitely contributed towards the pre-war fame of Amsterdam as a research centre for topology. Hurewicz was an unusual man, the son of a rich industrialist in Lodz (Poland). In Amsterdam he made himself useful in many respects. Like Freudenthal he refereed manuscripts for Brouwer, supervised students, taught various courses, and so on. His lectures were polished and very clear. Brouwer assigned various tasks to his assistants, e.g. they conducted seminars for advanced students, refereed papers for Brouwer's new journal, presented new results in private seminars, etc.[7]

When Freudenthal had settled in his new job, he wrote two extensive reports on his new surroundings, one to Heinz Hopf, and one to the organization for student grants in Dahlem. The letter to Hopf is of interest, as it represents the first impressions of a young mathematician in his first job. We can do no better than quote part of his letter,[8]

> At the moment I am almost exclusively occupied with dimension theory; and, as a matter of fact, as a part of my duties (*dienstlich*), I should indeed help to 'arm' for the 'Menger war'. Brouwer is planning all kinds of new battles against Menger. When he speaks of Menger, he makes an almost pathological impression. I am not as belligerently minded as Brouwer, but I can understand if somebody, who has known Urysohn personally, even if his name is not Brouwer, gets into such moods when faced with the intrigues of Menger; for having now occupied myself intensively with the papers of Urysohn and Menger, I have gained the opinion that Urysohn was far superior to Menger. Urysohn makes in all his papers the impression of a great genius, whereas Menger appears only as a talent (through a very gifted one). But whether a continuation of the conflict

[7]According to Freudenthal, Hurewicz presented Gödel's incompleteness theorem in a seminar. He also refereed foundational papers (including Heyting's big logic papers).

[8]Freudenthal to Hopf 22.XII.1930.

> serves the case, is very doubtful. But with these arguments one cannot come to Brouwer!

We have seen that no new actions were undertaken by Brouwer, but the above passage illustrates how much Brouwer was obsessed by Menger's tactless, to say the least, operations.

Hans Freudenthal (Courtesy Miriam Freudenthal)

The letter to the *Deutsche Studentwerke* contained more information on academic life; for a German the Dutch universities had some surprising aspects. Freudenthal's observations may have been a bit one-sided, but there was certainly some truth in them. Students, he said, heard lectures for some 6 to 8 hours a week, and it was no exception that a full professor lectured 2 to 4 hours a week. Nonetheless the students complained about an overloaded programme. But perhaps, this was exactly the consequence of the low teaching load, for it could force students to learn subjects from books. Freudenthal was not terribly impressed by the Dutch student:

> The level of the Dutch student is lower than that of the German one; that has to do with the somewhat passive national character. The Dutch student certainly works diligently, is, however, very slow in grasping things. One hardly sees original productive work of him.

Freudenthal, with his Berlin past, saw that there was some room for an improved curriculum. He indeed was quite active in stimulating the students, and in modernizing the programme.

Freudenthal found to his surprise that topology was not a standard part of the mathematics programme at the birthplace of modern topology; when he and Hurewicz—at Brouwer's suggestion—wanted to start a seminar on topology, it turned out that nobody knew the subject. The initiative of Alexandrov and Menger to establish a basic topology curriculum had not been followed up. And so they had to start elementary courses on topology; Hurewicz taught set theoretic topology and Freudenthal combinatorial topology. Brouwer lectured (one hour a week) on foundations of geometry to roughly 30 students and likewise on canonical differential equations to a dozen students.[9]

16.2 Intuitionistic logic

After the twenties, ironically enough, intuitionism became respectable, mainly through Heyting's formalization, which appeared on 20 February 1930 in the Proceedings of the Prussian Academy.

Already in 1929 the Belgian mathematician De Donder had urged Brouwer to write an exposition of intuitionistic logic.[10] The reason for this request was a series of papers on intuitionistic logic by Glivenko, Barzin, Errera, P. Lévy, presented by De Donder to the Royal Academy of Belgium. De Donder had serious doubts about a recent paper of Barzin and Errera 'On the principle of the excluded third' (1929) and was eager to have Brouwer's comments. In June 1930 Brouwer finally got round to the request, and wrote that he was about to write an exposition, but that he had not been able to get hold of some of the relevant papers, could De Donder provide him with those?[11] De Donder answered by returning post that he would send the requested papers. And there the matter unfortunately ended, for in October Brouwer informed De Donder that 'while preparing a note on intuitionism for the Bulletin de l'Académie Royale of Belgium, I was pleasantly surprised to see the publication of a note of my student Mr. Heyting which elucidates in a magisterial manner the points that I wanted to shed light upon myself. I believe that after Heyting's note little remains to be said.'[12] And so Brouwer's view never appeared in print. Indeed, Brouwer left logic entirely in the hands of Heyting. Heyting had represented the intuitionistic viewpoint at the famous Königsberg meeting, September 1930;[13] upon the receipt of Heyting's account of the meeting Brouwer wrote to him: 'Many thanks for your letter of the 16th of this month; I conclude from it that you must have been satisfied with your talk at Königsberg,

[9] First semester of 1930/31.

[10] De Donder to Brouwer 26.X.1929.

[11] Brouwer to De Donder 13.VI.1930.

[12] Brouwer to De Donder 9.X.1930.

[13] [Heyting 1931b].

and I share in this satisfaction with all my heart.'[14] In the same letter he requested Heyting to write an exposition for *Forschungen und Fortschritte*, which Heyting duly did.[15]

The logic discussion carried on by Heyting with Barzin and Errera, mentioned above, was characteristic for the confusion that intuitionism created among superficial readers. E.g. Barzin and Errera were convinced that Brouwer in effect introduced a 'third' into logic (namely $A \vee \neg A$ which is in their opinion neither true nor false). Heyting effectively refuted the two Belgians, and—rather exceptional for a cautious man like Heyting—indulged in a mild verbal wordplay; 'the classical logician resembles an imaginary mathematician who claims that all abstract spaces admit a metric and who reproaches those who demand a proof of that theorem for wanting to attack the liberty of science.'

In the exchange of views Barzin and Errera finally accepted that it is the interpretation of the logical connectives that makes the difference; e.g. they summed up the differences as follows, 'So we have arrived at this, the intuitionists call true what the formalists call demonstrated.' On the accepted reading of 'demonstrated' this shows a lack of understanding of Brouwer's position. Heyting, reconciliatory as ever, had stated, 'One or the other, either mathematics, consists of human thoughts, or it is purely formal. Brouwer's goal is to draw all the consequences of the first alternative.'[16]

What Heyting might have added is that Brouwer considered this the (only) right way to practice mathematics, but Heyting was in a way a better statesman than Brouwer—he knew how to sugar the pill. Barzin and Errera could thus remark: 'How meagre is this outcome, for what should have been a grand revolution.' The Barzin–Errera criticism of intuitionistic logic sparked a wider discussion in which Wavre, Lévy, Glivenko, Khintchin and others took part.

This small episode illustrates the fact that there was a rather confused conception of Brouwer's enterprise. And not only minor mathematicians, but even the best did not quite grasp the full extent of the intuitionistic programme. Herbrand considered intuitionism as something like finitism; he, so to speak, only took the discrete part of intuitionism into account.[17] Goldfarb in his edition of the collected works conjectures that Herbrand derived his knowledge of Brouwer's intuitionism from Hilbert, a quite plausible claim, since Herbrand was in contact with the Göttingen group and spent some time there. A similar explanation may cover the case of Von Neumann, who, at one point, also identified intuitionism with some form of finitism.

[14] Brouwer to Heyting 20.IX.1930.

[15] [Heyting 1931a].

[16] [Heyting 1932]. Note that the discussion actually took place in 1933. For the Barzin-Errera – Heyting discussion. see also [Hesseling 2002].

[17] Cf. [Herbrand 1971], p. 273,282 ff.

16.3 The Sodalitas affair

In 1931 the Amsterdam mathematics department watched with surprise and some mild horror how Brouwer embarked on a crusade against financial tricksters. It seems that somewhere in the late twenties Brouwer acquired some shares in a Spa (a complex organization of baths, a hotel, machine room, music pavilion, etc.), and before long got wind of unsavoury practices.[18] In the Brouwer Archive there is some fragmentary documentation concerning the matter, and newspapers from that period contained from time to time reports on the affair; from this, one can piece together a birds-eye account of the proceedings. There may be a fairly complete file somewhere, but for our present purpose a sketch will do.

In 1927 the Austrian prior Robert Koch bought some property in Budapest with the intention to train young man of limited means as missionaries. For this purpose the *Sodalitas-Medicorum Oblatorum Ordinis Sancti Benedicti* was founded. The Cardinal-Primate of Hungary acknowledged the project with the words: 'We inform all and sundry that the Superior Robert Koch with our authority and the authority of his bishop is occupied with the training of doctors for the mission, and as superior has the right to manage the affairs of this order which is registered and legally accredited by us.'

Koch had met a Dutchman, Isidoor Leenes, a director of both the Burgers & Co Bank and the Trust Office for Investment and Administration; from this union the idea sprung to start financial operations on a big scale, no doubt with the best intentions, for the missionary activities of superior Koch. In May 1927 the Sodalitas-Medicorum bought the *Elisabeth Salzbad* in Budapest, a considerable complex based on a health spa. In June Leenes issued a loan to the Sodalitas of 133% of the price paid for the bath; the duration of the loan was 40 years. Another loan was given by a Hungarian bank. One condition of the loan was that Koch should sell the building land belonging to the bath. Koch, however, entered into all kinds of speculations, so that at a certain moment there was a civil claim on him of the size of almost half the value of the bath. In spite of this, the trustee Leenes did not take any action. When it finally became known that most of the money of the sale of the building land had disappeared and that the books of Sodalitas were a shambles, the Dutch creditors took action and asked the Primate of Esztergom to appoint a new superior. The answer was as surprising as it was brief: the order had never been approved by the Church and did therefore not exist according to ecclesiastic law. The Primate saw therefore no reason to interfere. Even worse was the reaction of the Arch Abbey of Pannonhalma, whose coadjutor had given a deeply moving address, praying for the Lord's blessing over this new order, at the consacration of the new chapel. Their representative stated that the matter did not regard Pannonhalma, it did not know the order, which was not registered there.

[18]The matter is mentioned in a letter from Henri Borel to Brouwer in December 1928. From the wording it appears that Brouwer already had grave suspicions.

Thereupon Leenes travelled to Hungary and pointed out to the Primate that if he disbanded the order, the assets and liabilities of the order became his responsibility. Thus confronted, the Primate answered that Sodalitas 'then should exist again.' After some negotiations the bath was bought by a Hungarian bank for 8819 Pengö. The bank undertook to pay 7500 guilders a year to the Dutch investors for interest and repayment, and subsequently went insolvent. On March 20, 1930 Leenes bought the remaining parts of the bath for 1000 Pengö, and so the matter became a Dutch affair.

The financial mismanagement, which had not drawn much attention in Holland, was brought into the open in the press in 1930. One investor complained in a letter to the editor[19] that investors had so far believed that, in spite of the changing position of currency, a loan sanctioned by the Roman Catholic Church was relatively safe, 'Roma locuta, causa finita.' From then on the matter got in high gear; on July 7, 1930 the first big meeting of the bond-holders in Holland was held. The meeting was described as 'tumultuous' in the press. At this occasion also Brouwer's name pops up; amidst the general uproar Brouwer tried to restore a minimum of order—'let the chairman speak!' The chairman, Leenes, gave an account of the history of the Sodalitas. The lawyer of the Hungarian clergy vigorously denied Leenes' allegations of mismanagement and irresponsibility of the Benedictine order; he, in turn, accused Burgers Bank of overcharging and of suppression of facts. He exclaimed that 'a mission should be sent to Budapest to investigate Leenes' actions' (applause). After a good deal of discussion and against the wishes of Leenes the meeting eventually appointed a committee, called the Asch van Wijck Committee, after its chairman. The committee got a mandate from the Trust office for Investment and Administration, but when its investigations started to yield results the Trust office regretted its generosity and decided to withdraw the mandate. Hence a conflict between the committee and the Trust office (i.e. Leenes).

In the following weeks the Burgers Bank and the Hungarian clergy exchanged accusations. On July 21 a meeting was called in the Hague, which was so well attended that the company had to move to a larger hall of the hotel 'Hollandais'. The committee had come to the conclusion that the role of Leenes was far from perspicuous, in a combination of functions he was in fact the only opponent of the bondholders. The meeting, like its predecessor, was tumultuous, in the general disorder—to which Brouwer had contributed by submitting a number of motions—no sensible decisions were made and the lawyer of the Hungarians did not even get an opportunity to present his views, views, which he subsequently ventilated in the press, identifying Leenes as the main, if not only, culprit.

Looking back, one must conclude that both the Hungarian clergy and Burgers Bank were to blame for the financial disaster. The Roman Catholic authorities had not for a moment cared for their responsibility, the prior Koch had, to express it

[19] Nieuwe Rotterdamse Courant 2.VII.1930.

kindly, been guilty of poor management and Leenes had displayed a conspicuous lack of business scruples (again, to say the least). One should, of course, bear in mind that the Sodalitas project was founded in the worst period of global crisis. Under normal circumstances Church and Bank would probably have got away with their mismanagement. The bath itself was declared bankrupt on January 9, 1931, and the Asch van Wijck committee, in the meantime supplemented with Brouwer, founded on February 13 the 'Dutch-Hungarian Company for the Exploitation of the Bath and Terrain'[20] (HHBEM).

Brouwer immediately took his responsibilities seriously, he corresponded, travelled to Budapest, conferred with lawyers, communicated with individual bondholders. Already in 1930 he spent some time in Budapest and in 1931 he visited Budapest seven times as a representative of the HHBEM. Freudenthal reported in a letter to Hopf that Brouwer was intensely occupied with the bankruptcy of a Hungarian Spa—'after only just escaping from his own bankruptcy'. He dryly commented that 'Brouwer had already succeeded in getting some bishops into jail, they suffer, so to speak, the punishment that Brouwer had in mind for Koebe, Hilbert and Menger.'[21]

It is incredible how much time Brouwer wasted on the Hungarian affair; in a private note he added up his files in 1944 to a total of 67 kilo's, and he estimated the time spent on the affair at 'many thousands of hours.' Students of that period reported of lectures that were not given, of classes that were waiting for Brouwer who did not show up. What brought Brouwer to this unholy activity can only be guessed, probably it was a mixture of his exaggerated sense for justice and a form of academic vanity: as one of the sharpest minds in mathematics, I should be able to clean this financial Augean stable.

What started as a fast action to sort out the affairs of Sodalitas, quickly bogged down in a protracted warfare between Leenes and the committee. The war was fought with varying success. Where the committee carried out the first attack, Leenes fought back tenaciously. Both sides brought accusations of fraudulent behaviour, of selfish spending, of misleading, etc.

At a meeting of the bondholders of October 29, 1930, the committee read a report on Leenes' activities in Hungary, aimed at undermining the credibility of the committee. Leenes, in turn, accused the committee of forgery. At that point Brouwer, who chaired the meeting, summoned Leenes to leave the meeting, at his second summons, Brouwer gave a sign 'and two plainclothes men came up, whereupon Leenes left the room, while the audience applauded.' Thus excluded, Leenes defended himself in the newspapers, attacking the committee for smearing the good name of the bank and thus damaging the interests of the bondholders. His comments on Brouwer were surprisingly mild: 'Everybody should appreciate the fact that Prof. Brouwer was satisfied with a couch in the private house of Mr.

[20] Hollandsch-Hongaarsche Bad- en Terrein Exploitatie Maatschappij.

[21] Freudenthal to Hopf 2.IX.1931. The reference to bishops in jail is probably a bit of irreverent gossip; the notes in the archive do not mention such a thing.

Pogany,[22] and eats from his kitchen, so that the expenses involved can really be called marginal . . . '

The press considered the matter interesting enough to devote some space to it. A financial weekly published a report with the headline

> '**PROF. DOCTOR L. E. J. BROUWER NON-ACADEMICUS**.
>
> Not all are chairmen that wear gowns. The most learned man as a dictator and as power-mad.

It is said that this edition was handed out by Leenes himself in the courtyard of the *Oudemanhuispoort*, the central building of the university, to students coming from their classes.

PRIJS 10 CENT.

ADMINISTRATIE-ADRES:
WAGENSTRAAT 189 - DEN HAAG
ABONNEMENTEN:
ƒ 1.25 p. 3. mnd. (bij vooruitbet.)
REDACTIE-ADRES:
ZWETSTRAAT 60 - DEN HAAG
Advertenties worden niet opgenomen.

HETGELD

Van komende dingen.
(ZIE PAG. 3)

Ie JAARGANG No. 7 — HET WEEKBLAD VOOR DEZEN TIJD — 9 NOVEMBER 1931

PROF. DOCTOR L. E. J. BROUWER NON-ACADEMICUS

Het zijn allen geen Voorzitters, die Professorale Toga's dragen.

Zijn Hooggeleerde als Dictator en Machtswellusteling.

How a financial journal saw Brouwer's role in the Sodalitas affair.

From then on the affair developed along classical lines, attempts to settle the dispute were made and almost immediately frustrated. Third parties were engaged to give a opinion, but the third parties as a rule soon joined the melee. Brouwer's position, both in the social and in the financial sense, became precarious. He kept shuttling between Amsterdam and Budapest, no doubt spending a substantial part of his own money. In a letter of June 3 to Pogany, we read:

> In the meantime my personal position is catastrophic, maybe I can extend my (economic) existence for a few more days, perhaps weeks or months, but without a great change in Hungary I am certainly lost and each day the end can come. Every initiative of my conduct of life has been wrested from me, *other* men determine more than 80% of my actions and my time.

[22] A Hungarian co-director of the HHBEM.

> In addition fear, shame and distrust with respect to my hangmen, animal abusers (*Tierquäler*).

No doubt all this was more than Brouwer had bargained for; what had started as a simple crusade for justice, had become an obsession, nobody could be trusted, even his own party fell to bickering amongst themselves. At one occasion Brouwer felt so much bodily threatened by one of his fellow committee members, that he more or less saw himself forced to leave the room. Some members of the Dutch Association of Bondholders, started a counterattack at the Asch van Wijck committee and one of them brought forward claims to the effect that Brouwer had received as an advance fee bonds to the value of 10,000 guilders and 5000 guilders in cash. Brouwer at a next meeting gave a full account of all expenses and claimed that the members of the committee had in fact received a negative fee.

In 1933 it is mentioned in the correspondence that Leenes 'was already in jail for some time,' probably for some unrelated fact, the Sodalitas file does not mention particulars. More and more the whole affair dissolved into a cloud of in-fights, lawsuits, new loans, bankruptcies, ... In 1935 Brouwer, in a desperate attempt, brought the end of the affair within reach, but failed. Finally in 1938 a settlement with the Bank (Centresco) was reached, and in the same year the Bath was sold. The payment of the bondholders, however, had to wait again—Leenes and others saw their chance to present their claims first. Only in 1940 the HHBEM came to its end, and the bondholders could collect their money. In 1951 the last correspondence concerning Sodalitas was conducted, and the last remittance took place in 1971!

The Sodalitas affair doubtlessly asked too much from Brouwer; we have already witnessed his mental and physical reactions to stress, in this case Brouwer was dealing with powers that were alien to him, lawyers, bankers, speculators on one hand, and honest small investors on the other. Already in 1935, Brouwer had proposed to quit the committee on medical advice.[23] His physician had given strict orders to spend the summer vacation in the mountains, and he felt that he could no longer rush to Hungary at each tiding of new disasters. A letter to Kérékjarto confirms this intention.[24] Somehow, he did, however, not succeed in extracting himself from the lions' den.

When explaining Brouwer's scientific inactivity, one must certainly take into account the immense claims made by this rash involvement in the world of finance. The files of Brouwer were partly destroyed by two fires, hence the account of the affair is far from complete; one little note lists the various trips to Hungary: 1930—twice; 1931—seven times ; 1932—eight times; 1933—five times; 1934—four times; 1935—none; 1936—three times; 1937—three times; 1938—three times ; 1939—twice. By themselves these trips already represent a serious drain on a person's valuable (and limited) time resources. It is, in a way, surprising that in spite of this

[23] Brouwer to De Kanter 3.VIII.1935.

[24] Brouwer to Kérékjarto 3.VIII.1935.

financial tragedy, which by itself was sufficient to destroy a person, Brouwer still found time for other activities.

Brouwer would not have been Brouwer if he had not found an opportunity to mix some pleasure with the dreary Sodalitas matters; he befriended a beautiful Hungarian lady. One day, to the great surprise of the Van Anrooij family, he showed up at their vacation cottage in Switserland with this new friend.

All the time that the Sodalitas affair claimed his attention, Brouwer had to carry on his normal duties. As we have seen, the university suffered most. Nonetheless we find Brouwer's name regularly in the minutes of the faculty meetings. In 1930 he initiated an action for improved salaries for faculty members and for a form of sabbatical. Needless to say that in those days of financial hardship, the proposals were rejected. In a next meeting Brouwer came back with a proposal to introduce a salary differentiation via a new function of 'chairman of a department'.[25] When that too failed, he suggested that the matter should be taken to the senate. The minutes recorded that 'Brouwer asks if we, like the curators, should agree that a raise of the assistants' salary is more urgent than that of a professor's income. We have here a nice occasion to point out the insufficient character of the present salaries.'[26] In another meeting a proposal of Brouwer is mentioned to add 1000 guilders to the faculty budget for guest speakers—turned down by the curators. The proposal of a sabbatical every ten years was, however, 'considered favourably.'

1931 saw the publication of an intuitionistic topology paper of Brouwer: 'On free enclosures in space', it took eight years before the next mathematical paper appeared!

Freudenthal was settling in, and on Brouwer's proposal (faculty meeting of 18.III.1931) he was admitted as a private docent; on May 28 of the same year he gave his inaugural address.[27] That same year Brouwer repeated one of his old grievances; in the faculty meeting of December he complained that the faculty was not consulted about the philosophy appointments.

The University of Amsterdam celebrated its fiftieth birthday in 1931, with a rich variety of ceremonies—a reception in the Rijksmuseum, another one in the Royal Palace, a concert by the Concertgebouw Orchestra, conducted by the famous Mengelberg, . . . In the same year Korteweg was congratulated by the Senate of the University on his golden doctorate (he was in 1881 the first doctor of the University!)

The mathematics section of the faculty in Amsterdam had already lobbied for some time at the Board of the University for a separate Mathematical Institute, so far lectures were held in the centre of Amsterdam in the Oudemanhuispoort, where the mathematicians had a reading room and a cloak room. It was felt that, following the developments abroad, a building was required to function smoothly.

[25] Departementshoofd.

[26] Assistants earned at that time 1000 guilders a year, the proposed raise was 200 guilders.

[27] The so-called 'openbare les', i.e. 'public lecture'.

The Board informed Brouwer and Mannoury in 1932 that their demand could not possibly be met. A serious setback, but no reason for Brouwer to despair. He kept actively reminding the Curators of the desperate situation and finally his efforts were crowned with success, but not after many a disappointment.

When it was the faculty's turn to propose an honorary doctorate, the choice was between Levi-Civita, Féjer and Sierpinski. The latter was awarded the doctorate in the formal session of 25 April 1932, together with Korteweg, thanks to Brouwer's active influence.

At the same ceremony Brouwer's uncle Poutsma, the English teacher, was awarded an honorary doctorate. Although Brouwer loved to promote the interest of members of the family, it is not likely that he influenced that particular choice.

Over the years the minor (and not so minor) duties of Brouwer started to accumulate, e.g. somewhere around the thirties Brouwer had become a commissioner of the *North-Holland Publishing Company*. Generally speaking, the time of great achievements seemed over, Brouwer's activities present a picture of the rather mixed bag of duties, jobs and hobbies of the common professor. One should, of course, keep in mind that the Sodalitas affair was always at the background during the thirties, and that it emotionally and financially drained him. The thirties also brought Brouwer the recognition as the formidable scholar he was. Hopf, as one of the organizers of the international congress of mathematicians in Zürich, invited him as a main speaker in the philosophy section. Brouwer seems to have wavered between accepting the invitation and staying away; Freudenthal reported to Hopf that Brouwer was rather inaccessible (i.e. often absent), but that he had said not to go to Zürich—for 'he was afraid that he would lay one on Menger' (*Menger eine runterzuhauen*), but that he would send Heyting. Probably Hopf eventually talked Brouwer into visiting the conference, but just before the conference he cancelled his attendance, 'serious personal circumstances prevent me to give a talk.' Curiously enough, he did after all attend the conference—the proceedings mention him as the chairman of section III (topology).[28] From Budapest he sent a card to Hopf, thanking him for the hospitality during the conference, 'although I am perfectly aware that I attended somewhat as an outsider.'

In Holland a certain routine had established itself for Brouwer; his student Belinfante produced research papers on intuitionistic complex analysis and Heyting, after his participation at the Königsberg meeting, was making a name for himself as a logician. Indeed, the Springer Verlag engaged Gödel and Heyting to write a book in the *Ergebnisse* series. As it turned out, Gödel never got round to write his part of the projected book, so eventually Heyting produced by himself the monograph *Foundational Research. Intuitionism and Formalism* in 1934.[29] The book gives an impartial survey of the foundational activities in mathematics, and even today it is well worth reading. One novelty was his presentation of the so-called *proof interpretation*. This interpretation went back to Brouwer's ideas on logic, and was based

[28] Speakers: Alexandrov, Borsuk, Hurewicz, Kaufmann, Knaster.

[29] *Mathematische Grundlagenforschung. Intuitionismus, Beweistheorie*.

Hurewicz and Brouwer. (Brouwer Archive)

on the notion of 'construction'. He had been aware of this notion already at the time of his prize essay in 1927.[30] In a way it stands to reason that he had to possess some kind of test for intuitionistic validity, after all, he went through Russell's logical axioms, weeding out the non-intuitionistic ones.

There is little doubt that the proof interpretation goes back to Brouwer. Already in his dissertation of 1907, Brouwer discussed logic and its role in mathematics. Indeed, he paid a good deal of attention to the problem of 'hypothetical argument', and there indications for the 'construction'-aspect of implication can be found. In Brouwer's correspondence with his Ph.D. advisor, Korteweg, the role of construction is even more explicit.[31] It does not require too much imagination to trace the construction-meaning of implication in Brouwer's famous 'jump from ends to means', as I have argued elsewhere.[32]

As Brouwer was parsimonious with elucidations in his writings, one has to see how Brouwer actually handled implication, see for example [Mancosu 1998], p. 286, 290. In daily practice Brouwer must have mentioned the 'construction' aspect of implication freely. There is, as a matter of fact, a set of lecture notes of his course on Intuitionistic Order in 1925, which mentions explicitly that certain axioms in the form of an implication should be read in the *intuitionistic sense*, i.e. that there should be means of construction (*Konstruktionsmittel*) yielding the conclusion from the premises.[33] Heyting's contribution consisted of two parts—he found an axiomatization of intuitionistic logic, and he gave the proof interpretation its elegant and correct formulation.

[30] Heyting, oral communication.

[31] Brouwer to Korteweg 23.I.1907.

[32] See e.g. [vanDalen 2004].

[33] The student who took the notes was David van Dantzing, who was to contribute to the foundations himself much later.

The mathematical landscape in Holland was nominally determined by the professors at the various universities, including the *Technische Hogeschool* (Institute for Technology) in Delft. In the late twenties and the thirties chairs in mathematics were occupied by J.A. Barrau, L.E.J. Brouwer, J.G. van der Corput, Hk. de Vries, J. Wolff, J. de Vries, J.F. Koksma, W. van der Woude, R. Weitzenböck, G. Mannoury, J.C. Kluyver, J.A. Schouten, and B.L. van der Waerden. Furthermore there were a number of lecturers, e.g. D. Kloosterman, J. Droste, H.B.A. Bockwinkel, A. Heyting, and numerous *privaat docenten*. But the mathematical community was by no means exhausted by the above list; there were capable researchers, such as H.J.E. Beth, D. van Dantzig, E.J. Dijksterhuis, H. Freudenthal, W. Hurewicz, F.R. van Kampen, H. Looman, J. Ridder, G. Schaake. Some of these names will appear in later pages. For the history of mathematics around Brouwer, we turn now to an ingenious and original young man, David van Dantzig.[34] David was born in 1900 in a Jewish family. His father owned a modest factory of a chemical-pharmaceutical nature. Van Dantzig attended the HBS,[35] and enrolled at the age of seventeen in the university to study chemistry. Financial problems forced him to give up his study prematurely. Several years later he studied mathematics in the teachers' curriculum, following in Mannoury's and Korteweg's footsteps. Having obtained the desired teacher's diplomas, he entered the university again, this time to study mathematics. He was almost immediately captivated by Brouwer's topology and intuitionism, and by Mannoury's significs.

It was topology that was to make his name; he wanted to write a dissertation under Brouwer's supervision, but something went wrong, there must have been some friction of a personal nature. Brouwer, who was not eager at all to accept Ph.D. students, told Van Dantzig point blank, that 'he had no idea of mathematics, and that Van Dantzig had taken up more than enough of his and Mannoury's time with all his questions.'[36] The cause of this sharp rejection is not known. One may guess that Van Dantzig, for whom tact was not a natural asset, had in some way ruffled up Brouwer, and Brouwer had after all those years little patience to foster human relations that were not his choice. Van Dantzig drew the right conclusion, and after consulting Mannoury he turned to his friend Van der Waerden, who had been appointed in Groningen in 1928. The result was more than satisfactory; he wrote a beautiful dissertation on topological algebra; the term 'topological algebra' was Van Dantzig's. It is no exaggeration to consider Van Dantzig as the creator of the subject; there had already been publications on topological groups, but the general treatment of topological rings and fields was largely lacking. The actual dissertation was a very condensed version of Van Dantzig's work, the full exposition appeared subsequently in the *Mathematische Annalen* and in *Compositio Mathematica*.

[34] Most of the historical information on Van Dantzig is taken from Alberts' biography [Alberts, G. 2000].

[35] See p. 4

[36] Van der Waerden to Schouten 22.IV.1932.

Van der Waerden had in the mean time accepted a call to Leipzig, where he would spend a large part of his career.

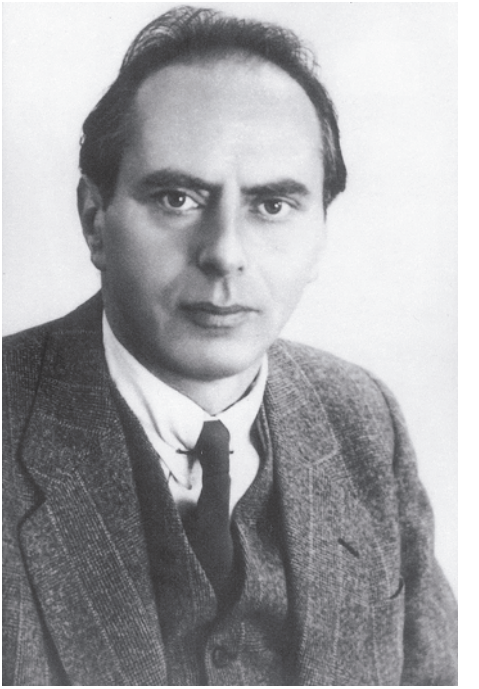
Brouwer seems to have acted with a certain lack of consistency. He had turned down Van Dantzig, and yet felt led down the garden path when Van Dantzig found a Ph.D. advisor elsewhere. The fact that the choice fell on Van der Waerden, and not on the advisor preferred by all students who could not make up their mind, Hk. de Vries, or on Weitzenböck as a fair second, must have contributed to his discount. Later, when Schouten was looking for a candidate for a chair in mathematics in Delft, there was a rumour that Brouwer claimed part of the results and methods of Van Dantzig's dissertation. Van der Waerden, when asked, vigorously denied the rumour.

Topologists will of course recognize Brouwerian traits in Van Dantzig's topological work, but that should not come as a surprise. It simply was a fact that Brouwer's investigations had set the tone for research until the fifties.

In 1932 Van Dantzig was appointed as a lecturer in Delft. This brief episode shows a pattern that will be recognized in later relationships. Brouwer had an enormous charisma, students naturally fell under his spell. But since he jealously guarded his privacy, conflicts between the opposing forces of attraction and protection were not unusual. Van Dantzig acknowledged Brouwer's genius and insight, without sacrificing his right to be critical. Brouwer maintained normal relations with Van Dantzig, but never came close to him.

In the thirties the older generation of significists had mostly withdrawn; Van Dantzig, inspired by Mannoury, provided new blood. He published and lectured on the topic. But even an outstanding mathematician like he could not get the subject going. He was definitely interested in the foundations of mathematics; he presented, for example Gödel's incompleteness results in the meeting of September 26, 1931 of the Dutch Mathematical Society, *Wiskundig Genootschap*. He did, however, not take part in the development of the formalization of intuitionistic logic, that area remained exclusively Heyting's domain in Holland. Only after the Second World War did Van Dantzig publish in that area.

Van der Waerden's departure from Groningen was a loss for Dutch mathematics. At that time, however, Groningen was often considered as a temporary sojourn, many a leading scientist moved from Groningen on to greener pastures. As a matter of fact there were forces that tried to keep Van der Waerden in Holland. It was in particular Paul Ehrenfest who made an effort to get Van der Waerden appointed in Leiden. Ehrenfest had a certain personal interest, as abstract methods, for example of group theory, were becoming prominent in theoretical physics. On 8 October 1928 he wrote to Van der Waerden that he would appreciate his assistance, as 'the group pest has broken out in our physics journals.' Two years later he was already negotiating with Van der Waerden, 'how nicely and soundly the problem of the mathematics appointments here in Leiden has developed.' He was aware that Leiden could not compete with Göttingen, 'The idea that in the fall you will start to work here, and that Leiden will develop into one of the centres of mathematics has



David van Dantzig. (CWI Archive)

been so much fixed in my head ..., that I would be totally discouraged if you were snapped away in the last moment.'[37] How serious the option was, appears from the fact that Hilbert had at Ehrenfest's request written a recommendation for Van der Waerden.[38]

Although Brouwer was no longer active in topology, he was still held in reverence by those who knew the notorious pitfalls of the subject. He was consulted, received reprints, etc. Alexandrov, who had personally experienced the influence of the old master, sent him a copy of his lovely monograph *The simplest fundamentals of topology*,[39] an elegant presentation of the basics of combinatorial topology. The booklet, originally intended as an appendix to Hilbert–Cohn-Vossen, *Anschauliche Geometrie*, was the outcome of lectures of Alexandrov in Göttingen. For a long time it was on the desk of every self-respecting mathematician, many learned their modern topology from it. Alexandrov carried out an admirable balancing act by keeping on the right side of both Brouwer and Hilbert. The latter wrote an appreciative introduction, and Brouwer was lavishly praised for his epochal shaping of mod-

[37] Ehrenfest to Van der Waerden 6.II.1930.

[38] Courant to Ehrenfest 9.II.1930. Courant informed Ehrenfest that Van der Waerden was number 3 on the list for Hilbert's succession.

[39] *Einfachste Grundbegriffe der Topologie* [Alexandrov 1932].

ern topology, 'confronting these two extreme wings[40] the monumental building of Brouwer's topology spires, ... There are hardly problems in grand style in modern topological research, from which no thread would lead to Brouwer's publications, for which there were no tool—often ready for use—to be found in Brouwer's stock of topological methods. ... With Brouwer the period of stormy development of topology begins ...'[41]

The gift reached Brouwer from Ascona, the centre of artists, social and spiritual guru's, health freaks, millionaires and their usual corona. It says something about Alexandrov that he found time, and had the courage to mix with that crowd. Brouwer subtly showed his appreciation for Alexandrov's gift and for his choice of location, 'Thank you for your beautiful booklet and for your card from Ascona, the Southern branch of Laren.' He added sadly: 'I am still contemplating in astonishment the process of disintegration of my life, which takes place with admirable universality and thoroughness, and I am curious if yet another season will germinate.'[42] The card underscores the general state of mind of Brouwer in the middle of the Sodalitas muddle, which would not leave the strongest untouched. Moreover, a sharp observer like Brouwer could not be misled about the stagnation in his scientific career.

Lize, for whom her husband had no secrets, viewed Brouwer's depression with deep concern. In particular at the height of the Sodalitas affair, when Brouwer had, often on short notice, to visit Budapest, Brouwer despaired of his scientific future. Would the well of his miraculous mathematical mastership ever flow again? In one of her letters to Bertus in Budapest she tried to put his mind at ease, 'In particular don't be sad about mathematics. I know that you can do it all like before, if only tranquility returns to your mind. And even if you could not do it, you have already done enough.'[43]

In the middle of all the worries and disappointments of the *Grundlagenstreit* period, marred by the legal and financial problems of the Sodalitas affair, there was one event that put balm on the scars of the recent past: it had pleased Her Majesty Queen Wilhelmina to confer the knighthood of the *Nederlandse Leeuw* (the Lion of the Netherlands) upon Brouwer. The knighthood was conferred on 27 May 1932, at the palace 'Het Loo' in Apeldoorn (the favourite palace of William and Mary). This in recognition of his outstanding contributions to science. It is not unthinkable that, when his name occurred on the honours list, Brouwer cast his mind back to his student days. As a student he had wondered if he would end up in the 'coarse mansions of society,' and 'light its chandeliers and grace its doorposts.'[44] The knighthood, pleasing as it might be—and there is no doubt that

[40] Combinatorial and set theoretic topology.

[41] [Alexandroff 1932] p.26.

[42] Brouwer to Alexandrov 20.X.1932.

[43] Lize to Brouwer 7.IX.1932.

[44] Cf. p. 83.

Brouwer cherished the honour—undeniably confirmed that the establishment had asserted its rights.[45]

In spite of the distractions of his financial and other projects, Brouwer carried on his intuitionistic project, albeit at a modest scale. He lectured on the subject and taught from time to time a class on intuitionistic mathematics. His activity in this direction could best be termed as 'polishing'.

In 1932 Brouwer had, probably at the invitation of Van der Corput, given a semester course on intuitionism in Groningen. This course was attended by a good sized audience, and the professors Van der Corput and Schaake had taken it upon them to produce a set of notes. Van der Corput proposed to have the notes typed and published in semi-official form; he also invited Brouwer to give another series of lectures. The proceeds from the sales of the notes, he suggested, would go to the faculty student association. Brouwer answered that he was perfectly willing to oblige but that his present financial state was so desperate that—after consulting informed colleagues—he thought that it would not be unreasonable if two-thirds of the proceeds would go to him.

> Whereas I deplore that my state of emergency forces me to adopt this commercial attitude, you must on the other hand take into consideration, that in the absence of this state I would not easily decide to give lectures away from my own post (nor to travel during the summer for two months as an examiner), as all this is at the expense of the continuation of my scientific researches, for which the vital conditions have been so strongly decimated since theft a couple of years ago, of my scientific documents.[46]

In 1935 the Groningen lecture notes were mentioned once more, but nothing came of the publication.[47]

The general acceptance of intuitionism as a part of the scientific world was a fact in Holland by 1930. This did not mean that mathematicians were contemplating a switch of allegiance to the stern Brouwer, but they came to consider it as just another school in mathematics, and there was a certain amount of quiet pride in the fact that a fellow countryman had apparently found recognition in the international world. As for the philosophers, it seems highly doubtful in how far they understood Brouwer's philosophy.

In the circle of Dutch philosophers, Brouwer's intuitionism was not a topic that reached the agenda. There is a notable exception; a philosopher from the Free University, Cornelis van Vollenhoven, wrote a dissertation *The philosophy of mathematics from a theistic point of view*[48] in which intuitionism plays an important

[45] It should be pointed out that a knighthood in the Netherlands carried no side benefits; there is no title attached to it and it does not lend social or legal status to a person. It is no more and no less than a sign of royal appreciation.

[46] Brouwer to Van der Corput 23.XI.1933. Cf. p. 652.

[47] Brouwer to Van der Corput 5.X.1935.

[48] [Vollenhoven 1918].

role. The Free University was (and still is) a protestant university based in Amsterdam. Theistic philosophy was introduced by Dooyeweerd.[49] Van Vollenhoven found much in Brouwer's intuitionistic mathematics that fitted his philosophical views. Curiously enough, nothing is known about contacts between Brouwer and Van Vollenhoven.[50]

The younger men, mostly on the borderline of mathematics and philosophy, did take up the ideas and issues from intuitionism. In particular E.W. Beth, H. Freudenthal and A. Heyting played a role in the study and development of modern intuitionism.

Brouwer himself kept a rather low profile. He gave his lectures in Groningen (see p. 690), and featured as a speaker in a series of lectures at the University of Amsterdam under the heading 'The way of expression of science'. On 12 December 1932 he delivered an abbreviated version of his Vienna lectures. The title of the talk was *Will*, *Knowledge*, *Speech*, a somewhat lame translation of the pithy Dutch title *Willen*, *Weten*, *Spreken*. This title has the dynamic feature of the activity rather than the act, it refers to willing, knowing and speaking rather than the frozen concepts. We may drawn attention, in passing, that Brouwer had a penchant for three part titles—*Leven*, *Kunst en Mystiek*; *Wiskunde*, *Waarheid*, *Werkelijkheid*; *Mathematik*, *Wissenschaft und Sprache*; *Consciousness*, *Philosophy and Mathematics*.[51]

The series was intended as an exposition of the epistemic character of the sciences, practised at the university. Apart from Brouwer, the physicist J. Clay (a former follower of Bolland, see section 2.4.), the theologician A.H. de Hartog, G. Mannoury, the philosopher H.J. Pos, the psychologist G. Révész, the economist J. Tinbergen, and the physicist J. Van der Waals jr., took part.

Révész' participation was probably the result of Brouwer's influence. He was a Hungarian scholar, who fled Hungary in 1920 under the regime of admiral Horty. He settled down in Amsterdam, where he became a *privaat docent* in 1923. Brouwer must have met him in the twenties, a time that universities in Holland were still of a modest size, with lots of social contacts. He apparently was sufficiently impressed by the Hungarian psychologist to find it worthwhile to find him a permanent position. In 1932 the opportunity offered itself, when the physics and mathematics faculty showed interest in a chair for philosophy. One can imagine that Brouwer, with his philosophical reputation, took the matter in hand and actively promoted Révész' case. This turned out to be a non-trivial matter. There were objections concerning Révész' teaching capacity. His inaugural lecture as a privaat docent had made such a poor impression that after all those years it was still brought up as an argument against the appointment. Moreover, two serious candidates showed up: J. Clay and H.J. Pos.

[49] Herman Dooyeweerd (1884–1977) was a law professor at the Calvinist *Vrije Universiteit*. His fame rests on his philosophy, as presented in *De wijsbegeerte van de wetsidee* (1935–36).

[50] For a discussion of the role of intuitionism in Van Vollenhoven's thinking, see [Blauwendraat 2004].

[51] [Brouwer 1905, Brouwer 1919c, Brouwer 1929b, Brouwer 1933a, Brouwer 1948a].

At this point Brouwer decided to assert his influence. At the faculty meeting he explained that philosophy and psychology were of great importance for his faculty, 'for example for the insight in the notions of space and time, for which the psychology of the tactile sense, the sense of hearing, etc. is necessary; where for example developments in mathematics yield indeed in psychology a clarifying insight.' And Révész, he said, was a prominent figure. Here he was confronted with a web of rumours. Eventually the faculty chairman had to bring the dark rumours from Budapest up in the faculty meeting. Brouwer remained firm. It was just a matter of slander, he said. A personal enemy of Révész, professor Hekker, had misled the literary faculty. The Ministry of Justice had no negative information about Révész.

> During the white terror, which succeeded the communist period, 30% of the professors in Budapest was brought before an investigation committee. Révész also had to appear before the committee; he demanded that the minutes of the interrogation should be sent with his signature to the minister. When this demand was turned down, Révész left Budapest and went abroad.

There were more questions to be parried. Were there not certain criminal acts of which Révész was accused, asked the chairman. Brouwer, who had done his homework, replied that there were three accusations: machinations with the jewels of a Hungarian family, misappropriation of a grand piano, and unlawful moving of books. In all three cases Révész acted correctly. Few candidates for a chair will have been subjected to this kind of opposition. Justice (or Brouwer) triumphed however, and Révész got his chair. The choice turned out to be fortuitous; Révész was extremely active and he founded the Amsterdam School of Psychology.

Brouwer and Révész became close friends, and it should be mentioned that during the Sodalitas affair, which was about the same time as the appointment of Révész, it was very convenient to be able to fall back on the advice of a born Hungarian. Révész, who greatly admired Brouwer, dedicated his book *The creative-personal aspect and the collective* to 'My friend L.E.J. Brouwer'.[52]

In 1932 Brouwer's old friend Frederik van Eeden passed away. His last years had not brought the contented tranquillity that old age traditionally seems to promise; instead it visited Van Eeden with an increasing loss of mental power. He was in a way fully aware of the phenomena, which is not surprising with his trained background. Mannoury recounted an incident during a walk with Van Eeden—suddenly Van Eeden stopped and said to Mannoury, 'I know it is not true, but there is a snake here on the street and we must be careful not to tread on it.' Van Eeden passed away on June 16, while Brouwer was out of the country. The widow wrote sadly: 'Yes, Bertus, I could guess that you were travelling, for I must honestly confess, that this silence astonished me and even hurt me.' Two years earlier Van Eeden had turned seventy, and this was celebrated with some ceremony. In particular a committee of intellectuals and artists from all parts of society presented a Liber Amicorum to

[52] [Revész 1933].

the grand old man. Brouwer had joined the committee, but he was not one of the contributors. This may perhaps be seen as a subtle distinction between his relation with Adama van Scheltema and Van Eeden. The committee had its quotum of celebrities, e.g. Van Anrooy, Berlage, Sigmund Freud, Van Ginneken, Gutkind, Mengelberg, Henriette Roland Holst, Romain Roland, Rabindranath Tagore, and Stefan Zweig. The list of contributors also contained names of the flower of Dutch (and European) culture. The significists were represented by Mannoury, Giltay, Van Ginneken, and Henri Borel,

16.4 Göttingen's fate under the Nazi's

The year 1933 brought Hitler and his NSDAP to power in Germany.[53] The consequences belong to the darkest parts of modern western civilization. The Third Reich was to bring terror, death, destruction, and suffering to Europe on a hitherto unknown scale. The symptoms soon became painfully visible in Germany. Almost immediately Hitler started to carry out his designs to rid Germany of Jews and left wing opponents. The civil service was one of the first targets, and the universities were soon under siege both from outside and inside. On April 7 the Law for the Restoration of the Civil Service[54] was passed. This law was the lesson drawn by Hitler from the Weimar Republic, which suffered continually from the lack of loyalty if not worse of the civil service. The law with its Arian-paragraph was the first systematic legal step in eliminating Jews from public life. Under the law non-arians were no longer eligible for civil service, but also political unreliability was a ground for dismissal. Anybody who could not be trusted to 'defend at any moment the national state without reservation' could be dismissed as well, and finally there was the overall clause that for the purpose of simplification of management individuals could be pensioned off.

With such a law the government virtually had a free hand. The old president Hindenburg registered a protest, and as a compromise non-arians, who had been in the civil service before 1914 or who were veterans (*alte Frontkämpfer*) of the war were exempted from the dismissal rule. The authorities could, however, force them into early retirement. In fact, the latter often functioned as a waiting room for later actions. All universities fell victim to 'cleaning up'-action of the new government, the *Gleichschaltung* as it was called euphemistically. For the losses in mathematics one may consult the reports of Pinl and Furtmüller.[55] Göttingen, for a long time the home away from home for Brouwer, was hit particularly hard. There had always been a large proportion of Jewish mathematicians, and politically the climate had

[53] After the present book was sent to Oxford University Press, I received copies of papers by Volker Remmert that dealt with mathematics, mathematicians, and politics in the Third Reich, [Remmert 2004b, Remmert 2004a]. The material is relevant for our picture of the Nazi period, but no use could be made of it.

[54] *Gesetz zur Wiederherstellung des Berufsbeamtentums*. Cf. [Schappacher–Kneser 1990], [Craig 1981] p. 579.

[55] [Pinl, M, L. and L. Furtmüller 1973].

been fairly liberal. Strictly speaking the Göttingen mathematics department should have had a reasonable portion of exempted professors: Felix Bernstein and Edmund Landau were pre-war appointees, and Richard Courant was a war veteran. Hermann Weyl and Gustav Herglotz were not of Jewish descent. Emmy Noether was at the time an extra-ordinary professor without a regular appointment, so the new law did not apply to her.

Nonetheless the authorities hastened to suspend Bernstein and Courant; they were informed by telegram of their immediate suspension. Emmy Noether was also suspended per telegram. As Schappacher put it, 'it was the only time that she was treated by the department of education in accordance with her scientific status.'[56]

Göttingen also lost Paul Bernays, Hertz, Hans Lewy, Neugebauer, Prague and Hermann Weyl. From the younger generation (assistants) Busemann, Fenchel, Heilbronn, F. John, K. Mahler, Steinhaus and Warschawski were dismissed.

Hermann Weyl could have stayed on, but he chose to give up his chair. Having a Jewish wife, he felt that under the new regime the circumstances were becoming unbearable. He emigrated to the USA, where he was appointed at the newly founded Institute for Advanced Study in Princeton. Bernstein was at the time of the political tragedy in the USA; when it became clear that he chose not to return, he was discharged. Courant, who was the managing genius of the Göttingen department, remained in Göttingen, hoping for a favourable development in the new academic policy. In the end he was disappointed, he retired voluntarily and emigrated in the summer of 1935 to the United States.[57]

The man who put the new regime, and in particular the Nazi students, to the test was Landau. He decided to resume his teaching in the winter term of 1933. On the second of November he had taken the brave step to start his lectures, but at the institute he was met by a guard of SA members who denied him the entrance. In a way it was a piece of luck that he got away without bodily harm. One of the students in charge of the boycott of Landau was the brilliant Oswald Teichmüller, (1913–1943) a gifted algebraïst.[58] Landau subsequently handed in his resignation and retired. He died in 1938, and was thus spared the fate of many of his colleagues.

Europe watched the events in Germany with horror, although in some quarters the violence was viewed with fascination. Whereas the terror of the storm troopers, and the mediaeval laws against Jews and political opponents shocked the population at large, there was here and there a certain sympathy for the authoritarian leader, who dared to take cruel and drastic steps in order to dispel real or

[56] [Schappacher–Kneser 1990], p.27. Schappacher pointed out that the authorities might have been in a hurry to handle the Göttingen mathematics department, in order to forestall possible student actions against the institute or individual mathematicians. The institute was viewed as a 'bastion of Marxism'. For background information on the political landscape and the ensuring developments, see also the above mentioned exposition.

[57] See [Reid 1986], ch. 15.

[58] Teichmüller would have become one of the top algebraïsts of his generation, if he had not fallen victim to his radical national socialist ideals. He later volunteered for military service and fell at the eastern front. For mathematical and historical information see [Schappacher-Scholz 1992].

imagined evils. Ever since the communist doctrine that you cannot make an omelette without breaking eggs, the limits of 'necessary violence' had been probed. And indeed, in most countries there were minor ultra-right or even fascist political parties, that usually made a lot of noise, but were not very successful in attracting the voter. Germany with its economic crisis, inflation and Versaille-syndrome was an exceptionally receptive nation for state terrorism. But the brief transition period from Weimar Republic to Third Reich offered enough to convince right-minded citizens that Germany was reverting to a barbaric stage. The boycott of Jewish business, the banning of Jews from civil service, the institution of concentration camps, book burning, Hitler's termination of German membership of the League of Nations, the Reichstag fire—all signs that spelled a collapse of civilization.

In Holland, where in spite of various political and economic crises, war and civil strife had been unknown since the Napoleonic wars (with the minor intermezzo of the campaign against rebellious Belgium (1830)), the German events were watched with incredulity and disapproval. There were a few fascist movements, the largest of which, the *Nationaal Socialistische Beweging* (NSB, National Socialist Movement), could count on a certain sympathy in middle class circles; it was represented with varying success in parliament. That party was, however, by no means as radical and brutal as its German sister party. In fact it was in its early stage not even anti-semitic.

Dutch scientists were as a rule prepared to support their German colleagues, and once the refugees started coming, many passed through Holland or stayed there. There are no records of Brouwer's activity, as far as national politics is concerned.

The only immediate trace of these events found in Brouwer's files concerns a German female mathematician, Marie Torhorst, who had a Ph.D. in topology. Her sister, who had left Germany, had begged Mannoury to assist Marie to get permission to visit her. Mannoury, who had not shrunk from belabouring Stalin by telegram, had practised the same technique on Hitler; he cautiously concluded that therefore he was not the right person to sponsor Marie's request because 'she could thus get in trouble, as I have sent a kind telegram to Hitler, that possibly was not taken kindly.'[59] Brouwer indeed invited Marie and in July he had the pleasure to receive her in Blaricum.

Dutch universities also acted on behalf of the German refugees; in the faculty meeting of July 7 a circular letter concerning Jewish students and scientists was tabled.

16.5 Bieberbach's conversion

The Third Reich created deep rifts in the German mathematical community. On the one hand there were the anti's, the Jews and the political victims, on the other hand there were the pro's—idealists and opportunists alike. In as far as there was

[59] Mannoury to Brouwer 17.VI.1933.

any discussion after 1933, it was a distorted one—freedom of the press and freedom of speech were things of the past. In Hitler's conception there was no place for dissenting opinions. *Gleichschaltung*[60] was carried by him and his followers to its ultimate form. The Nazi doctrine of *Ein Volk*, *Ein Führer* did not tolerate alternative political views, let alone criticism. Some mathematicians had been eagerly awaiting the hour of 'Germany's waking up', although, generally speaking, political extremism was not prominent in mathematical circles. In fact, quite a number of mathematicians had leftist views and some had even been active during the short-lived Weimar republic, Courant and Bernstein were examples. On the right wing of German politics there had been a loose confederation of '*Deutschnationalen*', not to be confused with national socialists. After the Nazi take-over these small groups were quickly and efficiently abolished by Hitler, some just disappeared, and some became part of the omnipresent Party. There are two mathematicians who became the paradigm for Nazi-scientists: Theodor Vahlen and Ludwig Bieberbach.[61] Vahlen's name has already come up; he was cited by Brouwer in his dissertation, and Bieberbach was the young brilliant analyst who recognized Brouwer's topological ideas at an early stage, and who joined Brouwer in his fight against the boycott of German scientists. Since both play a role in our history, let us have a look at their political past.

Theodor Vahlen was already an established mathematician at the time that Brouwer entered the profession. He had acquired a reputation in the area of Hilbert's Foundations of Geometry through his monograph *Abstract Geometry*.[62] He initially felt drawn towards the then fashionable and innovative axiomatic theory and also to set theory. Max Dehn, in a review, butchered Vahlen's *Abstract Geometry* as inexact and conceptually weak.[63] Although the book had obvious deficiencies, the review was in the eyes of some colleagues a bit too sharp; there were rumours that the Göttingen group used the occasion to prove its own superiority and to chastise an outsider. So much is certain, that it did not endear the Göttingers to Vahlen, something they would come to regret later. After his brief career in pure mathematics Vahlen turned towards applied mathematics. He worked in traditional areas and stuck to traditional methods. His publications and studies cover descriptive geometry, geodesics, technical mechanics, ballistics and the like. In 1911 he published a book *Constructions and approximations*,[64] which was well received.

During the First World War he saw active service as commander of an artillery battery. His book *Ballistics*,[65] which probably was inspired by his wartime expe-

[60]The National Socialist regime carried the political use of language to unknown heights; such terms as *Gleichschalten* had a normal everyday meaning, but under the regime it acquired a new one: 'following the Nazi doctrine,' or even cruder, 'eliminating opposition and deviant ideas and practice.'

[61]Vahlen's life and career is discussed in [Siegmund-Schultze 1984]. For Bieberbach see [Mehrtens 1987].

[62]Abstrakte Geometrie, [Vahlen 1905].

[63][Dehn 1905], [Vahlen 1905].

[64][Vahlen 1911].

[65][Vahlen 1922].

riences and which was dedicated to the memory of the fallen comrades in arms, was certainly not an up to date exposition; modern results and methods were often overlooked. At that time Vahlen had already made up his mind to enter politics. After Hitler's *putsch* he decided to throw his lot with the prisoner at Landsberg, and he became a prominent Nazi. One of his early functions was that of *Gauleiter* (chief of the district) of Pommern. As pro-rector of the University of Greifswald he demonstrated his rejection of the Weimar Republic by bringing down the flag of the republic in 1924 on the day of the Constitution. Vahlen was immediately suspended, and—quite exceptionally for a rather reactionary judiciary—eventually convicted.

After Hitler's triumph in 1933, Vahlen quickly rose in the ranks and became chief of the academic section of the Prussian department of culture. In this capacity he had a considerable influence on the nazification of the universities.

Where Vahlen was a middle of the road mathematician, respectable, but not original or deep, Bieberbach was a totally different case. He was a versatile mathematician with a lively presentation. His research interests were mainly in the area of function theory, and they bordered on many other areas. His career was fairly traditional, he studied in Heidelberg and Göttingen. In Göttingen he studied with Koebe and Klein. Koebe was his mentor in function theory and Klein influenced his view on mathematics—he became a lifelong adept of the geometrical spirit. His early research dealt with automorphic functions, in 1910 he wrote a dissertation on that topic, and in 1912 he was one of the invited speakers at the automorphic function-symposium in Karlsruhe (cf. section 5.3). His habilitation was on the topic of Hilbert's 18th problem; it dealt with groups of Euclidean motions. In 1910 he became a *Privat Dozent* in Zürich; the next year he moved to Königsberg.

After a brief spell as a professor in Basel he got a chair in Frankfurt, where Schoenflies was in charge of the new mathematics department. Bieberbach's star was rising fast. After Carathéodory left for Smyrna, Brouwer was offered his chair in Berlin. The offer was turned down, and subsequently also by the next candidate, Weyl. In the following search Bieberbach emerged in the end as the most promising candidate. From then on he remained in Berlin, where he was considered a stimulating teacher, albeit a bit sloppy. His books, on a large number of topics, were popular, and went through many editions. In the mathematical community he played an active role, since 1920 he was a secretary of the DMV and editor of the *Jahresbericht*. Furthermore, as we have seen, he was one of the active members of the board of the *Mathematische Annalen*. He was generally appreciated by his students and colleagues, his tendency towards mild vanity was accepted with a forgiving smile. Einstein wrote on one occasion to Max Born's wife, 'Bieberbach's love and devotion for himself and for his muse is quite priceless. May God keep it with him, this is the best way to live.'[66] Before the advent of the Third Reich Bieberbach was somewhat ambiguous about politics. On the one hand he thought '*deutschnational*', as appears from his resistance against the *Conseil* and the *Union*

[66] Einstein to Hedwig Born [Einstein-Born 1969].

mathématique (cf. chapter 9), on the other hand he freely mingled with leftists and communists. His house was open to guests of all political persuasions, and he had a reputation among the students for unorthodox open-mindedness.[67]

Almost overnight Bieberbach changed into a fervent adherent of National Socialism. An account and analysis of Bieberbach's conversion can be found in Mehrten's influential paper 'Ludwig Bieberbach and Deutsche Mathematik'.[68] Reading the story of Bieberbach's life and convictions, one cannot but conclude that Bieberbach's political extremism was not dictated by an iron fate, but rather the result of a mixture of personal preferences and ill-founded political views. Be that as it may, it remains a fact that Bieberbach joined the Nazi movement lock stock and barrel.

Now the interesting thing about Bieberbach is that he was not an opportunist, who sought more power or comfort. After all, a professor of good standing in Berlin was already quite high on the social ladder. He sought to harmonize his political views with his professional expertise. In fact, Bieberbach had developed views about the nature of mathematics which he eventually tried to bring into line with the Nazi ideology. It is not easy to be categorical about Bieberbach's alleged 'lust for power'. There is no doubt that vanity came to him naturally, but in a stable society his brand would be viewed with a smile. He did hold a number of prominent positions in the mathematical community, and there were no significant complaints about his functioning. His personal and scientific history after 1933 seems to be more the result of unconditional acceptance of the Nazi doctrines, than of clever schemes to accumulate power. How far he was driven by political correctness, or by a desire for personal advantages and honour, remains a matter of conjecture. Viewed from the perspective of the German citizen of that period, the convolutions of Bieberbach fell under the traditional 'not running away from one's responsibilities', unfortunately that phrase is, more often than not, a cover-up.

Early in his career Bieberbach took a view of mathematics that was in line with the modern trend of axiomatics. He quite explicitly endorsed formalism as the proper approach to mathematics, 'that does justice to the factual state of mathematical science [...]. We do not want to create knowledge of the truth but merely methods to gain knowledge [...]. The truth in mathematics rests solely in its logical correctness and consistency.'[69] But slowly he started to shift his allegiance. A few years later he started to take a broader view and defended in his books the significance of external problems, by pointing out that mathematics could sever the ties between mathematics proper and its applications only at its own peril.

He spoke of the 'living spirit of external stimuli which created mathematics, and to which it constantly and abundantly owes new life.' If anything, Bieberbach was developing and expressing ideas along the lines of Felix Klein's socio-cultural reflections on styles, the real world, and the like. He never actively engaged in

[67] Communication of H. Freudenthal.

[68] [Mehrtens 1987] Our presentation makes substantial use of this paper.

[69] Bieberbach's inaugural lecture [Bieberbach 1914], the translation is Mehrtens'.

professional, technical foundational work, nor did he display an inclination towards philosophical analysis.

One can observe, however, in Bieberbach an increasingly negative attitude towards formalism. In 1924 he spoke of the 'late rampant growth of Hilbert's axiomatics in the field of physics'.[70] Still later Bieberbach felt attracted to the ideas of Pierre Boutroux' book *The mathematicians' scientific ideal*,[71] which stressed a dichotomy: 'synthetic–analytic', or 'intuitive–discursive', or 'order of invention–order of proof'. Boutroux sided with the 'synthetic', which he claimed to be the rule rather than the exception with mathematicians. He had little sympathy for the 'algebraico-logical' artistry of mathematicians, which he considered a form of 'juggling'. In a lecture, held for the *Förderungsverein*[72] in 1926, Bieberbach expressed his views on the nature of mathematics. The title, *On the mathematician's ideal of science*,[73] already betrays the grand view the speaker was to unfold. This was not a philosophical discussion of the foundations of mathematics, a pedagogical guideline for teachers, or a historical orientation, but all of these at the same time. Bieberbach saw in Felix Klein the ideal type of a mathematician, who combined physical heuristics with mathematical intuition; Klein could be viewed as a mathematician in the old tradition of Newton, Laplace, Gauss and many others who were rooted in a global scientific idea, one in which mathematics was inspired by physics, and in which physics adopted the virtues of mathematics. On Bieberbach's view, the new exactness of the nineteenth century had separated mathematics from its external inspiration. Function theory, for example, had in the hands of Weierstrass become a showpiece of pure mathematics; Klein on the other hand found his inspiration in the tradition of Riemann—a function theory with a geometrical and physical flavour. Klein's work is indeed permeated with his intuitive geometrical spirit. It is not the logic and the formalism that determines mathematics for him, but rather the wealth of structures and their interrelations. Klein's style of non-Euclidean geometry and his group theoretical view of the notion of transformation was in the first half of the twentieth century common practice in the mathematical world.

After extolling the virtues of Klein's intuitive mathematics, Bieberbach went on to contrast it with the formalistic tendencies of his day. In the wake of Weierstrass' novel counterexamples to traditional mathematical intuitions, an understandable distrust of intuitive considerations became the rule: 'Now we have the psychological foundations for the tendency, which shows itself in the most extreme form in Hilbert's axiomatics, and that wants to have nothing to do with that what is intuitive object-oriented.' And so 'What is more plausible than the radical cure, to eliminate substance and meaning from mathematics. There are only objects of thought, which mean nothing, and with which one operates according to certain rules.' This was, in Bieberbach's opinion, Hilbert's ideal of science.

[70] [Bieberbach 1924].

[71] [Boutroux 1920] Bieberbach read the German translation.

[72] A society of friends and supporters of education in the exact sciences.

[73] *Vom Wissenschaftsideal der Mathematiker*, 15.II.1926.

The choice for Bieberbach was not difficult, mathematics should remain a meaningful subject, its objects should have substance and meaning. Moreover, Hilbert's ideal was automatically 'hostile towards the demands of the applications.' The formal, logical approach to mathematics would be disastrous for a harmonious building of mathematics, and worst of all for the teaching of mathematics.

Weierstrass had found the uncompromising Kronecker on his path, who prophesied that 'the day would come that [mathematics] would wake up from Weierstrass' analysis and Cantor's set theory, as from a horrible dream.' 'And now,' Bieberbach said, 'it is intuitionism that would aspire to herald this day.' The intuitionistic ideal of science, as characterized by Bieberbach, is in a way an amalgam of Klein's and Hilbert's ideals, it takes into account the points of Klein's philosophy, and it has a sharper notion of exactness in common with the Weierstrass–Hilbert ideal. However, where the formalist needs an extra-logical domain, which 'can only have come into their possession by unintended chance,' 'the intuitionist tries on the other hand, to do justice to the circumstances that it is people, who practice mathematics and he tries to take systematically, and everywhere in the construction of its theories, the intuitive-concrete aspects into account.' In particular, the intuitionist accepts the existence of an object satisfying certain conditions, only if a construction can be given. Considering the circumstances of this lecture for a mixed audience, Bieberbach managed to give a reasonably fair presentation of the situation in the fundaments of the mathematical building. It is quite clear that he sides with the intuitionists, Brouwer and Weyl—'Thus indeed in intuitionism the fresh air of spring is blowing,' he observed enthusiastically.

One would be wrong to see Bieberbach's lecture as part of the *Grundlagenstreit*. Bieberbach was rather campaigning for the normal activity of the 'working mathematician' and the teacher, for a meaningful mathematics as opposed to the axiomatic games of the formalists. It is not unlikely that he misinterpreted some of the signs on the wall. Whereas in the early days abstract algebra *à la* Emmy Noether and Van der Waerden, was seen as part of the axiomatic trend, we now know that 'modern algebra' is dealing with very concrete structures from the practice of mathematics. Views of this kind tend to change in time. We dwelt expressly on Bieberbach's views and his 1926 lecture, as it shows how intuitionism could be drawn into a political debate, where Brouwer would have been very reticent.

Politically, Bieberbach had been drifting towards the right. In the Riemann affair, the Bologna affair and the *Annalen* conflict, he had showed considerable affinity with the German nationalists. This by itself did not predestine him to become a Nazi. Not all German nationalists flocked to Hitler after the fall of the Weimar republic.

Bieberbach, however, seemed to have made up his mind in favour of the National Socialist Party. In 1933 he marched with his sons in one of the notorious SA[74] processions from Potsdam to Berlin. Soon after he joined the SA, and faithfully discharched his duties as a member. He strongly supported the new national awakening, and condoned, if not praised, the actions of nazi-student organizations against

[74] *Schütz Abteilung*, the storm troopers of the Party.

the old guard. Bieberbach's political choices and his apparent faith in the new movement surprised his colleagues and students. On the whole he had been a somewhat vain, but well-liked and friendly colleague and teacher; there was no clearly discernible cause for resentment or feeling of unfair treatment. He had been slighted by Hilbert in the Bologna affair, but certainly that would not have been enough to seek revenge.

One is inclined to view Bieberbach's alliance with the Nazis, as a deliberate, argued choice for a specific political and social system, of which he expected great things on all levels. He acted as an idealist, who, once he has chosen a particular conviction, decides to close his eyes for all negative aspects and to follow the party line right or wrong. There is no doubt that Bieberbach was totally serious about his political commitment. Mehrtens paints a vivid and disturbing picture of a man who not only follows the party line, but develops new initiatives.[75]

In fact, all of Bieberbach's activities were centred around mathematics. His conceptual views of mathematics had become more pronounced and had acquired political overtones. In particular he started to see the matter of mathematical styles in a political, and especially racial, frame. He applauded, for example, the above mentioned boycott of Landau's class, 'A few months ago differences with the Göttingen student body ended the teaching activities of Herr Landau ... This should be seen as a prime example of the fact that representatives of overly different races do not mix as student and teacher.'[76] In another lecture before the *Förderverein* Bieberbach launched in 1934 his classification of mathematics and mathematicians along racial types.[77] He had borrowed these types from a psychologist, Erich Jaensch, who had introduced two basic types. The 'S-type' with unstable psychic functions internally generated synaesthetic perception, and a tendency towards disintegration. The 'J-type,' on the contrary, had stable psychic functions, and strongly integrated perceptual imagery and conceptual thinking (the *integration* type). In view of his earlier statements, it is not surprising that Bieberbach's heart was with the J-type, with great Germans like Gauss, Weierstrass and Klein as representatives. It would take us too far to discuss the types in full detail, the reader may consult [Mehrtens 1987], [Mehrtens 1980], [Lindner 1980] for further details. It may suffice to mention the proliferation of subtypes, and—of course—the classification of Jewish mathematics under the S-type. The S-type, according to its creator, led to a 'cognition liberalism,' which was hostile to science, 'The cognitive-liberals basically hate truth, because it means commitment and limitation.'[78] It should not come as a surprise that Bieberbach drew his conclusions for the formalism–intuitionism clash. He saw the *Grundlagenstreit* as racially determined. Note, however, that he chose a fairly safe, academic formulation: 'By itself

[75] [Mehrtens 1987].

[76] Ibid. p. 227.

[77] [Mehrtens 1987], p. 224. [Bieberbach 1934].

[78] [Lindner 1980], p. 95.

the J-type will tend towards intuitionism or the style of Klein, whereas formalism seems to belong to the S-type.'[79]

It would be logical therefore to classify Hilbert as an S-type, but that would go too far, even for Bieberbach. On account of Hilbert's Baltic origin, and his wide ranging publications in areas other than his formalism, 'Hilbert cannot possibly be taken as an S-type.' But, he added 'a form of J-type is known which tends to be open to the influences of the S-type.' Brouwer and Hilbert would be '*bien étonné de se trouver ensemble*,' but Bieberbach had no problem explaining this phenomenon, 'The difference is quite compatible with the fact that both Hilbert and Brouwer should be classified under the psychological type J3/J2. The fact that two men approach their science with an ideal norm, does not necessarily imply that it has to be the same norm in both cases.'[80]

Bieberbach was not the only person to see the formalism–intuitionism debate in the light of the politics of the day. In mathematics there were a few exponents of this trend. However, even the hard-line Nazi mathematicians were not inclined at all to have their foundational views dictated by politics. Certainly, there would be an amount of lip service to the Nazi doctrines, but as a rule that would be gratuitous formulas. It would not make much difference for handling, say, differential equations. No, the intuitionism–formalism considerations flourished best in certain marginal philosophical circles. The term 'marginal' is in a way rather misleading, as the strange mixture of psychology, mathematics, intuitionism, anti-semitism went down with the Third Reich. If the Nazi's had on the other hand made good their claim on a thousand year domination, 'marginal' might have turned out to be the wrong adjective. As it is, we can from our point in history only express our amazement at the curious activity and arguments of the Nazi foundationalists. The names that come to mind first are those of Dingler and Steck, both philosophers with a habilitation in mathematics. These scholars made determined attacks at decadent formalists and logicists. Not even Hilbert escaped the wrath of the national socialistic inspired philosophers, albeit that the grand old man at his Olympus was out of reach of their arrows. Brouwer, who should have been fairly safe, got his share of criticism too; Steck set himself to prove that neither Hilbert, nor Brouwer, could capture the essence of mathematics.[81]

Bieberbach's excursions into the field of psychology hastened his downfall in the mathematical community. His lecture of 1934 caused some stir abroad; Hardy, for example, sent a scathing 'letter to the Editor' to *Nature*.[82]

> It is not reasonable to criticize too closely the utterances, even of men of science, in times of intense political or national excitement. There are many of us, many Englishmen and many Germans, who said things during the war which we scarcely meant and are sorry to remember now.

[79] [Mehrtens 1987], p. 228.

[80] [Mehrtens 1987], p. 228.

[81] See [Menzler-Trott 2001] Ch.4. and the literature cited in that book.

[82] [Hardy 1934]. Also in *Mathematical Intelligencer*, 6 (1984).

> Anxiety for one's own position, dread of falling behind the rising torrent of folly, determination at all costs not to be outdone, may be natural if not particular heroic excuses. Prof. Bieberbach's reputation excludes such explanations of his utterances; and I find myself driven to the more uncharitable conclusion that he really believes them true.

The Danish mathematician Harald Bohr, who had close ties with Germany, in particular Göttingen, strongly criticized Bieberbach in a Danish newspaper. Thereupon Bieberbach made the capital error to respond to Bohr's article, by means of an 'open letter to Harald Bohr' in the *Jahresbericht*, against the wishes of his fellow editors. In doing this he had overplayed his hand. The DMV did not want to antagonize its numerous foreign membership, and Bieberbach had more or less given the impression that he represented the views of the DMV. When the matter was put on the agenda of the Bad Pyrmont meeting of the DMV (1934), Bieberbach decided to organize a coup, or at least to put pressure on the meeting by marching into the meeting at the head of a group of students in SA uniforms. The chairman, Hecke, showed considerable courage by calling his bluff; he appealed to the statutes of the DMV to exclude all non-members, including the uniformed students, from the meeting. The Nazi-sympathizers were, however, in so far successful, that they turned the tables on the DMV. Bohr, they claimed, had insulted and attacked the German state, which Bieberbach then had sought to defend. The rest of the assembly did not stand up to this arm twisting, and thus Bieberbach's actions were only 'regretted', whereas Bohr was 'condemned' for his attack on the German state.

But when Bieberbach subsequently proposed to introduce the 'leadership principle' (*Führerprinzip*), with Tornier as leader, he was flatly voted down. Apart from a great deal of embarrassment, the matter could have ended here with a red-faced Bieberbach. The latter, however, in his function of secretary of the DMV tried to thwart the procedure. This was indeed the last straw; his colleagues demanded his resignation. When his political patrons showed no inclination to step in, Bieberbach resigned. Having lost his foothold in the German mathematical community, he created his own journal and his own entourage. The new, heavily subsidized, journal went by the name *Deutsche Mathematik*. It never acquired the status of the journals that Bieberbach had edited in the past. The journal was intended as a platform for politically correct authors and papers, it also published news items about Nazi-sponsored activities, such as summer camps. A certain amount of propaganda was printed in its columns, but one would not be justified to consider every author who published in the *Deutsche Mathematik* as a politically suspect character. Paul Koebe published for example one of his expositions, *Wesen der Kontinuitätsmethode*, in the 1936 volume, probably on request of, and to please his student Bieberbach.

16.6 Compositio Mathematica

Bieberbach and Brouwer had been on the same side in the affair of the Riemann volume, and in the War of the Frogs and the Mice. After that there had been little

contact, if any at all. Both parties went their separate ways, Bieberbach to pick up his daily routine, and Brouwer to lick his wounds. The loss of the *Mathematische Annalen* had hurt; the plan to found his own journal, mentioned to Springer,[83] perhaps for tactical reasons, had not been forgotten. Springer himself had at the time judged this a fair solution; he may have had his doubts about the feasibility, but that clearly was not his business. Indeed Brouwer cautiously inquired with the Teubner Verlag, if it was interested in founding a new journal. The reaction was predictable but disappointing. After consulting Bieberbach the publisher reached the conclusion that a new journal was a highly doubtful business proposition, and that it was questionable whether enough subscribers could be found to make the journal profitable. One must keep in mind that the world, but in particular Germany, was involved in one of its worst economic crises, so any publisher would think twice before starting a new journal. The loss of the status journal, *Mathematische Annalen*, to Springer must still have rankled, for the spokesman wrote that they would not gladly run the risk of another debacle, 'after the *Annalen* had already been wrenched from our hands.'

However discouraging this might have been, Brouwer did not give up so easily; he approached the publisher Noordhoff, a firm with some experience in mathematics, as it published the journal of the Dutch Mathematical Society, *Wiskundig Genootschap*, and it brought Dutch language mathematics textbooks on the market. Already in 1929 Brouwer opened negotiations with Noordhoff. On 24 October Noordhoff and Brouwer met in the house of Wijdenes, the publisher's advisor in mathematical matters. The mathematician Wijdenes, also one of Korteweg's students, was an extremely successful author of mathematics textbooks for high schools; the acquaintance with Brouwer went back to his student years. The agenda for the meeting mentioned 'the Journal and further publications.' Noordhoff was no stranger to Brouwer; his publishing company had already marketed the commercial versions of Brouwer's dissertation and his inaugural lecture, and it had published a small booklet containing the unreliability paper and both inaugural lectures.[84] In a letter of 18 March 1919 to Wijdenes Noordhoff reported about Brouwer's dissertation 'of which I just recently have received the stock. I had a number of copies bound in linen. There is still linen left for a small number of copies, and people usually prefer a bound copy.' Noordhoff marketed the dissertation for 4.50 guilders, with 50% royalty for Brouwer. In 1929 Noordhoff tried to convince Brouwer that a new edition of the dissertation should be published. Brouwer was, however, not too keen on the idea: 'The book is now out of date and

[83] Cf. p. 606.

[84] [Brouwer 1909, Brouwer 1912, Brouwer 1919c]. Noordhoff listed Brouwer's publications regularly in his catalogue. In 1922, 1926, 1928, 1933: De onbetrouwbaarheid der logische principes, Het Wezen der Meetkunde, Intuïtionisme en Formalisme (collected in *Wiskunde, Waarheid, Werkelijkheid*), Over de Grondslagen der Wiskunde, Luchtvaart en Photogrammetrie. In 1938, 1940,1942, 1948, De onbetrouwbaarheid der logische principes, Het Wezen der Meetkunde, Intuïtionisme en Formalisme. In 1949 only *De Uitdrukkingswijze der Wetenschap* (containing [Brouwer 1933b]), this item appeared for the last time in the catalogue of 1958.

it would have to be totally revised. In principle I am prepared to do so, but first there is a lot of other work to be done by me, among other things the publication as a book of the course on intuitionism I gave in Berlin, which I hope to submit to you soon, if in the meantime the journal has been realized.'[85]

Noordhoff was sufficiently interested in the publication of an international mathematics journal to give Brouwer the go-ahead. And so the preparations started; one of Brouwer's first decisions was the name: *Compositio Mathematica*. An important detail was the choice of editors for the journal. Brouwer decided to follow the example of the old *Annalen*, a modest board of managing editors and a large board of associate editors. Already in June 1930 the first letters went out to sound the prospective editors, and in October the definite letters of invitation were mailed.

The first list of mathematicians invited to join, contained the names of Alexandrov, Baer, Bieberbach, Borel, Cartan, Cech, Van der Corput, De Donder, Doetsch, Eisenhart, Feigl, Fréchet, Fubini, Fujiwara, Garnier, Hadamard, Hardy, Heegaard, Heyting, Hille, H. Hopf, Julia, Khintchine, Lefschetz, Levi-Civita, Lévy, Loewy, von Mises, Montel, von Neumann, Nörlund, Ostrowski, F. Riesz, M. Riesz, Saxer, Severi, Sierpinski, Süss, Szegö, Takagi, Tonelli, Valiron, de la Vallée-Poussin, Veblen, Wavre, Weitzenböck, Whittaker, Wilson, and Wolff.[86]

The board was in fact as international as one could possibly wish, and there was a judicious mix of the older, established generation, and the younger, coming generation.

One may well assume that most of the above, if not all, were aware of the motivation for the founding of this new journal. This is illustrated by Brouwer's old friend Hadamard. Their friendship went back to 1910, when Brouwer stayed with his brother in Paris (see p. 155). Brouwer had a very high opinion of Hadamard; he was eager to get him on the board, but Hadamard did not quite know what to make of the invitation. He wrote for advice to Einstein, saying that it was tempting to join a truly international journal, but that he was somewhat uncertain if he would in this way be used as a pawn against Hilbert.[87] Einstein replied a month later that there had indeed been a fell struggle,

> for which Hilbert, in my opinion, carried most of the blame. Brouwer, however, behaved at this occasion so excessive and obstinate, that he appears to me a man of pathological irritability.[88]

He advised Hadamard to steer clear of this new journal,

> I would unconditionally wash my hands of it, in spite of all respect for the subtleness and the honest character of Brouwer, who is not aware of the abyss of his temperament.

[85] Brouwer to Noordhoff 10.X.1929, cf. also p. 548.

[86] From Brouwer's letter to Veblen 11.X.1930.

[87] Hadamard to Einstein 16.X.1930.

[88] Einstein to Hadamard 15.XI.1930.

Einstein's dark predictions concerning Brouwer's handling of the *Compositio Mathematica* turned out to be unfounded, partly because Brouwer was a conscientious scholar, who could not sin against scientific norms, partly because his new assistant, Hans Freudenthal, first under Brouwer's guidance and gradually on his own, conducted the managing of the journal. Eventually Freudenthal just submitted each complete issue to Brouwer for his fiat, often Brouwer would not even answer, but he could also, suddenly, show interest in certain papers, and spend his time lavishly on the refereeing and the supervision of the corrections. Sometimes Brouwer noticed a particular point in a paper a year later, but by then it had already been published.

When the journal was about to be launched, the publisher sent out flyers with information. Among Freudenthal's documents there is a draft of the German text of the flyer; apparently he was asked to edit the final wording. It is interesting to read Brouwer's views on the role of a scientific journal in a time when in certain quarters the primacy of politics over science was taken for gospel. As a true internationalist Brouwer was not going to give in to new trends.

> Shortly the first issue of the mathematical journal Compositio Mathematica, edited by representatives of the mathematical sciences from 48 countries, will be find itself in print with the publishing house Noordhoff. It will be the task of Compositio Mathematica, not only to encourage the development of mathematics by accepting for publication valuable mathematical papers, but also to serve the international scientific cooperation, which is at present more than ever necessary. To do justice to this aim it is not sufficient to abstain from imposing any national or language-barrier; rather a, as far as possible, international composition of the editorial board is required in order to avoid any bias with respect to national aspects. In view of the nowadays often occurring specialization of mathematicians of specific nations on specific areas of research and methods of research, such a composition offers at the same time a guarantee against any one-sidedness with respect to the mathematical character of the published papers. . . .[89]

Alexandrov was confronted with a difficult problem; he fully realized that support from the Göttingen group was more valuable than the support Brouwer could give. Although Brouwer had got him a Rockefeller grant, the effective influence of Brouwer was limited and as things were in the world of mathematics, the backing of Hilbert's circle, including the publisher Springer, carried infinitely more weight than Brouwer's influence. So when he was asked to join the editorial board of the *Compositio*, he feared a clash of interest with the followers of Hilbert (who would, he thought, not welcome a competing journal). So he declined the invitation; Freudenthal, always a good observer of the mathematical scene and usually well informed, deplored Alexandrov's urge to ingratiate himself with the Göttinger

[89]This is a somewhat free translation of the German text. There is probably an English version somewhere in some archive, but I have not found any.

people, ' . . . who knows if they are so sincere. From the way they treat Noether, one might conclude that they will think twice to get him something in order not to loose him for Göttingen.'[90]

Brouwer did not take kindly to Alexandrov's refusal;[91] he was doubly disappointed as he had been using his influence attempting to get Alexandrov a chair in Groningen. As we will see, Alexandrov joined the board after all.

Compositio entered the mathematical world in a very awkward period. When Brouwer composed his first list of candidates for the editorial board, the political horizon was unclouded, but by the time real commitments had to made, the political landscape in Germany was no longer the same. Many competent mathematicians had been forced into exile, or silenced. The first list Brouwer had made contained the following German mathematicians: R. Baer, L. Bieberbach, G. Doetsch, G. Feigl, H. Hopf, A. Loewy, R. von Mises, J. von Neumann, and W. Süss.

In 1933, however, when the journal was about to be launched, this list had become rather suspect in the eyes of those Germans who followed the party line. And thus in 1934, when the first issue appeared, Bieberbach had developed strong political objections against the presence of some names on the list of editors. He wrote to Brouwer that the founding of *Compositio Mathematica* (*Compositio* for short) must have given satisfaction for the brutal dismissal from the board of the *Annalen*.[92] He for himself had considered his membership of the editorial board—he had indeed become a managing editor—a good thing, for it made certain that the name of a man of German spirit appeared on the cover of an international journal.

> I assumed that one would recognise this as an example that the new Germany, notwithstanding its fight with the international Jewry, would gladly co-operate with other nations, that meet us, if not with sympathy, then at least with loyalty. Instead people now see often the crucial point in the fact that Jews occur on the cover of Compositio.

And, he continued, this was explained as a sign of my cooperation with Jews. He could accept the fact that names of Jews figured on the cover, to show that he was prepared to tolerate the presence of Jews at the board as a defect, in view of the demonstrated willingness to join the international community. Other nations, he assumed would in the end recognize the necessity of the German actions. To his disappointment he had been subjected to hostile reactions from all sides. And so 'I feel obliged to make the disappearance of the Jews from the editorial board a condition for my presence in the editorial board of Compositio.' He hoped, he wrote, that the old alliances in matters of international cooperation, would make it easier for Brouwer to carry out the necessary steps. The letter ended with the barely veiled threat, that the present composition of the board would cause difficulties for the distribution of *Compositio* in Germany.

[90] Freudenthal to Hopf 22.XII.1930.

[91] *Brouwer schimpft jetzt auf Alexandrov*.

[92] Bieberbach to Brouwer, 21.VI.1934.

Bieberbach's appeal does not seem to have had much effect on Brouwer, who did not want to strike candidates from the list of editors because someone had personal or political objections. And thus in the beginning of 1935 Bieberbach resigned from the editorial board. It seems that Brouwer had tried to find a face saving formula for Bieberbach. He had sent a circular to all editors with the message, that in view of the delicate character of the present international situation, 'Science seems more than ever called to form a refuge for mutual understanding.' And hence the editors were advised that 'any editor's public participation in manifestations which could harm the mutual esteem of people and nations was incompatible with his function.'[93] This message has its own irony. Would Hilbert not have said the same thing in 1925? Responsibility apparently breeds prudence.

Bieberbach remained, however, adamant. One can see that the message of Brouwer offered Bieberbach in a sense protection against insults, but it equally condemned his attacks on Jews and emigrants. His letter repeatedly referred to the international Jewish community and its subversive war against his fatherland. 'My national feelings forbid me, to belong to an editorial board, in which so many representatives of international Jewry and in particular also emigrants are found.'[94] If Bieberbach had hoped that Brouwer would back down when threatened with his resignation, he had made a miscalculation. Brouwer did not even try to keep Bieberbach aboard. He replied: 'I hardly have to say, that your decision upset me *very* painfully, but on the other hand is completely respected by me, as I know that it was dictated by your conviction and your conscience.'[95] And that was the end of a long association between two persons who had shared a mathematical interest for many years, and who had fought the *Conseil* and its boycott shoulder to shoulder.

For a Nazi the large proportion of Jews and émigrés among the German editors must have been an insult. Indeed, Baer, Loewy, Von Mises and Von Neumann were objects of the racial persecution; Heinz Hopf, although not Jewish, had emigrated to Switzerland, and hence could from Bieberbach's point of view not be relied on. Baer and Von Neumann had already left Germany, and Alfred Loewy was still in Freiburg, where he died in 1935. After Bieberbach's resignation four German editors were listed: Artin, Doetsch, Feigl and Süss. Of these four, Gustav Doetsch and Wilhelm Süss were national socialists. The fact that after all German editors belonged to a board of dubious character was in Bieberbach's eyes a serious blemish on the honour of German science, a blemish that could only be removed by the resignation of the remaining Germans. He decided to put pressure on them, at first just in the form of a suggestion, 'In agreement with *Ministerialdirektor* Vahlen, I am asking you to follow my action.'[96] Brouwer, on the other hand, did not wish

[93] Cf. [Remmert 1999], p. 18.

[94] Bieberbach to Brouwer 8.I.1935.

[95] Brouwer to Bieberbach 15.I.1935.

[96] Bieberbach to Doetsch 19.I.1935.

to lose the editors that he had selected with so much care. He asked Doetsch to fill Bieberbach's place in the board of managing editors.[97]

Doetsch clearly found it difficult to determine his position. In July 1934 he was still on Bieberbach's side, as appears from a card to Feigl,[98] in which some scathing remarks about Jewish mathematicians and reviewers are to be found. The card ends with comments on the position of the Germans in the board:

> If Bieberbach resigns from Compositio, then it would be most desirable that we remaining German editors and editors of German descent act unanimously. Only you and Süss are to be considered. Szegö is a Jew, isn't he? Anyway, he is married to a Jewess. I will just wait for the result of the discussion between Bieberbach and Brouwer, but I am very much inclined to join Bieberbach. Heil Hitler

On the other hand Doetsch apparently did not want to give up a prestigious editorial post, and, as Remmert points out, he may have wanted to dissociate himself from Bieberbach, who by now had lost quite a bit of prestige.[99] He tried to bypass Bieberbach by getting himself a permission to remain on the board of *Compositio*. At the same time he attacked Bieberbach for his inconsistent actions in the *Compositio* matter, and for his bungling of the Bologna affair. The Prussian ministry of education, after some deliberation, decided that German scientists could publish in *Compositio*, but not join the editorial board. This clinched the matter. Doetsch, Feigl and Süss resigned from the board. Probably none of them would have objected to stay on. Feigl, who was not a Nazi, might have wished to side with Brouwer, but in the young totalitarian state one could not disobey without risking serious penalties. So in 1936 the only German on the board was Artin, who by then had emigrated.

In March 1936 Brouwer wrote to Hopf and Alexandrov, thanking them for the 'marvellous book, with which you have connected my name'[100] and informing them at the same time that all the German editors had resigned from the editorial board.[101] He asked if one of them would be prepared to join the executive board. In fact both of them did.

Compositio thus certainly had its share of difficulties at the start. But once the journal was on its way, things ran smoothly. Although Brouwer was the responsible editor, most of the work was done swiftly and competently by Freudenthal. Those who had judged Brouwer incapable of running a journal properly, turned out to be wrong. All the fears that Hilbert claimed to have for the disastrous influence of his Dutch opponent were after all ill-founded. Brouwer did not do any of the things

[97] Brouwer to Doetsch 20.III.1935.

[98] Doetsch to Feigl 16.VII.1934.

[99] See [Remmert 1999]. Bieberbach had failed in his coup at the meeting of the DMV, [Schappacher–Kneser 1990].

[100] Alexandroff-Hopf, *Topologie*. Dedicated to Brouwer.

[101] '*. . . aus der Redaktion vom Compositio Mathematica sämtliche reichsdeutsche Mitglieder ausgeschieden sind*, . . .'

he was suspected of; he did not stop French or Belgian authors or editors, he did not turn his journal into a vehicle for intuitionistic mathematics, he did not reject Russian Jewish authors. In short, *Compositio* became a normal respectable journal. Intuitionistic mathematics did not play an important role; until the Second World War only 6 such papers appeared, written by Belinfante, Freudenthal, Heyting and Johanson.

The scientific journal landscape, in particular in Germany, was changing dramatically in 1933. The new regime did not lose time to infiltrate existing journals; whenever possible and convenient, the *Führer* principle was enforced. This meant as a rule that political motives could, and often did, overrule scientific standards. The *Mathematische Annalen* was no exception. There is a pressing letter from Blumenthal to Hilbert in November 1933, in which he painted in vivid colours the dangers that lay ahead. The worst effect of the new times was the uncertainty that surrounded the Göttingen faculty. 'If Göttingen becomes a desert, or is populated by professors who discard tradition, then we have to open up new wells, or we come to nought.' In the light of the present threats, the founding of *Compositio*, which seemed so harmless at the time of the Annalen conflict, assumed ominous proportions: 'On the other hand the Annalen are threatened by Brouwer's newly founded Compositio Mathematica, in which, in numbers, a very large staff of international associate editors (*Mitarbeiter*) is brought together. Since Bieberbach and Feigl have joined this staff, it is clear that we cannot hope for the cooperation of the Berlin school for the Annalen. It is even more worrying that also Heinz Hopf (Zürich), with whom we always have worked well, has committed himself to this competing enterprise.'[102] Blumenthal's conclusion was that the Annalen urgently needed an expansion of the editorial board. He suggested Van der Waerden as a perfect candidate.

The relatively weak position of the *Annalen* at this point in time was a consequence of an over-confident decision in the past: to minimize the editorial board, and to make the *Annalen* even more exclusively a Göttingen matter. Nobody could have surmised that a momentous decision, taken for the wrong reason, would be regretted so soon.

16.7 Göttingen reconsidered?

After the wholesale elimination of staff members that did not conform to the requirements of the new government, only a few mathematicians of the old school were left to keep the Göttingen department going. Herglotz took care of the institute until a new director would be appointed. Eventually Hasse and Tornier were appointed. Hasse was a number theorist of the first rank; politically he was *deutschnational* and he apparently accepted the national socialist domination as a not unwelcome fact. Schappacher characterized Hasse's position under the Nazi regime as 'Just as he liked to identify himself during both world wars with the

[102] Blumenthal to Hilbert 11.XI.1933.

glamour of the German Navy, his army unit, it was for him a personal concern to contribute in the thirties to a reborn strong Germany. In the process he apparently did not hesitate to contact the more traditional Prussian side of the national socialist regime, rather than the …'revolutionary's.[103] Tornier was no outstanding mathematician, his main interest became probability theory. He was an active Nazi and he followed the party line in all respects. Soon Tornier and Hasse fell out, and finally after an internal struggle, Tornier was transferred in 1936 by Vahlen to Berlin. There he lost his job and membership of the party in 1938. Tornier must have cultivated strange company; Constance Reid reported that 'he embarrassed the mathematics faculty by being pictured in the newspaper walking on a fashionable boulevard with a notorious prostitute on his arm and a tame tortoise on a leash.'[104]

Hasse and Tornier had to staff the mathematical institute and for whatever reason they must have considered the famous Dutchman Brouwer with his record of activities for the re-admittance of German mathematicians into the international scholarly community a good catch.[105] One may guess that Brouwer was partly selected in order to embarrass the old Göttingen school. In view of the fact that Brouwer had for all practical purposes left topology, one can rule out that the need for a topologist motivated the choice. Neither does it seem likely that Hasse envisaged the founding of an intuitionistic tradition in Göttingen. So, in all, it seems most likely that the two directors saw in Brouwer a mathematician with universally acknowledged status, who moreover had shown himself to be an ally in the cause of rehabilitation of German science. The first letter to Brouwer mentioning the vacant chair came from Tornier, but one may be certain that he wrote to Brouwer with Hasse's approval. So there must have been some discussions in May 1934, or perhaps earlier. Tornier's letter has all the characteristics of the academic upstart. He did not bother to hide his satisfaction over the successful elimination of the Jewish mathematicians.[106]

> As you well know the flooding of Germans with alien races, in particular also in the teaching staff of the mathematical institute here, has resulted in intolerable situations.
>
> I permit myself to put the question, if you, considered by me and many German mathematicians already for a long time as one of the greatest scholars of a typical German disposition, would be prepared to help to restore the old reputation of mathematics in Göttingen.[107] […]

[103] [Schappacher 1987], p.354.

[104] [Reid 1986], p. 4-2. See also [Schappacher–Kneser 1990].

[105] Brouwer was in German circles described as *deutschfreundlich*. This term acquired after 1933 a very specific meaning: pro- Nazi. But before 1933 it just meant what it said: sympathetic towards Germany and Germans. It is not unusual for commentators to confuse the two meanings.

[106] Tornier to Brouwer 19.VI.1934.

[107] *Ich erlaube mir nun die Anfrage, ob Sie, den viele deutsche Mathematiker mit mir schon lange für einen der grössten Forscher von typische germanischer Prägung halten, bereit wären, den alten Ruf der Göttinger Mathematik neu begründen zu helfen.*

> I beg you to be so kind to tell me if you are in principle inclined to enter into negotiations about a call to Göttingen. I may add that my joy to see you perhaps for always in Göttingen, is shared by the responsible department of the ministry, that is Herr Ministerial-Direktor Professor Dr. Vahlen.

Considering the circumstances and the political overtone, this was not an offer to be pleased with. What would a man like Brouwer, jealous of his privacy, and a poor organizer, do in the hornet's nest of a mathematics department in Nazi Germany?

One would have expected a firm but polite rejection of the offer, but Brouwer decided to play the game. All available evidence seems to point to a financial motivation. Already before Tornier's letter there had been some correspondence with Karl Kerkhof, the man in charge of the *Reichszentrale für naturwissenschaftliche Berichterstattung*. Brouwer must finally have succeeded in finding a buyer for his house in Berlin-Zehlendorf. Considering the fact that he had tried to sell his property already for years, this was a most welcome event for Brouwer, who was at that time involved in the Hungarian Spa affair, and was threatened by nothing less than a financial disaster. But now he was confronted with interminable bureaucratic rules on housing, on sales, on the transfer of currency. Handling bureaucratic problems from a distance was (and still is) not easy, so he had enlisted the help of Kerkhof; and the latter had made his enquiries at the foreign office. On June 8 he wrote Brouwer that his 'request was granted under certain conditions.' The vagueness of this formulation was typical for the whole procedure. Whatever Brouwer tried, and whoever he begged to intervene, each time some local government organization or a bank would promise that the matter would be solved in a few days, and each time a new blockade would appear. Brouwer was tenacious as a bulldog. He repeatedly asked Kerkhof and Vahlen to intervene, and each time success seemed just round the corner. Brouwer found the Foreign office, the Reichsbank, the foreign currency office (*Devisenstelle*), the Department of Finance of Berlin, the Land Registry, the Dedibank, and probably a few more institutions in his way. A correspondence that ran from before June 1934 until far in 1935 pictures a Kafkaesque struggle against an impervious bureaucracy. Kerkhof and Vahlen acted a number of times on Brouwer's desperate calls for help, but more than friendly promises could not be obtained. It is not known if in the end Brouwer got his money out of Germany. Dependable information on Brouwer's strategy in the matter of the Göttingen vacancy is hard to come by. It seems highly unlikely that he seriously considered a chair in Göttingen. At the heydays of Göttingen's mathematics he had declined a call, and now a professorship under Nazi-rule was not attractive at all, to put it mildly. Apart from any other motive, his attachment to his property in Blaricum, to the pharmacy, was enough to keep him in Holland. Moreover, he would rather 'build a Göttingen' in Amsterdam than in Göttingen and a call to Göttingen, even then, came in handy in negotiations with the City Fathers of Amsterdam. Taking all arguments into consideration, one must conclude that he was using the call to Göttingen as a means of putting pressure on the various government and financial institutions that played a role.

However, he had to negotiate with the Göttingen representatives in order to keep up his credibility. Any sign of a hidden agenda might have jeopardized his financial plan. From the letters that have been preserved, the picture of a feet dragging Brouwer appears. He wrote to both Tornier and Hasse (the former soon disappeared from the scene) in such a manner that one would be inclined to discern a sincere interest in the Göttingen chair. He postponed his visit to Göttingen, however, as long as possible. On July 27 he wrote to Tornier, telling him how despondent he felt for not having come to Göttingen so far. Tornier was elated, he felt it a great honour that the famous Dutchman would probably within a week come over and visit him, although no such promise was made by Brouwer, who just mentioned the possibility. At this point Hasse stepped in, he clearly did not want to leave Brouwer in the hands of a Nazi-adventurer. On August 11 Brouwer informed Tornier that he had been bitten by a dog, and that medical complications had arisen that he would be confined to his bed for some weeks. The following months Brouwer claimed that he could not leave because of threats of financial disaster.

The fact that the appointment of Brouwer in Göttingen was seriously considered did not escape the old guard. It understandably caused much anxiety among those who had known Göttingen before 1933. Harald Bohr reported Veblen about the situation in Göttingen in vivid terms.[108]

> Tornier's next idea is to get Brouwer to Göttingen. Schmidt told us that Brouwer was extremely pleased about the idea and was very much inclined to come; in this moment he discusses the financial terms with the Nazi-government. All who have had the privilege of coming in closer contact with Hilbert feel the idea of bringing Brouwer to Göttingen perhaps the dirtiest trick, especially as those people do not really believe in Brouwer's ideas of a new foundation of mathematics, but simply wish to liquidate completely the époque of Hilbert and his school.

Bohr's letter makes it quite clear that the appointment of Brouwer was seen as the ultimate insult. Apparently Bohr and his fellow Göttingers took Brouwer's words serious, forgetting that usually in such a situation, information looses part of its accuracy in its transfer from one person to the next. Bohr overlooked the fact that anybody who wants to use an offered chair for some specific benefit, is likely to show some interest. Without it the leverage would not amount to much. Far away in Princeton, Hermann Weyl had also heard of the offer to Brouwer; in a report to the Rockefeller Foundation one reads '...the authorities attempted to obtain Nevanlinna [...] and the Dutchman L.E.J. Brouwer. W. considers that B. has definitely passed his period of maximum productivity. The negotiation collapsed however, because B., as a good Dutchman, insisted that his salary be paid in gold.'[109] In the absence of further confirmation, we will probably have to take Weyl's statement as a tongue in cheek comment on Brouwer's preoccupation with money.

[108] Bohr to Veblen 11.VIII.1934, cf. [Segal 1986].

[109] [Siegmund-Schultze 2001], p.191.

Hasse, who apparently sincerely wanted Brouwer to join the Göttingen department, was running out of time, he dearly wanted Brouwer's advice on the matter of the further vacancies in Göttingen, and Brouwer replied that he would be happy to give his views on the various candidates, but that this was more a matter for a personal discussion, so it had to wait for his visit to Göttingen, which—he said—could now take place any minute. In the end he sent a written advice. In October Hasse again prodded Brouwer into action; he desperately needed Brouwer's information on the course that he was supposed to give. From Hasse's letter (12.X.1934) it appears that in the meantime Brouwer had been in Göttingen, and promised to give a course on intuitionistic mathematics. Subsequent correspondence shows that Brouwer had managed to remove the full time appointment from the agenda, and to promise to come to Göttingen, for the time being, as a visiting professor. But even that did not work out. His attempts to get a transfer of his money (from the sale of the house) did not yield any results, and apparently even Vahlen reacted no longer to his entreaties. The last letter in the archive is resigned in tone; on February 5, 1935 Brouwer wrote to Hasse that he could not accept the visiting position as his financial situation made a prolonged absence impossible. His letter to Hasse ended with the sad words, 'Hopefully the expected rescue action from Berlin will come in time to prevent the definitive destruction of my spiritual activity. A scientific corps will be of no use to Göttingen.'

The persecution of Jews and persons of mixed marriages could not escape anybody who read his newspapers, but for many outside the immediate sphere of influence of the Nazi's it remained an abstract injustice in a central European country. It becomes a different matter, however, if one is confronted with an individual case, with a friend or relation who has become the victim of the regime. This happened to Brouwer when he learned in 1934 that Schoenflies' son-in-law Kaemmel was threatened with dismissal. Brouwer immediately set himself to support Kaemmel by collecting testimonials of Schoenflies' importance. In a letter to Veblen[110] he wrote 'There seems to be a chance to save him, if the German Government could be convinced that Professor Schoenflies has been at the time not only a functionary of loyal national feelings, but also a scientist of importance, having played his part in the development of German science.' Veblen almost immediately complied, adding that 'I very well remember the world-wide impression that was made by the publication of Schoenflies' famous "Bericht" on point-set theory. This, as you well know, was for a long period the chief handbook of all those who worked in this field, and there is no doubt that it contributes largely to the success of German science in that direction.'

It is unknown if Brouwer's action made any difference. The history of Nazi-period makes us fear the worst.

[110] Brouwer to Veblen 20.X.1934.

16.8 Dutch affairs

1935 saw the beginning of another parallel episode in Brouwer's life: local politics. In that year Brouwer was elected in the council of the town Blaricum. He had joined '*The Neutral Party*', which stood for local interests. Political or social issues played almost no role in the party program. For a beginner in politics Brouwer did very well, his party (no. 2) obtained 235 votes, of which 117 were for Brouwer. There were altogether 1372 votes, and the Neutral Party came out number 3. On 3 September Brouwer was installed as a member of the council, and right away made temporary acting alderman under the chairman of the council, mayor J.J. Klaarenbeek.

Given the multitude of projects and functions of Brouwer, he was surprisingly faithful; he attended all council meetings in 1935 and most of the meetings in later years. Looking at the minutes of the meetings—and the reports in the local newspaper '*De Bel*' one cannot say that Brouwer had a striking record. He did what he was supposed to do: look after local interests. So we see him pleading for the preservation of the local heath (a nature reserve), advocating better access for walkers while keeping out cars. He proposed a raise for the local police (*veldwachters*), added improvements to a plan for the construction of a cycle path (he had done his homework and quoted a report of the ANWB[111]), pleaded for the improvement of the soccer field—'sport has to do with national health.' Apart from the routine management of the town, incidentally a matter of principle cropped up. When, for example, the committee of action 'For God' demanded the barring of the periodical of the freethinkers from the public library, Brouwer remarked that according to the state's instructions all groups must be represented. Moreover he deemed it incorrect to act as a censor in the absence of dependable information. At one point he came into head-on collision with the mayor when defending the right of the Committee of Advice and Support to meet without the mayor and aldermen being present. The mayor was terribly upset and blamed Brouwer for displaying an unfriendly attitude in supporting the motion. The minutes record that: 'Mr. Brouwer did not wish to enter into the degree of friendliness which should be displayed by the council members. In his opinion a council member has reached the age at which he no longer needs to be educated in friendliness!' Brouwer won. For an individualist like Brouwer party lines were not the ultimate political wisdom. He was, for example, fairly close to alderman De Klerk, a shoemaker. Although they did not belong to the same party, De Klerk often sent his son with drafted proposals to Brouwer, who then went over the text and returned the documents typed and all.[112]

The elections of 1939 returned Brouwer to the council with the highest number of votes—310 out of 1601, while his party had become the second largest on the list. To run ahead of our tale, Brouwer remained councillor until the German

[111] Sister society of the AA and AAA.

[112] Interview H.P. de Klerk, junior.

Brouwer as seen by a local artist

occupation authorities disbanded all city-, town-, and village councils[113] (1941). After the war he did not return to politics.

Brouwer's council activities have been the subject of some speculation, 'why should a man like Brouwer stoop to enter town politics?' Evil tongues have suggested that there was something in it for Brouwer—after all he owned large tracts of land in and around Blaricum. There is nothing on or off the record that supports these views. Probably the simplest explanation is the true one: Brouwer cared for his town and its people, so when the local politicians appealed to him, what was more natural than allow himself to be put on the list of candidates for the council? In view of Brouwer's strong ecological views (cf. *Life, Art and Mysticism*) it is not surprising that he opted for a local party, without the power structure of the established parties, without the directives from above. There is no doubt that Brouwer was, in spite of his international orientation, thoroughly part of the town. Many of his friends were from village middle class circles, one of his close friends was the

[113] *gemeenteraden*.

shoemaker Bus (not the above mentioned alderman), with whom he could spend hours, talking about everything under the sun. When Brouwer died the shoemaker was inconsolable, 'my friend Brouwer is dead.' Brouwer was sincerely concerned with the families of the town people. At one occasion, when he heard that a child was stricken with a disease that kept it in bed, he went up to the house with a book under his arm, stepped in and sat down at the bedside of the child, reading to it for an hour.

Brouwer did not feel too high and mighty to take an interest in the lives and problems of his fellow villagers. On one occasion, one of them, a local shopkeeper, by the name of Oversteegen, confided to Brouwer that he found it difficult to assess the potential of his bright son; what should the boy study? Brouwer invited the boy over to his house and gave him a couple of private tutorials on mathematics, and concluded that no great mathematician would be lost to the world if he did not enrol in the mathematical-physical faculty. So the boy entered the university to study languages and literature, in due time became a well-respected professor.[114]

A fair number of friends and (even) relations lived in Laren-Blaricum. His own brother Lex lived ten minutes from his brother's house, one of the Poutsma relatives lived across the street, and some colleagues lived in the neighbourhood, e.g. Weitzenböck. The latter had become one of the townsmen, he shared in their pastimes and was for some time the president of the chess club of Blaricum. He was a fervent chess player, so maybe it was no wonder that Max Euwe, the one-time world champion was his Ph.D. student. Without actually going public, or trying to influence his students, he, a former Austrian military man, fostered virulent revanchist feelings. For some reason—probably the same reason that was put forwards by the majority of right wing German-Austrian citizens: the treaty of Versailles—he had become violently anti-French and in a later stadium Nazi. There is a curious manifestation of his Francophobia: in his well-known book *Invarianten Theorie*[115] the first letters of the sentences of the Introduction spelt '*Nieder mit den Franzosen*' (down with the French). Weitzenböck's book, by the way, became something like the bible for students in Amsterdam, his courses were well attended, and he had a substantial number of Ph.D. students. His students and the villagers remembered that before the Second World War he displayed no political activity at all. During the war he openly showed his political sympathies.

Brouwer got another taste of the new political masters in Germany in 1937. The story was told by Elias Balke, a German artist, who kept a diary from 1933 onwards, and published fragments in his '*Chroniknotizen*'.[116] Brouwer is mentioned in this diary. It appears that Brouwer owned some land (and probably a cottage or a cabin) in north Prussia, in the neighbourhood of Fischerkaten (now in Poland).

> In the spring of 1937 Prof. Luitzen Brouwer came one evening in the Forest house, to ask me how I saw the political developments and whether

[114] Private communication J.J. Oversteegen.

[115] [Weitzenböck 1923].

[116] [Balke 1973] p.101.

> he should accept a call to Göttingen. I advised against it. Brouwer was a striking visitor. Extensively informed, travelled, world-wise. A bronzed head with a bold profile, a tropical skin like parchment, eyes that see in the distance. He had property at Java, where I just came from. As the founder of the intuitionistic school he was the publisher of an international journal for mathematics. We got into a marvellous conversation. In a tense, pointed dialogue that gave pleasure to each partner. We had common friends in Java and Germany. In the meantime we had also heard a great deal from each other, as he had, at my request, immediately given shelter to Helen,[117] when I had to bring her in 1935 to safety for the Gestapo. He was—to my joy—strongly impressed by this talented young woman, and we knew to praise her. At the break of dawn Brouwer left. The bottles were empty, the sun was rising, the blackbirds sang and one could hear the breakers. When he was already at the bottom of the stairs, he once again looked up to me and took his leave in such a charming way that this little scene remained especially dear to me.

Helen Ernst was a gifted young artist who had been imprisoned by the Gestapo in 1933, was released with great difficulty, and when she was about to be arrested again, Balke took her to Dantzig where he eventually put her aboard a Dutch ship and when she arrived in Amsterdam, Brouwer took her to Laren. She found work at the *Nieuwe Kunstschool* in Amsterdam. In 1940 she was arrested and taken to the concentration camp in Ravensbrück. Helen survived the war, and died of tuberculosis in 1948. The story of Helen is also the subject of a biography of Hans Hübner.[118] Hübner mentions 1937 as the year that Balke and Brouwer met, but that was probably taken from Balke's book. Brouwer had offered Helen one of the little cottages on the grounds,[119] before she moved to Amsterdam.

The dates mentioned in the story are rather puzzling. In 1937 Brouwer did not get a call to Göttingen. Brouwer may have told Balke about his call to Göttingen in 1934, and Balke may have mixed up the years. The fact that Helen Ernst is mentioned adds evidence for 1937 as the date of meeting. Brouwer had become an almost compulsive talker, he enjoyed the attention of an audience, and when he got an opportunity, he could easily draw from his large stock of stories, both from his own experience, and from others. So he may have told about his brother's time in Java, who worked there as a geologist, and Balke may have got the impression that he was talking about himself. The real surprise is to find Brouwer in an east Prussian forest at the Baltic sea. Would it not be natural for him to sit at home and indulge in self-pity? The urge to move around was probably stronger than his feelings of despair over financial matters. Brouwer's habit to buy real estate had probably at some time resulted in the acquisition of a piece of forest in East Prussia,

[117] Helen Ernst, an artist.

[118] [Hübner 2002].

[119] *eine Hütte "mit bohème-artiger Verpflegung"*.

Irmgard Gawehn, Brouwer, Dolly Kiehl, Willem Langhout, Tine Langhout-Vermey. (Brouwer Archive)

and, while seeking solitude in this quiet corner of Germany, he had met Balke and his wife.[120]

Life at the Brouwer household went on along the established lines. Lize usually spent her days in Amsterdam, supervising the pharmacy, and Brouwer stayed in Blaricum or Laren, as the case might be. After the first influx of topologists had left to return to their respective home universities, the company at Brouwer's place consisted mainly of personal friends and relations. He had a particular fondness for female company and, generally speaking, the feeling was mutual. A small but devoted group of ladies visited Brouwer regularly and basked in the sun of the wit and the wisdom of their idol. In Brouwer's house tea was of great importance. There was always an ample choice of plain and exotic tea. He would, when he had visitors, pay special attention to the selection and mix of the various brands. He would look out of the window, and say, 'It is grim day, let us have Lapsang Suchong, and . . . ' The tea ceremony was an elaborate affair with cookies and sweets. But the main attraction was the conversation.

Although the times had changed, Brouwer still kept up his practice of healthy living, including swimming, open-air 'bathing', dieting, ... The son of his friend Ru Mauve, reported that he once was present at a discussion in the garden between Brouwer and Cor Jongejan, Irmgard Gawehn, and Tine Langhout. The topic of the discussion was 'being afraid in the dark,' a theme that appealed to the lively imagination of the ladies, and on which Brouwer could easily keep the company spellbound, being gifted with a perfect memory of important and less important

[120]There are some post-1945 documents in the archive that refer to land property in Poland. No details are given.

bits of lore, and with a vivid imagination. Indeed, the ladies were moved to tears. Suddenly Brouwer disappeared into the house, a little later he reappeared stark naked, picking up the discussion where it was left. No comment was made, no suppressed smiles; the ladies were were completely wrapped up in the conversation.

The years between the end of the Grundlagenstreit and the Second World War are almost devoid of creative mathematics in Brouwer's life. A great deal of time was taken up by all kinds of non-mathematical activities, e.g. the Hungarian investment adventure and the town council in Blaricum. Among the more mathematically oriented activities, the founding and organisation of the new journal *Compositio Mathematica* was far out the most prominent. Brouwer, after his initial enthusiasm, soon withdrew from the editorial tasks that he had so conscientiously carried out for the *Mathematische Annalen*. He did handle a number of papers himself, and corresponded with the referees and authors. He refereed for example the notes of Heyting and Freudenthal on intuitionistic logic and the meaning of implication. The result was a succinct approval:

> Report on the discussion Freudenthal-Heyting. Interesting discussion on the meaning of the implication of a theorem by another, when nothing is known about the correctness of the latter.

Both papers were duly published in *Compositio*, [Heyting 1936], [Freudenthal 1936]. Unfortunately that is all Brouwer said about the discussion, it would have been of historical value to know his position in the matter. Freudenthal defended an interpretation of the implication that came close to Brouwer's original views of 1907. Heyting took the modern view, known now as the 'proof interpretation'. As Brouwer's statements in his dissertation are far from precise, and leave room for various exegeses, some comments would have been most welcome. On the whole, Brouwer seems to have sided with Heyting's interpretation, but there are no explicit statements to the effect. The undesirable effects of Brouwer's overly strict view of 1907, are more likely the result of a lack of precision, than of an ultra constructivistic position.[121]

Freudenthal handled the editorial matters so diligent and efficient, that one might wonder why he had not been made an editor. Indeed, he was promised a place in the board, but it never came to anything before the war. It should be pointed out that prestigious journals insisted on prestigious editors, and the fact that Freudenthal was already making his name in mathematics, did not compensate the fact that he was not a professor. Brouwer attached a good deal of value to these formal matters.[122] Indispensable as Freudenthal was, his rise in the academic levels in Amsterdam was by no means exceptionally smooth. He arrived as an assistant, and remained in that position until 1937. In 1931 he was admitted as a '*privaat*

[121] Cf. [van Dalen 2004], [Kuiper 2004].

[122] The fact that Heyting was an associate editor may be explained by the distribution of the specialisms. There was already ample topological expertise in the board, but Heyting was the only foundationalist.

docent',[123] but that did little to boost his income. The salary was modest, and under the pressure of the economic crisis it was even reduced. In the middle of the summer vacation of 1935 Brouwer wrote him a letter as director of the Mathematical Institute, informing him that the board of the University had decided to reduce all salaries retroactive from January 1! He sadly added: 'And this will probably not be the last reduction, now that our country is slowly economically drying up, where arts and sciences are always abandoned first to whither.'[124]

In the history of Brouwer's life there are some controversies that derived from, intentional or unintentional, attempts to cheat or belittle him. We have seen the conflicts with Lebesgue, Koebe, Menger and Hilbert. These conflicts did have a ground, although it is true, generally speaking, that offence is not only given, but must also be taken. Among all the conflicts there is one, however, born out of ordinary friction, which is harder to explain on the basis of hard facts: the conflict between Brouwer and Freudenthal. It started, innocuously enough, in 1936.

In that year Hurewicz left Amsterdam and a vacancy occurred. The assistants, lately, had been topologists, and apart from Irmgard Gawehn, of good standing. Since there was no prominent candidate with a record in the foundations of mathematics in sight, it seemed not far fetched that the successor of Hurewicz should have good antecedents in the heartland of mathematics, and one might even conjecture that a topologist of some sort would be a plausible choice.

However, Brouwer made a curious choice, he offered the position to Miss Geldof, a student of nondescript merit. Freudenthal was not informed about the matter at all, although he was by far the first interested party. Just by chance the porter-caretaker of the Mathematical Institute, Koppers, told Freudenthal that Hurewicz would have a female successor.

The choice, as I said, was rather surprising, Brouwer could have easily picked stronger candidates, certainly so if he were to consider foreign mathematicians. Indeed, Freudenthal had already drawn Brouwer's attention to Erich Rothe, a friend of his and a fellow topologist, as a possible successor of Hurewicz. One might suspect that Brouwer's partiality for female company could have played a role, but there is no indication of that sort. Freudenthal conjectured that Brouwer chose Miss Geldof just because he wanted to make his own decision. Whatever the explanation, Freudenthal was incensed, and he wrote a polite letter that could not fail to annoy Brouwer:[125]

> Dear Professor, I heard from Mr. Koppers that a 'young lady'[126] has been appointed as a successor of Hurewicz; he did not know her name. If you have given up your original plan to replace Hurewicz by Erich Rothe, then that is something I immediately accept. That I was not informed about it, that my advice was not asked in this, for me so important,

[123] Inaugural address 28.V.1931.

[124] Brouwer to Freudenthal 20.VIII.1935.

[125] Freudenthal to Brouwer 12.VIII.1936.

[126] In Dutch, a '*juffrouw*'. Today it would be insulting, it was not so in the old days.

> appointment matter, and that I only through an chance conversation with Mr. Koppers was informed—these facts have not failed to make a strong impression on me. I have till now enjoyed your trust in so many matters, that I may hope that you will find my inquiry into the motives for this treatment not indiscrete. I am sorry, that I have to write to you about those things, I would have preferred a discussion with you. I hope, however, that you will grant me a moment about these things as soon as possible; I am prepared to come to Laren for this conversation. Looking forward to your reaction, I am, Yours truly,
>
> Hans Freudenthal

This was definitely not the kind of letter Brouwer enjoyed receiving—from any body, least of all from a *privaat docent*. His answer was, predictably, testy:[127]

> Dear Freudenthal, Since the assistant appointments regard the joint professors in mathematics, I cannot enter into answering or discussing your letter of the twelfth, without consulting my colleagues.
>
> Yours,
>
> L.E.J. Brouwer

Freudenthal immediately reacted and tried to find out what had gone wrong.

This time Brouwer answered in extenso, writing that nothing had happened until the letter of the twelfth that could have changed his opinion with respect to Freudenthal, but that he considered Freudenthal's letter an unseemly attempt at pressure, based on unjustified assumptions. Here the matter stopped. Freudenthal felt himself not taken seriously, and Brouwer had suffered from, what he considered, the impertinence of a young subaltern.

This was no reason for Brouwer, however, to think less of Freudenthal as a mathematician, and so when Mannoury and De Vries retired in 1937, he duly considered means to promote Freudenthal. In the faculty meeting of March 19, Brouwer proposed to replace the chairs of Mannoury and De Vries by a lecturer's position plus the position of '*conservator*' of the mathematical institute. It may seem strange to replace two professors in this manner, but one should bear in mind that this was the time of the economic crisis, and the university tried to balance the budget by downgrading positions, when possible. Already some time ago, in 1930, Brouwer had joined the discussion in the Senate on the topic of the introducing of the new position of conservator. For laboratories and clinics a conservator was generally accepted, but a conservator for a mathematics department was a rather surprising phenomenon. The authorities, nonetheless, went along, and Brouwer proposed to make Freudenthal conservator and Heyting, who was at that time a high school teacher in Enschede, a lecturer. Against some mild opposition of the non-mathematicians, the proposal was accepted and thus Freudenthal moved up in

[127] Brouwer to Freudenthal 17.VIII.1936.

Brouwer entering his mathematical institute (1937). (Brouwer Archive).

the hierarchy, doing exactly the same things he did before. His teaching duty (as a *privaat docent*) consisted of Analysis, Theory of Groups and Topology.

Brouwer had spoken about the matter to Freudenthal in his warm, charming manner, 'I would have made you with pleasure a lecturer too, but there are difficulties. We will first make Heyting a lecturer and then we shall see what we can do for you. But you will not suffer financially.'[128] Freudenthal, however, observed at the same time that Brouwer was taking over a few lectures from the former teaching duties of De Vries and Mannoury, for which he got some extra pay—exactly the difference between a lecturer's and a conservator's salary. 'I never talked about it, but he knew that I knew. From that moment on he had a bad conscience, and the problems between Brouwer and me began. More from his side than from mine, for I considered him a great man.'[129] Indeed, Brouwer had taken over the lectures in mechanics. The correspondence of the Curators of the University of the University shows that the assignment was extended in 1943, and that Brouwer touched 1125 guilders a year for it.

Suddenly, in November 1937, tempers flared up again. Brouwer, as the director of the institute, had decided that 'at the request of the Van der Waals Committee' there would be no classes on November 25 and 26,[130] and he had informed the staff in writing. In the words of Brouwer: 'Thursday, December 2, Freudenthal, after my class is over, shows me that letter with the words that I 'have no right' to issue such announcements ..., stating that he finds this note most unfriendly.' Freudenthal probably realised that he did not have a very strong case and wrote a soothing letter, but the harm was done, Brouwer was extremely sensitive to these

[128] Freudenthal, oral communication.

[129] Ibid.

[130] To commemorate the centenary of Van der Waals' birth.

little incidents that threatened his peace of mind, and questioned his authority; he did not forget them.

The above skirmishes were, objectively speaking, of no consequence. If anything, they showed that the personalities of Brouwer and Freudenthal had too much in common. Freudenthal's aggressive ambition did not differ much from that of Brouwer 30 years ago. The relation Brouwer–Freudenthal was in no way comparable to the relation Brouwer–Heyting. Freudenthal was a fair approximation to a universal mathematician, imaginative, geometrically minded, with literary, philosophical and historical talents to boot, and above all, without any hesitation to speak out; feared by his opponents, but easy company for his friends and students. Heyting, on the other hand, was certainly gifted, but lacked that exuberance that made Freudenthal stand out. He was cautious, a miracle of precision and formality. He would not expose himself to Brouwer's wrath during the latter's academic years, and so Brouwer came to see in this quiet introverted young intellectual a useful and docile follower, who could be promoted in due time.

Shortly before the war Brouwer and Freudenthal clashed again, this time there was a scientific issue. On Saturday April 24, 1939 Brouwer had presented a talk at the monthly meeting of the Dutch Mathematical Society, his topic was, quite unexpectedly a topological one: 'On the triangulation problem.' This problem had a long history, and can be described roughly as follows: given a surface of some kind, is it possible to subdivide it into triangles? The formulation for higher dimensional manifolds is similar.[131] The problem was one of the many fascinating challenges to topologists, and it is likely that at some time in the past, Brouwer had investigated the problem. However, already in the late twenties Brouwer had expressed his loss of interest in the glamour of impressive publication lists. So he carried out his research in a somewhat eclectic manner; when his curiosity was roused by a nice problem, he sat down and applied his still powerful mind to it. Although he had exchanged his topological research for the dream of an intuitionistic mathematics, it was generally recognized that he had not lost his touch in topological matters—be it that he persistently clung to his own brand of 'geometrical' topology. Rumours had it that his drawers contained many an interesting (possibly fragmentary) topological gem. The results on the triangulation problem may well have been in one of those drawers, pulled out when a topic for a talk was required. At that particular meeting Brouwer presented a solution for the triangulation of differentiable manifolds, without the intention to pursue the matter further, let alone to publish it. The lecture probably was not quite appreciated by the audience, not only because the subject had its own intrinsic difficulties, but also because Brouwer had long ago shed his classical feathers, and presented mathematics whenever possible in an intuitionistic framework. The result was a presentation that was hard to follow by traditional topologists.

If Freudenthal had not been present, that would probably have been the end of it, but he was, and he started to turn over the problem in his mind. Brouwer's idea

[131] For a survey of triangulation see [Kuiper 1979], also reproduced in [James 1999].

was new to him, but no matter how he tried—he could not use it. In his own words, 'I never understood it.'[132] But his reflections bore fruit; in due time Freudenthal came up with his own solution, which he wanted to publish—of course giving proper credit to Brouwer.[133] At this point Brouwer started to raise objections, the trauma of earlier priority conflicts was so deep that he could not extricate himself from the awful dilemma—either Freudenthal could publish his paper, but then the reader had to take Freudenthal's word for the existence of Brouwer's proof—or he could ask Freudenthal to hold back his paper for a while, so that he could publish first, but then he would have to do some work right away, for which he claimed to have no time.

The matter became urgent when Freudenthal had finished the first draft of his proof, and offered to send Brouwer a copy. Brouwer's reaction was to play for time, 'I think that after all it is more correct, that I do not take notice of your proof, until I have sent you the proof that I lectured about at the time. Be so good as to keep your proof for a short while longer.'[134] In spite of the moderate tone, Brouwer had already started to see the triangulation matter as a real priority conflict. He desperately looked for a way out of the impasse, when it occurred to him that Rosenthal was a suitable referee; he was sufficiently familiar with the problem and the techniques involved, and he was a friend of both parties. It is not fair to speak of contestants here, for Freudenthal did not wish to lay down any claim to priority, but he was more or less forced into the role of contestant by Brouwer's overprotective attitude. Rosenthal came to Holland (on account of the political situation he was planning to leave Germany anyway, so Holland served as a halfway house), and on 4 September Brouwer and Rosenthal met. Rosenthal reported that Freudenthal's paper had been finished at the end of August–beginning of September. On September 14 both met again and Brouwer launched his proposal that both parties should hand over their manuscripts at the same time to Rosenthal, who then would pass them on to the other party. On October 4 Brouwer was ill, he asked Rosenthal to change the procedure so that Freudenthal would hand over his manuscript in a closed envelope, addressed to Brouwer, whereupon Rosenthal would visit Brouwer and exchange the envelope against his own proof.[135] He wrote on the same day to Freudenthal that he was much pleased that a recent letter from Freudenthal had opened the road to a repair of the mutual relations, damaged by a previous letter of Freudenthal. The letter then went on to outline a procedure for exchanging manuscripts, and to justify his somewhat extreme caution.

> Already the national honour forbade the acceptation of your letter of 1 September. For such an acceptation would create in the Netherlands a situation, where a chairman of the Mathematical Society, who, in an address at the General Meeting of this Society, puts on record a scientific

[132] Oral communication Freudenthal.

[133] Freudenthal slightly extended the results by showing that C^q-manifolds allowed C^q-triangulations.

[134] Brouwer to Freudenthal 8.VII.1939.

[135] Brouwer to Rosenthal 4.X.1939.

> result with an exposition in extenso of a proof, which is then not immediately published, would be obliged as a protection against theft of priority by his audience, either to record at the meeting his address by means of a dictaphone, or to depose the sealed manuscript, used at his exposition at the meeting, forthwith at a notary or at a qualified body, under penalty of having to protect later his priority at the high cost of a chemical analysis in order to determine the age of his manuscript.[136]

One sees to what length Brouwer was willing to go to draw the lessons of his painful clash with Menger. Freudenthal, who knew the ins and outs of the Menger affair, must have recognized the symptoms of that conflict, and he could not possibly be amused by such a comparison.

Furthermore Brouwer pointed out the inconsistency in Freudenthal's intentions—in July the mailing of the manuscript to Brouwer was seen as a private matter between scholars, whereas on the first of September the manuscript was prepared for publication. To this he added an elaboration of his letter of July 8:

> I have immediately underlined that a written communication of these considerations to third parties can only be allowed posterior to a publication of my lecture, and together with a reference to such a publication or with an account of the contents of this lecture.

Freudenthal replied that he was quite willing to cooperate in the 'Rosenthal-procedure', but that he deplored the fact that in their mutual relations trust had been replaced by legal formulations. He further protested against passages in which implicit accusations had crept; indeed, certain arguments of Brouwer had a general ring, but could only be construed as referring to Freudenthal. Finally, he disagreed with the general tenor of the passage on the protection of non-written communications.

'The consequence of your view would be that one could block whole areas of research by giving out oral communications about one's research, without publishing them, and thus preventing others from publishing their researches.' Nonetheless he was quite willing to go along in this special case.

In most exchanges of the sort described above, there usually is an argument behind an arguments, behind an argument, ... In this case, one may guess that old wounds had started to burn at the prospect of another priority conflict. There was, moreover, an extra ground for Brouwer's feelings of discomfort not mentioned in the discussion so far, but cropping up in Brouwer's next letter.[137]

> It is not the case that my objections in this matter came up only on 1 September at the receipt of your letter, in which I missed every similarity with the intention you announced on July 8, but already with your remarks on July 8, which I could not bring in connection with the reply which I gave you in March or April to your statement that—as you con-

[136] Brouwer to Freudenthal 4.X.1939.

[137] Brouwer to Freudenthal 25.X.1939.

> cluded from a conversation with a foreign colleague, mentioned by you—the topic of my talk in April 1937 was seen as extremely important in another country, and which therefore had begun to occupy intensively your thoughts. Wondering about this foreign appraisal, I answered at the time, that I could only interpret your information as an exhortation directed to me to publish; but that I could hardly make the fragmentary results of the lecture public without their natural completion, and that it would, alas, take many more months before I could find any time at all to accomplish this completion. I had to conclude after our discussion of 8 July that you felt, after this answer of someone, to whom you have after all been officially assigned as an assistant (*medewerker*), and whose handicapped operational potential was known to you, like undertaking yourself the completion of the results of the lecture of your superior, which were put by him on *his own* work schema. And this was the situation, which I had to observe after our conversation of July 8. And this (as I am prepared to believe, unjustifiably, but in any case understandable) brought gradually in me such memories of experiences with earlier gifted assistants out in me, that I lost any ground to me to make myself familiar with your promised written communications, and also, subsequently, to send you the manuscript of April 1937 ....

Clearly Brouwer was thinking of the horrors of the Menger episode; he had no wish to go through such an ordeal again. This may explain why he in a sense overreacted to Freudenthal's interference in the privacy of his mathematics. What he chose to ignore was that after a public lecture the topic had become part of the public domain, so that one could not put up a sign 'Trespassers Keep Out'. We must add that the personalities of Menger and Freudenthal were not in the least similar, so that Brouwer's fears were largely imaginary.

Freudenthal was not altogether satisfied with Brouwer's account of the matter, he replied:[138]

> My reflections on the triangulation problem date back to October 1938. I actually arrived at the triangulation when I was preparing the theory of the volume of curved surfaces for my lectures and looked for a simple proof for the transformation theorem of multiple integrals. That is when I have found my proof, and reconstructed *your* proof, of which not all details had become clear to me during the lecture—and as I have observed in the meantime—reconstructed it faithfully, whereby the two activities, the original and the reproductive influenced each other to the extent that I would not be able to separate them anymore.
>
> I have not told you anything about my research at that time: but I have in chance conversations told some mathematicians about the existence of your proof, among them as you know, Lefschetz, at the end of February 1939. According to Lefschetz, Whitney has also looked into the problem,

[138] Freudenthal to Brouwer 27.X.1939.

> but he had not yet succeeded. Hearing this I thought it almost unavoidable that Whitney , who works at the borderline of topology and analysis, and the author of some twenty papers, and the outstanding expert of my generation, would publish in a couple of years or months, such a triangulation proof. I therefore decided to talk to you about this matter; the conversation took place—I think—half a week after my visit to Lefschetz. I cannot understand how you could have got the idea that I could have started and finished my study of the triangulation problem in 3-4 days. Lefschetz' and Whitney's interest in the problem was the cause, not of my own research, but of my attempt to get you to publish yours.

Basically, here the triangulation conflict ended; there was some more correspondence, mainly about corrections and formulations, but the dispute seemed, to all appearances, closed.

The story of the Brouwer–Freudenthal triangulation theorem got an unexpected twist, when Freudenthal received a bundle of reprints from the American topologist Cairns; these showed that he had already solved the problem in 1934/35. Freudenthal duly informed Brouwer,[139] who studied the papers of Cairns and wrote back that he felt a 'residue of suspicion' 'whether his proofs could pass the test of a precise, detailed critique.'[140]

Brouwer's paper appeared in the *Indagationes Mathematicae* of 1939 with an apologetic footnote, mentioning the reason for its publication, ending with the sigh that the circumstances had made him publish this paper which he otherwise would have considered premature.

The position of both parties can be understood. Whereas the younger party was on his way up and moved in circles where things were changing fast, the elder mathematician had no urgent drive to publish, moreover, with his status he could not risks a fiasco—either through a mistake or by publishing below his level. There was moreover Brouwer's conviction that the only good result is an intuitionistic result, so he cast the paper in his intuitionistic terminology. This by itself must have robbed him of a possible audience. Curiously enough, the paper is republished in the collected works in the intuitionistic part, whereas its main interest was of a topological nature. Even more curious: Freudenthal's paper, [Freudenthal 1939], is missing in the references.

Life in the thirties did not exclusively consist of conflicts and scientific or other business, Brouwer always found time to receive and visit friends. Among his friends, Peter van Anrooy took a special place. Van Anrooy was the conductor of the Residentie Orchestra in The Hague, and Brouwer from time to time visited the Van Anrooys in their home in Scheveningen. He used to stay with them during his usual examination spree if he was assigned to schools around The Hague. His behaviour was unconventional to say the least, entering the living room, he would often make straight for the couch, lie down and conduct the conversation in the horizontal

[139] Freudenthal to Brouwer 18.III.1940; Cairns to Freudenthal 14.I.1940; [Cairns 1935].

[140] Brouwer to Freudenthal 30.IV.1940.

position. Nor would he shrink from admonishing Mrs. van Anrooy about her cooking.[141] Van Anrooy was not overly impressed by Brouwer's Beethoven renditions on the piano, he used to refer to it as 'hammering'. As a young girl, Van Anrooy's daughter Fien occasionally stayed with Brouwer in Blaricum.

At one particular occasion Brouwer publicly came to Van Anrooy's support; in 1937 the wedding of Princess Juliana, the daughter of Queen Wilhelmina, to Prince Bernhard von Lippe Biesterfeld took place in The Hague. The marriage was received with great enthusiasm in The Netherlands, all kinds of festivities were organised. There was, however, one detail that displeased a good many Dutchmen; since Prince Bernhard was a German subject, representatives of the German State had to be invited, and the usual honours had to be observed. This meant that at the wedding not only the Dutch national anthem, but also the *Horst Wessel* song, the official song of the Nazi party, that had been elevated to the rank of German national anthem, had to be played. It fell to the *Residentie Orchestra* to play the national anthem and the *Horst Wessel* song; its conductor, Van Anrooy, however, and twenty five members of the orchestra bluntly refused and left. The Dutch authorities, who had to reckon with the whims of its neighbour on the eastern border, were not pleased at all—the German Government of the day was usually quick to construe an 'insult of a friendly head of state' out of such events; Van Anrooy's step found fortunately a good measure of popular support. Brouwer also came to the defence of his friend, together with 38 other 'prominent intellectuals' he sent a telegram in support of Van Anrooy.[142]

Should one interpret Brouwer's support of Van Anrooy as a political signal? That is indeed hard to say, it seems likely that an injustice committed to his friend counted more than the political gesture. One cannot see this kind of action isolated from the mood of the times; the opinions on national socialism and its German form were the subject of heated discussions. There was a Dutch national socialist party, the *Nationaal Socialistische Beweging*, *NSB*, which was actually a rather heterogeneous collection of all kinds of political convictions, mixing socialist ideas with right wing nationalist doctrines. The rich fabric of political movements in Holland generated a certain atmosphere of hostility, which induced the more cautious elements to avoid statements or actions that could invite repercussions. Brouwer was not inclined to hide his opinion, as we have seen. That the luxury of a political opinion could have consequences, is illustrated by one of Brouwer's colleagues, Bonger, a one-time fellow student of Brouwer and then the professor in sociology. Bongers was a fervent anti-Nazi; his activities were carefully watched from Germany, and radio Bremen openly threatened him with reprisals.[143]

1937 brought also the 25th anniversary of his professorial appointment; Brouwer was flooded with letters and well-wishes. He had become a well-known person in his own country, revered by some and feared by others. His former

[141] Oral communication Mrs.J.F. Heyting-van Anrooy, the later wife of Arend Heyting.

[142] Published in *De Tribune*, a communist journal, 16.I.1937. See also [Fasseur 2001], p. 144 ff.

[143] Bonger took his life when the Germans invaded Holland.

Hendrik de Vries leaves the mathematical institute after the farewell party, 1937. (Brouwer Archive).

students, generally, worshipped him, he was an easy talker when he felt inclined so, and he did not easily forget the name and face of a student. Somewhat surprisingly for a man who started his career with a definite lack of care for conventions, he had acquired the respectability and admiration that go with the conscientious fulfilling of one's duties.

In the same year two of his old friends, Hendrik de Vries and Gerrit Mannoury had reached the retirement age. Their valedictory lectures were given on the same day in the *Mathematisch Instituut* in the Roeterstraat. Brouwer, as the director of the institute addressed them in name of the faculty, and Max Euwe spoke on behalf of the students. The newspaper *Het Volk* mentioned that their old teacher, Korteweg, was also present at the occasion.

The Brouwer of 1937 certainly was a far cry from the young idealist who was master in his own kingdom. The metamorphosis from rebellious moralist to respectable professor had been thorough—although the professor had retained the mystic and radical views of the student. The road from withdrawn student to international celebrity was a long and lonely one. Real friends were few, and already two of these had fallen by the wayside, both artists—Adama van Scheltema and Frederik van Eeden.

The physical and mental health of his teacher Korteweg, was slowly detoriating; he had in 1932 been awarded an honorary doctorate by his own university, but since then the grand old man of Amsterdam mathematics withdrew more and more from the professional scene. He was still able to celebrate his golden doctorate and to accept an honorary membership from the grateful Mathematics Society, at his 90th

birthday at April 30, 1938. At that meeting of the *Wiskundig Genootschap* Brouwer, gave the presidential address with the title *Intuitionistic discontinuous functions of a real variable*.

The major event in the faculty was Zeeman's death, the consequences were hotly discussed. Zeeman's chair would automatically bestow a great measure of prestige and influence on its occupant. Brouwer vigorously campaigned for Zernike, but the majority went against him and Zernike. The general opinion was that Zernike lacked the authority and discipline to be the head of the physics department.

All this time the sky over Europe was darkening, in 1939 Poland was overrun and divided between Germany and the Soviet Union; the contacts with Polish colleagues were no longer possible. The dramatic fate of Poland did not strike everybody as necessarily a bad thing—e.g. Pontryagin wrote: 'recently the number of Soviet mathematicians has been greatly increased. In Lwow the following are present: Banach, Schauder, Knaster, Mazur and a number of others. Sierpinski and Kuratowski remained in Warsaw and are now in concentration camps.'[144]

Rosenthal was fortunate enough to leave Holland in March 1940. Not everybody was as lucky. Brouwer wrote to Freudenthal that the group theorist Remak was in imminent danger to be extradited on the grounds of misbehaviour.[145] The psychologist Frijda thought that the only way to save Remak was to have him admitted into an psychiatric hospital. Remak was indeed not an easy person to get along with; he had the reputation of being difficult and rude.

Another unfortunate German in exile in Holland was Blumenthal.[146] He had been discharged on the ground of political unreliability from Aachen on 22 September 1933. Till then he had expected, to be exempted according to the rules as a First World War veteran (*Frontkämpfer*); but apparently the rules did not particularly worry the authorities. Blumenthal had in the meantime made friends with Professor Burger from Delft. In the 'old days' they exchanged comments on literature and discussed a wide variety of topics. In one letter Blumenthal told Burgers that he had procured the best-known book of Frederik van Eeden—*The little Johannes*, a moralistic literary fairy-tale parable. In spite of the treatment he had received from the Nazi government, he would not allow nasty comments on the regime. It seemed as if he could not understand the political climate in the later years of his life, as if he refused to accept the possibility of an ultimate and systematic evil. In 1934 Blumenthal visited with his daughter Burgers in Delft; the latter managed to scratch up some money to support him, but he could not provide him with a job, and so he had to remain in Germany. From the correspondence with Burgers it appears that Blumenthal was a fairly regular visitor to Holland before his final emigration from Germany.

[144] Pontryagin to Lefschetz 29.XII.1939.

[145] Brouwer to Freudenthal 2.III.1940.

[146] See [Behnke 1978].

Blumenthal had put his life in the service of Hilbert's Göttingen, where he played, even from a distance, a role as the managing editor of the *Mathematische Annalen*. Surprisingly enough, he could carry on this duty until he was forced to resign in 1938. In a letter to Bernays (17.II.1938) we read that Behnke had taken over his position at the *Mathematische Annalen*. Behnke was a member of the younger generation, without hangups about the past of the *Annalen*. He told that he was rather surprised, and embarrassed, when, in Amsterdam in 1938, after he had given a talk, Brouwer and Weitzenböck came up to him, and asked why Blumenthal was still officially active as an editor of the *Annalen*.[147] In spite of the anti-semitic policy and atmosphere, Blumenthal could still move around freely in Germany, and had visited Göttingen in January. Hilbert, he wrote, only dreams at the sofa. His loyalty to Hilbert was not a matter of calculation, but of sincere devotion; nobody was therefore better qualified to write the biographical note for Hilbert's collected works.[148] Although his children managed to get away to England, Blumenthal had difficulties to get the required visa. In 1939 he tried to get to the United States, Weyl was prepared to intervene, but to no avail. He was able to visit his children in England, but he had to return to Holland. Expecting the worst, he asked Burgers in case of a new war, to relay his letters to his children in England. Finally he moved to Utrecht where the war caught up with him. After moving from one place to another he was 'housed' for some time in a camp in the neighbourhood of Utrecht; Freudenthal visited him there. After the rounding up of Jews had started, he was moved to Westerbork,[149] where his wife died of pneumonia. Blumenthal was eventually transported to Theresienstadt, where he died on 12 November 1944. Various people had tried to do something for him, in particular Burgers, Wolff, Mannoury, and Van Dantzig. There is, however, no report of a renewed contact between Brouwer and his former friend and later adversary. Under the circumstances one would have thought that the agonies of the present obliterate the bitterness of the past. But generally speaking, the Dutch were so much used to their traditional neutrality, that they almost considered it a natural right to be left alone in times of war; thus they were inclined to consider the exiles from Germany as reasonably safe. Coupled to the earnest promises of its German neighbour to respect the Dutch borders, the Dutch could not easily share the fears of the German immigrants—Jewish or not.

We cannot end the account of Brouwer's pre-war actions without mentioning an event that was already at the time considered a grave faux pas. In 1939 Brouwer published a short note in *Forschungen und Fortschritte* for the 70th birthday of Theodor Vahlen. The fact that Vahlen had joined the national socialist movement with heart and soul had not endeared him to his colleagues. For most German mathematicians his name was anathema. One had to accept the reality of his rise

[147] Behnke to Van Dalen 27.XI.1976.

[148] [Blumenthal 1935].

[149] A transit camp from where Jews were sent to the camps in Germany.

to a high function in the Third Reich,[150] but one would avoid contact with him whenever possible. The only correspondence between Vahlen and Brouwer in the Brouwer Archive concerns financial problems surrounding the call to Göttingen, There is no evidence that the two ever met, other than possibly at some meetings of the DMV in the old days. Whatever could have induced Brouwer to write such a dedication is unclear. Freudenthal conjectured that Brouwer protected Vahlen (as a mathematician) to annoy the Göttingen crowd. It is a fact that Dehn had written an extremely negative review of Vahlen's book Abstract Geometry,[151] and it is said that Vahlen never forgot the humiliation. Brouwer on the other hand, recommended Vahlen's book in the university guide right until the end of his career—and his successor Heyting kept the Vahlen book on the reading list. With the Vahlen dedication, Brouwer had succeeded at one stroke to astonish both the pro- and anti-Nazi groups in the German mathematical community.

The small note opens with a philosophical introduction, which concentrates on the dichotomy 'subjective–objective', or 'man–outer world'.

> ...we must go back to the fact, that man directs the operation of his impulses of will by means of subjective thought, his subjective with objective thought and his objective with mathematical thought, and that these impulses of will are activated by the believe in the interaction of thinking and acting man with an outer world (*Anschauungswelt*), which contains himself, his thoughts and his actions, including its laws, and which exists independent from thought.
>
> The thinking that incorporates the thinking subject into the outer world now necessarily leads to objective self-contemplation. This self-contemplation leads first to the insight that the laws of the outer world are products of the objective thought, and this amazing insight generates for the time being a paralysing doubt, first of the the existence at all of an outer world, independent of thinking, next of the seriousness of the private impulses of will that presuppose the outer world. In this situation thinking can only be practiced as a game and a sport, and indeed with thought as its only object.

Brouwer then went on to point a way out of these problems by using the notion of responsibility. The most notable fact about this brief discussion of Brouwer is that he does not just brush away the outer world, but looks at its inherent problems.

After describing the successive phases in the mathematician's development, Brouwer continued 'The above mentioned norms have become especially apparent for the present generation of mathematicians, in the figure of Felix Klein, which today radiates undiminished from the past; they will again become visible in the figure of Theodor Vahlen.' This piece of traditional flattery is all the more remarkable

[150] Cf. p.697, see also [Siegmund-Schultze 1984].

[151] In Brouwer's words '*sein tiefsinniges und suggestives, viel zu wenig gewürdigtes Werk über geometrische Grundlagenfragen; "Abstrakte Geometrie*" '

as Vahlen cut a rather poor figure when compared to Klein, not even considering the political aspects.

The private world of Brouwer was drastically changed some years before the war. Brouwer had managed to get Cor Jongejan a position as an '*adjunct-assistant*' (the lowest rank of assistant at the university) in 1925. She was 32 years old and still a highly attractive young woman, witty and with a lively spirit. Living in the Brouwer household did not keep her from male company, there was no lack of admirers. Even Paul Alexandrov appreciated her charms, to the extent that Brouwer showed covert signs of jealousy (cf. p. 522). For Cor life at Brouwer's had the fascination of contact with this wonderful spiritual man and the added attraction of all those casual contacts with a rich variety of visitors, but it had become clear to her that the relationship with Brouwer stood in the way of the fulfillment of her own life and talents; her place was just that of a member of the court of a great man.

Although the reports picture her as an unfettered, easy going person, she worked hard enough for Brouwer. As an inofficial secretary and an official assistent at the Mathematical Institute, she had quite enough to do with all the copying (often by hand, later mostly in type writing) of manuscripts—not only Brouwer's, but at the time of his editorial activities for the *Mathematische Annalen* also of submitting authors—and the correspondence in the many affairs Brouwer was involved in. It was not unusual for Brouwer to request five or more copies of his own letters, but he also had incoming correspondence retyped by Cor.

Brouwer's place was a regular meeting point for a number of female admirers, and a good deal of intrigue was going on among the ladies. Apart from Cor the main claimant for Brouwer's attention was Tine Langhout-Vermeij, an intelligent lady of good taste, who earned a living as a free-lance interior decorator. Tine quite often accompanied Brouwer on his travels in Holland and abroad. At the time she met Brouwer she was married to Willem Langhout, a man of no particular merit, if one does not count his financially independent position. Willem cultivated the art of philandering to the extent that Tine divorced him. She made a successful effort to become a member of Brouwer's entourage, and fought occasional fierce battles with Cor Jongejan. Cauliflowers, it was said, went crashing through the windows. Tine was a person of good taste, she in due time became a well-respected interior designer.

None of this bothered Lize Brouwer, she was well aware of the fact that Bertus loved to bask in the sun of (in particular female) admiration, but she was equally convinced that none of these romantic episodes or flirtations would come to anything serious. When all was said and done, Lize was in actual control of the household, her invisible hand arranged the daily affairs by and large. She often used to stay in Amsterdam, taking care of the pharmacy, her main problem being her awkward position between her husband and her child. There was an unmistakable aversion on Brouwer's part to Louise, and, not surprisingly, the feeling was mutual. Lize was in the position of the unfortunate ferryman, who had to carry a goat and

Brouwer and Tine Langhout-Vermey. (Brouwer Archive)

a cabbage, she always had to arrange things, so that Louise could come home when Brouwer was absent. This, indeed, was her real cross; she had to choose between husband and child, but whichever choice she made, she would feel guilty.

In this hectic environment, where her young life was simply running away, Cor suddenly reached the decision to leave and lead her own life. After living with Brouwer from 1915 on, she left him to live in Zandvoort, a fashionable seaside resort. Zandvoort could not satisfy her for long, so soon she moved to Amsterdam, where at the instigation of a friend[152] she rented a big apartment at the Stadhouderskade 136 and took in a number of girls. One of the girls was Janny van Wering, a music student who later became one of the leading cembalo-players in Holland.[153] The household at the Stadhouderskade was run by Cor, who could finally display her social and organizational talents; the girls felt happy and were quite attached to Cor. Of course, the change of address did not mean an complete separation from Brouwer, for she still was an assistant at the Mathematical Institute where Brouwer was her boss, but at least she had regained a measure of independence.

The house at Stadhouderskade eventually (and inevitably) attracted a fair number of male visitors, among them was Brouwer's brother Aldert, the geologist, who

[152]Wim Bierens de Haan.

[153]Most of the information on this episode was provided by Mrs. Van Wering.

at that time was at odds with Bertus. Aldert had a keen eye for female attractions, and he did not waste time to get familiar with the renowned Cor. Eventually he hardly missed an evening meal in the cosy circle of Cor and her friends.

It took quite a while, but eventually Bertus Brouwer also started to frequent the house at the Stadhouderskade. This lasted until well in the war. At one occasion Brouwer and Max Euwe (the chess master, one time world champion.) were in the middle of a mathematical discussion when the air-raid-sirens sounded; they both moved out of the room, sat themselves on the stairs and continued their discussion as if nothing had happened.

At one occasion Brouwer dropped in with Tine Vermeij, and Gerda Holdert; when Cor went to the kitchen, Brouwer stretched himself on the couch and said: 'I want to be caressed.' Gerda who was at that moment not in the mood for any of the Brouwerian ceremonies curtly said—'Tine, why don't you do it, the way I do it is not good enough anyway.' Janny van Wering thought the exchange so silly that she burst into laughter, so that Gerda could not suppress a curt 'I would like to know why that little spook is laughing?'

Eventually Brouwer got a small room in the house, where he could have his siesta, or stay the night if he had late business in Amsterdam (e.g. senate meetings or concerts).

In the end Cor moved back to Blaricum. Janny van Wering spent one summer during the wartime in the Padox—one of the little cottages at Brouwer's property, she lived there with her cembalo and gave some house concerts for the Brouwers and the guests. She remembered one sunny afternoon in Blaricum that Cor Jongejan came on her bike home in Blaricum with a box of cakes to celebrate her assistant-pharmacist diploma. The pharmacy was indeed always in Brouwer's mind, and so he had with some foresight suggested that Cor should make herself familiar with the running of the pharmacy and get herself a diploma.

Among the extra-academic activities of Brouwer there is also one that had a remote relation with his university teaching; he, like a number of his colleagues, was involved in the organization and supervision of secondary education. The contacts between secondary and higher education was more or less automatically ensured by the existing regulations of the final examinations of high-schools and gymnasia. Brouwer always faithfully took part in the examination tours, which could take him to any town in The Netherlands. He and many of his colleagues, considered this one of the self-evident duties of the academic world. The fact that there was a modest fee connected with the function of examiner might have had some influence in a time when the economical crisis was still acutely felt. On the whole, Brouwer took his responsibilities for secondary education seriously, even to the extent that in 1938 he became a member of the board of trustees (a *curator*) of the Gymnasium at Hilversum,[154] which was a municipal institution in the sense that the town of

[154]Mayor of Hilversum to Brouwer 15.VI.1938.

Hilversum was the responsible authority (subject to general rules of the national government).

Brouwer's appointment was renewed from year to year, also during the war years. A gymnasium was an institution with a considerable status, as a rule teachers had a university degree and quite a number had obtained a doctorate at one of the universities. The gymnasium teachers (and to a lesser degree those of the HBS) formed a pool of future professors; the rector was a man of consequence and the trustees were carefully selected from the upper strata of society. In this circle Brouwer felt perfectly at home, he always enjoyed the company of the literary minded, in particular the classicists. Even in this select board, Brouwer could not avoid conflicts. A minor one took place in his second year; the board of trustees had made a short list of candidates for the vacancy of a mathematics teacher, but after some consideration the council of Hilversum changed the order switched the numbers 2 and 3. Number 2 on the list was a young physicist, Dr. E.M. Bruins. Both the rector and the conrector were of the opinion that Bruins lacked the experience to qualify for a full-time appointment at the gymnasium. This was seen by the trustees as an infringement of their rights, and they protested.

Brouwer, who was in a better position than most to judge the mathematical merits of the candidates drafted an answer,[155] saying that 'the trustees recognise in Bruins such an excellent personality of special qualities of talent and character, who can play a role in the cultural life of the town, and that they feel justified to do so, as the trustee Prof. Brouwer in the first place had admired, during the years of study at the University of Amsterdam, Dr. Bruins' versatile way of thinking and his often unselfish devotion, in the second place he has seen him excel subsequently, not only as an independent scientific researcher, but also in charge of courses of the *Volksuniversiteit*[156] and as a tutor.'[157]

One might wonder why such an excellent mathematician chose to apply for a job at a gymnasium, instead of following an academic career. The answer is simple, there were hardly any tenured jobs below the rank of professor at the universities and in view of the magnitude of unemployment a position at a gymnasium was definitely to be preferred to that of ticket collector or to the fate of the unemployed. We will meet Dr. Bruins again in our history.

[155] Trustees to council 13.VII.1939.

[156] A semi-official institution providing courses in various areas

[157] repetitor.

17

WAR AND OCCUPATION

17.1 Occupied Holland

War came to the Netherlands on a beautiful clear day in May 1940; on the tenth the weather was exactly as any supreme command could wish for an all-out attack. The German forces crossed the borders, and German paratroops descended on vital points in the west of the country. The country was not totally unprepared; the army, consisting mainly of conscripts, was mobilized in 1939, measures had been taken to protect the civilians—bomb shelters were erected in the parks and the public gardens of the cities, black-outs were ordered, air-raid wardens were appointed, the proverbially impregnable waterline was activated by means of its system of inundations, . . . But still, hardly anybody had believed that the Germans would violate the neutrality of this harmless little country.

It took the Germans only five days to break the resistance, and after the atrocious and large scale bombing of Rotterdam, the army capitulated and the government including the Royal family fled to England.

The legal situation after the capitulation was confused, the Dutch government in exile, residing in London, considered itself the real government, but the German occupation authorities were the de facto governing party. The Dutch democratic system comprised quite a number of parties; the main parties being the Socialist Party, the Liberal Party, the Catholic and two Protestant Parties, followed by a number of smaller parties, including the Communists and the National Socialists. The latter, the National Socialist Movement (NSB) had tried unsuccessfully to imitate their German-Italian models, but remained small and rather marginal. The NSB welcomed the German invasion, and hoped to play a major role in the new constellation. This is not the place to present a history of the Dutch under German occupation, but it may suffice to say that the overwhelming majority of the Dutch resented the brutal invasion and instinctively distrusted every move and statement of the new authorities. There was also a group that, stunned by the military superiority of the German armies, took a 'realistic' view of the new situation in Europe, and were prepared to adept itself to a future under German supremacy and there were those who actively gave aid and comfort to the enemy, to the extent that the Germans succeeded in attracting volunteers for civil, paramilitary and military organisations. There was a great demand for labour force in Germany, there was a Dutch division in the SS, there were places to be occupied in the civil service, the police, the secret police, etc. The Dutch being neither better nor worse than the average nation, produced both men of principle, who refused to bow for the

usurpers, and 'collaborators' or downright Nazis, some out of political motivation and some for opportunist reasons.

After a brief period of military authority, the Netherlands were put in charge of a *Rijkscommissaris* (State commissioner) Seyss-Inquart. This new head of state was a seasoned national socialist of Austrian descent. Seyss-Inquart's plan was to keep the Dutch civil service in working order and to use it as a tool in the nazification of the country. In particular he tried to keep the civil heads of the government departments, called *secretaris-generaal* (secretary general), in function. One of his first acts was to demand from civil servants, judges and teachers a declaration of loyalty, to the effect that they would obey the instructions of the State Commissioner and his staff, and abstain from anti-German activities.

Since any promising strategy for nazification had to build on the education of the nation's youth, Seyss-Inquart almost immediately set out to put a national socialistically minded person in charge of the department of education. The most suitable man for the job was an Amsterdam professor in German philology, J. van Dam. Already before the war Van Dam had become attracted to the national socialist principles, so when he was offered the influential position of secretary general of the department of *Education, Science and Culture Protection* (the new name of the department) he had soon made up his mind. Van Dam was basically a harmless man, but with a strong dose of ambition and vanity. As soon as he was appointed, he was of course the object of intense contempt. During and after the occupation the two adjectives 'goed' and 'fout' – 'good' and 'bad', acquired a political meaning. 'Good' was patriottic and anti-Nazi and 'bad' was pro-Nazi. These labels persisted until long after the war. Van Dam definitely was bad. The new NSB mayor of Amsterdam, Voûte, also belonged to that class. As president curator of the University of Amsterdam he plays a role in the wartime history of the university.

When the Germans took over the civil government, they did not immediately set about revising the existing traditions and rights; the subversion of the Netherlands was a gradual, sometimes imperceptible, sometimes brutal process.

A few examples may suffice: the Germans introduced the summertime–wintertime system (now universally accepted in Europe); in the country this was seen as a pernicious German trick, people doggedly stuck to the pre-war time and referred to 'the new time' as something evil, and to 'the old time' as the right thing. Another, and more serious example is the system of social care that the Germans introduced after the pattern of their own '*Winterhilfe*' (winter care). This Dutch system, *Winterhulp Nederland*, was persistently and with great tenacity boycotted by the Dutch. The average Dutchman distrusted all regulations introduced by the enemy. One did not give money to organisations with National Socialist leanings.

A major visible effect of the occupation was the ever intensifying persecution of the Jews, for whom Holland (in particular Amsterdam) had, ever since it became a free republic, been a haven.

At the university the consequences of the war were at first rather modest, and life went on much as usual. Freudenthal, Alexandrov and Hopf had started in 1939

the preparations for a *Festschrift* for Brouwer's sixtieth birthday; even after the occupation of Holland, correspondence with the Soviet Union and Switzerland was carried on as usual. Hopf had reacted positively to Freudenthal's suggestion to honour Brouwer in this way, adding 'Moreover, Brouwer's work is not sufficiently valued; it would therefore be doubly advisable to demonstrate the contemporaries how much he is appreciated.'[1] The plan to dedicate a volume of *Compositio Mathematica* to Brouwer was frustrated by the war. In the end the initiators advised the prospective authors to submit their paper for the Brouwer *Festschrift* to a journal of their choice.

The sixtieth birthday itself was a low key affair, as to be expected in occupied Holland. The newspaper *Het Handelsblad* mentioned the scholar in its columns, and that was it.

Freudenthal complained in June that it had become very difficult, not to say impossible to reach the editors of *Compositio*,[2] and he asked Brouwer what to do. Should one appoint editors that could easily be reached by mail? The publication of the next issue had become problematic. Brouwer replied that the first issue of volume 8 could be published, but that in view of the difficulties it would be better not to start any new typesetting.[3] A month later Wijdenes told Freudenthal that Brouwer and he had decided to stop the publication of *Compositio Mathematica* for the time being,[4] and a couple of weeks later Brouwer wrote that no permanent closing down of *Compositio* was intended.[5] In the beginning of September censorship of newspapers and journals was introduced with respect to information with military significance, including a large number of civil topics, e.g. the building of roads, bridges. Even *Compositio Mathematica* received the instructions of the Military commander in the Netherlands.[6] In view of all the problems and uncertainties, Brouwer, after some deliberation, decided to end all activities of *Compositio Mathematica*.[7] Obviously, authors of already submitted and refereed papers should be completely free to resubmit their papers elsewhere. In view of the fact that the first issue of volume 8 had not yet appeared, five months after the announced date of appearance, he also decided that that issue should be cancelled altogether.[8]

17.2 Weitzenböck's choice

In the first year of the occupation an event took place in Blaricum that so far has not been explained. This curious incident has baffled experts and the villagers alike. We recall that the Austrian mathematician Roland Weitzenböck joined the Amsterdam

[1] Hopf to Freudenthal 21.XII.1939.

[2] Freudenthal to Brouwer 15.VI.1940.

[3] Brouwer to Freudenthal 26.VI.1940.

[4] Wijdenes to Freudenthal 27.VII.1940.

[5] Brouwer to Freudenthal 9.VIII.1940.

[6] *Wehrmachtbefehlshaber in den Niederlanden. Militarische Zensurstelle*. 9.IX.1940.

[7] Freudenthal to Hopf 10.XI.1940.

[8] Brouwer to Freudenthal 17.X.1940.

mathematicians in 1921, after Hermann Weyl had turned down Brouwer's offer. Weitzenböck had settled in Blaricum, where he become a fully accepted member of the community. He was a man of few words, without observable political views. Appearances are often, however, deceptive, and in this case the solid imperturbable exterior hid a considerable amount of frustration resulting from the disastrous course of the First World War. As so many German and Austrian ex-service men, Weitzenböck became a hard-core revanchist, and an implacable enemy of France. But whereas Brouwer actively campaigned for the rehabilitation of German scientists, Weitzenböck refrained from political activity. However, after the '*Anschluss*' of Austria in 1938, he started to vent his approval of Hitler's policies in private conversations.

Walter Ledderman recalled that in 1938 in St. Andrews he ran into Turnbull and Weitzenböck walking in the street. 'Turnbull tried to introduce me to Weitzenböck, who, however, refused to shake hands with me.' Shortly before the war Weitzenböck wrote to Turnbull that Britain would certainly be defeated in this war, and that the British empire would be destroyed. 'But he would like to express his condolence to Dan [Turnbull], whom he liked since his student days in Amsterdam.'[9]

Weitzenböck's son Willy had joined the Dutch air force and was trained as a pilot at the airbase Soesterberg, but when the war broke out on May 10, 1940, his superior and his fellow pilots did not trust him well enough, so he was transferred to a non-combatant place. Whether he felt so insulted that he had to make a gesture, cannot be said with certainty, but after the capitulation he joined the SS. Weitzenböck himself joined the NSB, as a result the whole village knew that the nice mathematics professor had become an enemy. In reaction he and his family were completely ignored, nobody greeted him, the chess club did not want to have any further contact. After the war, when Weitzenböck was arrested and was awaiting trial, Brouwer wrote a memorandum for the district attorney (*officier van justitie*); it is instructive to read Brouwer's view of the case. Here is a fragment:

> He began to acquire a reputation of 'greater Germany'-inclinations, when at the time of a possible war and political tension between the Third Reich and the Austria of Dollfuss and Schuschnig he applied for Dutch naturalization; the purpose of this application, was to prevent that his sons, who were approaching the age of military service, could be forced to take up arms as Austrians *against Germany*. This naturalization must have been invalid if Weitzenböck at the time was still bound by his oath as an Austrian reserve officer. This probably has been the case, as during the occupation his naturalization was annulled.
>
> Only after the annexation of Austria in 1938, Weitzenböck's 'greater Germany' disposition became undeniably clear when friends and colleagues offered their condolences and he, while rejecting the condolences,

[9] Ledderman to Dyckhoff 39.I.2004.

> declared himself openly a supporter of the German empire. He has, since that time, in conversations, always frankly given expression to this disposition in such a manner that almost all his earlier relations stopped seeing him, or at least restricted their contact to the necessary professional contact. At the same time Germans and members of the NSB naturally sought his company; but he never became an active party member. On the contrary, he withdrew more and more into himself, and made, even more than before, all his working energy available for his scientific researches.

We may assume that Brouwer's view was correct and it is largely borne out by the available evidence.

In the early morning of 6 October 1940 the house of Weitzenböck was hit by a bomb, which killed his wife and son. Weitzenböck and the housekeeper survived the attack. The house was completely destroyed. The local paper wrote 'an English bomb attack, carried out in a dive, claimed two civilian victims'. The local population saw in the act the revenging hand of a villager, who had fled to England and joined the RAF. Weitzenböck himself blamed the Dutch government, and he thought it possible that one of his son's former mates had carried out the attack. An attack by the resistance must be ruled out, at that time it was not up to such a job, moreover, the police, or the German military, would have noted. The idea of a precision bombing in those days on a villa hidden, together with numerous other villas, among trees in half dark, seems preposterous. According to the archives there was no English squadron flight over that part of Holland at the time, but of course a plane could have lost contact, and got rid of its bombs. Finally, there may have been anti-aircraft fire, and an unexploded grenade could have dropped at the house. It all adds up to a chance bomb or projectile. The bombing became a sort of legend. Stories were told about a secret meeting of high military officers, about Weitzenböck's work for military research, but none of it holds water.

17.3 Freudenthal dismissed

The mathematics department had two staff members, who, as Jews, were in actual danger, Belinfante and Freudenthal. Belinfante belonged to the old Portuguese Jewish community. He was a quiet, conscientious man. Since 1924 he had been teaching intuitionistic topics on a part-time basis. In addition he was a high school teacher in mathematics. At the outbreak of the war he taught at a girl's college in Amsterdam (*Lyceum voor Meisjes*). Belinfante did not play an important role in the mathematical institute, he restricted himself to his courses and his research in intuitionistic real and complex analysis.

Freudenthal, on the other hand, was the centre of activity in the institute. He was in fact after Hurewicz had left, the only modern mathematician, with ideas about almost all topics. His courses were notorious for their merciless use of new methods and level of abstractness. Nonetheless he was extremely popular with the (better) students. Nobody who took his courses, or enrolled in his seminars, and

grasped what the young magician with his German accent was doing, would ever forget the thrill of being shown the mysteries and vistas of mathematics.

Freudenthal came to Holland in 1930 at the age of 25, but for some reason he had never acquired Dutch citizenship, although he had become completely assimilated. He was known to his mathematical colleagues all over the country, and he was at home in Dutch cultural and artistic circles. Since Jews who left Germany for a longer period, automatically lost their citizenship (they were '*ausgebürgert*' (denationalized)), Freudenthal was at the outbreak of the war stateless. The Dutch authorities had in the months preceding the war, in view of the increasing tension, started to check on persons without the Dutch nationality. As a consequence, Freudenthal had to report in March 1940 at the police station, where he had to show his working permit. Freudenthal, who cared more about topology than bureaucratic documents, had completely forgotten about the permit. Brouwer, when asked by Freudenthal, made some inquiries, with the result that Freudenthal had to apply for an extension at City Hall.

Freudenthal soon became the target of small bureaucratic harassment. Although Brouwer had personally approved Freudenthal's next term as a *privaat docent*, the authorities (no doubt as part of the plan to eliminate Jews from public life) had not automatically extended this appointment, and requested him to apply for a new one. Fourteen days after the capitulation he was already asked to appear at some later time at the office of the head of education of the city council in order to submit a request. Freudenthal flatly refused this: 'It surpasses all reasonable limits, to demand from me that I submit a new request for a *privaat docent* position, and I will inform the head of this.'[10]

Brouwer gave the request a more neutral interpretation, according to him this was a normal procedure, carried out every five years. It did not take long, however, for the Germans to start the elimination of Jews from public life. The first steps were of an administrative kind, not directly visible out on the streets, but the open discrimination followed soon, the banning of Jews from public places, parks, cinema's, pubs, swimming pools, ... One of the first steps in the direction was the so-called 'declaration of Aryan descent'; all civil servants had in October 1940 to fill out and sign a form specifying their racial descent. A month later all Jews were dismissed from the civil service, including the educational sector. On November 23 Freudenthal lost his position as a conservator, and at the same time Belinfante's permission to teach as a *privaat docent* was withdrawn. For the mathematics curriculum the loss of Freudenthal was serious, he was in charge of a number of central topics. Belinfante's teaching had always been restricted to parts of intuitionistic mathematics, with him the institute lost a conscientious teacher, albeit of somewhat marginal topics. So at one stroke Freudenthal and Belinfante had joined the class of the unemployed, albeit that they received a form of payment by the state. Not only were the Jewish staff members dismissed, they were no longer allowed on the

[10] Freudenthal to Brouwer 6.VI.1940.

premisses of their former institutions; for a scientist that was a serious restriction of his professional activity.

When a number of students approached Freudenthal with the request to conduct the examination of the parts of the curriculum that used to be his responsibility, Freudenthal asked Brouwer to take care of their interests, since he was in no position to do so.[11] He ended the letter by assuring Brouwer that he was, of course, willing to assist him in matters that were not of an official nature.

In his reply, Brouwer conjectured that the authorities would, before the end of the year, propose a solution to the problems mentioned.[12] He thought that there was no possibility for Freudenthal to continue further his duties as a privaat docent (as such he had not been fired), since Belinfante had already been dismissed, expressing his sympathy—'May you hold your own in life, and may new perspectives open up for you before long.'

In the eyes of the Germans, unrest had to avoided, 'business as usual' was an important principle in occupied territory. So the universities, in as far as they had not been closed down, as was the case with Leiden and Delft, had to carry on the normal teaching duties. The Amsterdam mathematics department had to find a replacement for Freudenthal, who had been de facto responsible for the analysis-topology curriculum. The faculty, no doubt prompted by Brouwer, soon found a replacement: Evert Marie Bruins, the same man who was considered by Brouwer as a mathematics teacher in Hilversum (cf. p. 733). On 10 January 1941 Bruins was appointed to teach the analysis course. A few days later Brouwer informed Freudenthal of the new appointment, and asked him to provide Bruins with any information he might need. In his letter to Freudenthal, Brouwer spoke of 'temporary replacement' (*waarneming*).

Why the 'temporary'? Like many Dutchmen, Brouwer probably trusted that the Third Reich would not last. In 1941 the outcome of the war was no longer a foregone matter. The Germans were seemingly invincible, but nonetheless the majority of the Dutch clung to the conviction—perhaps slightly irrational at the time—that the German superiority would eventually break down. History, so to speak, was on their side, had they not survived the King of Spain and Napoleon? The military events of 1941 carried the germ of an eventual downfall; the German army was dangerously overextending itself. The successes in North-Africa and Russia were bought at a price. Moreover, the outcome of the battle of Britain and the continuing support of the United States were reasons for cautious optimism. So Bruins' services might well have been of use for a short period only.

Under the circumstances the Dutch were cautiously optimistic about the final outcome of the war, although they suspected a great deal of misery before the Third Reich would be a thing of the past.

[11] Freudenthal to Brouwer 30.XI.1940.

[12] Brouwer to Freudenthal 27.XII.1940.

Bruins, was perhaps not the optimal choice for a mathematics department; by training he was a physicist, with a taste for down-to-earth calculus, algebra and geometry, *à la* Clebsch. He had a solid acquaintance with the traditional subjects before the advance of modern algebra, topology, functional analysis and the like. But in the area of his choice he was an accomplished expert. While still a student, he had found his first results in applied mathematics at the chemistry laboratory of Büchner. Subsequently he wrote a dissertation under Jacob Clay on cosmic rays.[13] Clay was one of the physics professors in Amsterdam; he got his physics training in Leiden under Kamerlingh Onnes. In his early years he had been a follower of the philosopher Bolland, be it only temporarily. He kept a lifelong interest in the philosophy of the sciences (in particular physics). His fame was based on his work on cosmic rays (1927). In addition to the standard package of mathematics for physicists, Bruins had made himself thoroughly familiar with Weitzenböck's theory of invariants, in the style of 'symbolic notation'.

At the age of 31 he became a teacher at the mathematics department. Needless to say that he was in no way the person to bring Amsterdam to the forefront of modern mathematics, as Freudenthal did. Even in his applied mathematics he cultivated an almost nineteenth century taste, books like Courant–Hilbert's *Mathematische Methoden der Physik* were not his cup of tea. For a temporary position Bruins was not a bad choice, but soon he was upgraded to a permanent position; on 6 September 1941 he was made conservator, Freudenthal's old rank, and on 1 July 1942 he was promoted to lecturer in analysis, a position that Freudenthal had not attained after ten years in Amsterdam. On 7 July 1943 Bruins gave his inaugural lecture '*Mathematicians and Physicists*'.

The appointment and the quick rise through the ranks surprised the insiders. Although nobody denied that Bruins was an intelligent and resourceful man, he would have been placed better at the physics department or at the institute of technology. In addition it was considered 'not done' to take the position of a discharged Jew. One might help out with the teaching, but to accept the vacancy on a permanent basis, and to seal it, so to speak, with an inaugural address was a demonstration of poor taste, to say the least..

After the war Freudenthal conjectured that this promotion would have accorded with Brouwer's wish to make a return for him harder,[14] for although at that time there was no animosity between Bruins and Freudenthal, it was unthinkable that Freudenthal would serve in his old rank, as a conservator under Bruins, who was mathematically no match for him. The inaugural lecture had not escaped Freudenthal, and he mentioned it as an example of Bruins' curious sense of humour. Commenting on the present circumstances, Bruins told his audience, 'And finally this, the only fact, whereby the events of war could have had an inhibiting influence on the activities of a mathematician, is in fact provided by the stories about Archimedes'.

[13] [Bruins 1938].

[14] Freudenthal to Comm. of Restoration 9.VII.1945; not sent. Freudenthal to De Groot 17.IX.1945.

As Freudenthal viewed it, 'a phrase, which contains an insult for all mathematicians of reputation, who were at that moment in camps, and for the memory of those, who were murdered by the Germans, for the students too, who as a consequence of the war and the German usurpation had to interrupt their study, and finally, an insult for our whole nation, where Dr. Bruins draws a line between the *misera plebs* on the one side and the mathematicians on the other side, who cannot be affected by the war.'

17.4 University—resistance or survival

For the present biography the wartime history of the University of Amsterdam is of course the first subject of interest; one should, however, not forget that the universities did not act in isolation, nor were they treated by the authorities on an individual basis. Events at one university usually had their repercussion at other universities. The University at Leiden was the first to protest on a large scale against the anti-Jewish laws of the occupation authorities, in reaction it was closed down in November 1940. The other universities had avoided open action; the views of the student bodies and the boards of the universities were diametrically opposed on this issue. The history of the universities during the war is to be found in the series of books of L. de Jong[15] For the University of Amsterdam the books of A.W. de Groot and P.J. Knegtmans should be consulted.[16]

Once the Jewish staff members were removed from the universities, it was only a matter of time before the Jewish students would be the target of new decrees. Indeed, on 5 February 1941 secretary general Van Dam, in a confidential letter, informed the universities that Seyss-Inquart had ordered a numerus clausus for Jewish students. At first he had ordered the total ban of all Jewish students, but after a protest of the universities he had allowed them the compromise of a *numerus clausus*. He had practiced the standard tactics of the Germans in Holland: first ask too much, and then grant the other party an illusion of a reasonable deal. The Dutch had to learn the hard way that negotiating with the Nazi government was not really possible, the government would change the rules at will.

When the Amsterdam rector, a man with a name that might invite confusion: Bertus Brouwer—no family, was informed about the plans, he immediately set to work in order to prevent strikes, protest marches and the like. He was by no means pro-German, or pro-Nazi, but, like many of his fellow authorities, he thought that by avoiding open clashes, and hence the closure of the university, he would be able to salvage the main part of the Dutch educational system. The matter was almost immediately discussed in a special session of the senate of the university. After a lengthy debate, the senate decided to send a letter of protest to Van Dam. But since this letter was sent to his private address, it had hardly any public effect.

[15] Het Koninkrijk der Nederlanden in de tweede wereldoorlog, [Jong 1969].

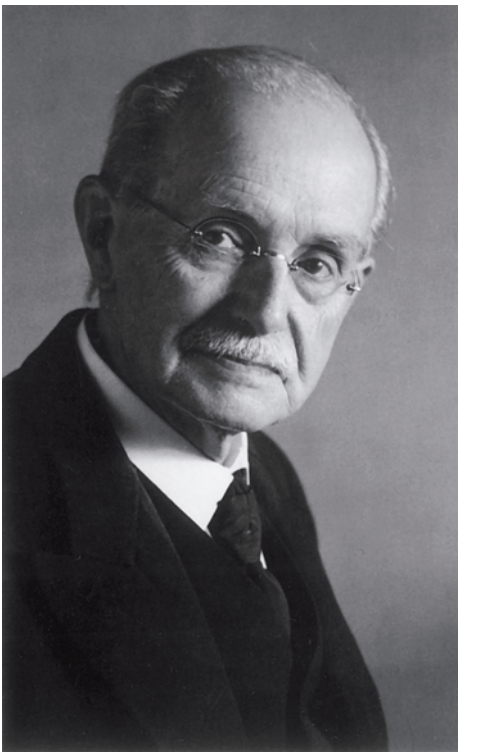
[16] [Groot 1946], [Knegtmans 1998].

The retired, but still active, Mannoury had considered it his moral duty to stand up for his Jewish students, in his archive there is a draft of a motion he introduced in the senate:

> The senate is of the opinion that by any discrimination against the academic rights of the Jewish students in comparison to other students, not only the interests of the University and of science is harmed, but also the higher interests of society and humanity are seriously harmed, and it resolves to bring this notion to the attention of occupation authorities of The Netherlands.

In the meeting he argued that the rejection of the announced discrimination is based on two principles: that of freedom of thought and of justice. The principle of justice carried more weight because,

> Justice is itself a limit, a demarcation and a balancing of material and cultural interests of the individual and society, of group and subgroup, of majority and minority, located in the sub- and supra consciousness of human beings. The enforcement of this principle of justice demands a high level of self-control, that prevents us from acting under unrestricted passionate emotions of anger, hate or fear; and which allows us to test our



Gerrit Mannoury. (Courtesy J. Mannoury)

> feelings with reason. And that this self-control and testing is completely lacking in the mentality of which I spoke, is shown in every line of the documents, in which this mentality is advocated.

Mannoury's fearless defence of the values of our western culture in the face of the new barbarism was a brave act indeed. After all, even the senate contained members who were not exactly opposed to the new order.

The local political activity of L.E.J. Brouwer came to an end on July 29, 1941, when the public council meetings were banned by the occupation authorities. His membership of the council had not resulted in spectacular facts that would engrave his name in the hearts of the town people, rather he had been an ordinary. dependable member, just like thousands other council members all over the country. There was one fateful event, however, that seemed innocuous, and even beneficial at the time, that was to cost him dearly after the war.

In their effort to eliminate the old traditional political, cultural and social institutions of Holland in order to create new institutions that would be at best indifferent towards the Nazi politics, and at worst actively support the National Socialist goals, the Germans had gradually suppressed all kinds of organizations, ranging from Scouting to the trade unions, from sanatoriums to welfare organizations. The next step was the founding of new centralized institutions, that either were actively or passively supporting the German war effort and Nazi policies, or at least were easier to control than the myriads of societies, institution, clubs, charities, … that were part of the rich fabric of Dutch social and cultural life.

In Germany charity had been monopolized by the *Winterhilfe* organization, this supplanted all previously existing organizations, and as such it was an example of successful *Gleichschaltung*. What seemed more logical than copying this idea in Holland. Indeed in June 1940 *Winterhulp Nederland* was founded. In spite of large scale propaganda actions, the organization was immediately viewed as an enemy institution, and no Dutchman was going to support the street collections of it, if he could help it. In fact, the fund raising power of *Winterhulp* was negligible.

There was however another way to get control of the charity market, and this was another German creation, the *Nederlandse Volksdienst*, NVD.[17] The NVD was designed after the German example, which had replaced the traditional religious and social organizations. It should provide the care and support that was of old part of the program of the churches and the labour organizations, such as mother care, childcare, health care, holiday camps for the needy, support for the unemployed, etc. With the help of the confiscated property of secular and religious institutions, such as parish youth clubs, trade unions, the NVD, founded on 28 July 1941, started to compete for the hearts and minds of the Dutch population. Whereas the *Winterhulp* was a professional organization, the NVD was intended to involve the population in a direct way: it was an organization of members. As might be expected, this was inviting defeat, in the first four months of its existence

[17] The people's support organization of the Netherlands.

the NVD, it attracted nation wide no more than three thousand members, and most of these were members of the NSB.

When it became clear that the NVD would under normal circumstances never develop into a healthy organization with enough members to demonstrate national support, the authorities decided to put pressure on bodies and on persons in prominent places to promote the NVD. Brouwer was confronted with the phenomenon in a letter from Mr. Klaarenbeek, the Mayor of Blaricum. By what means Klaarenbeek, who was not an NSB sympathizer, was ensnared in the nets of the NVD remains unclear; usually some sort of veiled threats were involved. On November 21, Klaarenbeek, who had been made the head of the local *Nederlandse Volksdienst*,[18] sent out a circular letter, informing the readers of this new organisation, which in his words was 'not connected with any political movement or group', and which solicited the support of 'all persons with a warm heart for their fellow men' and which was to 'reach out to all Dutchmen in need of its help.' After describing the (in itself laudable) social goals of the NVD, the letter continued with the request to join of the organisation. Klaarenbeek (naturally, one would say) did not invent this membership campaign himself, he was 'asked' to carry out a membership drive among the leading citizens of the town. The letter ended with the words 'In the (unlikely) event, that you should be of the opinion, that you will not be able to join as a member, I will appreciate to be informed by you, preferably with the reasons that led you to this decision.'

Confronted with this embarrassing request, Brouwer contemplated the matter[19] and came to the conclusion that Klaarenbeek—'whose patriotic feelings were above suspicion'—should in the general interest remain as long as possible in function during the occupation. As a (suspended) council member and deputy alderman, he felt that he had to support his mayor.

To run ahead of our history, Klaarenbeek remained in function until 1944, when he was arrested by the Germans, and at his release after the liberation, reinstated as mayor.

When Brouwer observed, moreover, that a charwoman of his institute, who was a hard-core communist, and active in the resistance, had received financial support from the NVD, he concluded that the organisation did not pose a serious threat: 'I was gradually strengthened in the opinion to see in the *Blaricum Volksdienst* a charitable institution, although modelled after the German pattern, nonetheless operating according to Dutch standards, that, since all our previous charitable institutions were suspended, in spite of its noxious origin, *après tout* more or less filled a social gap.'

After ample consideration Brouwer decided to join the NVD, he duly paid his membership[20] until 1944, as he stated after the war, when he found out that the

[18] Buurtschapshoofd van de NVD te Blaricum.

[19] Cf. Brouwer to College van Herstel 30.VIII.1945.

[20] The dues were one and a half guilder a month, not a big deal. It was rather the symbolic value of the membership that the authorities were after.

control of the NVD was completely in the hands of the National Socialists—but by then he had already paid his dues for the whole year, so he could not withdraw from the NVD before the end of 1944. The matter will come up again in the next chapter.

The war years were certainly not easy or pleasant in any sense, one tried to muddle through as well as possible. In 1941 the hardships were more of a political than of a material nature. Food and clothing were rationed, one had to stand in queu, keep en eye on the sudden appearance in the shops of goods such as vegetables, fish, toilet paper, . . . , but one could survive if no emergency occurred. And this was exactly what happened to Brouwer. On March 11, 1941 his cottage in Blaricum was almost totally destroyed by a fire. At 8 o'clock in the morning a curtain close to the gas heater caught fire. Brouwer quickly acted and extinguished the flames in the curtains. But a few minutes later a penetrating smell told him that the fire had already reached the thatched roof. The fire alarm was raised and within ten minutes the fire brigade was on the spot. Brouwer was already carrying his valuable possessions, in particular his scientific archive and books, out of the cottage. The commander of the fire brigade saw the house as his first priority; he judged that he could not spare his men for the salvaging of all this paper. He ordered his men to direct their hoses at the cottage. As a result the fire was extinguished, and the cottage was reduced to the four bare walls. Brouwer bitterly remarked, trying to save what he could from the soaking remainders, 'So I will leave no *Nachlass*'. Indeed, the collection of letters and papers that was eventually left after his death was far from complete, and some sheets still show traces of the fire.

The builder who inspected the remains judged them a total loss. Fortunately, Brouwer could move into the villa 'De Pimpernel' on the grounds, which he had bought in 1937. One can, however, easily imagine what a traumatic experience it must have been for a 60-year old scientist to see his house and scientific papers become the prey of flames.

On May 10, 1941 Korteweg, the founder of the Amsterdam Mathematics Department, died at the age of 93. Brouwer spoke at his grave.[21] Korteweg had played an important role in the faculty in Amsterdam, and in mathematics in Holland. He was a well-respected man, whose judgement was equally appreciated in the Faculty, the Senate, the Royal Academy and the Mathematical Society. Of the latter, he had been president three times, and he had been on its board for almost 60 years. A whole generation of applied mathematicians was educated and shaped by him. He was already a legend before he died, and . . . soon forgotten. Eventually he was only known by the first name in the Korteweg-de Vries equation and as the PhD adviser of Brouwer.[22] The mathematical work he did for his PhD adviser J.D.

[21] [Brouwer 1941].

[22] Even his own department in Amsterdam thought it safer to add 'de Vries' to the name for its 'Korteweg–de Vries mathematics institute', when a name had to be chosen in the eighties.

van der Waals is nowadays also fully recognized.[23] Korteweg was an able and original mathematician, he was magnanimous and modest. He fully deserves his place in the gallery of great Dutch scientists.

The effects of the occupation were clearly visible for anyone who wanted to look around. It was not just a matter of a military occupation with its logical military consequences, but it was a regime that wanted to impose its political views, in particular those concerning the extermination of the Jews.

17.5 Freudenthal's fortunes

One would expect Brouwer to be pleased that Freudenthal had offered his personal assistance in mathematical matters. In the past Freudenthal had carried out many duties that relieved Brouwer from a dreary routine. He had, for example, written a report on the state of topological algebra, and the result came in handy when one of Brouwer's students, Frans Loonstra, asked for a topic for his dissertation. Again, in April and May 1941 Freudenthal sent Brouwer a report on Loonstra's manuscript for his dissertation.[24] The report is a rather cautious attempt to let Brouwer know that it was not a masterpiece, but more or less a competent survey and completion of known work. The last sentence, for example, leaves little doubt of Freudenthal's view, 'Loonstra succeeded in understanding the paper of Schnirelmann, which is partly written is a very condensed manner, to supplement missing proofs, clarify some obscure points, and put the text into an easily readable form'. It is doubtful how much Brouwer had seen of the manuscript; Freudenthal said that he would not be surprised if Brouwer had not read it at all.[25] The doctorate was awarded to Loonstra on 11 July 1941 for the thesis *Analytische Untersuchungen über bewertete Körper*.

The persecution of the Jews went through various stages, each time the screw was turned a bit further. On June 29, 1942 the authorities announced their decision to deport *all* Jews to the eastern part of Europe, and on July 14 the first trains rolled from the Central Station in Amsterdam to the camp at Westerbork[26] (a small place in the eastern part of the country) to deliver the Jews that had complied with the summons.

The first intuitionistic follower of Brouwer, Belinfante, was also in jeopardy. He was Jewish, and somewhat like Blumenthal, he could not imagine that any government could wish to harm him. He was a conscientious citizen, and nothing could be said against him. In a way, he stood a chance of getting a special treatment. He belonged to the group of Portuguese Jews whose fate was not yet settled. There were casuistic discussions among the German experts on Semitic affairs about this

[23] [Levelt Sengers 2002].

[24] Freudenthal to Brouwer 28.IV.1941, 23.V.1941.

[25] Interview 22.XI.1988.

[26] *Durchgangslager* (transit camp).

group. Were they real Jews, could they be given permission to emigrate to Portugal? In that typical Nazi thoroughness the matter was discussed by an assortment of departments and groups, and in the meantime the Portuguese Jews were granted deferment of deportation.[27] Eventually the group was put into camps anyway. As it happens, Belinfante was noted by a Dutch author, Durlacher, who was an inmate of the camp Theresienstadt at the same time as Belinfante. In his book , *Quarantine*, Durlacher recalled his encounter with Dr. Belinfante. A small man with glasses, who noted the author during a lecture of a philosopher on Hegel, and offered to teach him mathematics. Belinfante taught the young man his favourite subject, infinite series. 'In the following weeks simple series acquire meaning for me in a few evenings, in spite of the tiredness that rests on my eyelids, and in spite of the gnawing hunger. The thoughts of my teacher often wander, and his wife excuses the silences with a reference to his health. But I know that it is the disease of which we all suffer: fear and sorrow.'[28] Belinfante spent eight months in Theresienstadt, on 12 October 1944 he was transported to Auschwitz, where he died two days later. In his commemoration Max Euwe wrote that Belinfante did not use the opportunities he had to go underground, distrust was no part of his world of thinking. In Euwe's words, 'A great child, but a great man.'

The fortunes of Freudenthal, one of the central figures in this period of Brouwer's history, are by no means exemplary for what happened to the Dutch Jews at large, but they illustrate well the unpredictability of events under a systematic persecution. Freudenthal had married a Dutch wife of non-Jewish descent, thus he was exempted from deportation (although a wise man did well to doubt the permanence of such laws). This did not mean that he was out of danger; anybody could be arrested at any moment, and the Germans had no moral scruples to deport a Jew against the rules. A justification could always be found.

Freudenthal was attempting to lead a life that preserved a certain degree of normality, he did mathematics, talked to colleagues and students, submitted papers; all this as a private citizen, who stayed away from the mathematical institute.

In spite of the mixed marriage, there was no certainty for Freudenthal under the German regime; in general there never was, not for Jews and not for non-Jews. This was brought home to Freudenthal when, as a foreign Jew (with a mixed marriage), he had to file for an emigration permit. The *Obersturmscharführer* who happened to check the passports discovered that in Freudenthal's the stamp 'J' was missing. This was perfectly understandable, as the J's dated back to 1938, whereas Freudenthal's (German) passport was older, and living in Holland he had, of course, not felt the necessity to get a stamp somewhere in Germany. But a sin against the legal rules could not be tolerated. So Freudenthal had to present himself at the

[27] The matter is discussed in [Presser 1965].

[28] [Durlacher 1993].

Gestapo headquarters in the Euterpestraat in Amsterdam.[29] There the officer in charge hardly looked at him, and said 'six weeks'. As a result from 17 February till 30 March Freudenthal was incarcerated in the central prison at the Weteringschans, which was partly used as an SS prison. So, all of a sudden, Freudenthal had disappeared from the face of the earth; nobody, not even his wife, was informed. Fortunately she was a determined woman, with a perfect command of the German language. As a matter of fact she had studied German language and literature with the same professor Van Dam who was now the secretary general of the department of education. Telephoning all police stations and German agencies she could find, she finally got by chance through to the secret telephone number of the Gestapo office. Her German impressed the man at the other side of the line so much, that he gave the '*gnädige Frau*' all the information, including the promise that her husband would be out after six weeks. And indeed, after six weeks of fear and uncertainty Freudenthal was a free man again.[30]

Soon Freudenthal was his usual active self again; he reported in April and May on Loonstra's manuscript, he recommended a paper of J. de Groot on extending certain homeomorphisms for publication in the *Indagationes*. De Groot was a student of Schaake and he worked on topology under Freudenthal's supervision. Schaake, one of the Groningen mathematics professors, was not an expert in the area, so it was a natural move for De Groot to turn to Amsterdam for support. Brouwer, as a rule, left these things to Freudenthal. As a consequence, Freudenthal, although not a professor, was *de facto* the PhD adviser of a number of young mathematicians. Brouwer was prepared to submit the paper of De Groot to the Academy modulo some definitions and references. On July 6 he wrote Freudenthal that De Groot had adopted not all of his suggestions, and that De Groot had failed to react to Brouwer's subsequent letter. Could Freudenthal perhaps check what was the matter? Freudenthal's mediation must have been effective, for Brouwer confirmed to Freudenthal the receipt of the corrected manuscript, which he intended to submit to the Academy.[31]

For a long time no letters were exchanged, until on 24 May 1942 Freudenthal received a somewhat testy letter in which Brouwer demanded the return of the books that Freudenthal had taken out of the library of the mathematical institute *against the rules*. The library had always been Brouwer's pet, he had exerted considerable pressure to get an separate library for the mathematicians, which was, after the example of the Leiden reading room, only for consultation on the spot. Only on special request a book could be taken out. Furthermore he asked Freudenthal to instruct the students to return their books, borrowed with Freudenthal's permission, and finally to turn over the complete Compositio administration to Bruins.

[29] The name '*Euterpestraat*' became during the war a synonym for all the Gestapo horrors, that the word itself held terror for the average citizen. After the war the street was renamed after Gerrit Van der Veen, a renown member of the resistance.

[30] See [Freudenthal 1987b], p. 205 ff.

[31] Brouwer to Freudenthal 26.VII.1941.

After the war, when Freudenthal was fighting for his rehabilitation, he recalled the gradual erosion of his relation with Brouwer.[32] Until the spring of 1942, he wrote, the relation was with the exception of a few incidents, good. But after Freudenthal's imprisonment the situation seemed no longer the same. In spite of the fact that Brouwer was informed about Freudenthal's arrest, he did not contact or comfort Freudenthal's wife (who had three small children and was expecting the fourth). His first letter after Freudenthal's discharge from prison was the formal demand of May 24—no word of personal concern. 'After receiving that letter from professor Brouwer, it was clear to me, that he had not presented my note in the April meeting of the Academy, and that he wished to provoke a quarrel by means of a sharp letter, in order to free himself from the unpleasant task to present my note'.[33] Determined to ignore the provocation, Freudenthal wrote thereupon Brouwer that he had called the Academy to inquire after De Groot's note, and that it apparently had not been submitted in the April meeting. Out of concern for De Groot (and Loonstra) he therefore asked Brouwer not to postpone the submission any longer, as 'one of the interested parties could or would possibly make other arrangements.'[34] If Freudenthal had planned to alienate Brouwer, he could not have followed a better strategy. If there was anything at all that Brouwer could not tolerate, it was interference with his publication policy. Freudenthal's letter reached Brouwer in Roosendaal, where he was conducting the annual examinations of gymnasia and high schools; he replied, 'You and the two other gentlemen involved seem to forget that the members of the Academy constitute an editorial board for the Proceedings, and that editorial boards of journals are no printing automata!'[35] This letter was followed a day later by an envelope with the manuscripts, and a brief note, 'Hereby the manuscripts of you and the two other impatient gentlemen are returned for "making other arrangements".' Freudenthal interpreted the letter as the definite end of their relationship. In his reply to the letter of May 24, he observed that he had immediately returned the books of the library, but he could not recall having given students permission to take books home. As to the *Compositio* archive, it was all the time in a cupboard in the assistants room, but without the permission of the *Reichskommissar* he could not enter the institute to hand it over to Bruins. That, roughly, was what there was to say. Freudenthal just added that he had not passed on the message of May 29 to the other authors. 'They might perhaps feel little appreciation for the insults, that one can only bear out of respect for a great mathematician'.

After the war, when confronted with this episode, Brouwer, described the events as follows: 'just before a meeting of the Academy I received some 100 pages of manuscript. I took these on my trip. A bit later I got a letter with a reprimand for my negligence. Subsequently he threatened to publish elsewhere. Then I have returned

[32] Freudenthal to Van der Corput 24.VIII.1945.

[33] Ibid.

[34] Freudenthal to Brouwer 26.V.1942.

[35] Brouwer to Freudenthal 29.V.1942.

it.' The accounts are not all that different, be it that Brouwer and Freudenthal interpreted the events totally different. Freudenthal had in fact been supervising three PhD theses; he was not a professor, so he could not officially act as an advisor, but in practice he had been the one who showed Loonstra, De Groot and Van Heemert their way through the intricacies of modern topology. In a way the style of the topology involved, was very much that of Brouwer and Alexandrov–Hopf. The new, more algebraically oriented topology had not yet become familiar in Holland.

Loonstra had already obtained his doctor's degree under Brouwer, and it would have been natural enough if the two other candidates would also have chosen Brouwer as their PhD adviser, he was after all the proper expert to handle topology dissertations. The difficulties with the manuscripts for the Academy had however cast enough doubt on a speedy and efficient procedure, so that De Groot had chosen to obtain his doctorate in Groningen with Schaake. On 13 October 1942 he defended his dissertation, *Topological Studies. Compactification, extension of mappings and Connectedness*.[36]

Van Heemert, a student of Brouwer, had also written his dissertation with hardly any consultation of Brouwer. He had worked with Freudenthal on a topic that was studied first by Brouwer: indecomposable continua.[37] On 1 June 1942, Freudenthal had sent a strongly positive report on Van Heemert's work to Brouwer, and it looked as if the doctorate was within reach. A couple of weeks later Freudenthal wrote a disconcerted letter, in which he informed Van Heemert of the unfortunate matter of the manuscripts of De Groot, Loonstra and himself for the *Indagationes*. He advised Van Heemert to present the manuscript of his dissertation directly to Brouwer. When Van Heemert showed some hesitation, he urged him not to antagonize Brouwer, and just submit the manuscript to him. Brouwer, Freudenthal wrote, had developed a strong dislike for him. The reason he could not fathom. 'This incident surpasses in intensity and candour all earlier ones, and I would, when asked, have to refuse resolutely to take up my old position, although this incident has—in my opinion—nothing to do with politics, but it is one of his normal tantrums, only stronger than before.'[38]

Taking Freudenthal's advise, Van Heemert wrote to Brouwer that he would like to send him the manuscript of his dissertation, explaining that his military service and a teacher's position had caused a delay in the research as discussed in 1939.[39]

After the reminder of 13 July, Van Heemert sent a second request to Brouwer. He had hoped to finish the discussion of the content of his dissertation during the summer vacation. But in case Brouwer had no time for him, would he give him permission to take his dissertation to the Groningen faculty? Brouwer replied on 28 July that he had a large backlog of all sorts, and that he had no time to read the manuscript for the next half year. He therefore advised to send the manuscript to

[36] [Groot 1942].

[37] Cf. p.144.

[38] Freudenthal to Van Heemert 26.VI.1942.

[39] Van Heemert to Brouwer 29.VI.1942.

Loonstra, who had agreed to read it. Van Heemert had in the meantime contacted Van der Corput, and explained the situation. Apparently he had little hope that Brouwer would oblige, and thus he expressed his preference to defend his thesis in Groningen.

Brouwer's suggestion, to ask Loonstra to read the manuscript, did not appeal to Van Heemert. It was obvious that if Brouwer chose not to rely on Freudenthal's expert opinion, but what could be won by involving someone who was only marginally informed. Van Heemert obviously wanted to keep his options open, so he called Brouwer and informed him that further deliberation was required before he could decide what to do. This clearly piqued Brouwer; four days later he wrote that he was amazed that Van Heemert had not reached a decision. 'I see now in this an argument to advise you to look for a Ph.D. adviser whose suggestions you can follow without protracted deliberation.'[40] Now Van Heemert was free to go where he wanted. When asked for advice, Van der Corput told Van Heemert that the proper advisor in Groningen was Schaake. The latter had no wish to alienate Brouwer, so he urged Van Heemert to stick with Brouwer. But eventually he accepted the dissertation, and on 5 February 1943 Van Heemert received his doctorate in Groningen.[41]

The conflict between Brouwer and Freudenthal—for a conflict we may call it—was of a totally different nature than earlier conflicts. Looking at the relation between the two, one gets the impression that here is a classical case of *incompatibilité d'humeurs*. Both men had too much in common to live in peace together; both were spiritual, highly intelligent, well read, ambitious, both had a quick wit and a sharp tongue. Brouwer, who in spite of his many public obligations, jealously guarded his private domain, resented attempts from others to intrude on his privacy, no matter how slightly. Freudenthal came dangerously close to the impregnable citadel of Brouwer's personality. There is, by the way, no doubt of Freudenthal's sincere admiration for Brouwer, nor of Brouwer's recognition of the younger man's gifted personality. And so the classical Saul-David situation arose, more or less triggered in 1937, at the time of Freudenthal's conservator appointment (cf. p. 719). And as experience shows, these things get worse, unless one has the greatness to make a clean breast and admit one's own shortcomings. In a way Brouwer had been Hilbert's David, but the two could easily avoid personal contact, in the case of Brouwer–Freudenthal this was far more problematic. Moreover, the war had made it improbable that Freudenthal could be promoted to a professorship elsewhere, so Brouwer and Freudenthal were condemned to each other's company—even the unintended German intervention did not really separate them, as Freudenthal remained in Amsterdam and continued his activities.

There is one example of Freudenthal's persistent presence in Dutch mathematics; the Dutch mathematical society traditionally published prize questions of a purely academic nature. The crowning of a prize essay was considered a sufficient

[40] Brouwer to Van Heemert 3.VIII.1942.

[41] [Heemert 1943].

honour, no prize money or any other benefits were attached. Participants sent in their essays, which were judged blindfolded. Only after an essay was crowned, the envelope with the name of the author was opened. During the war Freudenthal had submitted a solution to one of the prize problems, and his submission was inspected by the committee and awarded. This highly displeased the government department for education, science and culture protection. An disapproving letter followed;[42] the department informed the mathematical society of its displeasure in threatening terms: 'Freudenthal, who is a Jew, has participated in a Prize contest, and been awarded a prize. The Commissar-General finds it remarkable that your board lacks the perception to abstain from granting the award in such a case, and he reserves the right to investigate the case further.' In fact the matter was not followed up, but it shows that at every level in society the authorities kept a tight watch. It also showed that Freudenthal had no intention to make himself invisible.

Confronted with Freudenthal's undaunted spirit and art of survival, Brouwer probably started to think of means to transfer Freudenthal to a place where he could make a career without further clashes; later developments are consistent with that possibility. The question to ask here is, did Brouwer wish Freudenthal ill? Again, one can only guess, but the answer appears to be negative. It is more likely that he felt uncomfortable, or even threatened by the presence of a person who wanted things—including Brouwer—to get moving. In plain words, Freudenthal got on his nerves. The situation is comparable to that of the rabbi in the musical Anatevka who was perfectly willing to bless the Czar, if only the Lord 'kept him far away from here'.

In 1942 Brouwer resumed his publication activity for a brief spell. He communicated in March, April and September three short notes to the Academy.[43] The first note, *On the Free Genesis of Spreads and Functions*, was a translation of a letter to Heyting, written on 30.III.1936, which dealt with an extension of the notion of function. At various places in the early publication on intuitionistic mathematics functions were defined as 'laws', but since then Brouwer had realized that the procedure of generating points (say in the plane) as choice sequences was equally applicable to functions (think of functions as points in a function space).[44] The paper also contained an explicit formulation of the higher-order restrictions in the definitions of the notion of choice sequence.

[42] Department of Education to Wiskundig Genootschap 11.I.1944. Note the stubborn way in which Nazi authorities concentrated on side issues, at a time that there were more serious (military) survival issues to be considered. Perhaps this is a universal characteristic of bureaucracies.

[43] [Brouwer 1942c, Brouwer 1942b, Brouwer 1942a].

[44] Brouwer seemed to have forgotten that he had already established basically the same facts in [Brouwer, L.E.J. et al. 1918] (p. 14,15). There he showed that the set of continuous functions from [0,1] is of the same cardinality as Baire space, but overlooked the fact that the left-hand side consisted of lawlike elements and the right-hand side of choice sequences. The argument is correct in [Brouwer 1925b], p. 253, where the lawlike part of N^N is considered.

The next paper, *The Representing Spread of the Continuous Functions of the Unit Continuum*, carried out the construction, foreshadowed in the preceding paper (this, by the way, is literally the construction of [Brouwer 1918]).

Finally, the last note, *Proof that the Notion of Spread of Higher-order is not appropriate as a Fundamental Notion of Intuitionistic Mathematics*, showed that one could reduce a generalized sort of choice sequence which allowed the chosen items of a choice sequence to be choice sequences themselves, to ordinary choice sequences. All of these publications had, so to speak, been waiting in a drawer to be called up. In this particular case the publication had an external cause, i.e. a passage in a recently published book of E.W. Beth, *Introduction to the philosophy of mathematics* (2nd ed.).[45]

Although a certain amount of international activity was taking place, the Dutch mathematicians became more and more isolated; Brouwer was no exception, his international contacts had been modest before the war, but now he maintained a self-chosen isolation When in 1942 he was selected by the Secretary General of Education, Professor van Dam, to represent the Netherlands at a conference in Italy, he flatly refused to go.[46]

The Academy kept up its activities, albeit in a smaller scale, and the section mathematics and physics continued their monthly meetings. Brouwer, apparently, was sufficiently impressed by his most successful intuitionistic follower, Arend Heyting, to start lobbying for his election in the Academy. In 1942 these efforts were crowned with success. Heyting was not without worldly wisdom, he took no steps to occupy his place in the Academy. A judicious decision, as the academic community did not appreciate appointments by a government that was only an extension of the occupying forces. The historian of mathematics, Dijksterhuis, was not as cautious as Heyting, and his membership of the Academy was cancelled after the war.

Let us now return to the fortunes of the university during the occupation. No systematic attempts had been made to turn the universities into Nazi, or even pro-German, institutions, but the elimination of the Jews had been systematically carried out already in an early stage.

The moment of truth had arrived on November 26, 1940 in Leiden, when after the dismissal of all Jewish professors and staff members the eminent Law professor Cleveringa held his famous address in the auditorium of the University of Leiden, summing up the events and the lack of legal basis for dismissing Jewish colleagues, going even further by pointing out the violation of the Geneva rules. The students lost no time to conclude that a strike was the only possible and justified answer. The German General Commissioner for Law and Administration, Dr. Wimmer, immediately retaliated by closing down the University of Leiden. The other universities were spared that fate, but the occupation authorities had made it crystal clear that they would not tolerate student unrest. That was, certainly in the beginning of the

[45] [Beth 1940].

[46] City Council to Secretary General 23.IX.1942.

occupation, not difficult. The Dutch were as a rule a law abiding nation and the students too were not used to political issues and actions (the Socialist or Communist Student Associations made up a small portion of the total student population); it was, however, a fact that the majority of the students was downright opposed to National Socialism and to the occupation forces.

Since our story is mostly concerned with the events at Amsterdam, we will as a rule not comment on Dutch universities in general. When the Jewish students were excluded from the student corporations, the corporations considered this unacceptable, they decided that this was a reason for their dissolution (October 1941). Apart from confiscation of the property of the corporations, no further action was undertaken by the authorities.

17.6 The declaration of loyalty

The first violence in the Amsterdam student world erupted on January 22, 1942, when a bomb exploded in the building of the NSB student organisation. The German representative in Amsterdam concluded (without proper evidence) that the attack was carried out by students, and the highest German authority, Dr. Seyss-Inquart, took the matter in his own hands. He decreed that six professors should be dismissed without pension, and sent to a concentration camp,[47] where also ten students should be incarcerated. The University judged that such a draconian measure made it impossible to carry out its task any longer and the rector had informed the authorities that he himself and the leading professors would resign if the plans were carried out. After fervent protests and hectic conferences of various groups of authorities, the Germans reconsidered the sentence.

The Mayor of Amsterdam, Voûte, an NSB member and an appointee of the occupation forces, had in the meantime been granted an audience with State Commissioner Seyss-Inquart, who had subsequently modified the sentence:

1. Nobody would be fired at this moment. Possible dismissals at a later time would not be connected with the bomb attack.
2. Fifty citizens of Amsterdam would be arrested.

In addition it was promised that a certain professor (Hazewinkel-Suringa), who had already clashed with an NSB colleague, would not be arrested.

The reaction of the State Commissioner was typical for the Nazi way of handling opposition, a tactical shift would make the sentence cosmetically more acceptable, and the opposition would lose much of its thrust and coherence. The revision of the verdict could, however, hardly be considered a victory for the university, as some later claimed it to be.

Eventually the Germans arrested a large number of citizens of Amsterdam, including four professors and three students (the arrest of three more specially designated students failed). Pannekoek, the retired professor of Astronomy, was also

[47] Romein, Tielrooy, Hazewinkel-Suringa , Kollewijn, Kranenburg, H.J. Scheltema.

arrested, but he was soon released—his communist past was not sufficient reason to keep him in custody.

Soon, however, the Nazis ordered the dismissal of nine professors. A meeting of the Academic Senate was immediately called, and long and earnest discussions followed. One faction proposed that unless the dismissals were repealed, Seyss-Inquart would be asked to close the university. The rector and secretary of the university tabled another motion: unless the dismissals were repealed, the university would issue a strong protest.

The arguments pro and contra were many and varied, but in the end the view of the radicals boiled down to: we and our students cannot and will not accept dismissals unrelated to the basic tasks of the university. Therefore we must act and shield our students. Hence we should, if necessary, discontinue the activities of the university. The other side pointed out that at the closure of the university, staff and students risked forced (or at least non-voluntary) labour in Germany. In the end the moderates won.

At the opening session of the university, looking back on the academic year 1941–42, rector Brouwer could report only misfortunes. Six students had died (he did not mention that five of them had died in concentration camps), twelve professors had been dismissed, and since August 1942 Jewish students were no longer admitted to the universities. His successor, rector Deelman, had no pleasant prospects.

The next move in the battle for the universities was made again by the Germans. On 5 February 1943 General Seyffardt, who was in charge of the Dutch Legion of the SS, was shot by the resistance. That same evening Reydon, the secretary general for Mass Education and the Arts, was wounded in an attack, and his wife was killed. Almost immediately General Rauter, assuming that students had been involved in the attack, raided the Universities of Amsterdam, Delft, Utrecht and Wageningen. At half past ten on Saturday morning machine guns were mounted on the lecterns, and a large number of students was arrested and taken to the concentration camp in Vught (near 's-Hertogenbosch in the southern part of the country). In this case the rector of the Amsterdam university and his assessors took a brave and determined step: until further notice all lectures and practical instruction were to be suspended. Surprisingly enough, the Germans did not act on this provocation

The presentation of the intricate story of the universities in general, and in particular that of Amsterdam during the occupation cannot be told here in full detail. The reader is referred to the books of De Jong, De Groot or Knegtmans. For the following episode, the reader may recall that the battle of Stalingrad had been lost in January 1943, and that the Africa campaign was in serious trouble after the battle of El Alamein. The Germans were for the continuation of the war depending on labour from the occupied countries. And so after the attacks on Seyffart and Reydon, the policy of the Germans with respect to the universities had two objectives: the prevention of unrest and obtaining labour for Germany.

After rumours about German intervention had been circulating, causing considerable excitement, the Secretary General for Education Van Dam unexpectedly announced on 10 March 1943 the following *Act for the safeguarding of the order at universities and institutes of higher education*, which laid down that any student following courses or taking part in examinations had to sign what became known as the *declaration of loyalty*:

> The undersigned …, born at … at … living in … solemnly declares hereby that he will on good faith obey the laws, ordinances and other commands in force in the occupied Dutch territory, and will abstain from any action directed against the German Empire, the German army or the Dutch authorities, and also of actions and behaviour, which endanger the public order at the institutions of higher education, in view of the present circumstances.

This was a curious document indeed. It was a mixture of general civil jurisdiction with what traditionally belonged to the jurisdiction of the universities. The declaration seemed to be inspired by the wish to separate the 'friendly' students from the opposition. Furthermore, it was a reliable device to get people entangled in a web that could at any moment be made more complicated, more compromising. The registration of the Jews was a similar device—in itself harmless, but potentially lethal. Finally, and most people realized this, the relation citizen—authority had fundamentally changed. After 1940 all the advantages were at the side of the authorities, all regulations could be repealed, all civil rights could be abolished.

Almost at the same time the State Commissioner, Seyss-Inquart, issued an ordinance of even greater consequence, which supplemented and strengthened the status of the declaration of loyalty. It laid down that:

1. the Secretary General of the Department of Education can impose a numerus clausus for any subject (discipline) at any university,[48]
2. Every student who finishes his studies has to enrol in the *arbeidsinzet* [an obligatory period of labour for projects determined by the authorities, which usually meant projects supporting the German war effort, be it directly in military industries or as a replacement in other projects (e.g. hospitals, technical industries, …)].

The practical consequence of this ordinance was that any student who did not sign the declaration of loyalty had to terminate his or her study, and thus, being unemployed, automatically was drafted into the *arbeidsinzet*. The latter invariably meant a period of (forced) labour in Germany.

The publication of the above ordinances created understandably considerable unrest among the students, their professors, and advisors. The reactions were of two kinds: anyone who signs the declaration gives moral support to the enemy, or: the declaration is legally meaningless, signing does no harm. Even the spiritual

[48]Not just an entrance restriction, but applicable to any stage of an academic study.

guides of the Dutch were in confusion, e.g. the Archbishop of Utrecht declared: 'The declaration can be signed in good conscience; I consider, however, the signing as an anti-patriotic act.'

For the senates of the universities and for the individual professors the declaration posed a serious problem. Although it did not concern them, it concerned their students and in the end the position of the Dutch universities. Up and down the country the matter was debated in all kinds of boards—senates, faculties, laboratories, etc.

In Amsterdam an emergency meeting of the Senate was called on March 26, 1943. At the meeting a draft of a statement of the Senate, to be sent to the Secretary General of education, Van Dam, was presented to the members. It contained comments on the numerus clausus ordinance, mainly to the effect that if there had to be one, the senate should determine its size, and that no student, who had already enrolled, should be subjected to it. It remarked that no unanimous advice as to the signing of the declaration of loyalty could be agreed on. In addition it included the refusal to resume the lectures until all students had been released from the concentration camp in Vught. After pointing out that some students possibly could not sign the declaration on moral grounds, and that some members of the senate would feel it intolerable if they had to continue their teaching without these students, the draft concluded with the statement that if both ordinances were carried out in the given form, the members of the Senate had no choice but to resign.

The last sentence was a clincher, it actually amounted to the closing of the university of Amsterdam. A substantial debate ensued with arguments on all sides—which in this case basically boiled down to either a flat refusal, or a compromise of some sort. It is important to keep in mind that the meeting was attended also by persons of National Socialistic leanings such as Weitzenböck.[49] It must be pointed out that even though Weitzenböck was known to be pro-German, so far no indications or evidence have been found that he ever denounced or harmed persons. Yet, … one never knew!

At the time of the meeting 3600 forms for the declaration of loyalty had been stored in the safe of the university. In spite of the pressure of the President Curator (i.e. the NSB mayor) none had been laid before to the students to sign.

In the discussion about the draft and its contents a number of views were defended. It was argued that the draft admitted that the Senate could not issue a unanimous reaction, and that it was unwise to advertise this.

Brouwer, who had been unusually low key during the preceding Senate meetings, had this time decided to make himself heard. He had prepared a number of amendments, all designed to weaken the formulation of the draft and to improve the chances of a compromise. His main point, and indeed the only one that was discussed, exclusively dealt with the last line, with the threat, or rather—as the

[49] In fact, one lecturer sent reports of the meetings directly to the chief of the *Sicherheitsamt* (security service) [Knegtmans 1998], p. 181.

Rector said—the factual statement of a collective resignation. Brouwer clarified and defended his amendment with the words:

> My version [of the draft] differs from the one of the Rector mainly with respect to the final passage, which in the form of the Rector, strikes me as objectionable, and this for two reasons, which I would elucidate as follows: We want to try to get a favour of an authority that has an absolute power over us. This is a reality for which none of us, no matter how he assesses this situation, can close his eyes. If we really want to see our wishes granted and our expectations fulfilled, then we first have to make an effort to create the indispensable atmosphere of goodwill and understanding. And for that no worse method can be imagined, than—at least indirectly—to send a request to an authority that considers the withdrawing of one's manpower from the community as one of the most unbearable crimes, ending in an ultimatum-like threat of strike. A second ground for finding the final passage objectionable, I deem even more serious, namely that, when the threat of strike by their teachers becomes known among the students, it will inevitably lend support to possibly already existing agitation for the refusal to sign the declaration of loyalty, and which therefore would cast a serious co-responsibility on the Senate for the indescribable misery, which this refusal could bring over our students.
>
> The new version of the final passage proposed by me intends to express to some insights, wishes and expectations, just as the original one, with the deletion of the strike threat.

Reading Brouwer's arguments, one gets the strong impression that he had no illusions about the willingness of the authorities to act or think fairly and that therefore—in view of the ruthlessness of the occupation forces, as demonstrated in the brutal suppression of the so-called 'February strike'[50]—one has to take care not to antagonize them. His information about the policies of the Nazis in Germany may have influenced his views, but he failed to take into account the absolute and calculated unreliability of the German authorities. What good does it do the mouse to please the cat?

After a long list of speakers had addressed the Senate, the Rector asked for a vote. Brouwer's amendments were rejected by a solid majority.

With the approved draft the Rector went to a meeting of the rectors of the universities to discuss a collective reaction. This meeting did not yield much; it proposed to deal with the numerus fixus matter, in the hope that this would yield a bargaining position with respect to the declaration of loyalty and its consequences. The rectors clearly misjudged the situation, they were not dealing with polite Dutch civil servants, but with the State Commissioner himself. After a discussion of their proposals with the Secretary General, the State Commissioner was informed of the views and proposals of the universities. Seyss-Inquart ignored the proposals of the rectors and simply insisted on the complete introduction of the ordinances,

[50] February 25 and 26, 1941

including the signing of the declarations of loyalty. So suddenly the universities found themselves short of time, their adversary was not so naive to play the game on their terms.

The Senate in Amsterdam reconvened on April 8 to discuss the new situation, but in the meantime the students had organized themselves and they informed the meeting that roughly 1400 students had signed a letter to the Senate to the effect that they would not sign the loyalty declaration. This meeting hardly achieved anything, the firm resolutions of the previous meeting seemed to have been forgotten. The promise of collective resignation was forgotten by all but a few. The central question, 'should the students be advised to sign the declaration of loyalty?', proved too tough for a clear 'yes' or 'no'. When no firm view on this issue emerged, Brouwer proposed to leave the matter to the discretion of the rector: [51]

> The Senate, taking into account that neither the law of higher education or any other law or ordinance, lends it authority to undertake any action in the matter of the declaration of loyalty submitted to the students, expresses its confidence in the Rector Magnificus, and passes on to the business of the day.

Eventually no decision was taken.

The above simple resumé of the crucial meetings of the senate does no justice to the sincerity and emotions of the participants. There were basically two views on the matter of 'signing or not': that of the rector and his assessors—signing is permissible and advisable, and that of the group called '*het hooglerarenverzet*' (the professors resistance group)—under no circumstances sign the declaration. Between these two opposites there was a scale of opinion. Apparently the two extremes were not able to get a majority. Brouwer's modest contribution had steered a kind of middle course. For some reason the professors resistance group had taken Brouwer's interference badly, it was explained as obstruction of their resistance. The reason is obscure; perhaps Brouwer behaved provokingly with respect to the group, perhaps his words were taken more seriously than the group considered proper. It may be remarked that the group was considered with a measure of reservation by by some of the colleagues; the unwavering righteousness of a number of members of the group could easily be confused with arrogance.

On April 6 the Secretary General of education, Prof. van Dam, went on the air to address the nation, explaining the meaning of the declaration and exhorting the students to sign. One day later, Bolkestein, the Dutch Minister of Education in exile,[52] presented the views of the legitimate government on the declaration in a

[51]The fact that Brouwer's role in the Senate is discussed here, should not give the reader the impression that he was a prominent member. Her used his right to speak and to submit amendments like everybody else. That he is mentioned so often in the context of the Senate, is because he happens to be the subject of the present biography.

[52]Not to be confused with Brouwer's fellow student H. Bolkestein, the later history professor in Utrecht, cf. [van Dalen 1984], [Wiessing 1960].

broadcast from London. Bolkestein was quite explicit in his address, 'The country (*het vaderland*) and the government also demand that the president curators and the rectors of the universities abstain from any cooperation concerning the acquisition of the declarations', and to the students he said: The country demands that you don't do this. The general interpretation was that the London government went further than a fatherly advise, that it in fact ordered the rectors and students to ignore the loyalty declaration. In how far this is a legally correct view is debatable, but those who did sign the declaration probably viewed the London parole as just a patriotic advice. At this point there was at least no longer any misunderstanding, clear instructions were given, albeit by London radio and imperfectly received[53] by a part of the Dutch population.

Reading accounts of the Senate meetings discussing the declaration of loyalty, one should not get the mistaken idea that it was a topic of calm debate among the Dutch population. Every measure, be it directly proclaimed by the German authorities, or indirectly by the Dutch, acted divisively. A good patriot rejected it out of hand and if you did not, you were almost certain to be considered guilty of high treason (*landverraad*). There was good and bad, and nothing in the middle. Brouwer, the individualistic thinker, either was not aware of this wartime dichotomy, or he chose to value his intellectual integrity over the wartime national feelings.

As we have seen, Brouwer thought that it was of the greatest importance that the students should not be deported to Germany, moreover he considered the signing of the declaration an act without legal justification, and hence without consequences. In the opinion of a number of members of the Senate, the declaration could at any moment be renounced by those who had signed it. It is in this respect instructive to note that even some staunch Marxists thought the signing unobjectionable, and even good strategy.

Brouwer went so far as to go beyond the advice of the Senate, he posted on April 8 at the bulletin board of the Mathematical Institute a note informing that in his opinion and that of his colleagues, the students could sign the declaration without scruples.

> The undersigned are of the opinion that the obvious interpretation of the declaration of loyalty given to sign to the students, is such that students can sign this without any essential objection, and that their own interest and that of the country requires that they sign the declaration.
>
> In the hypothetical case that students who have signed, feel that they can no longer endorse it, they can always revoke their declaration and discontinue their studies.
>
> L.E.J. Brouwer
> E.M. Bruins
> A. Heyting

[53]The broadcasts from England were continually jammed. Listening to the English radio was moreover strictly forbidden by the Germans.

The note was removed at Brouwer's orders on the 12th; on the 19th the note was again put on the bulletin board. On that day the Senate was convened to discuss a letter to the Secretary General of Education, dealing with the ordinances of the Commissioner of State and the declaration of loyalty. The authorities forestalled a possible protest by the Senate by simply forbidding the meeting. This did not stop the rector to dispatch the letter that had been drafted, but no discussion and no amendments had been possible. The letter stressed that since no answer had been given to the preceding letter, the Senate saw no grounds to resume teaching. Brouwer had, as he stated later in a letter to the postwar tribunal, expected on this particular senate meeting would settle the 'sign' or 'not sign' advice. So when the meeting was cancelled, he decided to stick to his old position. Apparently he was so poorly informed that he was not aware that the deadline for signing was April 13. Brouwer could not have known the contents of the letter of the rector, but he felt that the students had every right to expect an advice from their mentors, and so he reissued the old advice, which represented his sincere opinion. He did not consult Bruins and Heyting at this occasion.[54]

It was his bad luck that Brouwer, although he, Bruins and Heyting had heard about the radio address of Minister Bolkestein, did not, according to a later statement of his, know the contents. Indeed, most professors who advised the students to sign, contrary to the instructions of the government in exile, later pleaded ignorance of the details of the radio speech. Given the difficulties to listen to '*Radio Oranje*', the London broadcast of the government in exile, it was indeed quite likely that a large portion of the population did not learn the details of the address, but the general contents must have been discussed extensively, if not freely. Nonetheless, a legalistic person could (and perhaps rightly) say that in such an important matter one could not act on the ground of mere hearsay. It remains a matter of conjecture, in how far all these professors and academic authorities were truthfully ignorant of the content of Bolkestein's address.

The rest of the story of the University of Amsterdam in wartime is simple. The authorities ordered the university to resume its activities in June 1943. Some 19 professors and eight lecturers resigned in protest; the Germans simply refused to accept the resignations and General Rauter, known for his ruthlessness, telephoned the university on May 12 at 11.30 that the resignations should be withdrawn before three o'clock. Otherwise summary justice would be carried out. All resignations were withdrawn.

Heyting had at an earlier stage informed the senate that he would under no circumstances resign, had he foreseen the German reaction? There is a Dutch proverb '*Beter blô Jan dan dô Jan*' (Better a living coward than a dead lion), and at this stage most academics followed that adage. The reader should, however, be aware, that there was a sizeable part of the academic community that joined the resistance and took considerable risks, often to a heroic and tragic end. The skir-

[54] Brouwer to College van Herstel 30.VIII.1945.

mishes of 1943 ended in a kind of stalemate.[55] The Germans had ordered the universities to carry on as usual, be it that only part of the former student body was allowed to attend classes and work in laboratories. The London government, on the other hand, did not allow any teaching to students who had signed the declaration of loyalty, and the resistance made it clear that teaching to 'signers' was a form of treason. In the confused situation that followed, professors sabotaged their courses, student members of the resistance intimidated signers, ... When professors were put under pressure by NSB students or German authorities, they reluctantly put up a show of compliance, but no successful courses were offered. The students who had not signed were obliged to enlist in the labour service, and, as a rule, to go to Germany. Quite a number of those went underground, hiding wherever they could. Some professors organised illegal classes or private instruction for the non-signers. Brouwer and Heyting played double roles, they kept the mathematical institute going, lectured for signers, but they also instructed non-signers, and conducted illegal examinations. At a certain point, when the public transport had more or less broken down, they rode together on a tandem to Amsterdam for illegal teaching and examinations.[56] The conduct of the universities and their various governing bodies, and of individual scientists as well, was of vital interest to the student resistant movement. The discussions and decisions in the matter of the declaration of loyalty were therefore closely watched, and commented in the underground student periodical '*De Geus*'.[57] In the issue of December 7, 1943 it critically commented on the general attitude of the universities and their reactions to the numerous violations of international law and the Geneva conventions, such as the dismissal of Jewish staff members and the declaration of loyalty. 'They think their honour is not involved when they are treated as schoolboys by a patented puppet. They are all the more sensitive to vague threats, in which he [Van Dam] has shown himself to be a master, and which they immediately are prepared to give a concrete form in the eternal "Pistol on one's chest", the only ratio to which the Dutch professor yields ...'

Summing up the situation, *De Geus* listed the names of some professors who carried on their lectures as ordered by the Secretary General. From the Mathematics and Physics faculty in Amsterdam Brouwer, Bruins and Weitzenböck were mentioned by name;[58] this was not just a piece of information for interested readers, but rather a warning, who could, or could not be trusted, and it held a message for the inevitable day of reckoning after the war. In patriotic circles there were bitter feelings indeed against those who collaborated with the Germans. The political picture was for most people a black and white one. Seen in that light the student

[55] See [Knegtmans 1998], p.191 ff.

[56] Oral communication A.J. Abels, L. van den Brom.

[57] 'Geuzen' was the name for the group of freedom fighters (in modern terminology) who resisted the Spanish forces during the dark hours of the eighty-year war, when according to the military rules the official opposition to the Spanish had been decisively defeated.

[58] Weitzenböck had in fact already been put on a black list in 1941.

underground press was on the whole remarkably objective, there was a dignified and earnest effort to rise to a level of serious discussion and comment. It was as if this most serious and desperate situation brought out a higher level of responsibility and moral awareness. The student resistance of the war years bears in no way a resemblance to the general atmosphere of welcome divertissement of post-war student revolts.

Knowing that *De Geus* stuck to a low key review of facts, one might assume that Brouwer (and Bruins for that matter) was sharply criticized by the Amsterdam underground student population.

It is unknown if Brouwer himself was aware of the mentioning of his name in the underground student press, if so, he had reasons to worry. The mood of the Dutch population turned more and more grim and revengeful at every new injustice, and there was a general looking forward to the '*bijltjesdag*' (hatchet day).

The faculties of law and medicine had great difficulty and little inclination to keep the teaching going, the faculty of the sciences decided to carry on as well as possible. In the mathematics department the usual routines were still carried out.

Mathematics at Amsterdam did not fare well in those years. Whereas a certain superiority over the rest of Holland was tacitly recognised before the war, the loss of Freudenthal seriously reduced the status of the department. In fact, it had lost its only modern researcher—Freudenthal, and furthermore the *privaat docent* Belinfante, who was at the time quite productive. The remaining mathematicians were not able to fill up the gap. Brouwer had since long given up the publishing-industry and Heyting was far too busy teaching and taking care of his large family. Weitzenböck was the only remaining productive man, but even he flagged after 1943. Moreover he practised a kind of mathematics that was on its way out. The particular Clebsch–Gordan–Weitzenböck theory of invariants was already past its heyday. Bruins was to be the main exponent of that school in the Netherlands, but resuscitation did not prove feasible.

In general, university teachers tried to survive the war as well as possible. With the exception of those who were dismissed or arrested, they, as a rule, discharged their duties when possible. Research was impossible for the laboratory scientist working at home, but the 'paper-and-pencil' scientists could at least privately pursue their research. This was far from easy, as the daily routine of obtaining food and fuel became ever more time consuming.

A number of mathematicians went underground, among them Van Dantzig. Curiously enough Freudenthal was not directly in danger. He took nonetheless the wise precautions not to expose himself. In general it was wise to avoid trouble spots and to remain invisible at times of razzias.[59] Freudenthal practiced mathematics in private, and he also tried his hand as an author. Under the pseudonym F. Sirolf he wrote a prize winning historical novel. As a Jew he could not go and collect the prize money. Fortunately Willy Bloemendal, who later became a professor in

[59] In Holland the name 'razzia' has since the war been reserved for the raids and manhunts during the occupation.

Delft, was willing to pose as the author, and to collect the prize money. Under the circumstances this required a good deal of courage. The prize, by the way, was five-thousand guilders, a considerable sum for these days (1944).[60] In the spring of 1944 Freudenthal was commandeered to Havelte, a place in the eastern part of the country; there he had to work at an airfield which was under construction. In his autobiographical book '*Write that down, Hans*' he related both the dark and light sides of the camp. One anecdote may be included here: there was a simple trick that Freudenthal applied to get a bit of rest, he picked up a spade or some tool and walked purposeful from one end of the field to the other. Any guard who saw him pass, thought 'this man is on his way to a specific job.'[61]

On *Dolle Dinsdag* (mad Tuesday) September 5, 1944, the day that a panic broke out among the Germans and their followers in the Netherlands, because of persistent rumours that the allied forces were about to liberate Holland, Freudenthal decided that he had quite enough, and he absconded. In Amsterdam where the worst was still to come during the so-called hunger-winter, he and his family waited for the liberation.

Of Brouwer's scientific activities little or nothing is known. In view of his post-war publications, one may conjecture that he returned to his old intuitionistic research and started to grapple with loose ends. In 1944, on April 28 he fell again victim to a fire in his house in the Torenlaan.

In 1943 the man who in Brouwer's words had 'added many mathematical theories, as monuments of crystalline simplicity to the spiritual property of humanity,' and who regrettably had become totally estranged from his former Dutch admirer, died in a Germany that had undone his life's work. His Göttingen had been replaced by a caricature of the once so superior mathematical establishment that was largely his making. Hilbert had no longer played an active role in mathematics in the Third Reich. He had remained the chief editor of the *Mathematische Annalen*, but rather more as a symbol. Surprisingly, Blumenthal had been allowed to carry on in his function as an editor until 1939. The last big enterprise of Hilbert, the completion of the comprehensive exposition of mathematical logic and proof theory, had seen its second and last volume in 1939. The scientific-political views in Nazi Germany were not exactly positive on Hilbert's formalism (cf. p.699), but the support of the majority of the mathematicians, and the immense status of Hilbert, seems to have outweighed political opposition to his programme.[62] This was made clear on Hilbert's eightieth birthday, when he was awarded the Goethe Prize by the Führer Adolf Hitler.[63] Süss, the politically prominent mathematician in the Nazi period, only saw the negative interpretation of this gesture: according to him it

[60] [Freudenthal 1987b], p. 214, 215.

[61] The anecdote has a follow up. When he told this story at one of the many workshops in Oberwolfach, some of his German colleagues found it difficult to accept that one would not carry out given orders.

[62] Cf. p.700.

[63] Thus sharing this dubious honour with Vahlen, cf. [Inachin 1960].

illustrated the lack of status of mathematics, as 'the most famous mathematician, David Hilbert, was not honoured by the applied for "Adlerschild" ' (Insignia of the Eagle), but, just like 'hundreds of respectable and diligent professors, fopped off with the Goethe medal.'[64] Knowing the preoccupation of the Nazi party with political correctness, this event can only be interpreted as a clean bill of health for formalism, signed by the highest authority. In view of the aggressive criticism of formalism, this Goethe Prize has its political significance, cf. p. 700. It would be totally wrong to draw any personal conclusions in this case. Hilbert was already for some time out of touch with the world around him, and one may wonder if he was conscious of the political background of the prize. After seeing his mathematics department ruined by the Party, there is little likelihood that he could be placated by an honour from the hands of the perpetrators. In scientific circles the event drew little attention, only Lietzmann mentioned it in the paper dedicated to Hilbert's 80th birthday[65] In 1943 Hilbert died and he was buried in Göttingen.[66] The sad news travelled fast, and all over the world his death was commemorated.

17.7 The Brouwer family in wartime

If one forgets for a moment the hidden and open atrocities perpetrated against Jews and all forms of opposition, the war years presented the view of a gradually worsening economic and civil crisis. A citizen who held no political or social responsible position, and who did not read the underground press, nor listen to the London radio, had no Jewish friends, would only see the decline in consumer goods, personal facilities and the like. In the first two years the food rationing restricted the variety in the shopping basket, but still one did not have to go hungry. It became difficult to buy shoes, clothing; vacations became a rare luxury; spare parts for almost everything became scarce. But in the beginning of the war years, frugality had already become a way of life to a population that had already gotten used to the hardships of the big Crisis of the thirties. Later, public goods, such as gas and electricity became scarce, and eventually were cut off altogether. Curfew restricted social contact; harassment by all sorts of all kinds of military and semi-military organizations increased. And finally, in the last year, survival became the immediate concern. In addition there was the systematic confiscation by the authorities of private cars, horses, radios (it was not only a crime to listen to the BBC, but even to possess a radio), bicycles, brass objects, church bells, and so on. Parts of the country were made restricted military areas, Nazis swaggered on the streets looking for victims.

And after the great railway strike (called by the London government in exile in September 1944, after the invasion in France) almost the total work force of the Dutch railways went on strike until the end of the war. The Germans retaliated with large scale robbery. Against this background one has to imagine daily life. And

[64] [Remmert 2005A].
[65] [Lietzmann 1942].
[66] Cf. [Braun 1990].

it was not different for the Brouwer family. Lize was still spending most of the week in the pharmacy and Brouwer commuted between Blaricum and Amsterdam, until finally public transport broke down. Already in 1941 the trip to Amsterdam was an uncertain and arduous adventure. Lize, in one of her letters to Louise, wrote that travelling back and forth cost Brouwer sometimes five hours a day.

The running of the pharmacy absorbed a lot of energy, finding new assistants, interviewing applicants, maintaining a reasonable stock of medicaments, it all took a lot out of Brouwer. Another urgent problem was the rebuilding of the hut left to the mercy of rain, wind and snow; there would soon little be left of the remnants of Mauve's creation. Brouwer pointed this out to the mayor. The preservation, he wrote 'anticipating a later rebuilding may, thus be considered in the interest of Dutch economy'.[67] Therefore he asked permission to install a temporary roof. Brouwer was not in luck, because the State Commissioner forbade from July 1 on, any private building activity.[68] Nonetheless the mayor must have considered Brouwer's cottage of economic interest, for on July 29 he got his building permit. Of course, between the permit and the rebuilding of the cottage, there was a long way to go.

During the war Brouwer senior, who was almost ninety years, had moved in with his son. Brouwer never got on with his father, so the arrangement did little to brighten his life. The old gentleman indeed did not spread much happiness. Although a female help was engaged for him, he spent his days grumbling. Even the warm-hearted, serene Lize found it difficult to put up with the demands of the old gentleman. But her own husband was sometimes also a handful. Once, when Brouwer returned from Amsterdam, he caught a cold and ran a high temperature. But Brouwer was not the person to exercise patience and take time to recover. In letters to Louise Lize complained about the short temper of her husband, 'Dad still has a fever, but he had absolutely to go to the institute'. And a bit later 'Daddy is not yet well, but he behaves so silly. Goes out for a half hour's walk with a temperature, receives visitors, eats all sorts of things, and then flies into a temper about underpants that have not been properly mended, and then dead tired to bed and complains about his heart. He is terribly tiresome'.[69] The war and the consequences of the fire in the hut made life extremely complicated; the Brouwer couple had more than before to live in Amsterdam, as Lize wrote in a letter to her daughter. Commuting between Blaricum and Amsterdam had become so difficult now that the usual connections were scarce and uncertain.

Tensions ran high in the small family; Brouwer could not suffer his step daughter under normal circumstances, but the close contact that was the result of wartime restrictions, was more than he could bear. It came to explosive scenes between him

[67] Brouwer to Klaarenbeek 30.VI.1942.

[68] To be specific, 'Requests for exemption, which are not clearly motivated by economic or military arguments, according to art. 1 of the regulation of the Führer concerning the protection of armament industry of 29.III.1942, are under penalty of detention, and in particularly serious cases of death'.

[69] Lize to Louise 16.XII.1941, 18.XII.1941.

and his stepdaughter, each blaming the other for all kinds of misery. A complicating factor was the financial motive: Louise's father had, when he died, left 10,000 guilders to Louise. Lize and Bertus had invested the money in an annuity, and not trusting Louise's worldly wisdom they had built in a clause against pressure from third parties to hand over (part of) the annuity. Apparently Louise had not quite understood what had happened, and she had become convinced that her stepfather had used the money for his own purposes. Lize did her utmost to calm the child, 'I am so afraid that you will get insane from whipping yourself into a frenzy all the time. For Dad has not appropriated your money, he is not using it for the hut, but he has invested it and that has always been his intention.'[70] Lize was in the unfortunate and uncomfortable position of the conflict between her feelings as a mother and as a wife. She tried, with moderate success, to intervene between Bertus and Louise; but even she could not prevent clashes. Moreover, even to her Bertus could be quite unreasonable.

Mother and daughter had a regular correspondence, albeit that Lize was the more active writer. The letters confirm that the relation between Brouwer and his stepdaughter had not improved over the years. Lize always tried to improve the relation, 'And, my dear child, try, for once, not to be so angry at Daddy. It is so bad for you and it causes you so much stress which blocks your recovery.'[71] Indeed, Louise was the permanent victim of all imaginable inflictions, the legs, the stomach, the throat, …, it was all in a state of perpetual ill health. In how far some afflictions were imaginary or exaggerated is hard to say; some of them were doubtlessly genuine. But being a martyr suited her wonder well.

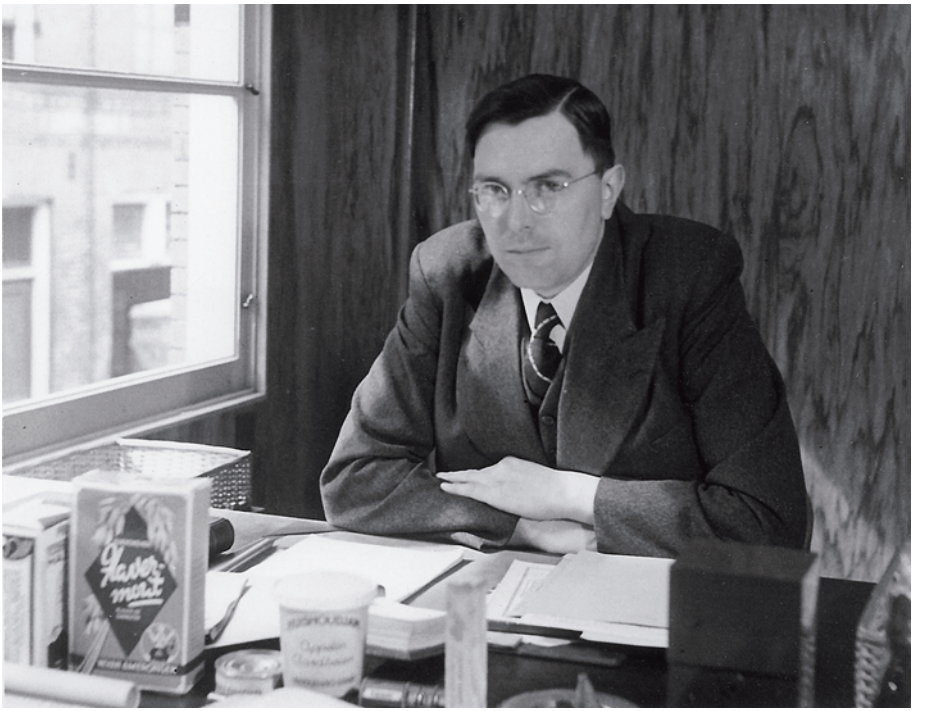
During the war no more international trips were possible, and so in the summer of 1942 the Brouwers took their vacation in a guesthouse in South Limburg, where they made short walks. Lize had to take it easy with her seventy-two-years. Brouwer accepted the limitations, trying to work in spare moments; 'Daddy is quietly doing his mathematics in the small dining room', Lize wrote to Louise.

The final years of the war brought extreme hardship to a considerable section of the population. It became quite normal for city dwellers to make long trips into the country to obtain food from the farmers either for money, or by barter. Under these circumstances not much time was left for intellectual activity. Heyting, e.g., reported that the quest for food and fuel took so much of his time and energy that he simply had to give up mathematics temporarily. In an area like Blaricum trees were rather abundantly present and an inviting commodity for the villagers who had no coal to heat their houses. It is characteristic for Heyting's unparalleled honesty that he asked and obtained Brouwer's permission to chop down a few trees at his estate. Brouwer himself, of course, was also heating the various houses and cottages with wood of his own trees.

These same little houses were used for hiding persons who were at risk, be it for evading labour service in Germany, for being Jewish, for belonging to the

[70] Lize to Louise 20.VIII.1942.

[71] Lize to Louise 20.VIII.1943.



Euwe, during the war, as president of the grocery chain Van Amerongen. (Courtesy Mrs. S.M.P. Brouwer-Euwe).

resistance movement, or simply for being a male of the appropriate age group. Under the wooden floors hiding places were dug out and one got access to the space underneath by lifting the planking. Riet Musters, the daughter of a neighbour, recalled that during the last years of the war these hiding places were used (often at night); it fell to her to clean the houses during the day.[72] Her memories of Brouwer are those of a kind, somewhat introvert man, who walked his grounds reading a book. The children of the adjoining houses used to play on Brouwer's grounds; if they met Brouwer, he would greet them and immediately warn them to stay on the paths. There was a modest sized wood behind Brouwer's house, called *Brouwer's copse* (Brouwer's bosje). Sometimes, the weather permitting, Riet's father would get out a pair of folding chairs, and sit down with Brouwer in the copse to discuss whatever subject came up. Lize, as recalls it, was rather strict with the children; Brouwer had given them permission to come to the back door, however, if Lize found them there, she invariably wanted to send them away. Lucky for them, Brouwer would often come to their rescue, and say, 'you cannot send these kids away without candy!'

[72]Interview R. Musters.

Brouwer was lucky enough to have good friends and ex-students to keep an eye on him. Max Euwe gave up his job as a mathematics teacher at a Lyceum for girls and became the director of VANA, a chain of grocery stores in Amsterdam. In that capacity he could from time to time provide the Brouwer household, just as he had done for the Freudenthal family, with groceries that had become hard to obtain; the right that coupons were supposed to give was often illusory.[73]

Also in his quality of trustee of the Hilversum gymnasium, Brouwer saw the new authorities at work; Jewish students were expelled, teachers were fired or taken hostage, the building was eventually requisitioned by the German army. In the middle of the war there was even a brush with the local authorities; the council wished to reduce the power of the trustees to that of a mere advisory board. It goes without saying that Brouwer without reservation took the side of the trustees.

17.8 Weitzenböck in uniform

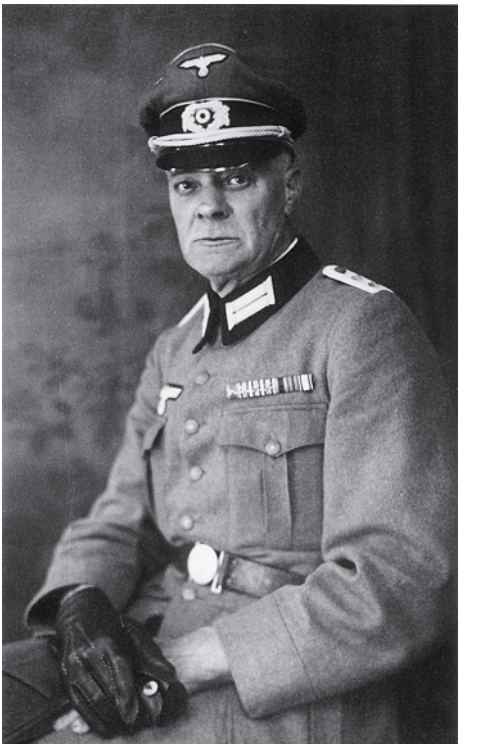
There was one mathematics professor in Amsterdam for whom the war held a vista of redressed injustice. Weitzenböck had welcomed a reborn Germany under Hitler and the union of Germany and Austria. His political conviction, which he did not try to hide, had made him extremely unpopular in Blaricum, and at the university students and staff members had started to avoid him. There were no open conflicts, nor were there any reports of unfitting manifestation on Weitzenböck's side. He did not betray Jewish students or colleagues, nor did he report any resistance activities that came to his knowledge.

His position in the faculty had always been loyal and neutral. Although he owed his position to Brouwer, he was not owned by Brouwer. When he thought Brouwer was wrong, he did not hesitate to withhold his support in the faculty. Max Euwe, who was politically above any suspicion, and who was actively involved in the post-war 'cleansing' (*zuivering*), pointed out that the Marxist Mannoury and National Socialist Weitzenböck were friends before the war. During the war Weitzenböck used his influence to protect Mannoury.

When in May 1940 the German army invaded the Netherlands, Weitzenböck was already known as pro-German. In his own words, '... initially it was a strongly unpleasant surprise for me, but later I had to reconcile myself with the necessity of the attack as part of the campaign against France. At that time I saw the realization of a German victory as of necessary importance for Europe'. After the capitulation in May, he joined the NSB, from which he resigned again in September 1941. In 1942 Weitzenböck acquired German nationality. As a former officer of the *(k. und k.)*[74] Austrian army, he became a reserve officer in the German army.

[73] Euwe also provided each week a meal at his house for Karel van het Reve in payment for lessons in Russian. After the war Van het Reve became well-known in Holland as an author; he became the professor of Slavonic Languages in Leiden. Euwe profited from the lessons in Russian when he became the president of the FIFA, the international chess association.

[74] Imperial and Royal.



Roland Weitzenböck at the end of the war. (National Archief, Den Haag)

Although he was from that moment a German subject, he did not join the NSDAP.[75] In 1942 a *Schützgruppe* (guard unit) was formed, and Weitzenböck was detailed to it. He did not take part in the various exercises of the group. In 1944 the war had reached a stage where the army started to scrape the bottom of the barrel. The unit became active, uniforms were issued and Weitzenböck had to report in Hilversum, where he had to replace the commander of the quarter office (*Quartier Amt*). In this quality he had to requisition housing for army units. This was of course a rather painful matter; nobody enjoys being expulsed from his house for the enemy, but it amounts to adding insult to injury when the requisition-officer was a fellow villager. It hardly needs to be said that Weitzenböck was hated by the local population. He did not go out of his way to provoke the villagers, but he considered it, for example, his right to put out the Swastika flag at Hitler's birthday, and according to the records of his post-war interogation, he once did so. It was clear at the end of 1944 that it was only a matter of time before the German army would collapse, nonetheless Weitzenböck stuck to his obligations as an Austrian-German subject. After Hitler's death, the small group of Weitzenböck's unit, consisting of the older

[75] National Socialist German Labourer's Party.

generation (Weitzenböck was almost sixty at the time) was ordered to take the oath on Hitler's successor, admiral Dönitz, in the reading room of the public library in Hilversum on 4 May. The next day the capitulation of the German army in the Netherlands was signed, and Weitzenböck's deplorable adventure had come to an end. He had lost a son and his wife in the bomb attack of 1940, and his second son had fallen on 13 February 1944 in Tserkassie on the eastern front—a high price for a dubious political conviction.[76]

All testimonials confirm that Weitzenböck preserved a high standard of integrity. No reports of acts against persons is known. In the village and at the university he was given the cold shoulder, and so he must have felt rather desolate. He was probably aware of the underground activities that were going on at the mathematical institute, indeed some witnesses at the post-war trial said as much. 'He was definitely aware that in the mathematical institute things were going on that were connected with the illigal movement. From certain statements it even appeared that he knew that Jews were hidden in the institute'.[77]

He never alluded to the activities of the resistance movement in the institute; he may even have given (unintentional) protection, as his presence lent the institute a certain respectability in the eyes of the Germans.

Brouwer also ran a certain risk through the underground activities. If the Germans had found the explosives, counterfeiting material and whatever was stored and used, it is unlikely that he would be spared. The illegal activities are confirmed from various sides, the building with its nooks and crannies offered ample space for hiding. The janitor, Koppers, was the key figure in the whole organization. It is reported that he was a bit careless where security was concerned. Eventually he was arrested, taken to Germany from where he did not return. Bruins told in his valedictory lecture the other part of the story. He was in Limburg, conducting examinations, where the news of Koppers' disappearance reached him. He hastened by train back to Amsterdam and managed with another person to remove the explosives from the institute's coal cellar. The Germans, who raided the building found nothing. There is a private note of Mannoury on the incident: 'From Mathematical Institute ammunition removed in as far as possible. Later there appeared some to be left.'

[76]The information on Weitzenböck's political and military activities is drawn from the file in the *Algemeen Rijks Archief* in The Hague.

[77]E.M. Bruins in 'Proces Verbaal, 24.VII.1947.

18

POSTWAR EVENTS

18.1 Purging the university

The winter of 1944/45 is remembered as the 'hunger winter' (*hongerwinter*); it reduced the western part of the Netherlands to a state of malnutrition unparalleled in modern Dutch history. It also brought a fiercer repression. And finally the long awaited capitulation arrived: on May 5, 1945, the German commander General Blaskowitz, signed the capitulation of his army in the Netherlands, and three days later the German army as a whole laid down its arms. The liberation brought rehabilitation to the victims of wartime persecution, and retribution to the traitors, members of National Socialist organisations, and the like. Up and down the country the former resistance forces, their ranks swelled with many an opportunist, filled the jails and camps with the creatures of the defeated enemy. Tribunals were everywhere installed and the process of bringing this strange assortment of suspects to justice commenced. Indeed the variety was perplexing, among the new population of the prisons and camps there were notorious politicians, secret agents, Jew hunters, collaborationists, builders of fortifications, but also quite simple souls who had joined the NSB for naive political reasons and who lacked the degree of opportunism to leave the sinking ship in time.

The end of the war also ended the isolation that was caused by a gradual and almost complete breakdown of the postal services. All over the world people were anxious to learn about the fate of their friends and relations. There was a great deal of correspondence between German emigrants and their colleagues who had stayed behind. A large number of letters was exchanged, bringing sad news of the victims of the war or happy news of the survivors. Brouwer's contacts with the world beyond the Dutch borders were also revived; his former student De Loor from South-Africa, for example, inquired after the fortunes of his old Ph.D. advisor. In his answer Brouwer wrote that a horrible period had come to an end, and that fortunately he and his family had been spared death and imprisonment. Mannoury and De Vries[1] had survived, but, he sighed, Johan Huizinga had died during the war—'His death is a great loss for the Dutch nation; of which he with his three aspects of humanist, literator and scientist, was one of the most representative elements.'[2]

Unfortunately his house had been on fire twice, and in addition part of the furniture was stolen,

[1] Hk. de Vries had moved to Palestine before the war.

[2] Brouwer to De Loor 20.VI.1945.

> as a consequence my scientific archive and with it the manuscripts of my unpublished research have been lost. And in order to be able to reconstruct the content of those from memory (which is already gradually darkening at my age of 64), a period of leisure, and free of care, should be my share before long; there won't be any chance for that in my native country which will only very slowly raise itself from the state of material and spiritual upheaval, resulting from five years of plunder and terror. Where, as I believe, the interest of science would be served by my temporary expatriating, the plan, which we discussed in the past, to create at some time a temporary position in South-Africa, is again in my mind (and this time much more vivid then at that time) and I would even greet a speedy realisation of it as a delivery.

The reaction to this letter is unknown, but little did Brouwer know what the future had in store for him. A similar sentiment was expressed in a reply to Arnold Dresden's inquiries after the war;[3] it sheds an interesting light on Brouwer's global political views:

> . . . Yes, the securing of peace here also absorbs a large part of the available intellectual capacity. The first requirement seems to me the abolishing of the splitting of the earth into various areas with separate centres of military power.
>
> In anticipation of the realisation of this, the United States together with the other American States, the British Empire, Scandinavia, Switzerland, Austria, Italy, Spain, Portugal, France, Belgium and the Netherlands, should in my opinion without delay be united into a single state as far as military matters are concerned . . . In the meantime, I would, fearing that the unified state will take some time, at the moment (unlike before) gladly emigrate from the Netherlands. If now the possibility of a position was offered in America, I would eagerly grasp the possibility with both hands. For the confused situation during the first months after the liberation has here (also in scientific and academic circles) brought persons to power. . . [4]

The reader may guess the rest of the sentence. When Brouwer wrote to Dresden, he had learned the painful lesson that the status quo had in some respects completely changed. As soon as the Germans had evacuated Holland, special courts and committees started the process of 'purging' (*zuivering*, the equivalent of the German de-nazification).

The general impression of this purge leaves us with rather mixed feelings. The simple harmless followers of the National Socialist banner, whose only fault had been that they had made an objectionable political choice, complained at the severity of their punishment and members of the underground movement grumbled at

[3] Brouwer to Dresden May 1946, draft.

[4] The rest of the draft is missing.

the same time that the guilty often got off far too lightly. A few leading National Socialists were sentenced to death, but the clever operators escaped justice. Later historical research has fully confirmed these facts. On the whole, the practice of exploiting the confusion for personal advantages was frowned upon, but nonetheless, a certain amount of fishing in troubled waters was practised by those with a taste for it. In the process people with enemies could easily suffer. The academic world was no exception; Brouwer, in particular, felt that his treatment was largely the effect of old feuds and jealousies. Looking back at Brouwer's postwar treatment, it is indeed not so certain whether there was not a poorly veiled desire to isolate the undoubtedly great, but also inflexible, man.

The purge of the universities was energetically undertaken by so-called 'Committees of Restoration' (*College van Herstel*). At the University of Amsterdam this Committee consisted of the following persons: C.T. van Valkenburg, G. van Hall and W. Bruin. Van Valkenburg was a neurologist who had been active in the resistance, and who, during the war, had been in prison for some time. He had been passed over a few times for a professor's appointment. Academic circles entertained doubts as to his appreciation of the operation of the universities. Wieger Bruin was a well respected artist; he was appointed by the Chief of the Military Authority, Christiaan Kok, to counterbalance the hard liners Van Hall and Valkenburg.

The Committee was assisted by a committee of professors and a similar committee of students. Its task was to advise the temporary authority, the so-called 'Military Authority' (*Militair Gezag*), which for the time being exercised all military and civil authority in the Netherlands, with respect to the purging of the members of the University of Amsterdam. The legal steps, such as dismissal, reprimand, loss of pension, were in the hands of the Minister of Education, so the Military Authority in turn had to report to the minister. As we will see, the minister and the Military Authority were not always of one mind. The *professors' resistance group* was quick to form the advisory committee and the students followed suit. These committee's were in a fairly good position to provide information and to collect evidence on their colleagues and fellow students.

In the excited atmosphere that followed the liberation, the thought of justice and revenge dominated the minds of most Dutchmen. Having experienced five years of perversion of the citizen's rights, there was a strong tendency to give Nazis, collaborationists, black marketeers and in general all those who had shown a lack of patriotism, a taste of their own medicine. On the whole the special courts of justice observed the law well enough, but outside the courtroom often popular justice was meted out.

The advisory committee of professors had a precarious task, it was so close to the persons it had report about, that objectivity became a precious commodity. Not being trained in legal traditions, the committee could not agree on a policy on confidentiality. Professor Derkje Hazewinkel-Suringa, a member of the Law Faculty, attached importance to it, but her view was not generally shared. Borst, a prominent professor in the medical faculty, for example, was of the opinion that the purge as an expression of the national conscience was of such an importance that

each individual injustice became insignificant in comparison.[5] In fact, from a legal point of view, much about the purges, carried out in haste and in a grim mood, did not meet the standards of normal justice.

The first act of Brouwer's part in the purge opened on July 3, when the commander of the Blaricum squad in charge of arresting suspects with a questionable war-time record (the *arrestatieploeg*), dispatched the membership cards of the *Nederlandse Volksdienst*[6] of Brouwer and Weitzenböck to the Mayor of Amsterdam, in his function of president curator of the University of Amsterdam. These records only added to the complaints that were being brought against Brouwer. Both the professors and the students had weighty objections to Brouwer's wartime conduct.

Already on July 7 the Student Contact Committee, in answer to questions posed by the Committee of Restoration, informed the Committee that 'it would greatly astonish the students of the faculties concerned if [...] H.A. Brouwer, L.E.J. Brouwer, [...] would without further consideration be put in a position to resume their teaching.' 'A large number of students', it added, 'considers these professors by definition as unacceptable (these numbers are not negligible).'

The Committee of Restoration wished to push ahead with the investigations, a long drawn procedure would stand in the way of a speedy recovery of the university. The Committee of Restoration, once it had collected incriminating material and accusations, started to summon some twenty staff members. Brouwer's brother Aldert was interrogated on July 11 and two days later it was the turn of the mathematician Brouwer. On the seventeenth he had to appear again before the Committee; Bruins and Heyting followed on that same day.

The interrogation was a summary affair, in fact four questions and their answers were recorded. The actual conversation may have been more extensive, but the Committee apparently concentrated on these particular key points.[7]

The first question dealt with the declaration of loyalty. Brouwer stuck to the position taken in the Senate. He argued that the signing of the declaration had material advantages for the Netherlands. Not signing would have resulted in large scale forced labour (*Arbeidsinzet*), with disastrous consequences for the health of the students. Moreover, a student who signed had a better opportunity to take part in underground activities. He declared, that after the bulletin board episode he had never personally advised any student to sign. He insisted that at the time he did not know the precise contents of the London broadcast.[8]

The second question concerned his position in the Senate. Had he agitated against the resistance movement of the professors? This clearly referred to the Senate Meetings in March and April 1943 when the declaration of loyalty was discussed. Brouwer's reply here was: 'I and many others did disagree at times with the professors' resistance movement. In March 1943 a draft of a letter to the

[5][Knegtmans 1998], p. 241.

[6]Cf. p. 742.

[7]The records of these interrogations are kept in the *Gemeente Archief Amsterdam*.

[8]Cf. p. 757.

Commissioner of State was laid before the Senate, with the final clause, that a rejection of our demands would lead to the resigning of the Senate members. I opposed that, because in my opinion one could not request and threaten at the same time. The majority of the Senate agreed with me.' One might well question his argument. After all, bargaining traditionally consists of a certain degree of 'carrot and stick'. Perhaps he judged a collective resignation so unacceptable to the Germans (which it turned out to be), that one could no longer speak of bargaining. He probably felt that his argument could do with some elaboration, for in a letter he tried to spell out his reasons with more force,[9] the letter lined up all the socio-philosophical ideas from earlier days, in a defensive array:

The declaration of loyalty 1943.

> When a company of civilised travellers is overpowered by superstitious cannibals, their conduct, in particular their language, or gesticulation, exchange with their captors, will be exclusively aimed at their deliverance. Guile, deception and dissimulation will be their tools, both with respect to communications, and proposals and promises. Honesty, chivalry and demonstrative testimony will not only be practically improper because of its counter-effecting the goal, but also be without reasonable content: for the essential binding of the meaning of word gesture or sign, essential for honest understanding, is only possibly on the basis of tacit co-operation of the communicating parties as 'good listeners' and this co-operation can only derive its moral justification from a (in the case under consideration missing) common orientation of will.[10]
>
> A situation of that kind existed in the Netherlands during the occupation. The manner, in which the enemy had descended upon us and in which he subsequently had violated the good faith and trampled on the human rights, had on the one hand eradicated any common orientation of will or respect between the Dutch population and the occupation forces, on the other hand had directed the conduct of the Dutch population with respect to the occupation forces exclusively towards the following goals:
>
> 1. to serve the occupation authority as little as possible,
> 2. to harm the occupation authority as much as possible,
> 3. to safeguard as well as possible our national heritage against destructive interference of the occupation authority.
>
> And in this context guile, deception and dissimulation were for the language- and gesture- exchange of the Dutch population with this enemy with respect to communications, proposals and promises proper on the

[9] Brouwer to Comm. of Restoration, 17.VII.1945. Either written after the interrogation, or prepared beforehand.

[10] [Brouwer's footnote] Cf. my lecture '*Willen, weten en spreken*' (published in '*De uitdrukkingswijze der wetenschap*', Groningen, Noordhoff, 1933, in particular I.3

> grounds mentioned in the preceding paragraph; honesty, chivalry and demonstrative testimony had, on the contrary, become both practically improper and lost its rational content.
>
> In the light of this exposition, it is therefore incorrect that, as was claimed at the time, that the signing or not signing of the declaration of loyalty by Dutch students involved ethical or ideal goods of the Dutch community. On the contrary, there was the possibility, that a general signing would have the following consequences:
>
> 1. less Dutch labour would serve the enemy.
> 2. the students who worked in the underground movement would have better conditions for their work under the guise of loyalty.
> 3. the health and intellectual training of the Dutch students would on the whole be less harmed.
>
> At that time this had to imply, that a general signing of the loyalty declaration would have served both the interest of the students and of the nation. And this conclusion I could not, therefore, when it pressed itself on me, keep concealed, because the venerated tradition (in particular in the Dutch community), which not only designates the expression of a sincere opinion as an inalienable right, but also as an imperative duty in matters of the general interest, represents to me one of the most valuable goods, which made that I have felt lastingly bound to the Netherlands, also against personal material interests.

One wonders if the members of the Committee found time to read this document, and if they did, whether they accepted it at its face value, or whether they judged it an elaborate excuse of an old professor.

Reading Brouwer's statements at the interrogation and his written defence, one can only conclude that Brouwer had no intention of renouncing his war-time actions and statements. He did not wish to don the garb of penitence in order to mollify his interrogators. If anything, he was still convinced that at the time it was the best thing for the students to sign the declaration of loyalty, although he seemed to admit that had he known the text of the ministerial address on the topic of declaration of loyalty, he would have acted accordingly.

There is always the difficulty of analysing someone's statements in the light of past and present experience. Of course, there is a possibility that Brouwer invented the above justification after the events. It does not seem plausible, however. The arguments are too much in accord with his earlier philosophical considerations, and in the Senate Meeting of April 1943, he had already used a similar, but abbreviated argument. The true explanation is probably that Brouwer really had a convincing private social theory for the optimal strategy in the case of a formidable enemy. The weakness of his own argument is that he allows the population to practice deception to oppose the enemy. Why, then, should the enemy not do the same—as he indeed did. In this case, supposing all students had signed the declaration, what would stop the enemy from nonetheless deporting them if the war effort made such

profitable? In short, there is no reason to accuse Brouwer of insincerity, it is rather short-sightedness, or perhaps blindness, which he is to blame for.

The remaining two questions of the Committee were: 'Have you lectured for students who signed?' and 'Have you had contact with Professor Weitzenböck?' The answer to the first question was 'yes—but poorly!' Indeed, after the statement of the London Government, the students who signed the declaration had been marked as pariahs and it was considered unpatriotic to teach them. The general reaction at the universities had been mixed. Some professors gave classes for students who had signed, and at the same time conducted illegal classes for the others; some did not teach at all, but sabotaged their classes right from the beginning. Brouwer had also privately instructed students who went underground and examined them. Heyting went so far as to visit students at their home and offer his help, sometimes to great discomfort of the parent who opened the door[11]—how was one to know if this was a real academic, and not an agent provocateur?

Brouwer's answer to the last question was—'after he turned nazi, only professionally.' Brouwer did add that he considered Weitzenböck a great mathematician and a good person.

Early August the Committee of Restoration had sent a first tentative report to the Military Authority. It advised to dismiss a number of professors and lecturers (among them Weitzenböck and Bruins), to reprimand some of them (among them Heyting) and to suspend some for a period of further investigation (among them Révész and the brothers Brouwer).

The Military Authority acted promptly and took its decision on August 8. On the 14th the bailiff showed up on Brouwer's doorstep to serve him an order of suspension. Since nobody was at home, he left the order at the town hall. It must have been a shock for Brouwer to find out that he was still a suspect. He drew the conclusion that he was suspended from all public duties, so he also laid down his office of trustee of the Hilversum gymnasium. He rightly considered the suspension a serious blow, if not an insult. It was a painful social stigma, to be considered a Nazi-supporter or a war-time profiteer. The popular view at the time of the purge, was 'guilty until proved innocent.' The general public was not going to be bothered about the finer distinctions of 'being investigated' and 'being guilty'. Another detail that would add to his worries was that, the universities would have to open soon, and one did not have to be a genius to expect that a good deal of organizing and negotiating would follow. Not to be included in the discussions was almost certainly to have direct consequences for an academic, let alone for Brouwer, who had run the mathematics department for years.

Brouwer, who had probably not been worried about the outcome of the review of his wartime record, now began to fear that he might become the subject of a drawn-out investigation and political investigation. Most professors and lecturers, who were subjected to the post-war purge, had been informed on August 24 of the Minister of education's verdict, and subsequently reinstated. So he clearly was

[11] Oral communication J.S. Abels.

singled out for special treatment. He decided not to wait for future developments in subdued resignation, but to present the Committee of Restoration with additional information. Already on 20 August he sent a letter to the Committee motivating the teaching activities at the Mathematical Institute after the London government had forbidden such activities for students who had signed the loyalty declaration (the other students could not risk to attend classes). Had he known that the topic would come up during the interrogation, he wrote, he certainly would have prepared a note on the matter; therefore he had decided to send the motivation anyway.

And so Brouwer came into action in order to prevent a possible conviction and discharge. He started to send letters with additional information and arguments to the Committee of Restoration.

The first letter contained an explanation why the Mathematical Institute was not closed after May 1943, in particular why the lectures were continued (this topic, by the way, was not part of the interrogation on July 17).[12] He explained that after the ban on resignation in May 1943 the faculty members of Mathematics and Physics got together and agreed that in order to protect the instruments, books and the collections, the activities should preferably go on as usual. The argument did not quite fit the Mathematical Institute. This was problematic, for it was mainly used for lectures, which 'formed the camouflage for an underground operation, for which the director[13]had made available space and resources and to which the janitor devoted almost his whole working day, and in which a "docent",[14] as well as a female adjunct assistant,[15] took part.'

Emeritus Professor Mannoury also came to the rescue. He had probably heard that the Committee had not reinstated Brouwer and he felt that Brouwer was being punished for his frank views expressed at the Senate Meetings. He gave his views in a letter[16] which contained a psycho-signific analysis of the situation and the participants. His basic thesis was that in the early days after the announcement of declaration of loyalty the students and staff wavered between (a) an emotional and conscientious rejection of the declaration, (b) a rational and tactical inclination to accept it. The students themselves opted for (a). He argued persuasively that free discussion and forming of opinion are an indispensable good for the sake of upholding democratic principles in the group processes that lead to decisions of the sort involving the loyalty declaration. 'However, it would be in flagrant contradiction with this principle of unhindered opinion forming, to prosecute in retrospect, now that the power of the oppressor has been broken, a number of those who in the stage of deliberation have given their views in good faith and to the best of their

[12]Brouwer to Comm. of Restoration 20.VIII.1945.

[13]I.e. Brouwer.

[14]Bruins

[15]Cor Jongejan

[16]Mannoury to CvH 18.VIII.1945.

knowledge.' He added that 'the advice to sign, in the conviction that the declaration could be withdrawn at any moment, now seems odious to many, is understandable, but it may not be concluded that any sympathy with the enemy was involved.'

Brouwer must have realized by now that the Committee meant business, in view of the kind of advice it had been giving (and which certainly must have been known to Brouwer), everything was possible—ranging from rehabilitation to dismissal. Apart from the shame and humiliation ('there goes that professor who collaborated (was *fout*) during the war'), there were the ominous professional consequences. So to speed up matters and to clear his name, Brouwer started to collect evidence and witnesses of his innocence.

On August 30, he sent a letter with enclosures to the Committee of Restoration that served to explain the circumstances and background of his membership of the *Nederlandse Volksdienst* These, basically, have already been related in the preceding chapter (cf. p.742). Mayor Klaarenbeek confirmed the account that Brouwer had given of the *Nederlandse Volksdienst* episode; a copy of this letter was duly made and dispatched to the Committee of Restoration.

It is probably no exaggeration to use the word 'hostile' to describe the views of the Committee in Brouwer's case. Brouwer was not the person to feign deference for persons he did not respect. His icy politeness could, however, be more forbidding than harsh words, and it is likely that no love was lost between the Committee and the defendant.

That the mathematical institute was used for resistance purposes is an established fact. Apart from the oral tradition there is in this case even some written evidence. The janitor Koppers, who had been arrested on 6 July 1944, was sent to Germany, where he died in a hospital in Hamburg on 7 December 1944. This was reported to Brouwer by another former political prisoner. Brouwer informed the responsible alderman of Koppers' fate, and urged him to grant the widow of Koppers a pension.[17] Koppers had, he wrote, from the beginning of the occupation to his arrest been active in the resistance, 'and in that way saved the lives of hundreds of Dutchmen.' He enclosed the testimony of two former resistance members. One of them, Van der Hurk,[18] had declared in a letter to the Committee of Restoration that 'Professor Brouwer, by making the mathematical institute available for illegal work in the widest sense of the word, has created the opportunity for various acts of sabotage directed against the occupation forces, and for the saving of life and health of a great many fellow citizens.'[19] The obvious corollary was that Brouwer, the director of the institute and the direct superior of Koppers, could not reasonably be suspected of pro-German feelings or actions, since the institute was used with his tacit approval.

[17] Brouwer to City Council 12.9.1945.

[18] A former associate of the famous resistance fighter Van der Veen.

[19] Van der Hurk to Committee of Restoration 12.IX.1945.

After the letters to the Committee of September, there was a long silence. It took its time in the case of the few remaining difficult cases, the brothers Brouwer, Révész and five more members of the teaching staff.

The advice of the Committee was submitted to the minister. The minister had no time for long deliberations, the new term was about to start and teachers could not easily be missed. And so at the end of August he announced that a number of individuals would be *reprimanded*.[20]

Bruins and Heyting belonged to that group. On 3 September the suspension of Bruins and Heyting was lifted. The typical formulation of the reprimand was 'In view of [incriminating facts], the Minister of Education, Arts and Sciences, condemns the position taken by . . . ' In serious cases the minister could write 'strongly condemns', and in mild case such as Heyting's, he would add a sentence 'there is no doubt of the correct patriotic disposition.'

Heyting and Bruins were reprimanded in the light form; Heyting, said the reprimand, had acted wrongly, he had given guidance, but in the wrong direction. In fact, when Heyting was eventually considered for a royal honour, this reprimand was not seen as an obstacle. The official who handled the investigation into Heyting's antecedents, informed the mayor that the Committee of Restoration had allowed Heyting to conduct examinations, while suspended. 'This may illustrate that one did not attach too much importance to it.' Heyting took a very serious view of the purge. He did not have the poise of Brouwer, nor the brashness of his fellow lecturer, Bruins.

As Freudenthal wrote in a letter to Fraenkel, Heyting was not exactly a paradigm of courage during the war. After the liberation he came to Freudenthal, probably to keep him from negative comments. But Freudenthal had, as he wrote 'not for a moment contemplated to register a complaint.' The war had divided the scientific community in good and bad guys. Fraenkel, who was extremely sensitive to Nazi-tendencies, was apparently trying to find out which scientists were still acceptable. He must have asked Freudenthal for advice. 'There are no objections to resume your relationship with him', was Freudenthal's comment.[21]

Bruins was an altogether different fry. When the Committee of Restoration had summoned him, he was not in the least impressed; on the contrary, he went to the attack right away. He denied all charges, explained them as ill-founded or misrepresented, and he presented the Committee with an impressive illegal record. His own testimony shows him as an undaunted freedom fighter.

There is no doubt that he had the making of an enterprising and courageous man. He was very much the master of his own mind and actions; mathematically he chose his own style and topics without caring for the fashion of the times, socially he displayed considerable talents for useful relations, and later for making enemies.

[20]This is an official term (*berisping*) which in the civil service casts a serious blemish on a person's record.

[21]Freudenthal to Fraenkel 15.XI.1945.

He was an easy talker, with a fund of amusing and interesting (although not always true) stories. Fear was unknown to him, and a role in the resistance appealed to him. His wartime connections proved useful in the period of the purge; his sources at the Military Authority (for some time the only real authority in Holland) provided him with valuable information on the activities and plans of the Committee of Restoration. And so, when he was summoned to be interrogated, he was in possession of all the facts. By and large he was in the same situation as Heyting, he had taught students who had signed the loyalty declaration, and he had endorsed the note that was posted after the discussions of the loyalty declaration (cf. p. 758). But there was something else and worse; Bruins had been appointed as Freudenthal's successor. In itself there was nothing wrong with this, after all courses had to be given. The real problem was that he held the position of a dismissed Jew, and the Committee of Restoration had decreed that all appointments of that sort would be reconsidered.

Any Jew who returned after the war should be restored in his old position. In fact most wartime appointees gave up their position without being forced to do so. But Bruins was not prepared to step down, he felt no obligation whatsoever to give up his lectureship. His argument was that his present position was not Freudenthal's old position, and he had a case. After all, when Freudenthal was dismissed, he was a conservator, and Bruins had been promoted to lecturer. And so Bruins saw no reason to give up his position. In a way his argument only served to change the status of his case, for there was a general rule that *all* wartime appointments should be terminated, and only after further investigation the university might decide to continue the appointment. His information on the line of action of the Committee of Restoration was so accurate, that he had even prepared typed replies to the questions that were going to be asked.[22]

Although the Committee failed to impress Bruins, or to solicit interesting information, it was firmly convinced that he should be dismissed as a lecturer. So it advised the minister accordingly; the Military Authority, however, advised the minister not to adopt the Committee's proposal. When the Committee found out that the minister was not going to dismiss Bruins, it made a last attempt to outmanoeuvre the latter. It advised Bruins to step down voluntarily (August 22).

The Committee took the matter of Bruins' appointment in the position, left by a discharged Jew [Freudenthal] extremely seriously. When it asked if he considered laying down his lectureship, he did not even deign it an answer; instead he declared that he was well aware that Mr. Freudenthal was spreading nasty rumours about him. As a matter of fact, the university had immediately after the liberation adopted the policy that persons appointed during the occupation should be dismissed, with a possibility to re-apply for the job. The historian Dijksterhuis lost his position on those grounds. For some reason Bruins weathered the storm: he had found out that the case against him was not strong enough to dismiss him, and so, when the Committee advised him to step down out of free will, he knew that the Minister

[22][Bruins 1982].

would not endorse his dismissal, and therefore he simply ignored the advice. Two days later he got off with the lightest reprimand and a lecturer's post.

Most academic teachers who were appointed during the war, did in fact give up their positions, and those who were 'clean', fairly easily got (new or their old) jobs. Bruins' reasons for not doing the right thing can only be conjectured, but one may safely assume that Brouwer had a vested interest in preventing the return of Freudenthal, so he would certainly not have encouraged Bruins to step down.

A curious case worth mentioning here, was that of Professor Révész, founder of the Amsterdam School of Psychology.

After leaving Hungary, he had settled in Amsterdam, where he was strongly supported by Brouwer (see p. 686). During the war Révész, in the interest of the resistance, cultivated certain contacts with the Germans, and after the liberation he belonged to the small group of suspended professors. After some time the suspension was lifted without further arguments. He understandably insisted that his reputation should be restored; the faculty sent a supporting note to the Military Authority and the Committee of Restoration. Révész had in the meantime started some private investigations into the curious circumstances of his suspension. It turned out that a girl who consorted with a member of the Gestapo, had sometime during the war told Bruins that Révész worked for the Gestapo. The inveterate gossip Bruins had passed the rumour on, so that the Committee of Restoration and the organization, responsible for enforcing the political order (*Politieke Orde Dienst*) picked up the information after the war. It soon turned out to be unfounded. Révész was indeed known to have rescued people in trouble, as in the case of the daughter of the well-known history scholars, Jan and Annie Romein. In 1944 he managed to get her out of the notorious headquartes of the Gestapo in the Euterpestraat[23].

Eventually Bruins himself admitted that the accusation lacked substance.[24] Révész had managed to survive the hardships of the war reasonably well. In spite of his evidently non-Dutch name, and his heavy accent, he had been left in peace. His relationship with Brouwer had always been very close, Brouwer over the years became a good friend of the family, dropping in whenever he felt like it. During the war the frequency dropped and Freudenthal told that after the war he had a long talk with Révész. They (naturally) eventually came to discuss Brouwer. Not surprisingly, Freudenthal related the treatment he had received in the hands of Brouwer, but even Révész complained about Brouwer's behaviour during the war. According to Freudenthal, Brouwer had all but cut Révész dead. Révész was at a loss for an explanation, and conjectured that Brouwer might have thought that Révész was Jewish, and hence dissociated himself from an embarrassing relation. This story conflicts with the testimony of Révész's daughter Judith, who resolutely denied any such estrangement. She had recalled quite clearly that not only during the war, but also after the war, Brouwer visited the Révész family regularly.[25] The conflicting

[23][Romein-Verschoor 1970] vol. 2 p. 52.

[24]Faculty meeting 1.XII.1945.

[25]Oral communication, August 1994.

reports may be due to the stress of the post-war situation. One should also keep in mind that during the war it became harder to move around, and so the frequency of Brouwer's visits may indeed have dropped on the grounds of factual difficulties (e.g. curfew, break down of the transport system).

No progress was made in Brouwer's case; it was apparently no use to try to hurry the Committee. In the beginning of December Brouwer was still in the dark, and so he once more approached the Committee again with a number of facts.[26] He informed it that: 1. he had stopped the publication of *Compositio Mathematica*, 'because I could not possibly have my editorial policy controlled by the occupation authorities', 2. he had tried to stop the creation of a Corporation of Scientific Organisations in the Netherlands[27] and when that turned out to be impossible, had tried to exercise his influence to make the statutes as harmless as possible. When this corporation decided to publish a survey of Research in the Netherlands, he first tried to talk the Mathematical Society out of it, and finally boycotted it himself. 3. he did not attend mathematical conferences in enemy territory, and bluntly told the authorities so.[28] 4. he refused to ask for a reconnection of his telephone line in 1944, when virtually the whole network was closed down. It is a touching detail that Brouwer mentioned the fact that he was approached by someone who tried to persuade him to submit an application for the re-establishment of his telephone connection, and all that was needed was a sort of pledge of loyalty. Brouwer indignantly turned the advice down, he even convinced the man that such things were 'only allowed for the benefit of the community at large and never for the advantage of individuals.' After some discussion the person decided to drop his own request. 5. 'during the occupation I have adhered to the view that persons or groups, that had no direct or official relation to the authorities and did not possess any means of power (e.g. miners and medical men), should direct neither requests, nor admonitions to the occupation authorities. For such approaches could only derive their meaning and content from a basis of understanding between the parties, and indicated thus implicitly the existence of such a basis of understanding, which would have an encouraging effect on the ever present pursuit of penetration, and could furnish opportunities or pretexts for further measures of nazification. This view was the reason that during the occupation I have only been in personal contact with the occupation authorities, when under force, such as checks, search of the premises and police interrogations.'

Again, Brouwer had felt compelled to offer a long theoretical view. It was perfectly sound, but it is doubtful if the Committee would attach much importance to it.

The letter also mentioned, and contained as evidence a newspaper cutting, Brouwer's involvement in a protest in support of the conductor Van Anrooy in the conflict over the Horst Wessel song (cf. p. 722). This, together with the fact

[26] Brouwer to Committee of Restoration 3.XII.1945.

[27] *Werkgemeenschap van wetenschappelijke organisaties in Nederland.*

[28] E.g. the Italian Conference in Rome, 1942.

that all German editors had left in the thirties the board of *Compositio Mathematica*, should lend support to the claim of Brouwer's aversion to the Nazi spirit.

While the officials of the ministry were still pondering the remaining cases of suspended scientists, the student bodies had their own deliberations. The association of students of the exact sciences (*Natuur Filosofische Faculteitsvereniging*) had paid a great deal of attention to the purge of their teachers. Although most cases in Amsterdam had been decided, there were strong emotions as far as some of their professors were concerned. Two chemistry professors were strongly criticized, and their rehabilitation was violently opposed. But also Brouwer came under fire. In the student weekly one could read in the report about the meeting of 13 December that 'About Professor L.E.J. Brouwer little was known, but some indisputable facts were for the meeting sufficient to pronounce the "unacceptable." A resolution was passed to inform the person under consideration, as he had agreed to offer his resignation in this case.' No written evidence has been found to confirm this statement, on the contrary, in his letter to Wibaut of 12 January, Brouwer refers to the above report in Propria Cures as a 'total fabrication'. True or not, to offer his resignation would have been a clever ploy. There was a fairly effective anti-Brouwer lobby among the students, which dominated the meeting. Those who remember the post-war years know that the meetings were not always the theatre of rationality. It is not clear what exactly motivated the anti-Brouwer mood. It certainly was not helpful for his case that Brouwer strongly resented fools and pretentious persons. He could be merciless in his reactions. Somehow he managed to inspire either love and admiration, or hatred and repugnance. Apparently there were enough students in the first category to tip in the end the scales in his favour. The weekly *Propria Cures* of 4 January 1946, which reported about the meeting in December, wrote on the same page: 'The accusations brought against Prof. L.E.J. Brouwer have for the time being not been sufficiently confirmed as a consequence of contradicting testimonials and a lack of witnesses, to justify further action in the sense of the resolution of the meeting.' The students showed, after all, to have a commendable respect for justice and procedure.

The City Council had in the meantime run out of patience with the Minister; it asked him to make up his mind in the matter of the eight suspended professors.[29] If in some cases there could as yet no decision be taken, could he at least give the names? The regular teaching required decisions.

A month later the Minister gave his verdict.[30] It apparently took some time to inform Brouwer of the decision, as he was still sending corrections and additional material to the Committee of Restoration on 30 December.[31] Brouwer was found at fault at three points:

[29]Mayor to Minister of Education Van der Leeuw 13.XI.1945.

[30]Minister to Committee of Restoration 11.XII.1945.

[31]In at least one memorandum to the Committee of Restoration (undated) Brouwer returned to the accusations that were made against him, referring to the letter of December 11 of the Minister. He argued that the Committee had not represented his case faithfully to the minister, and that the same accusations that were raised against him, were overlooked in the case of colleagues. In short, that the

1. the posting of the advice that students could sign the loyalty declaration 'without essential scruple' and re-posting it after the statement of the exiled government in London,
2. the opposition in the Senate to the resistance movements of the professors and of the students,
3. the financial assistance to the Nederlandse Volksdienst.

And thus, the statement ran, the Minister 'disapproves of the attitude adopted by Professor L.E.J. Brouwer.' The matter ended here, there was no possibility of appeal against the reprimand.

This was the end of the procedure, the University was free to reinstate Brouwer—which it did. Although one might consider this a light punishment, it was not considered so in Holland. A reprimand was an almost visible mark, like a 'scarlet letter'. It certainly cast a shadow on Brouwer's authority for the rest of his academic career.

The most problematic, to put it mildly, case of post-war justice was undoubtedly Weitzenböck. He had been arrested in March 1946,[32] and taken to the detention centre 'Oude Molen' in Naarden (not far from Blaricum). There his wartime record was thoroughly checked. Witnesses were produced, statements were compared, but apparently no actions against persons or institutions could be established. All that could be ascertained, was that he was pro-Nazi, that he had been called up for active army duty towards the end of the war, and that in his capacity of billet-master he had requisitioned housing in 't Gooi. After his interrogations he was interned in Camp Vught, the former Nazi camp. In March 1948 the Attorney General decided that Weitzenböck was to be released. His case was further handled by the courts in Amsterdam and Hilversum. The eventual outcome of the legal procedure was that his German nationality was annulled, part of his personal property and savings was confiscated. He lost his position and rights of pension. He was indeed barred from professional positions for life. In addition he was denied a passport, so that he could no longer travel abroad. Being a free man again, he settled in the village of Zelhem, not far from the German border, mainly because his daughter who at time was a nurse in Germany, could visit him more easily. In his excile in Zelhem, he occupied himself until his death in 1955 with mathematics; some papers were published, and the republication of his book on four-dimensional space was prepared—it appeared posthumously in 1956 with Birkhäuser. He was also a reviewer for the Zentralblatt. In 1949 the Free University in Berlin put out feelers to offer him a chair, the Dutch government, however, did not give Weitzenböck permission to leave the country. And so Weitzenböck, who gravely erred in his political choices, and who out of traditional military loyalty joined the German army (albeit as a member of a guard

principle of equality of rights had not been upheld properly. He thus carried on the defence when the authorities had already closed the case.

[32]According to Euwe he was no longer in Holland at the end of the war, after the arrest he was returned to Holland for trial. Interview 25.VI.1977.

group (*Schützgruppe*), had to bear the punishments fixed by the post-war courts. Was his fate inhuman or unfair? Certainly not when compared to the treatment of Jews and of the opposition in wartime; what may count in favour of him is that he did not go beyond the support of an odious political movement, there are no reports of personal actions of his against persons or institutions.

As to be expected, the rumours of Brouwer's fate under the post-war purge also travelled abroad. When Fraenkel inquired with Courant what the attitude should be with respect to people like Bieberbach, he also expressed his interest in Brouwer and Weitzenböck. Courant replied[33] that he had come to doubt the notions of collective guilt and the like, but 'As to individuals, I don't think that anyone behaved as crazily as Bieberbach. I cannot help feeling that he is and always was just crazy but not really dangerous. Much more dangerous are people such as Brouwer, who has been an ardent collaborationist and has been deposed.' Clearly, the grapevine did not get it right, but it certainly helped to further the rumour that Brouwer had been a Nazi, or at least had supported the Nazis. On the basis of the facts, one has to reject that idea; but also Brouwer's personality would make him quite unfit for either subscribing to the Nazi doctrines, with its *Blut und Boden*, anti-semitism, and the like, or for lending pragmatic support. Brouwer at any time despised rabble-rousers and cheap talk; even Bieberbach's political creed he could not take seriously, he mockingly characterized him as 'the white Negro'.[34] The utter vulgarity of National Socialist practice and its supporters, were sufficient to put a man like Brouwer off.[35] His philosophical justification of his views and actions during the occupation are too true to type, and too much a consequence of his earlier philosophical thoughts, to dispose of it as an argument of convenience.

There is no doubt that Brouwer suffered under the post-war accusations—he, one of the leading Dutch scientists, who had always come to the defence of the underdog, was classified as spineless collaborationist!

There are few documents that shed light on his plight; at the end of 1945 he re-established contact with his old friend Gutkind, who had moved to the States long ago. The letter is no longer extant, but we can guess from Gutkind's letter to Einstein[36] that it dealt with war and post-war problems. Indeed, Gutkind gave Brouwer's kindest regards to Einstein, and told Einstein that Brouwer had expressed a strong wish to settle in America. After mentioning Brouwer's support of illegal activities ('that Brouwer had hidden some Jews in his institute and took care of them', and 'Moreover, I recall that Brouwer travelled a couple of times to

[33]Fraenkel to Courant 5.IX.1945; Courant to Fraenkel 19.X.1945.

[34]The curious combination of Bieberbach's political views and the fact that he did not in the least fit the ideal of the German blond warrior, often elicited smiles.

[35]There are reports that Brouwer, with his curious sense of humour, sometimes answered professional mail from Germany which ended with the obligatory 'Heil Hitler' by ending his reply with 'Greetings from Queen Wilhelmina'. Oral communication Mrs. Sapir. I have, however, not found written evidence.

[36]Gutkind to Einstein 3.I.1946.

Germany to get German mathematicians out of German concentration camps'),[37] he asked Einstein to help Brouwer:

> Brouwer writes, that he would prefer a research position over a teaching job. He believes that he would fit well in the Princeton Institute for Advanced Studies. He would like equally well to go to one of the Californian Universities, where the mild climate would suit him better. Both Brouwer and his wife have, in spite of their great toughness, a tender constitution ...
>
> Surely, you, my dearest Professor Einstein, will understand my initiative to approach you, to save this great man and to deliver him from his present difficult situation.

Einstein did not dismiss the matter without some serious consideration. He wrote that he had discussed the matter with a colleague, but that, although everybody agreed that Brouwer had outstanding scientific merits, little could be done for him.

> It has become known that he has been suspended by the University of Amsterdam on political grounds, and it is clear that with such an important man one would not have decided lightly upon such a move. Therefore it seems to me that it is rather hopeless to take steps for his permanent appointment in this country, until the matter has been settled.[38]

There is ample evidence that the climate in Amsterdam had become too unpleasant, not to say hostile, for Brouwer, to be considered an acceptable place for spending the last few years of his professional career. Undoubtedly, the purge and its consequences were felt as unbearable insults by Brouwer.

The end of the war had raised Freudenthal's hope for a happy end to his involuntary exile. There was every reason for optimism. After the defeat of the invaders one could reasonably expect the rehabilitation of the victims of the occupation. Freudenthal and his family had survived the war as well as one could hope. There had been little contact with his colleagues during the last years. Koksma later told that at the entry of the Canadian forces in Amsterdam he was watching the long line of vehicles when he suddenly saw Freudenthal sitting on a Canadian tank,

[37] The first refers to the role of the institute in the resistance movement, since Brouwer's letter is no longer extant, it is hard to what exactly was its content. As to the second statement, Gutkind's wording suggests that he had independent information on this matter. It is not wholly impossible, cf. the book of Balke, but then it probably took place before the war. In the absence of further information, there is little to be said. Balke's book contains a reference to Brouwer's support of a Nazi victim (cf. p. 711), there may have been more instances. In fact in some interviews Brouwer's role in helping and hiding Jews and other victims of the regime have been mentioned; as no further confirmation could be obtained, it is no more than a rumour. There is similar, but more concrete evidence that Brouwer's cottages were used by people who had gone in hiding (*onderduikers*), cf. p. 766). Brouwer did not mention facts of this sort in his defence; it was not his habit to use in arguments facts that were not directly relevant to the issue.

[38] Einstein to Gutkind 10.I.1946.

'When I saw Freudenthal on that tank I was really convinced that the war was over.'[39]

What seemed more natural in those hectic days, full of optimism, then a speedy reinstatement of Freudenthal at his university? Of course, he expected more than just a reappointment in his old function; the person who had made use of the opportunity provided by the Nazi policy not only had succeeded Freudenthal, but also obtained the promotion that was promised to Freudenthal. In spite of the glaring injustice, nothing happened. In a letter to the Mayor[40] he drew attention to his treatment and expressed the hope that his 15 years of service to the University of Amsterdam and his international status would result in his reinstatement. Actually, Freudenthal as an unemployed citizen, living under dangerous circumstances, had contributed more to the wartime mathematical activities than all the official staff members of the Mathematical Institute together. He had, for example, supervised three Ph.D. theses: of J. de Groot, A. van Heemert and F. Loonstra. His appeal to the mayor asked for recognition, 'after the disqualification, which any outsider could discern in the career of my successor, who, as a physicist was a stranger in the realm of mathematics, and remained so, and who nonetheless was already after one year appointed as a lecturer, a position which was not granted to me after ten years, in spite of numerous promises.'

In spite of the treatment he had received from Brouwer, and of Brouwer's disputed war record, Freudenthal had expected some token from Brouwer, 'and I must say, I am not sufficiently vengeful not to have entered into a reconciliation',[41] but nothing happened. Brouwer and Bruins insisted that the most Freudenthal could expect was the position of conservator under Bruins.

The conflicts between Brouwer and Freudenthal had left Brouwer with the conviction that cooperation had become impossible. He wished Freudenthal no harm, but he could not bear to have him around, and so he pursued a double policy. He strongly opposed Freudenthal's reinstatement and promotion, and at the same time used his influence to get him a chair at another university. For Freudenthal that solution was unacceptable, to be denied the lecturer's position would be a grave insult.

Since Brouwer was under investigation, Freudenthal hoped that his rehabilitation would not be difficult. In order to show Brouwer that the personal conflicts of the past did not enter into his request for reinstatement, Freudenthal had sent Brouwer a copy of the letter to the Mayor. And Brouwer had passed the letter on to Bruins, a not uninterested party. Bruins immediately set out to refute Freudenthal's claims and statements in a letter to the mayor.[42] Apart from a rather artificial objection—Bruins denied the fact that he was Freudenthal's successor by pointing out that duties were not identical. He simply stuck to the argument that a

[39] Interview in Alberts, et al. p. 116.

[40] Freudenthal to Mayor 11.VI.1945.

[41] Freudenthal to Van der Corput 25.VIII.1945.

[42] Bruins to Mayor 20.VI.1945.

lecturer could not be the successor of a conservator. Bruins' letter consisted mostly of attempts to show that he was indeed a mathematician, combined with hardly veiled insults addressed to Freudenthal. A couple of letters followed, repeating arguments, providing new details.[43] Freudenthal's style was by far the more dignified one; he carefully avoided getting drawn into a vulgar match of abuse. Bruins' evidence for his strong mathematical background was easily dealt with. Most of his articles at that time were clever applications of (fairly elementary) parts of mathematics to problems in physics, chemistry and medicine, but they could hardly support his claim to his status as a mathematician. Likewise, Freudenthal added, not much weight could be attached to a prize essay (which was advanced by Bruins as a serious argument), that did not merit an award. In short 'If Dr. Bruins thinks, that on the grounds of a prize essay that was not successful, and some other minor papers he enjoys a reputation in the mathematical world beyond the limits of the University of Amsterdam, he is grossly mistaken.'

The only new fact that Bruins mentioned in a subsequent letter, was that his position was already agreed upon by Brouwer and his colleagues in 1938. On the whole this would not be impossible, but highly unlikely, and it was never mentioned by Brouwer. The subsequent letters of Freudenthal repeated and further underpinned his case against Bruins.

Both Bruins and Freudenthal appealed to the mayor in a number of letters, in which the given arguments were further elaborated. Freudenthal's arguments can be summarized as 'in essence Bruins has taken over my position, and he is not a proper mathematician.' Bruins on the other hand maintained that 'legally my position is different from Freudenthal's last position, and I am a good and proper mathematician.' On both counts Freudenthal's side was the stronger one. Anybody who compared Freudenthal's mathematical record with Bruins will agree that Freudenthal was already at the top of his profession, whereas Bruins was respectable middle class. And where the claim on the position is concerned, Bruins' argument appears as a legal trick, possibly useful in the hands of a slick lawyer in court, but definitely weak on the moral side. Freudenthal, as a persecuted Jew, clearly had the moral right on his side.

Bruins, bent on the defence of his position, apparently used curious means to thwart Freudenthal's claim. Freudenthal told Van der Corput that Bruins, with his taste for the bizarre 'had me lured to an address in Amsterdam-South where he had a simple minded acquaintance of his make a proposal to withdraw my claims on a restoration, for which he would compensate me financially; Bruins, by the way, was hiding behind the door when this proposal was made.'[44] Nonetheless, Freudenthal was optimistic that justice would be done, and most colleagues would have shared this optimism.

[43]Freudenthal to Mayor, 22.VI.1945; Bruins to Mayor 1.VII.1945; Freudenthal to Mayor 6.I.1945, Freudenthal to Comm. of Restoration 9.VII.1945.

[44]Ibid.

The matter of Freudenthal's rehabilitation was, as most people at the time agreed, and as we nowadays see it, clear enough for the university to make a decision; it would have meant some reshuffling of the budget, but it could be done. That, however, is not the way big institutions solve their problems. One could not take such steps without proper advice, and whose advice is one to ask? Of course that of the Director of the Mathematical Institute, suspended or not.

There is no doubt that the sympathy of the University officials and the mathematical world was on the side of Freudenthal; Bruins' legal position was, however, strong enough for him to brave all threats, but his greatest asset was the support of Brouwer.

The latter was invited to give his views on Freudenthal. On August 28 he had an interview with the official in charge. According to the note that Brouwer composed after the interview, the young, promising German mathematician Dr. Freudenthal had come to Amsterdam in 1930 to conduct research in areas relating to and supporting Brouwer's research. Since Freudenthal (and Hurewicz, for that matter) carried on their own research and did a bit of teaching as *privaat docent*, and since this could not fully justify their presence, Brouwer (according to this note) founded *Compositio Mathematica*, in order to supervise the editing 'with the help of both assistants.'[45] At the same time he had fostered hope that both would find suitable jobs elsewhere. Hurewicz did, but Freudenthal stayed. As we have seen, at the retirement of De Vries, Freudenthal was entrusted with the analysis courses and was appointed conservator. 'The professors at the Mathematical Institute [Brouwer, Mannoury, de Vries, Weitzenböck] expected, I think unanimously—and did not hide this from Freudenthal—that the natural course of things would automatically result in his inclusion in the ranks of the official teaching staff (*docentenkorps*), if his request for naturalisation, submitted already a long time ago, was granted, if it should turn out that he was well-suited for teaching other than advanced and specially gifted audiences, and if an adequate understanding and co-operation between him and the other mathematical staff members should develop.'

Then the grim conclusion followed:

> However, the fulfilment of this threefold condition until now has not taken place. Moreover, Freudenthal's attitude of impertinent irritation with respect to me, of which signs had been manifest already before 1937, has since then become more outspoken and had taken the sharper form of slander and aggression. Only the hope, that his activity in cultivating relations here and abroad, would after the war soon bring him an appointment elsewhere, combined with my patience with respect to psychopaths, has enabled me to maintain so far my patience with respect to him.

This letter confirms an earlier diagnosis of the conflict. Brouwer could simply not bear to have somebody around who criticized him, no matter how discreetly.

[45]The argument is a curious mixture of fact and fiction, clearly designed to add plausibility to Brouwer's views. Hurewicz and Freudenthal were indeed appointed in Amsterdam before Compositio began to appear, so their presence was convenient for the running of the journal, but nobody would maintain that Compositio was designed to keep Brouwer's assistants occupied.

Brouwer was no fool at any time, and he obviously realised that more than a decade of his scientific career had been wasted, but he could not bear to look into the mirror that Freudenthal held up for him. All this talk about aggression and psychopaths was a clumsy but nasty device to counter Freudenthal's unspoken criticism.

After all these years Brouwer was as high strung as ever and his urge to get rid of Freudenthal assumed alarming proportions. At the hearing itself he defended himself against Freudenthal's reproach that he had refused the papers of De Groot, Loonstra and Freudenthal, saying that in 1942, shortly before a meeting of the Academy he got more than 100 pages of manuscripts from Freudenthal.

'I took them along on a trip. Shortly afterwards I got a reproving letter. Then he threatened to publish them elsewhere, I returned the papers. I have not approved a dissertation of a student of Freudenthal just like that, as he demanded.'[46] And when asked if Freudenthal could be charged with the running of the institute,[47] Brouwer exclaimed, 'Not necessary, I will yield only for bayonets!' Brouwer probably managed to get his message accros, he could argue very persuasively, and charm or repulse at will. It is a fact indeed, that Freudenthal felt later that the chairman of the Committee of Restoration, Van Valkenburg, was rather cold towards him.[48]

Almost immediately after the interview with the Committee of Restoration, Brouwer must have learnt something that, in his opinion, confirmed his worst fears. On the thirtieth of August Brouwer reported a violation of the Mathematical Institute;[49] he had (with the permission of the chairman of the Committee of Restoration) gone to the institute, as a part of his private library and archive was in his room. On entering his room he perceived that his papers were disturbed and on closer examination a large number of the keys of various drawers and filing cabinets had disappeared. These keys were hidden between archive folders at spots only known to Brouwer and Cor Jongejan. The cleaning lady, Mrs. Peters, told him that she had found an unknown person in Brouwer's room who was inspecting papers, and who told her that from now on she had to deal with him as a Director of the Institute, and that furthermore only one of the members of the institute would be allowed to return.

On inquiry, the temporary supervisor Mr. Pater declared (according to Brouwer's report) that on Monday August 20, the chairman of the faculty, Professor Clay, asked him to give the key to the Mathematical Institute to Freudenthal. Pater objected that he could not do so without the permission of Brouwer, or his deputy, Bruins. On Clay's assurance that he would take full responsibility; Pater accompanied Freudenthal to the institute, who said that he was going to teach. After some time Freudenthal sent Pater away and when Freudenthal left, Pater saw that the briefcase of Freudenthal had considerably grown in size between arrival and departure. The matter was not only known to Brouwer, but also to Bruins, who

[46] Notes of the Secretary of the Comm. of Restoration.

[47] I.e. act as temporary director.

[48] Freudenthal to Van der Corput 17.IX.1945.

[49] Brouwer to Alderman for Education 30.VIII.1945.

immediately saw the value of the incident, either as interesting gossip or as convenient smear.[50]

For Bruins (and Brouwer) this was a welcome argument in the battle against Freudenthal. Brouwer left it at the above letter, but Bruins started to spread accusations against Freudenthal.[51] When Freudenthal got wind of this, he immediately asked the authorities to investigate the matter. The Wethouder duly started an investigation and asked Clay to report to him. Clay checked the facts, interviewed the persons involved. His findings were:[52]

- Freudenthal was authorised by an official at City Hall[53] to enter the Mathematical Institute, which he had to run for the time being.
- Clay had asked him to look for the examination administration of Weitzenböck (which was vital for the final exams). This was to be found in the room which Weitzenböck shared with Brouwer.
- Mr. Pater had left Freudenthal alone on his own accord.
- Freudenthal had only opened unlocked drawers and cabinets.
- The cleaning lady stated that the larger part of the keys was missing.

Clay ended the report by noting that nothing could be concluded against or in favour of Freudenthal however, he continued,

> As it became clear to me that there existed previous grievances between Prof. Brouwer and Dr. Freudenthal, it is unpleasant for Prof. Brouwer to learn that Dr. Freudenthal has been in his room without his knowledge; on the other hand it is most unpleasant for Dr. Freudenthal to be suspected of malicious intent, which is not confirmed by any of the facts that have been brought to my attention, and for which I cannot find any indication.
>
> There has been here an unfortunate coincidence, of which both gentlemen have become victims.
>
> A better understanding in general, or should this be impossible, a definitive separation of their areas of activity, shall have to bring the solution to the incident.

This incident is related here at length, to illustrate that the relation had really detoriated to the point of cloak and dagger stories.

Indeed Freudenthal had already asked Brouwer in July to be admitted to the institute, without success. Freudenthal was indeed authorised to run the institute, but this authorisation was mysteriously withdrawn. He bitterly complained to the chairman of the Committee of Restoration that it was surprising to say the least, that he—the only 'clean' staff member could not enter the institute and go about his business, while all the suspended members (Brouwer, Bruins, Heyting) were in

[50] Freudenthal to Van der Corput 17.IX.1945.

[51] Freudenthal to Mayor 21.IX.1945.

[52] Clay to Alderman15.X.1945.

[53] Ottens

the possession of a key.[54] Partial justice was done to Freudenthal, when on September 14 he was reappointed as conservator, while 'the possibility of honouring your request for appointment to lecturer is further investigated.'

For Freudenthal this was not enough, he saw the wartime appointment of Bruins as a trick to pre-empt any claims of his and he viewed the lectureship as his due. The fact that he should work under Bruins is in retrospect indeed preposterous, and Freudenthal completely ruled out this option. Roughly at that time various actions were taken to support Freudenthal. His students and former-students sent an address to the Committee of Restoration, and Van der Corput alarmed the Minister.

All of this was to no avail: the Faculty, although on the whole sympathetic towards Freudenthal, could not make up its mind to support Freudenthal. There were too many conflicting interests, and when Van der Corput, Van der Waerden, and Van Dantzig all wanted to be on the Amsterdam faculty, Freudenthal was no longer a priority. Furthermore, the faculty felt understandably that, even though Brouwer was suspended, no major decision on mathematics should be taken without consulting him. And so Brouwer could even at this time delay the decision process.

At the first post-war faculty meeting,[55] Clay suggested that Van der Waerden would be a good candidate for Weitzenböck's vacancy. The chairman (Weevers, appointed by the Committee of Restoration) objected on the grounds that Van der Waerden had been in Leipzig during the war. He suggested that in Brouwer's absence Freudenthal, who was close to Brouwer, should conduct Brouwer's exams.

18.2 Faculty politics

It was no coincidence that Van der Waerden was mentioned in the faculty meeting; this man, who was the brightest Dutch mathematician after Brouwer, had made a shining career. After his study in Amsterdam he had gone to Göttingen to study and work with Emmy Noether. In 1926 he got his Ph.D. in Amsterdam under Hk. de Vries, and in 1929, 26 years old, he became a professor in Groningen. Three years later he accepted a chair in Leipzig, and became the successor of Otto Hölder. During his Leipzig years he taught a large variety of courses and published on a wealth of topics. In that period he was considered the leading expert on algebraic geometry.

The advent of the Nazi regime brought him in conflicts that were almost predictable in the case of an intelligent, original man with a talent for speaking his mind. In 1934 he was reprimanded by the local government for joining Heisenberg in a protest in the faculty against the dismissal of four Jews,—a foreigner should stay out of German politics (unless he supported the party, of course)! He never compromised himself by supporting the Party; when the Germans invaded Holland, he was arrested, released after the capitulation and ordered not to leave

[54] Freudenthal to Comm. of Restoration 23.VIII.1945.

[55] 15.VIII.1945.

Germany. For all practical purposes he had become a high-class forced labourer. In 1943 the Faculty in Utrecht offered him a chair at the retirement of Barrau, a Dutch geometer, who like Brouwer wrote his dissertation under Korteweg. In the same year Van der Waerden's house and property was destroyed by bombing. After a difficult odyssey he returned after the war to Holland and found that the post-war government did not wish to endorse the appointment that the Utrecht University had arranged for him.[56] Freudenthal brought him in contact with Professor W.J.D. van Dijk, the director of the research department of Shell Oil, who was at that moment looking for mathematical expertise for his laboratory.[57] As a result Van der Waerden got a research position at the Shell establishment in the Hague, so that he at least had a decent income.

The reader may recall that Brouwer could not get along with the young Van der Waerden, who used to speak his mind no matter where and when. He had, however, supported him in going to Emmy Noether in Göttingen. But the fact that Van der Waerden became a fervent admirer of Hilbert certainly contributed to a further alienation between the two. As long as Van der Waerden was in Leipzig, things were fine, but a Van der Waerden in Holland represented a threat to Brouwer's emotional stability. Sooner or later Van der Waerden was going to play a role in Dutch mathematics, and worse—he might be interested in a position in Amsterdam!

Indeed, from Brouwer's point of view the situation was fraught with dangers and unacceptable consequences. He would have to fight a war at two fronts at least, and as we will see, the number of fronts quickly multiplied. When the faculty met again, the new chairman Clay had faithfully consulted Brouwer on the matter of Weitzenböck's successor. Brouwer had, he said, told him that the chair of Weitzenböck was a piece of luxury the faculty could dispense with—no successor was needed. Instead Brouwer pushed Heyting's promotion to full professor. The emeritus Pannekoek (an advisory member of the faculty) drew attention to Van der Waerden as a successor of Weitzenböck; Clay answered that in view of the well-known aversion of Brouwer to Van der Waerden, he had not undertaken anything in this direction. Nonetheless he insisted that the vacancy should be filled. There was an external reason for doing so, as we will see.

The situation in the faculty (and in all of Dutch mathematics, I may add) was indeed complicated by a new initiative of the Minister of Education; he had installed a committee to design plans for a reorganisation of mathematics in Holland. The records of the committee are somewhat elusive; the ministry has not been able to locate any minutes or recommendations in their archives, but from bits of correspondence and from various archives some information could be collected.[58]

[56]The Committee of Restoration and the 'Van der Corput' committee played a somewhat dubious role in the matter, see below.

[57][Alberts, G. 1987], p. 134.

[58]I am indebted to G. Alberts for putting the following facts at my disposal.

The committee, called the 'Committee for co-ordinating and reorganising higher education in mathematics in the Netherlands' was created by Minister G. van der Leeuw on 26 October 1945; it was disbanded in the beginning of 1947. There were two well-defined tasks of the committee: 1. to co-ordinate the appointments in the then existing vacancies in mathematics, 2. to investigate the possibility and desirability of a centre for scientific mathematical activities, and to consider means to make closer contact between pure mathematics and its applications in other areas.

The committee consisted of four mathematicians, one physicist and an astronomer: J.G. van der Corput (Groningen), J.F. Koksma (Amsterdam, Free University), J.A. Schouten (retired), D. van Dantzig (Delft), H.A. Kramers (Leiden), M.G.J. Minnaert (Utrecht). Three of the four mathematicians were not friendly inclined towards Brouwer, to say the least. The plain fact that Brouwer, the greatest Dutch mathematician of his day, was not a member is telling. Formally speaking, Brouwer could not very well have been a member, after all, at the time of the establishing of the committee, Brouwer was suspended, and one cannot blame a minister for disregarding a candidate with a problematic political profile. Factually, however, leaving out Brouwer could only be seen as a calculated insult, not so much from the side of the minister, as from the side of his colleagues. In a sense it was a good thing that Brouwer was not a member of the committee; at least he could not be held responsible for the havoc that the committee created. The committee did not contain a member from the Amsterdam faculty. This was in a way surprising as the University of Amsterdam had for a long time been at the top of the profession where mathematics was concerned. Counting out Brouwer, Freudenthal or Heyting might very well have qualified; both had a good reputation in the field. Technically it would have perhaps have been a bit awkward, as neither was a professor at the time, so they would have to discuss their own promotion. There is little documentation about the committee left (I have actually seen no original material), so most information is based on recollections of mathematicians who at that time were junior members of the community. The general impression was that Van der Corput had made a judicious choice of (mathematical) members of his committee (to which I will refer as the 'Van der Corput Committee'), and that to all appearances he had a hidden agenda.

The committee took its task very seriously; it went so far as to dictate faculties the choice of candidates for chairs. E.g. Van der Woude, who was a lecturer at Leiden, was commandeered to a chair in Utrecht—completely against his wishes. He wisely ignored the instructions of the committee and remained in Leiden. In Delft the committee ran into a vigorous opposition. The mathematician Bottema bluntly refused to continue the discussion with the committee, almost threatening Van der Corput with bodily harm.[59] The older generation of Dutch mathematicians could recite instances of undesirable interference and nepotism. Van der Corput and Van

[59]Oral communication N.G. de Bruijn. Van der Corput also wanted to transfer Blumenthal's reprint collection from Delft to the Mathematical Centre in Amsterdam. The request was turned down. In retrospect, it would have been better that his request had been granted, for eventually the valuable collection was lost, and not so long ago items of the collection turned up in antiquarian bookshops.

Dantzig were for example reported to have used the committee to get appointed in Amsterdam. Both of them had earlier offers from Utrecht, but Amsterdam apparently appealed more in those days. Indeed, Utrecht had no mathematics professor at the time. Julius Wolff was deported during the war; he died in a German camp. Barrau had retired in 1943, and the vacancy had not yet been filled. The faculty and the board of the university had not neglected the matter. As a matter of fact, they had sounded Van der Waerden, who was at that time still at the Leipzig university. Van der Waerden was in a difficult position, as a professor in Germany he was not free to accept offers without the government's permission. He had already received calls to Munich and Göttingen, but permission to accept had been denied by the authorities. It is highly doubtful if a call from Utrecht would fare any better. He replied the Utrecht faculty that he could not be certain to accept the offer under the present circumstances.[60] It should be added that his German colleagues would be very sorry to see him go. Carathéodory, for example, asked Van der Waerden if he could not temporize in the Utrecht matter. Van der Waerden later claimed that he did not want to accept a chair under state secretary Van Dam; a wise decision indeed, he was already a suspect person for his teaching in Germany, and to accept a position in wartime Holland would have been a real blot on his career. Once the war was over, Van der Waerden was repatriated to Holland, and the Utrecht job came in sight again. Already on 14 August 1945 the Utrecht University sent a list to its Committee of Restoration with names of candidates for the mathematics vacancy: 1. Van der Waerden, 2. Van Dantzig, 3. Struik, 4. Bottema.[61] These were the candidates for the geometry chair, simultaneously candidates were offered for the chair 'analytic mathematics'. There Van der Corput headed the list. The faculty did its best to convince the Committee that Van der Waerden was politically clean.

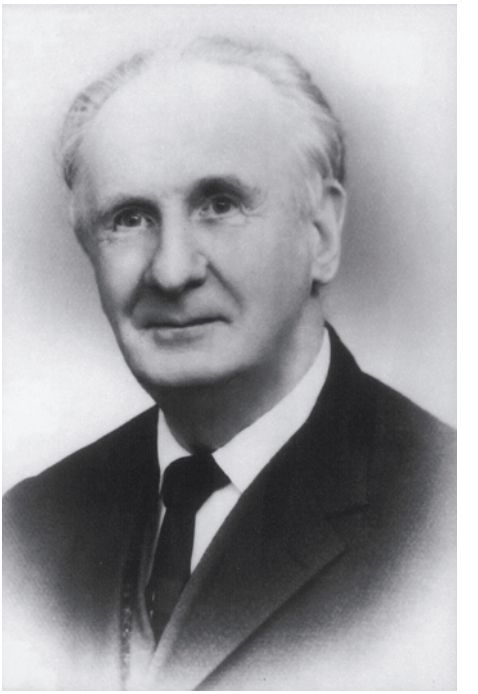
In the meantime Van der Corput's committee had considered the options for a national mathematics institute. Two issues were of immediate importance: the place and the staff. In The Hague at the ministry there were voices that favoured Utrecht. With Van der Waerden (and at least one more professor) it would have an extremely strong and competent mathematician. And Utrecht was in the geographical centre. The Van der Corput committee considered however Amsterdam the better choice. When the rumours about the proposed centre started to circulate, the Utrecht faculty again approached the Committee of Restoration.[62] It summed up the prospective distribution of mathematicians: Van der Corput opted for Amsterdam, Koksma stayed at the Free University, and Kloosterman in Leiden. Van der Waerden now had also an offer from Amsterdam, but he would prefer Utrecht if the appointment could be realized soon.[63] The faculty had no objection to Amsterdam as the seat of the new institute, but with Van der Corput and Van der Waerden

[60] Utrecht faculty to Van der Waerden 27.VII.1943, Van der Waerden to Utrecht faculty, 19.IX.1943.

[61] Van Dantzig and Bottema were professors in Delft, Struik was since the twenties at M.I.T.

[62] Faculty Utrecht to Committee of Restoration 13.XII.1945.

[63] In fact the Amsterdam faculty used a similar argument. Van der Waerden had apparently decided to sit on the fence.



B.L.G. van der Corput. (C.W.I. Archive).

both in Amsterdam, the Utrecht students would probably defect to Amsterdam. With Van der Waerden in Utrecht the balance would be restored. The Utrecht hopes were dashed when the minister announced that persons who worked voluntarily in Germany during the war, were not eligible for appointments (in any of the departments). Exit Van der Waerden.[64] There were persistent rumours (confirmed nor denied) that Queen Wilhelmina personally blocked Van der Waerden's appointment, and that the above formulation was an official arabesque.[65]

The requested advice about a research institute for mathematics eventually led to the founding of the Mathematical Centre in Amsterdam.

For the proper understanding of some of the delicate workings of Dutch politics in this particular case, the reader should know a bit more about the major actor, namely Professor Johannes Gualtherus van der Corput, a mathematician of good standing. He had a fine reputation in number theory and his publications had

[64]Van der Waerden's life has recently been the subject of a series of papers by A. Soifer—[Soifer 2005a, Soifer 2005b, Soifer 2005c]. On account of the advanced stage of preparation of the present book, no use could be made of this material.

[65]Until 1981 professors and lecturers at the state universities were appointed by the Queen on the recommendation of the Minister of Education. The Minister in turn was presented with an ordered pair of two candidates by the university. Usually the Queen followed the recommendation of the faculties, but even in ordinary times, politics could play a role and a candidate could be 'returned' to the faculty.

appeared in the leading journals. We have already met his name in connection with Brouwer—it was on his instigation that Brouwer gave a course on intuitionism in Groningen in 1932. Van der Corput's prominence in post-war Dutch mathematics was partially the consequence of his friendship with the first Minister of Education after the war. This Minister, Van der Leeuw, had been a professor in theology in Groningen; he was respected and known for his views and publications on the philosophy of culture. In the exciting period following the liberation, there was enough turmoil to admit people into the political arena who had not made a life-long career in politics. One of those persons was Van der Leeuw. Van der Corput, as a Groningen professor, happened to be on good terms with Van der Leeuw, and when the latter became the Minister of Education, Van der Corput had the minister's ear. Consequently, Van der Corput soon found himself chairman of the most important committee for Dutch mathematics. The idea for an independent Mathematical Institute had in fact been born during the war. Mathematicians in the forced period of inactivity said to each other, wouldn't it be nice if we had an institute where we could sit and talk mathematics, and work?[66]

Clay, the faculty chairman in Amsterdam, mentioned the plan for a national mathematical institute in the third post-war faculty meeting. In his view the plan foresaw a new Mathematical Institute in Amsterdam, provided the leading mathematicians of the University of Amsterdam should (at least partly) belong to the institute. He had already consulted Brouwer, and the latter agreed to the appointment of Van der Corput in Amsterdam. Clay added that one should not wait too long, for otherwise the new institute might go to Utrecht.

The discussions at the first post-war faculty meetings were confused; it is almost as if its members did not dare to make decisions 'while father was away.' Heyting's promotion was discussed, but there was little enthusiasm for a copy of Brouwer; moreover, one of the members thought it improper to put a recently suspended colleague up for promotion. The chairman reported that Brouwer had expressed the wish to work with Van der Corput, while at the same time making it clear that the appointment of Van Dantzig and Van der Waerden would strongly displease him. He had even mentioned to some members of the faculty that the appointment of Van der Waerden would be a reason for him to resign.

The discussions at the faculty meetings later that year were dominated by the prospect of securing the new institute for Amsterdam. The offers of Utrecht to Van der Corput and Van der Waerden were a serious setback, but if the faculty managed to outbid Utrecht, there was a chance. Van der Corput was willing to favour Amsterdam, provided the new institute came to Amsterdam and that Van der Waerden was to join him there. In view of this fact, Clay motioned that Van der Waerden should be put forward as a candidate at the next City Council Meeting, the more so as the Minister and the responsible councilman had in principle agreed to share the financial responsibility for the new institute. Although Brouwer was

[66] Quoth Koksma and Van Dantzig. (Oral communication N.G. de Bruijn).

known to have personal objections to Van der Waerden, the chairman said that apparently Brouwer was willing to give in if the new institute were to be situated in Amsterdam and if Van der Waerden were to be attached to the new institute. When the meeting continued to discuss the aversion of Brouwer to Van der Waerden, the physicist Michels remarked dryly that if Brouwer didn't like it, he should get out. After all, he was 65.[67]

The consequence of all these new prospects was that suddenly Freudenthal had become a side issue. The appointment of at least two new professors, it was argued, made it inopportune to promote Freudenthal at this moment to a lectureship, as the new professors should not be presented with a *fait accompli*..

Although the faculty deplored the injustice of the situation—Bruins sticking to his chair and Freudenthal left out in the cold—it did not go beyond a truly academic 'on the one hand . . . , on the other hand . . . ' message to the Committee of Restoration. The Committee urgently wanted a bit of practical advice, but the faculty did not give any. Freudenthal was certainly not mistaken when he wrote to De Groot,[68] 'how powerful Brouwer is, although he is suspended.'

In spite of numerous letters to City Hall, the Rector, the Committee of Restoration, the faculty, . . . , Freudenthal did not succeed getting justice done. Even worse, he shared the fate of so many men who simply wanted justice—when in May 1946 Freudenthal implored Clay, the faculty chairman, to see that he was finally rehabilitated, and made it clear that the sympathy of the faculty was no substitute, Clay replied testily, 'It is not clear to me why you keep speaking about rehabilitation. You have been *re-instated* in the position you occupied before the war. . . The feelings of distrust that you ventilate with respect to the faculty are not helpful to reach a just solution.'[69]

When Brouwer returned to the faculty after the suspension was lifted, he immediately set out to recover lost ground. His most urgent goals were (a) to remove Freudenthal, (b) to prevent an appointment of Van der Waerden.

He had already personally, and through Heyting, made it clear that the faculty could do very well without a successor of Weitzenböck. The chairman, Clay, faithfully informed the board of the university,[70]

> In addition to my letter of October 19, I inform you at the request of colleague Brouwer, that he and the lecturers Dr. Heyting and Dr. Bruins find that in Amsterdam, also after Weitzenböck's departure, there are enough mathematical teachers. They are of the opinion that the appointment of a successor of Weitzenböck—who (in order to meet a condition of L.E.J. Brouwer for turning down the offer of a chair abroad) was appointed as

[67] Brouwer had thwarted Michels' appointment in the thirties. Not much love seems to have been lost between the two [Maas 2001], p. 180.

[68] Freudenthal to De Groot 17.IX.1945.

[69] Clay to Freudenthal 15.V.1946.

[70] Clay to rector 22.X.1945.

> an extra-ordinary professor at the time when there was a strong influx of interested foreigners, – is under the present drastically changed circumstances not justified.[71]

Having complied with Brouwer's wishes, he added that the faculty, in view of the prospective institute, did not share Brouwer's views.

After the resumption of his regular duties, the first thing Brouwer did was to call a special meeting of the faculty to discuss 'proposals for teaching mathematics.'[72] No sooner had the chairman given the floor to Brouwer, than he embarked on a lengthy harangue. He had, he said, been absent against his will and on returning he had found undesirable persons in his scientific home. The three mathematical staff members,[73] according to him, had wished to promote Heyting to a professorship. That being taken care of, he had further proposals in mind. He was surprised by the plan to appoint Van der Waerden. After making it quite explicit that co-operation with Van der Waerden would be most painful to him, Brouwer cleverly argued that it would be the proper thing to appoint Van der Waerden at the new institute. The chairman, Clay, had no difficulty in grasping the immediate consequences of Brouwer's proposal; he vigorously pleaded that no time should be lost. The minister had been at the point of appointing Van der Waerden in Utrecht, he said, when some members of the Mathematical Committee begged him not to do so. Van der Waerden himself, it was said, was willing to accept an offer from Amsterdam provided it was made before the Utrecht offer. The founding of the new institute in Amsterdam was vigorous pleaded by Clay.

The discussion did not yield any particular results. Brouwer tried to push Heyting's promotion, but some members raised the objection that Heyting had broken the unanimity of the professors at a critical period of the occupation. In Brouwer's eyes this was unfair, taking into account Van der Waerden's past, etc. It was clear, however, that Brouwer was again casting his spell over the faculty. His arguments against an immediate offer to Van der Waerden—namely that no commitment could be made to Van der Waerden until the funds for the new institute had been granted, were in so far successful, that the chairman finally declined to send any letters about the appointment to the University Authorities.

Brouwer had not waited, however, for a faculty meeting to pay attention to Freudenthal's lecturer's position. He had found out in December that two lecturers in mathematics were proposed to the City Council, and it was plausible to suppose that one of these was for Freudenthal. He complained that neither he, nor Bruins or Heyting were informed of these proposals.[74] In discussions with members of the Van der Corput committee for mathematics, he had already pointed out that co-operation with Freudenthal had become extremely difficult for him, and

[71] The teaching staff consisted at that moment of Brouwer, Bruins, Freudenthal (who had been reinstated as a conservator on September 14), and Heyting.

[72] 24.I.1946.

[73] Brouwer, Bruins and Heyting.

[74] Brouwer to B & W (Mayor and Aldermen) 7.I.1946.

that it 'has had such a paralysing influence on his energy, that anybody, who can end this co-operation without personal disadvantage to Mr. Freudenthal, should do this in the general interest. Thus, in my opinion, the above mentioned Committee is faced with the duty, to further as best it can that the various vacancies at the Dutch Universities are filled in such a way, that a fitting position outside Amsterdam is allocated either to Freudenthal, if he turns out to be acceptable in Holland, or to me.' A member of the committee had, however, conjectured to Brouwer that Freudenthal might refuse to leave Amsterdam. If that were the case, Brouwer argued, Freudenthal's inflexibility would only be reinforced by an offer of the position of lecturer at the present moment. In the same letter to the City Council Brouwer maintained that Bruins should be made a lecturer in applied and introductory mathematics (*propaedeutische wiskunde*).

The subsequent developments show that Brouwer's power had eroded. The chairman of the faculty, Clay, received as a matter of routine, a copy of Brouwer's letter, and he firmly took Brouwer to task:[75] he assured that no proposals for lecturer appointments had been made; but, he said, even if Freudenthal were offered the position of lecturer in Amsterdam, it would make it easier for him to get offers from elsewhere. The letter closed with a telling remark: nothing was decided with respect to Bruins, but 'It makes me sad that at this moment he is active in ruining his own position by a smear campaign.'

Power is concentrated in decision making and advisory bodies, and the single most interesting body at the time of Brouwer's suspension was the Ministerial Committee for Mathematics. So what could be more natural for Brouwer than to request a seat on it? On the same day Clay sent his firm letter, Brouwer wrote to Van der Corput, 'Now that I have been re-instated in my position, don't you think it right and proper, and in the interest of further developments, that I am after all given a seat in the co-ordination committee, installed by the minister and presided over by you?' The answer is not known, but Brouwer's demand had no effect.

It must have dawned on Brouwer that the new colleague, Van der Corput, was no push-over. The domination of Brouwer had been challenged, and the decline of his empire had set in. Whatever his actions and protests were, he no longer could make the law, or keep City Hall under his control.

A demonstration was to come earlier than he expected; on January 16 Van der Waerden's appointment was discussed in the City Council. Brouwer was furious; the chairman of the faculty had kept him informed of the progress of the new institute, and he had managed to postpone Van der Waerden's appointment till later (indefinitely, he hoped), at the price of accepting Van der Corput as the successor of Weitzenböck.

He indignantly complained to the chairman that he was the victim of repeated breach of faith.[76] First, he claimed, the letters of advice re Van der Corput had been sent with a text not approved by him, at a time not determined by him—contrary

[75] Clay to Brouwer 10.I.1946.

[76] Brouwer to Clay 23.I.1946.

to recent promises. Next, Clay and Van der Corput had made plans to guarantee the participation of Van der Waerden in the new institute. And now the Van der Waerden matter had reached the City council. 'This cannot fail to fill me with mistrust with respect to the persons involved in this matter.'

The procedure of Van der Waerden's appointment did not make any progress that year, the urgency had also somewhat diminished as he accepted a visiting professorship at the Johns Hopkins University in Philadelphia. Nonetheless, Brouwer felt he had to be vigilant, for he started to see enemies and conspiracies everywhere (the latter could be right, by now people started to look for means to get around this formidable obstacle). It goes without saying that the suspension with its humiliation, including some attacks from the side of the students, did not leave Brouwer unaffected, apart from little scraps of paper with remarks on his persecutors, there is little known about his reactions. When the chemistry Professor Wibaut, who had been—on the whole, a good—rector in the last year of the university's war years, and who was to a far greater degree the target of student actions, wished Brouwer Happy New Year and said that he would not attend the Dies (the birthday of the university) as long as certain colleagues were treated so shabbily. Brouwer for a moment could not withhold his bitter comments:[77]

> I see in the actions of the aggressive groups in the Senate such a striking similarity with the Jacobin and Fascist clubs of the past in their as yet unbloody initial period, that I can neither believe in their early disintegration, nor in their early humanisation. Averting the dangers that threaten from that side will only have a chance of success, as I fear, through a militant union of the colleagues of good-will, … For this reason alone, the defeat of the fascist states had to be paid with so much blood and tears and such an endless aftermath of desperation, and brought so much desperation, because the militant union of the 'united nations' (in particular of the safe outsiders) has been realised so slowly.
>
> This team forming is not at all in my line, but it looks as if, after the collapse of the liberal state structures there will no longer be a place for isolated persons, and that one can only obtain the right and the means for existence through uniting.

Although Brouwer kept a stiff upper lip in his dealings with colleagues and with the faculty, he was privately extremely bitter about the treatment he had received at the hands of the new men in power.

Among the drafts in the Brouwer archive there are some drafts of letters to (presumably) Gutkind, in which Brouwer complained about his humiliations. The letters must have been written roughly at the time of the battles in the faculty. The reader will recall that in his early years, Brouwer reacted extremely emotionally to obstruction and unfairness. His brushes with e.g. Reiman (in the matter of

[77] Brouwer to Wibaut 12.I.1946.

the International School of Philosophy), Denjoy, Menger, almost produced mental breakdowns.

A few samples from the drafts will illustrate Brouwer's feelings:

> — I am still flabbergasted by your message that I have to make a "statement" in order to enter the U.S.A.
> — In the vacuum after the liberation, when democracy was temporarily suspended, a take-over has taken place under cries of the hysterical usurpers, who, in order to maintain themselves at the place where they don't belong, have to exterminate the liberal, and as long as they have not succeeded to do this, to detain them in order to prevent that they write their 'I chose freedom'.[78] As long as they cannot kill or imprison him, they have to slander him abroad, so that the flight abroad is prevented.
> — Here too, at the time of the military authority and the suspension of democracy after the liberation, a take-over has taken place by a section of the population consisting of the ambitious mediocre, who had no chance under a normal democracy. And who are now forced, in order to remain in their usurped positions, in the first place, to persecute by means of fraud, slander, insult and maltreatment the 'independent' persons, in the second place to erode democracy ...
> — The usurpers hate most, not the helpers and collaborators of the enemy, but those, who do not admire and never have admired, their so-called resistance style. And among these they carry out their persecution, and in this they are not checked, even though the persecuted have obstructed and hindered the enemy with any considerable risk *in another style*.
> — When a band of ambitious mediocrities, who do not have a chance in times of a solidly united democracy, have penetrated, in times of a damaged social structure, with a hysteric device in the centre of power, for which they have no competence, the structure cannot regenerate itself. And thus the usurpers are more and more forced, in the first place, to persecute the independent (i.e. those who think for themselves, who are honest and candid and do not belong to a clique), and to hinder their emigration; in the second place to extend state interference to dumb automatisms, because they are not fit for intelligent official positions, and to suppress the instinctive spiritual opposition (or criticism, at least) of the non-thinking masses, by propaganda (schools for politics, state information service), and curtail the freedom of speech, ...

It is not difficult to guess what Brouwer was referring to. His personal position was thoroughly undermined by the purge procedure, but he was equally upset about the changes in the university in general, and in mathematics in particular. Moreover, like any citizen he was confronted by the new elite, political or otherwise, a certain

[78] This was an popular book written by a Soviet defector, Victor Kravchenko.

portion of which consisted of opportunists and incompetent bunglers with a fair dose of political flair.

In his young years Brouwer, incidentally. was a victim of spells of nervous outburst; later in life there were fewer signs of those, and the glowing anger often made place for utter despair. The above quotations illustrate a mixture of anger and despair that was to mar his remaining years.

In the beginning of January 1946 Brouwer collected some items that he had entrusted to the safe of the *Nederlandse Handelmaatschappij*. With the uncertainties of the war in mind, he had deposited his most precious notes. One would think that a man like Brouwer would give priority to the essential research notes in as far as they were salvaged from the fire in his cottage, but in fact the safe held a curious collection:

Package 1	first part main course
collected 11.6.45	notes main course 1940 and additions
	notes main course 43/44.
Package 2.	Lecture notes mechanics
collected 12.14.46	id. canonical equations, + id. classical vibrations
	+ enclosures
Package 3	Basis notes main course + higher algebraic curves
collected 11.6.45	list of examinations with lecture reports.

and furthermore

Foreign lecture series on intuitionistic mathematics in folder,
16 quarto sheets (with 5 additional enclosures) on measure and one-parameter continuous groups in intuitionistic precision (in cover).

Annotated academy documents with enclosures (in cover), de la Vallee Poussin : 'Integrales de Lebesgue' etc. (with notes);
Lecture 30 April 1938 (in one package)

Debet accounts of Sodalitas committee. Advances Sodalitas.

It is surprising that Brouwer first and foremost thought of the security of the above material. It may well reflect his feeling of responsibility for his duties as a teacher.

For the time being Brouwer's first priority was the appointment of Freudenthal elsewhere. There seemed to be hope for him, for Utrecht was interested in filling the vacancies left by Barrau and Wolff. In fact, Utrecht was left with one lecturer, Bockwinkel, a man with a curious reputation; at one time he had shown interest in constructive mathematics, but he actually paid more attention to perfecting his

lecture notes than to research; needless to say that his teaching was conservative. So far the Utrecht faculty had not been very fortunate. Van der Corput and Van der Waerden had opted for Amsterdam. In the post-war negotiations Utrecht won at least one prize: the prominent mathematician David van Dantzig was appointed. When, however, Amsterdam made him an offer, he almost immediately left; he did, however, complete his one year course in Utrecht.[79] So the University of Utrecht began to look like the poor bride who was twice left before the altar, and once deserted in haste.

Brouwer, who was of course aware of the vacancy at Utrecht, probably sent a glowing recommendation of Freudenthal to the chairman of the Utrecht faculty, the astronomer Minnaert. Minnaert was not fooled for a minute, the goings on in Amsterdam were not kept a secret from the rest of Holland. He was, however, sensible enough to realize that Freudenthal would be an asset to the Utrecht faculty.[80]

For Freudenthal this was a perplexing situation; he did not object to a chair in Utrecht, but he felt that by accepting the Utrecht chair, he would also accept the injustice inflicted on him in Amsterdam. Moreover his students more-or-less appealed to him not to give up Amsterdam. The choice was difficult and painful. Freudenthal's students admirably summed up the argument: after such intensive campaigns for rehabilitation a sudden transfer to another university could give support to the idea that there was a reason after all for denying Freudenthal the rehabilitation. This was indeed entirely Freudenthal's sentiment; it was in this spirit that he wrote Minnaert.[81] He could not leave Amsterdam without at least an effective rehabilitation. Giving up would mean deserting all those who fought for his rehabilitation, or that of others.

> I would betray my patriotic duty, and would be labelled as a deserter from the ranks of thousands, who now fight for their rehabilitation or that of others, if I would acquiesce without very compelling reasons, (now in liberated Holland), for the second time in the orders of the enemy, i.e. my dismissal, with all its consequences. I could only have done so, if I had been able to learn the motives of Professor Brouwer, and if I could have convinced myself and my supporters, that by giving in I would not serve the interest of my successor, but that of a great mathematician. Professor Brouwer has to my deepest disappointment never made an appeal to the childlike respect and devotion, with which I have served him since 1930 ...
>
> Rather, by appealing to my hypothetical fear for a public discussion of my appointment, Professor Brouwer had made it impossible for me, to withdraw my candidacy with honour.

[79] Van Dantzig was appointed in Amsterdam on 22 May1946. Cf. [Alberts, G. 1998].

[80] In those days one did not apply for a chair. One was recommended by colleagues and sister faculties. After the traditional bargaining the Universtity submitted the name of the candidate to the Minister for approval.

[81] Freudenthal to Minnaert 6.V.1946.

> I don't reject hereby the possibility to convenience Professor Brouwer after my appointment in Amsterdam, under reasonable conditions, e.g. by accepting a possible appointment in Utrecht.

The letter was dispatched two days before the City Council of Amsterdam was going to discuss the appointment of a lecturer in analysis, theory of groups and topology.

Brouwer, who by now really feared that the City Council might appoint Freudenthal, had on May 1 again addressed the Mayor, praising Freudenthal's capacities: 'a mathematician of exceptional decisiveness, universality and productivity, to whom we thank important results in a diversity of fields, and who as a consequence, considering his scientific merits, is fully entitled to an independent position at a university.' But, he repeated, 'I for myself have to avoid any contact with Dr. Freudenthal for reasons of self-preservation ... As, furthermore, my relation to Dr. Freudenthal already for years paralyses my scientific production, the continuation of an official connection between him and me will probably destroy the remaining chances on a resuming and finishing of my lifework.'

The above two letters illustrate the tragic quality of the conflict, sought by none of the participants, but after the fashion of a classical drama, ordained by the Gods. Here *Schicksal*, to use the once fashionable German term, had brought together the young man, admiring and loving Brouwer as a father, only to find himself rejected, and the older spiritual giant, who has to free himself of the younger budding intellect in order to protect his vulnerable inner life. It is a tragic aspect indeed, that these emotions can be confessed to third parties but seldom to the object of love-hate.

Be that as it may, the struggle went on. Brouwer even went so far as to write to all the chairmen of the parties in the City Council, begging them to postpone the decision.[82] At the same time he drafted a letter from the faculty to Freudenthal, saying that 'The faculty of Mathematics and Physics would like to inform you, that it unanimously judges your position in the higher education [system] completely unsatisfactory. The faculty will thus remain alert, in order to bring this position in line with your scientific merits. For a speedy realisation of an optimal solution, the faculty counts on your personal understanding and co-operation.' This letter was duly signed and dispatched by Clay.[83]

The letter, coupled to the unsatisfactory outcome of the council meeting, incensed Freudenthal, who pointed out to Clay that the letter did not explain why he still had not been rehabilitated one year after the liberation, that his scientific merits had never been doubted, that the appeal to 'personal understanding and co-operation' rather seemed to indicate a prelude to the abandoning of the rehabilitation, and finally that the factual content of the letter was nil. He said that he had heard some rumours, that in the near future he would have to make certain

[82] Brouwer to political parties 6.V.1946.

[83] Clay to Freudenthal 8.V.1946 (cc. Brouwer).

choices which would solve all the problems for the faculty. The least the faculty should do, is outline these choices.

Of course, Freudenthal was perfectly aware that the choices involved the chair in Utrecht, but he objected to the strategy that made him the sole actor in this matter. The faculty, he said, was in honour obliged to acknowledge the fact. This was more than Clay could stand; he got so annoyed that he more or less gave up Freudenthal's cause.[84] Freudenthal was, understandably, disgusted about the treatment he had received at the hands of the authorities and his colleagues. Apart from the loyal support of the students little had been done to further his cause. Even his close allies offered scarcely more than verbal sympathy. In an interview in 1987[85] Freudenthal looked back at the painful events after the war: 'At this time, when my fight for rehabilitation was going on, I received neither from Van der Corput, nor from Van Dantzig any support. In words yes, but not in deeds, for they wanted a position in Amsterdam themselves.'

The whole episode is a classic example of the pressure a system can bring to bear on an individual who simply wants his grievances redressed. It is the more a tragic example, as a friendly solution was perfectly possible and as a minority of two could apparently dictate the actions of a, on the whole reasonable and fair, faculty. The affair ended with Freudenthal's acceptance of the chair in Utrecht. To Freudenthal's immense surprise, Brouwer, entering the Mathematical Institute in Amsterdam, when Freudenthal was just leaving, grasped his hand and warmly congratulated him with his appointment. On 9 December 1946, Freudenthal held his inaugural address in Utrecht and from that time on he was the driving power behind the mathematical activity in Utrecht. Sometimes his conceptions were taxing the imagination of his company too heavily. At the traditional interview between the Curators and the new professor, he was asked what in his opinion the size of the staff of the Utrecht Mathematical Institute should be. His answer was '11 professors'. This man must be out of his mind, was the general reaction. However, before he retired he had built an institute that surpassed the demands of his early days. He introduced, for that time, modern methods and topics in the mathematics curriculum, and since Freudenthal's appointment, mathematics at Utrecht has obtained a status that made it stand out in the Netherlands. He remained a great man till the end, he never tried or wished to take his revenge on Brouwer. As a matter of fact, his edition of the *Collected Topological Works of Brouwer* is a fitting monument to the man who gave topology a new lease on life. Freudenthal has played an important role in Dutch mathematics and education in general. He was a man overflowing with ideas, always ready for new initiatives. Yet he never became part of the 'old-boys-network', his outspoken opinions and his habit of voicing unpleasant truths (e.g. his criticism of the Mathematical Centre, which soon after its founding developed into an Amsterdam instead of a national institution) gave him the reputation of being 'difficult'.

[84]Clay to Freudenthal 15.V.1946.

[85]Alberts et al. p. 118.

18.3 Back to research

With Freudenthal safely in Utrecht, Van der Waerden on his way to, or in Philadelphia, Brouwer could have relaxed and enjoyed the last few years of his professorship. He had five more years to go, and according to himself, he had an ambitious programme for finishing and redoing his mathematics. There was even a return to his old research activity; from 1946 onwards papers started to appear with some regularity. The first post-war paper was a retrospection, *Synopsis of the signific movement in the Netherlands. Prospects of the signific movement.* The paper was the result of a lecture at the 'Second International Summer Conference of the Signific movement', held from 24 through 31 August. In it, he summed up the participants and aims of the *Signific Circle* and the *International Academy for Philosophy*, as discussed in chapter 10. He added also some remarks on the contemporary use of the same methods. After all, World War II had brought even more 'abuse of false slogans for the satisfying of dark instincts.' He listed the following desiderata 'serving self-realisation of the individual': public safety, public welfare, mental freedom, and as much as possible freedom of action for the individual. These desiderata were not surprising in a world that had been subjected to the Nazi domination, but even to-day they would stand unchallenged. Furthermore he indicated two necessary conditions: '1. The utmost moderation of the state influence over the individual and the utmost reduction of the possibility of domination of the individuals over each other; 2. the existence of a relatively harmless and innocuous mode of diverting ineradicable dark and frivolous instincts such as lust of power, sadism and gambling.' The role of language surfaced finally in the closing passages:

> ..., it is my opinion that in any case in a happy humanity, state intervention will have to be prudently handled, and the state will have to use a language strictly indicative. If it deviates from this duty and admits into its language vaguely spiritually tinged terms, such as *principles*, *attitude*, *character*, *moulding*, *firmness of character*, *resoluteness*, *leader*, *qualities*, *heroisms* (for continuation of the list see Göbbels), then inquisition, denunciation and man hunting will still have their chance, man will oppress man and man will mistrust man.
>
> The fight against abuse of hysterical devices, the fight for unmasking them in private, and for removing them from public life will in future remain a preponderant part of the business of significs.

All this is perfectly reasonable, and it summed up the subtle and less than subtle tricks of the totalitarian regimes that had then been toppled (and the remaining one that was to haunt mankind for another fifty years), but we know that Brouwer was also referring to the post war powers in the Netherlands, no doubt in the light of his own recent experiences.

Significs also figured prominently in the major first post-war event in the mathematical circle in Amsterdam: the honouring of the senior mathematician Mannoury. On September 11, 1946 Brouwer awarded in the familiar surroundings of the auditorium of the University of Amsterdam an honorary doctorate to his

former teacher and lifelong friend, Mannoury. Knowing how reluctant universities are to present honorary doctorates to their own staff, one may be certain that Mannoury had strong supporters in the Senate. Brouwer honoured Mannoury not in the first place as a mathematician, but most of all as significist. Mannoury was indeed revered by al his students and followers, and the honorary doctorate was not only well-deserved, but also widely applauded. Those who have known Mannoury treasured for always the memory of a kind, wise enthusiastic friend and teacher.

Brouwer's mathematical papers that appeared after the war were mainly reports on earlier work. They were in part triggered by publications of Griss and Van Dantzig. Griss, a student of Weitzenböck, was a mathematician with pronounced philosophic interests. His analysis of mathematical intuitionism led him one step beyond Brouwer: he denied the meaningfulness of negation (and falsity, e.g. 0 = 1). The argument being (basically) that one cannot have a mental picture of constructions of such things.

Brouwer, in his first paper in the series dealing with negative properties, [Brouwer 1948c], introduced the so-called *creating subject*,[86] a notion already present in his Berlin Lectures. He used it to show that there are real numbers for which one could not possibly show that they are not positive, but for which one (so far) has no proof that they are positive. In Brouwer's terminology 'order is stronger than virtual order.' His result can also be stated as 'apartness is stronger than inequality.' That is, 'inequality is essentialy negative.' The result is still not optimal, as it depends on the state of the creating subject's knowledge (or our knowledge, if you like); it is still a weak counterexample.[87] In a subsequent paper Brouwer really exploited the creating subject in full force, by showing that the identity of apartness and inequality is even contradictory, in symbols $\neg\forall xy(x \neq y \rightarrow x\#y)$.[88]

This was his ultimate evidence that $\neq$ is not a positive notion. Here again, Brouwer had demonstrated his ingenuity and insight. He also formulated this new result in a geometrical context in a somewhat provoking manner: Euclidean plane geometry is contradictory, [Brouwer 1949a]. Of course, this way of describing the result is somewhat misleading. What he meant was that a particular classical theorem turned out to be false in the intuitionistic setting. But that he had already established in the twenties, when he showed that for the intuitionist continuum the dichotomy property was false.

The use of the creating subject remained mysterious for some time.[89] Later meta-mathematical researches have shown it to be consistent with most of the

[86]Later called 'creative subject' by Kreisel.

[87][Brouwer 1949b].

[88]There is a thin line separating the weak from the strong refutations. Here the matter is particularly delicate, as one would like to exhibit a real that was distinct from 0, but not apart. However, $\neg a\#0 \rightarrow a = 0$, so such a straightforward example is not available. The weak counterexamples are usually of the form 'we have no evidence for ...', whereas the strong counterexamples are plain contradictions, 'it is not the case that ...'. Brouwer's new strong counterexamples used the full strength of his theory of choice sequences (including the continuity principle).

[89]Heyting, in his monograph [Heyting 1956], put the topic in the chapter 'Controversial subjects'.

traditional intuitionistic principles.[90] To be sure, allowing sequences to depend on the mental activity of the creative subject (or the *idealized mathematician*), comes to a revision of mathematical universe, and hence it is by no means trivial (or even true) that all the old principles still hold.

No mathematical activity, unfortunately, could keep Brouwer from academic politics. He felt that he had made Amsterdam mathematics what it was, and to a large extent that was true. In the twenties and early thirties Amsterdam had even been a prominent centre for topology, but later developments, partly caused by the economic crisis, partly by Brouwer's disinterest in, and emotional unsuitability for, active leadership. Nonetheless, he still clung to the old promise of 1920, when Amsterdam had bought his continued presence and activity. He did not realize that administrative promises have a hidden date of expiration, in particular administrations are not prepared to honour promises to parties that have not fulfilled their part of the bargain. In Brouwer's case one could argue that after the golden decennium of the twenties, Brouwer had done little to keep his centre in perfect shape. Weitzenböck's solid routine did little to enhance the prestige of Amsterdam mathematics; the succession of Hurewicz and Brouwer's own withdrawal into his private world of research were not helpful to boost its status—not to mention the replacement of Freudenthal by Bruins. And so his claim to an Amsterdam equivalent of Göttingen had lost much of its force.

He must have been vaguely worried when he was informed that the minister contemplated the founding of a solid, real research institute for mathematics. Was it going to be in Amsterdam, and if so, what would the consequences be for his mathematical institute? In the beginning of 1946 he still was not worried about the new institute (which eventually got the name *Mathematical Centre*, so we will, somewhat anachronistically, adopt that name from now on, although at this point in our history it was as yet nameless). It even could serve as a means to keep Van der Waerden at bay. The driving power behind the Mathematical Centre was Van der Corput, who was the chairman of the ministerial committee for mathematics, and who was to advise the Minister on the desirability and feasibility of the centre.

In December 1945 Brouwer had asked Van der Corput to bring him up to date as to the centre—he was still suspended and hence could not take part in the deliberations. We have seen that Brouwer considered himself entitled to a seat in the committee, and he understandably felt neglected.

A major step towards this Mathematical Center was taken on 11 February 1946, when a special foundation '*Het Mathematisch Centrum*' was created in Amsterdam.[91] The board of the foundation consisted of the members of the Van der Corput committee, J.G. van der Corput, D. van Dantzig, J.F. Koksma, H.A. Kramers, M.G.M. Minnaert, and J. Schouten, supplemented with the two chairmen of the science faculties of the University of Amsterdam and the Free University, J. Clay and

[90] It is however incompatibkle with $\forall\alpha\exists\beta$-continuity, [Myhill 1966].

[91] An account of the history of the Mathematical Centre can be found in Alberts' dissertation, [Alberts, G. 1998].

G.J. Sizoo. The active kernel of this group was made up of Van der Corput, Van Dantzig, and Koksma—the most powerful men in postwar mathematics. In October they formed, together with Van de Waerden, the Administrative Council of the Centre. Simultaneously the board was replaced by a Curatorium, where the leading industries, the City Council of Amsterdam, the Institute for Technology at Delft and both Amsterdam Universities were represented. Schouten assumed the central position of secretary.

In September 1946 there is a puzzling bit of correspondence between Brouwer and Van der Corput:[92]

> Amice, In a telephone conversation with Heyting (who had called me about something else) I recently let slip something about the consequences, which could possibly follow from deceptions which I have experienced lately. Since these deceptions have by no means as yet resulted in a definite plan, it would have been better, to practice complete silence about possible consequences, even towards a person whom I consider in the matter more-or-less a fellow.
>
> Having learned—somewhat to my surprise—that the content of my telephonic impulse has been communicated by Heyting to you, the above explanation had to be given.

Somewhat later he returned to the deceptions,[93] saying that as long as the deceptions had not turned into hard facts ('which could easily not just hurt me, but also go against the general interest'), he should try to prevent those facts.

Cryptic language! One does not have to be *clairvoyant* to guess what made Brouwer write these notes. It is almost impossible that rumours of the wheeling and dealing around the Mathematical Centre should not have reached Brouwer.

The conflict at the faculty, the threatening developments around the Mathematical Centre, could not escape Lize. She realized that her husband had become an isolated person in the Amsterdam mathematical community. In a letter to her daughter, Louise, she saw early retirement as the plausible solution.[94]

> We seriously think that Dad, now that he has reached the age of 65, should resign. That miserable hurrying, and the hasting back and forth should be ended now. He is now entitled to his full pension, and that is indeed quite a difference with his salary. But we should then live more economically. Dad can spend more time on the administration of the pharmacy, and pick up his own mathematics again. If he carries on until his 70th year, he will be totally exhausted, and no longer able to get something done. So that will probably happen before long.

It is surprising that a clever operator like Van der Corput had overlooked the consequences of withholding information from a close associé. In an open dis-

[92] Brouwer to Van der Corput 27.IX.1946.

[93] Brouwer to Van der Corput 8.X.1946.

[94] Lize to Louise 6.III 1946.

cussion some form of compromise could have been reached, but when Brouwer learned that the man whom he had welcomed as a likeminded colleague, was practically sidetracking him, he saw that Van der Corput was not after cooperation but rather capitulation. Desperate situations ask for desperate measures, so he decided to turn directly to the mayor and the alderman for education. On 8 October he explained in a long letter, how a center for mathematics, if it were to be established in Amsterdam, was rightfully his.

In the summer of 1946 the City Council had reserved a budget of 25,000 guilders with the motivation, 'It is proposed to grant for the year 1946 to the Foundation Mathematical Institute a subsidy to the sum of D.fl. 25,000.-, for the Mathematical Institute that will take the place of the European Center for Mathematics at Göttingen.' As there was only one Mathematical Institute in Amsterdam, which was in 1920 designated by the City Councel as to be organized, as soon as the city finances allow this, on the same footing as the Mathematical Institute in Göttingen, the councel must therefore have been under the impression that the old promise was now going to be fulfilled.

In Brouwer's words:

> One of the two promises has at the time been fulfilled, the realization of the second one was between 1920 and 1934 repeatedly initiated, but these initial steps led each time, through causes unknown to me, to nothing. In that period there was a pronounced influx of foreign mathematicians to Amsterdam, which I have welcomed for some time. But because in the absence of an institute and an appropriate equipment for directing a group of studying foreigners, the necessary personal, spiritual and financial sacrifices eventually became too much for me (in particular after an indispensible source of income, which was discussed as such in the 1920 negotiations, was strongly reduced after an expropriation by the City[95]), this hospitality had to terminate. Thus the board of the city had through its temporizing not only duped me personally, but it had also nipped the international center, which was evolving by itself in Amsterdam, in the bud.
>
> It is possible that the fact that the fulfillment of the promises made to me in Amsterdam, and the vanishing of my instruction of foreigners in Amsterdam had attracted international attention; once again a chair in Göttingen was offered to me in 1934. This was, in spite of the high remuneration and ample equipment, already because of the then in Germany established form of government ab initio unacceptable for me. Only after again explaining in extenso, to the president curator the disappointments that the City Council had caused me, and after hearing from his mouth that he seriously deplored the course matters had taken, and that from the side of the board of the city everything that could be done by way of redress, also *would* be done, I formally turned down the offer. [...]

[95] Brouwer refers here to the pharmacy, see p. 557

> On the basis of the above historical exposition, in combination with the passage of the concept budget for 1946, it is difficult, in my opinion, to interprete the granting of the mentioned budget item other than that the amount mentioned—whether or not via a foundation—should be spent for the benefit of the mathematical institute of the University of Amsterdam, for purposes to be determined in agreement with the director of this institute.
>
> Moreover, on the basis of the above account, it would in my opinion be inadmissable in whatever way, to take the authority out of my hands, after I had raised it to the present level under difficult circumstances and under the sacrifice of a variety of personal interests. Alas, there are indications, that in certain circles there is a design for such a step; also that the preparatory actions in the direction already took place under the protection of the smokescreen of confusion of the Liberation. If this scheme should succeed (quod consules avertant) I believe that a page in the history of science would be written that will not fail to arouse astonished interest with future generations.

The above document apparently caused quite a stir; City Hall warned Van der Corput that things threatened to take a turn for the worse, and Van der Corput was so alarmed that he had the whole letter read to a secretary, who took a stenographic report. A copy of Brouwer's letter was forwarded to the ministry of education. One gets the impression that the fathers of the Mathematical Centre were not so certain of their case as they professed to be.

Following the above mentioned telephone conversation with Heyting, which inadvertedly was conveyed to Van der Corput, Brouwer wrote to the latter that he was not quite ready to discuss the matter with him, but a few days later he spelled out his discontent to Van der Corput.[96] It was better, he wrote, 'if to begin with I give you a written exposition of the thoughts with respect to the new mathematical institute, which so depress me, and that you subsequently answer in writing. Thus we can at least lay a basis for later personal discussions.'

The promised exposition followed three days later, it was a bitter complaint about the steps that had already been taken in the matter of the founding of the Mathematical Centre. He was, he wrote, unpleasantly surprised when he found out that the State and the City of Amsterdam had taken steps to open a 'Göttingen' Mathematical Institute in Amsterdam, without consulting him—an institute which had been promised to him in 1920, as soon as the financial situation allowed such. But, he guessed, the 'insult of being ignored' was probably a consequence of the 'political persecution'.

Since Van der Corput had a large say in the preparations, Brouwer had felt assured that if the preparations were successful, the leading position to which he was entitled would not be withheld from him. Indeed, at the end of October 1945 he was even invited for a discussion with colleagues who had already been informed.

[96] Brouwer to Van der Corput 8.X.1946, 14.X.1946.

When, subsequently, Van der Corput had asked him if he would object to a possible cooperation with Van der Waerden within the framework of the new institute, he had concluded that the question could only be meaningfully posed under the assumption of his own participation in the board. That he did not hear of further developments did not overly worry him, but suddenly in July 'I had to hear from a member of the ruling group that dealt with the organisation of the new institute, that there was no intention of giving me a place in the managing bodies.' The reason given was 'to spare me the strenuous work that was connected with the leading position.' Brouwer bitingly remarked that he did not care for such a respect for his peace. After that he heard no more, until he happened to find out that the leading positions had been filled.[97]

> Now I have to count to my great disappointment with the possibility that people dare indeed to expel me from the leading positions, since long entrusted to me, of the mathematical organisation in Amsterdam. I say 'dare', because the history of my thirty-seven years of activity at the University of Amsterdam, will make it nothing less than an impropriety, if from now on a mathematical institute, modelled after Göttingen, will be supported, where I am excluded from a leading position.
>
> But, you with your influence, can still stop my being passed over, and eventually it will appear that you have served the interest of our country both scientifically and morally.

In a postscript he added a final private concern:

> An additional circumstance is the danger that the dark figures, who had already earlier discredited me abroad, will now make my exclusion from a leading position of the new institute public and argue that it is a token of my reduced status here.

He felt insulted; after being the leader of Dutch mathematics for more than 30 years, he was quietly pushed aside. The purge and Brouwer's reprimand cannot have played more than a marginal role; some Dutch mathematicians may have borne Brouwer a grudge, but there may well have been a general feeling that Brouwer had a disproportionate influence on Dutch mathematics. Kloosterman, the mathematician from Leiden, took a commonsensical view of the anti-Brouwer feelings: 'Brouwer is a better mathematician, and the gentlemen cannot stand that, but it does not worry me.'[98]

The letter had its effect, Van der Corput sent Brouwer a short note[99]

> I will use my influence to see that you get a position that you deserve on the grounds of your capacity and personality at the Mathematical Centre.

The first assault at the bastion of the Mathematical Centre had created a visible breach. The two men discussed the problems the next day, apparently in complete

[97] Brouwer to Van der Corput 17.X.1946.

[98] Oral communication T.A. Springer.

[99] Van der Corput to Brouwer 31.X.1946.

harmony, for Brouwer wrote, 'I have the feeling that our conversation yesterday might have been most clarifying, both on account of the new facts we heard from each other, and of the impression of our mutual disposition and opinions we have exchanged.'[100]

But Brouwer was too wily to let it go at that, he knew the game too well; so he added that he considered that no actions were promised, as long as they had not been given in writing. Van der Corput immediately sat down to compose an answer; this was a delicate matter, for Brouwer had a reputation for getting the most out of any written statement. The first draft was written on 4 November and the final letter was dated 13 November.

After agreeing with Brouwer that the conversation of 1 November had a clarifying effect, Van der Corput went on to enumerate 'the points on which we, as I think, have reached an agreement.'

> 1. There will be a friendly co-operation between the Mathematical Institute and the Mathematical Centre.
> 2. In the spirit of the team of the Mathematical Centre, personal sympathies and antipathies will have no influence where the interest of the Centre, or of mathematics, is concerned. Let me mention as an example the possible appointment of Van der Waerden at the Municipal University, which appointment should be not only of great importance for mathematics in Holland, but also be financially profitable for the Mathematical Centre.
> 3. You give a description of the work you think you will do for the Mathematical Centre.
> 4. Your letter of October 8 to B. & W. [mayor and aldermen] of Amsterdam will be withdrawn.
> 5. I have to add that the board of trustees considers it self-evident, that no one of its members will send on his own initiative complaints, advice or counter-advice to authorities such as the City Hall, the Government, industry or the Curators. A new member of the Board of Trustees is asked to commit himself to this line of conduct.
> As soon as I have your endorsement in writing of the above five points, without exception, expressis verbis, it will be a pleasure to propose you as a member of the Board of Trustees.

One wonders if Van der Corput was quite serious about appointing Brouwer in any position at all. He could not very well have expected Brouwer to agree—in particular to points 2, 4 and 5. Whether one agrees with Brouwer, yes or no, the passage concerning Van der Waerden is plainly in contradiction with the results of the faculty discussions, and point 5 would make Brouwer (and any member) a hostage of the rest of the company. As if pushing Brouwer yet a bit further over the brink, Van der Corput told him that on acceptance of the conditions, and hence

[100] Brouwer to Van der Corput 2.XI.1946.

on a possible appointment, the stipulation that only written agreements were valid, was to be abandoned in the interest of an efficient management.

Brouwer must have been stunned—what Van der Corput asked him was nothing else than an unconditional surrender; he had, as the expression has it, been adding insult to injury. Brouwer took his time and answered coolly a month later;[101] he pointed out that the conversation of 1 November had a purely informative character; nothing was agreed, ideas and views were exchanged. He wrote, that he had drawn attention to the fact that the activities of the Mathematical Centre seemed largely of purely academic nature and hence should be practised within the university, and further that 'the previous history of the mathematical activity in Amsterdam does not allow that I am included in an Amsterdam mathematical organisation in a way that places other officials above me.'

The letter of 13 November had completely perplexed him, he said, for no agreement was reached on any point, with the exception of the desirability of further consultations.

> How you could write 'the points, on which I think we reached on agreement' instead of 'which I had expressed as my wishes and proposals' escapes me completely. Approximative equivalence of these two utterances exists only for very egocentric or very dominating natures, but for the time being I have no right to classify you as such.

The letter illustrated that an altogether written communication would have been preferable; now all misunderstandings had to be repaired. 'In October you twice declared yourself unable to meet my wish for written negotiations. However, how much less time would a written formulation of your proposals have asked at the time, than the amount I have to devote now to correcting conclusions that you have drawn from oral communications!' Brouwer's letter is one big demonstration of his dissatisfaction with Van der Corput's manner of operating. Van der Corput was for example implying in his letter that personal discussions with Brouwer were a waste of time: 'Furthermore you impute me the tendency to consider given promises valid "when they are laid down in writing." I will consider these words as a slip of the pen. I have to bear myself too much grudge as a consequence of oral agreements that have been broken against me. However, it is something totally different to distinguish between negotiating and informative discussions, and likewise the stipulating that a certain discussion will have the character of the one or the other.' In order to answer Van der Corput's letter in extenso, he wrote, he would have to see the relevant files first. The final passage, too, bore the first signs of pessimism and resignation:

> So much is clear to me that if the supervision of the mathematical enterprise in Amsterdam should be taken away from me, only the parties inflicting the injustice should be responsible for the resulting situation and that therefore the suffering party should under no circumstances be subjected to conditions for the purpose of a possible repair. And also that the

[101] Brouwer to Van der Corput 14.XII.1946.

> position of the mathematical sciences founded in the past at the University of Amsterdam by the authorities, should be protected by those same authorities.

As far as the correspondence shows, this is the end of a possible accomodation between Brouwer and the Mathematical Centre. Brouwer could not expect any compromise from the leading proponents. Van der Corput did take Brouwer's opposition seriously, not in the sense that he would reconsider the position, but rather he wished to forestall any possible moves of Brouwer against the Centre. Already before Brouwer's reply he had shared some of his worries with G. Bolkestein, the former London Minister, who had put his mind at ease—no action of Brouwer against the Mathematical Centre had reached the ministry. Nonetheless, Bolkestein advised against further dealings with Brouwer:

> You and Professor Clay know better how to assess Professor Brouwer's attitude than I do, but in my opinion the M.C. should no longer seek contacts with him; whatever the consequences. There must after all be a certain feeling of self-respect on our side (sic), that should prevent us from negotiating with someone, who, like Mr. Brouwer, agitates against the M.C.[102]

From this point onwards the roads of Brouwer and the Mathematical Centre separated, Brouwer considered it an odious creation, which had stolen his 'Göttingen in Amsterdam.' There was a halfhearted attempt to mollify Brouwer by offering him the title of 'honorary chairman' of the *Mathematisch Centrum*. Needless to say that the suggestion was ignored.

One might get the impression that the post-war years offered nothing but misery to Brouwer, but fortunately there were also some brighter spots. In 1946 Brouwer had received an invitation from the University of Cambridge to give a course on intuitionism; as one can easily imagine, this appealed very much to Brouwer.

The (spiritual) sponsors on the Cambridge side of Brouwer's lecture series could very well have been the resident logician Steen and the topologist Newman. We find his course listed for the Michaelmas term of 1947 as 'Intuitionistic Mathematics.' Among the persons attending Brouwer's lectures was a man who was in due time to give a new impetus to the study of intuitionism, Georg Kreisel. He reported later that he asked Brouwer after a lecture, if he meant all he said. On Brouwer's reply that he followed Shaw's dictum, 'you have to exaggerate in order to make an impression.' Kreisel innocently pointed out that, 'nobody promised you that it would be a good impression. Incidentally, Brouwer was not amused. Apparently he did not like to be interrupted anyway; fittingly, for a good solipsist.'[103] For Brouwer the course was not only a welcome escape from what he considered a hostile environment, but it also gave him an opportunity to prepare an up to

[102] Bolkestein to Van der Corput 21.XII.1946.

[103] See [Kreisel 1987], p. 147

date account of his intuitionstic theories. After all he was lecturing to bright young mathematicians, and he was casting his lecture notes in English, the new lingua franca of science. The course notes were destined to be published by the Cambridge University Press, and Brouwer seriously worked and reworked his notes in order to get a polished exposition. He even made an appointment with Steen, for 'consultations on the intuitionistic vocabulary for the forthcoming book.'[104] But the Cambridge Lectures fared no better than the Berlin ones.

In the middle of all these conflicts and machinations, there was at least one event that assured Brouwer that not the whole world had turned against him. In 1947 it was forty years ago that Brouwer got his doctorate. Friends, students and colleagues who had not deserted the old master, organized a jubilee symposium on 19 February. The ceremony took place in the lecture hall of the Geological Institute of Brouwer's brother Aldert. Mannoury addressed his famous student and colleague; he sketched how Bolyai, Riemann and Peirce had broken the hegemony of the Euclidean and Archimedean axioms, how Einstein had 'widened the visual field of physical science', but how Aristotelian logic had survived all disturbances until Brouwer 'emancipated human thought from the authority of the logical principles and so ran down that stronghold itself.' He mentioned Weyl and Dingler who had recognized in Brouwer's ideas a revolution and chaos, without appreciating the positive element in Brouwer's programme.[105] He went on to appraise the fundamental contributions in the words of the paper *La question vitale: 'A ou B'*.[106]

Faculty politics did not wholly absorb Brouwer's scarce time, he gradually picked up his international contacts. In the summer vacation he took part in a symposium of the '*Institut des Sciences Théoriques*' in Brussels. There, in the Palace of the Academy, from 8 through 13 September a small select company got together to discuss the modern developments in the exact sciences. The Netherlands were represented by Beth, Brouwer Heyting and Pos. For Brouwer this meeting offered an extra bonus: after all those years he got together with his friend Hermann Weyl. Both parties were extremely pleased to find that they could pick up the threads of their friendship. Beth was one of the speakers, but Brouwer and Weyl only took part in the discussions.[107]

The story of Brouwer and the faculty now is speeding towards its end. Although Brouwer remained active till the last day, both in his teaching and as a Director of the Institute, his influence was waning. In fact, Van der Corput was laying down the

[104] Steen to Brouwer 27.1XI.1947.

[105] Mannoury did Weyl an injustice. In fact Weyl was one of the first who fully appreciated Brouwer's ideas, cf. [Weyl 1921], [Dalen, D. van 1995].

[106] [Mannoury 1943]; insiders knew that 'A or B' meant 'Aristotle or Brouwer.'.

[107] Recorded in the proceedings, *Problèmes de philosophie des sciences: premier symposium, Bruxelles*, 1947. Hermann, 1948-1950. 7 vols *Serie Archives de l'Institut International des Sciences Thoriques. Ser.A.Bulletin de l'Acadmie Internationale de Philosophie de Sciences*.

Celebrating the 40th anniversary of Brouwers doctorate. Front row: Clay, Brouwer, Mannoury. Two persons to the left of Clay, Van Dantzig; to the right of Brouwer, Lize; right behind Mannoury, Heyting, next to him Louise; third row from the right, Evert en Cor Bruins. (Courtesy R.A.F. Guasco).

law in Amsterdam mathematics, and all Brouwer could do was carry out rear-guard actions.

At the end of 1947 the faculty resumed its activity to secure Van der Waerden for Amsterdam. The faculty had called a meeting on December 10, knowing that Brouwer would be in Cambridge at that time and making sure that the invitation would not reach his home address until after the meeting.[108] At the meeting a plan to extend the mathematics section vigorously in the 'technical direction' was accepted. Brouwer found out what was going on when in Cambridge, he was—with good reason—furious, he immediately sent a telegram to Bruins,

> Please send stencilled circular to faculty members containing my protest against faculty adopting proposal without previous duly convoked discussion by first section.

[108]The faculty files show that Brouwer had duly requested the Minister's permission to obtain foreign currency at the rate of . 5,- a day, 3.XII.1957.

A letter to the faculty was dispatched the same day, it suggested that 'the question should be faced, if such extensions [in a technical direction] are not in conflict with the calling of the university, and that therefore the action cannot be allowed to be carried out in haste and without written preparation.'

Upon returning, he called the lecturers Bruins, Heyting, de Groot and the assistant Loonstra to discuss the matter and to present his views. The result was a joint letter to the Central Committee of the Faculty;[109] it stated that at the request of City Hall the wishes for an extension of the faculty had been registered, that some sections had expressed interest in an extension in a technical direction, but that the undersigned, not being involved in extra-universitary activities for the sake of their mathematical practice, 'Asked the committee to pass their desiderata on to City Hall.' It hardly needs mentioning that the desiderata coincided with Brouwer's views: Pure mathematics (including rational and stellar mechanics) should be brought up to a fairly complete strength, 'as already promised to the first undersigned in 1920.' This required two full professors. Furthermore it contained the hardly veiled recommendation that Bruins should get a chair for applied mathematics; the second chair was reserved for those areas of mathematical research out of which the rational and stellar mechanics had developed and remained interwoven with. However (strange switch of subject), the first priority was a renovation of the Mathematical Institute, including the addition of a coffee room for the students. The letter made one thing clear beyond any doubt: the undersigned could not agree with the change in a technical direction.

The chairman, Clay, did not care one bit for Brouwer's objections, and sent a list of desiderata to the Senate, mentioning Brouwer's personal wishes, without mentioning the support that Brouwer had found. The faculty wished, he wrote, a chair for applied mathematics, to be filled by a scientist of the first rank (exit Bruins, enter Van der Waerden), and a position for 'modern computation methods and computing machinery'— this could be shared with the Mathematical Centre on a part-time basis.

It may seem curious, from the viewpoint of fair representation, that the preferences of one mathematician (Van der Corput) in the faculty prevailed over those of the remaining four mathematicians. One should keep in mind however, that the Mathematical Centre was a fact since February 1946, and that Clay and Van der Corput had leading positions in it.

In February a new development occurred; J. de Groot (the topologist) was offered a chair at the Institute for Technology (*Technische Hogeschool*, TH) at Delft. Clay confronted the Curators with the fact that the TH had eight professors in mathematics and was to get two more in the course of 1948, whereas Amsterdam, the Dutch capital of mathematics had to offer a complete curriculum with two professors and three lecturers. 'It does not become the faculty to compare the position of a mathematics professor in Delft with that of a university lecturer in the same subject, but it should not be forgotten that the latter has the task to educate and

[109] Brouwer, Bruins, Heyting, De Groot, Loonstra to Faculty 27.XII.1947.

train mathematicians such that his students will be able to conduct independent and original research.'[110] Now that De Groot had an offer from Delft, steps had to be taken to secure the mathematical education in Amsterdam. The faculty did not think it necessary at this moment to promote all lecturers to professors, but at least De Groot and Heyting should be made extra-ordinary professors. The choice for De Groot and Heyting was not a difficult one. Both had a good reputation, and Brouwer had composed a flowery recommendation for Heyting. 'Science', he wrote, 'owes some fundamental discoveries to Dr. Heyting, which have also influenced philosophy, and in particular have thoroughly renewed epistemology.' After extolling Heyting's significance for non-Aristotelian geometry and for the formalisation of the Heyting-logic, originating from intuitionism, he concluded: 'Heyting is now 50 years old. It is more or less poignant that at this age, a man of his merits and his fame, still has the academic position of a lecturer, which previously had always been occupied by professors.'[111]

In March the battle over Van der Waerden's appointment was renewed. The faculty chairman pleaded in a letter to the Rector for the admission of a special chair in applied mathematics, to be administered by the 'Foundation for Higher Education in Applied Mathematics.'[112] A word of explanation may be in order. The Dutch academic system basically knew three kinds of professors (chairs), the full professor, the extra-ordinary professor and the special professor.[113] The second one is for all purposes a regular appointee of the university, but in a lower rank, with a lower salary and possibly with a part time appointment. The special professor is appointed and paid by an extra-academic institution, society, foundation, etc. As a result there are special chairs for a wide range of topics, ranging from parapsychology to theoretical physics.

The request of the above mentioned foundation was therefore nothing special, but in this particular case, it was but another attempt to attach Van der Waerden to the University of Amsterdam. Indeed the Foundation was just a derivative of the Mathematical Centre, with Clay and Van der Corput in the driver's seat. The Foundation did not beat about the bush, it straightforwardly proposed to appoint Van der Waerden, who had a visiting chair in the USA, and was prepared to resist some attractive American offers, provided he could pursue his research in Holland, combined with a special chair in Amsterdam, in order to train students and researchers for applied mathematics.

Brouwer immediately countered the proposal; he presented his objections in a five-point note to the Senate. For one thing, he said, we don't need any more applied mathematics than is already provided by Dr. Bruins; for another, this is a device to get around the earlier objection of the Minister to the appointment of

[110] Clay to Curators 27.II.1948.

[111] Brouwer to faculty 27.II.1948.

[112] Clay to Rector 13.III.1948.

[113] At present there are full, part-time and extra-ordinary professors. Apart from the change in title, the positions are the same as the old ones.

Van der Waerden (in Utrecht). Nonetheless the appointment of Van der Waerden is presented as so urgent, 'that it is not too high a price to pay for the definite insolvency of the City of Amsterdam with respect to promises made to undersigned in 1920, trusting in which he therefore remained in the Netherlands.'

At the subsequent Senate Meeting there was a lively discussion. When Brouwer was given the floor, he began by remarking that there was a small group of experts in the Senate, of which he was the oldest. He—to everyone's surprise—supported the faculty's request and warmly recommended Van der Waerden. There was just, he said, a difference of opinion between himself and the Faculty. It was his personal opinion that applied mathematics had no place at a university; it were not the merits of Van der Waerden in that area that had inspired the proposal, Van der Waerden was known for completely different reasons. He then suggested that he should formulate his views and present them to the Rector—Van der Corput could add a rejoinder if he wished to do so.

This move, clearly, had not been expected by Clay; he remarked that he was glad that Brouwer supported the candidacy of Van der Waerden, but he did not see how Brouwer's proposal could be carried out. The secretary of the Senate thought there was no problem: let Brouwer's views, as received by the Faculty, and Van der Corput's, be sent to the Rector. As to be expected, there was also opposition to the proposal of the faculty for a different reason. It was pointed out that not too long ago the Minister had refused to confirm Van der Waerden's appointment for political reasons. Could one support such a questionable candidate? But now, Clay countered, Van der Waerden is at Johns Hopkins and we will sound the Minister on the acceptability. These arguments, coupled with Brouwer's support, were for the Senate good enough to go along with the faculty's proposal.

Brouwer held his part of the bargain, he composed a memorandum for the Curators, in which he gave an exposition of his views.[114] Since the arguments have a definite Brouwerian philosophical flavour, it is worthwhile to reproduce them here:

> 1. Mathematics is an introvert science, as such it merges with philosophy, theology and reflexive psychology, but is to a higher degree than these, constructive. And the mathematical urge to create is therefore directed not only at inner clarification, but also at beauty, a beauty related to that of architecture and music, but more immaterial.
> 2. On these grounds the mathematical state of mind is usually indifferent with respect to natural science and definitely unfavourably inclined towards the promoting of the exploitation of nature and towards technique, which creates possibilities for it.
> 3. This does not detract from the generally known fact, that *and* technique *and* natural science *and* a manifold of other extrovert sciences have been able to reach their present status and size only, because they 'reckoned' (arithmetically or graphically), i.e. they operated mathematically on the

[114] Brouwer to Curators 2.IV.1948

> mathematical systems which were 'projected' on their activity, or their domain.
> 4. Although, in this manner, in the first place the technical, but furthermore all other extrovert sciences belong more or less to 'applied mathematics', they are in their essence fundamentally different from that of the introvert mathematics.
> 5. Where applied mathematics is mixed with the activity of a university as an all-permeating incidental circumstance, and almost totally represents the substance itself of the activity at an institute of technology, there is, exactly on the ground of this ubiquity, no place for a special curriculum at either of the two institutions. On the contrary, every extrovert science is interwoven with its own applied mathematics, and should remain inseparably connected to it in its teaching.

The reader will recognize a Shakespearean flavour in the argument, it recalls Anthony's great oratory in *Julius Ceasar*. After thus determining the place of applied mathematics in the general curriculum, he went on to argue that one could imagine a sort of education in applied mathematics, namely the very elementary part which belongs to the 'propaedeuse'. This, he added, is already taken care of by Bruins. The rest of the arguments (counting up to 11) is a repetition of his earlier ones—abuse of power by the Faculty, procedural mistakes, the violation of the 1920 promise, Van der Waerden's expertise in pure mathematics.

Not content with presenting his views on the proposed appointment of Van der Waerden to the board of the university, Brouwer went one step further and sent his arguments to the minister of education.[115] He repeated the above arguments, but added one that reflected on Van der Waerden's career in Germany:

> From a researcher like Professor Van der Waerden, who is only theoretically, but not experimentally active, the scientific influence is almost independent of personal presence. Thus, as soon as a materially and scientifically favourable position has been secured, the question of his presence here in the country loses all scientific and national importance, and it becomes almost exclusively a matter of national prestige. From a viewpoint of national prestige the motivation of his appointment here in the country seems however extremely weak to the undersigned. For if it is claimed that by the presence of Professor Van der Waerden in Amsterdam the strength of our nation is enhanced, the reply is forced upon us that in that case the national strength of the German empire has been enhanced during the whole period of the Hitler regime by the presence of Professor Van der Waerden in Leipzig. And if it is argued that if Professor Van der Waerden is not offered a suitable position by the Netherlands, this will be done by America, the reply is forced upon us that if at the moment there are positions open to Professor Van der Waerden in America, this should not have been less the case between 1933 and 1940, when many

[115] Brouwer to Minister of Education, 15.IV1948.

> prominent and right-minded German scholars and artists were welcomed with open arms in America, and that therefore one has to assume that Professor Van der Waerden had not felt the desire to turn his back on the Hitler regime.

There are no indications that the ministry reacted on Brouwer's letter. It is not unlikely that Brouwer intended to use Van de Waerden's past in the attempt to keep him away from Amsterdam—after all, the minister (and ultimately the Queen) had the last word on appointments, but the feeling expressed in this passage perfectly reflected the general opinion of the Dutch, and in particular the students, in this matter.

Clay and Van der Corput had once more been put on the defensive. They wrote a rather unconvincing letter to the Rector and Van der Corput was given the task to dissociate the three lecturers from Brouwer.[116] Van der Corput announced that the faculty was going to interview the lecturers, because 'In my opinion the enclosure added to the letter of April 2nd does not completely correctly represent the views of the gentlemen Heyting, Bruins, De Groot and Loonstra.'[117] In short, Brouwer was a fraud or the four gentlemen had signed something without reading it! Even a milder man than Brouwer would have resented such an imputation. A short note was the reaction:[118]

> Prof. Dr. J.G. van der Corput, Amsterdam. Your message of the ninth, received by me on the twelfth and your attitude, expressed in it, have deeply offended me and have changed things for me.
>
> Yours truly
> L.E.J. Brouwer

No 'amice' or 'friendly greetings'. The friendly relations between Brouwer and Van der Corput were a thing of the past. Did Van der Corput provoke this parting of ways on purpose, or did he not quite know how to formulate his thoughts? There is little to go on, he was a kind man with a good reputation with students and colleagues, but a man who knew to get what he wanted. In a mixture of post-war emotions and academic power play, there was a regrettable tendency to assume that in a confrontation with Brouwer one was allowed to bend the rules and the mores. This attitude even lingered years after his death.

Once Brouwer had accepted the inevitable, he stopped taking Van der Corput seriously. One way of coming to terms with persons he considered despicable, was to provide them with nicknames. And Brouwer was a master at the game. There are a few nicknames that have been handed down from generation to generation. Van der Corput's nickname was particular apt (in Brouwer eyes), he called him *Van der Corrupt*. There is an anecdote about the Brouwer–Van der Corput relationship, that has been vouched for by students and colleagues of the period. It had not

[116] Clay and Van der Corput to Rector 9.IV.1948.

[117] Van der Corput to Brouwer 9.IV.1948.

[118] Brouwer to Van der Corput 14.IV.1948.

escaped Mrs. Van der Corput that her husband and Brouwer were no longer on the friendly footing of the beginning. She therefore took the bold decision to go and see Brouwer about the matter. She hoped to accomplish what the men could not: a reconcilliation. So she spent an afternoon with Brouwer, who turned on his charm at full strength. The matter of the deplorable detoriation of the relations got all the attention it deserved, but no conciliation was accomplished. When she returned home, and ran into her husband, she gazed at him, and half seriously, half mockingly asked 'are you really so corrupt?'

One gets the impression from the surviving correspondence that at this point Brouwer had given up fighting the opposition any longer. There are some isolated letters, e.g. a curious one from Clay, who claimed not to understand how one could prevent in 1948 the fulfilment of a promise made in 1920.

The Secretary of the faculty, the physicist J. de Boer, tried to repair the rift between Brouwer and Clay–van der Corput; but Brouwer simply ignored the attempt and he refused to attend the faculty hearing of the lecturers Heyting, Bruins, De Groot and the assistant Loonstra. Being convinced that the Faculty was conspiring against him, he saw in the fact that the convocation, dated April 10, reached him on April 13, one day before the meeting, another indication that Clay c.s. intended to make it difficult for him to attend.

De Boer wrote Brouwer right after the Faculty Meeting, that three of the four who had signed the letter of 27.XII.1947, had declared that they did not wish their letter to be used at this moment against an appointment of Van der Waerden.[119] This, by itself, meant that Van der Corput's insinuation that Brouwer had made them sign something that they did not mean to say, had not been substantiated. In the same letter, De Boer informed Brouwer that the Minister no longer objected to Van der Waerden's appointment as a special professor.

At the Senate Meeting of July 5 Brouwer conducted a minor rear guard action, consisting of a correction of the minutes, so that his view point was properly recorded.

The events in the faculty might give the reader the impression that Brouwer was living in a state of permanent siege, with the ever-present prospect of utter destruction. In Brouwer's mind this may have been so to a certain extent, but there were also heart-warming signals of recognition. One of such spots of sunlight was his election to Foreign Member of the Royal Society. J.H.C. Whitehead had written a brief but forceful recommendation:[120]

> Brouwer revolutionised the foundations of mathematics by his critique of the notion of 'existence' in the mathematical sense and by his constructive theory of 'Intuitionism', which arises from it. Also he was the outstanding figure in topology during the twenty odd years which followed the publication of Poincaré's papers on the subject. His work on fixed points, on the degree of a mapping and on the concept of dimension opened up

[119] De Boer to Brouwer 15.IV.1948.

[120] 5.II.1948.

> some of the most fruitful fields of research in the subject. It is not unlikely that he will subsequently be considered the most original mathematician now alive.

Brouwer was elected on May 27, 1948 together with a number of other worthies (among them Linus Pauling, Kurt Mahler (the number theorist), J.M. Whittaker).

Brouwer's gradual return to the other fascination of his youth, philosophy, found an eloquent expression in 1948, when he took part in the Tenth International Congress of Philosophy at Amsterdam, which was held from 11 to 18 August. Brouwer used the occasion to spell out his views in the address *Consciousness, Philosophy and Mathematics*.[121] In a way this was an update of his Vienna lectures, be it that there was a notable shift in style and presentation. Where the Vienna lectures were the credo of a conqueror of the world in the strength of his life, compact, to the point, with little consideration for his readers, the Amsterdam lecture was more reflective and resigned in nature. The opening sentences faithfully expressed Brouwer's feelings,

> First of all an account should be rendered of the phases consciousness has to pass through in its transition from its deepest home to the exterior world in which we cooperate and seek mutual understanding. This account does not imply mutual understanding and in some way may remain a soliloquy. This can be said of other parts of this lecture too.

The first part of the lecture can be viewed as an elaborate justification and elucidation of the ur-intuition of the dissertation. The key notion, of course, is the subject, experiencing sensations. Through the *move of time*, already (namelessly) occurring in the Vienna Lectures, the whole world of the subject is constituted—both the egoic and the exterior. The jump from end to means (cf. p. 69, 560) is upheld in its full extent, here under the name 'cunning act' ('mathematical act' in the Vienna Lectures). The exposition expressly mentions an aspect that, although obviously part of Brouwer's views, was not mentioned before:

> In this connection there is a phenomenon of *play*, occurring when conative activity or causal thinking or acting is performed *playfully*, i.e. without inducement of either desire or apprehension or vocation or inspiration or compulsion.

Brouwer recognized three phases of the 'exodus of consciousness from its deepest home':

> – the *naive* phase, of 'the creation of the world of sensation',
> – the *isolated causal* phase, where the causal acts take place,
> – and the *social* one, in which 'cooperation with the individuals' finds its place.

[121] [Brouwer 1948a].

Given the phases, 'the question arises, whether and where, on and after the exodus of consciousness, *beauty*, *mutual understanding*, *wisdom* and *truth* can be found.' In the last two phases 'there is beauty in remembrance of the miracle of bygone naivety, remembrance evoked either by reverie through a haze of wistfulness and nostalgia, or by (self-created or encountered) works of art, or by certain kinds of science. Such science evoking beauty reveals or playfully mathematises naively perceptible forms and laws of nature, after having approached them with attentive reverence, and with a minimum of tools.' Here, and in later passages, play and playfulness are awarded a high place in the intellectual activity, in particular where non-pragmatic acts are concerned.

The subject recognizes in the 'outer world' object individuals, who behave with a certain similarity to the subject; this, according to Brouwer, cannot be derived from 'mind', as it would induce the subject 'to place in each individual a mind with free-will dependent on this individual, thus elevating itself to a mind of the second order experiencing incognizable alien consciousnesses as sensations. Quod non est! '*Plurality of mind*' is thus firmly rejected by Brouwer, and 'in default of a plurality of mind, *there is no exchange of thought either*.' In view of these passages there can be little doubt of Brouwer's solipsistic inclinations at that stage of his life.

Although Brouwer explicitly mentions the social phase, one should not be misled by this term; fellow (human) beings are no independent beings in a pre-existing outer world, they are, like the rest of the universe, creations of the subject. For logicians and philosophers the interesting topic is of course that of truth and its place in logic. '*Truth* is only in *reality*', quoth Brouwer, 'i.e. in the present and past experiences of consciousness.' The purpose of logic being to preserve truth, a certain reliance on logic in this respect developed; however, the conclusions of logic do not 'convey *truths* before there truths have been experienced', nor is it certain that 'these truths always can be experienced.' Brouwer's conclusion is that 'logic is not a reliable instrument to discover truths, and cannot deduce truths which would not be accessible in another way as well.' In other words 'there are no non-experienced truths.'

The lecture also contains creating subject arguments, albeit not in the strong form of his 1949 paper [Brouwer 1949b]. A trace of the defence against Griss' negationless criticism can be discovered in the modest claim 'there seems to be little hope for reducing irrationality of a real number $a \ldots$ to a constructive property.'

A number of more or less expository papers that followed, spelled out the same messages; but none with the clarity and explicitness of this philosophical lecture.

18.4 The Loss of *Compositio Mathematica*

The fight for *Compositio Mathematica* is one more drama in Brouwer's life—the last big one. It took place at the end of his academic career, and it is a vivid illustration of the erosion of his position in Dutch mathematics, his inability to build and maintain a sufficient support in mathematical circles. For the lone operator Brouwer it was no longer possible to defend his position. Even his considerable

command of argumentation and persuasion had lost its magic power. As Kreisel put it, in the obituary of Brouwer for the Royal Society: '. . . , while, . . . , solipsism seems an excellent first approximation for an analysis of mathematical reasoning, it would not be expected to be equally sound in public relations.'[122]

Most of the documents of the *Compositio* affair are to be found in the Brouwer archive. Unfortunately the publisher Noordhoff has not preserved the correspondence and documents pertaining to the matter.[123]

When life resumed its course after the war, many threads had to be picked up which were either dropped at the outbreak of the war, or which had become entangled in a number of ways during the war. In almost all organisations and companies there was a, sometimes subtle, sometimes not so subtle, power struggle between the forces of renewal and those of restoration. Next to the old political parties, new parties sprung up with new names and new programmes. In art young men eagerly waited for the fall of the establishment. New dailies and weeklies appeared, most of them the legal successors of the underground papers published by the various resistance movements.

In the universities one could also observe a mild echo of the social-political changes in the Netherlands. By and large the most significant phenomenon was a temporary speed up of appointments of professors. The war and the purge had left vacancies to be filled. On the whole one could speak in the case of the post-war developments in academia more of a restoration than of a revolution. A disruption like that of the sixties and its democratization was out of the question. The scientific organizations, as a rule, resumed their activities, their regular meetings and publications.

The publishing houses could not immediately join the upsurge of economic and cultural life, hampered as they were by the shortage of paper. This had consequences in particular for scientific publications; for a long time libraries, professors and students alike had to make due with second-hand pre-war copies and with books donated by (mostly American) universities.

As the man in charge, Brouwer had to consider the future of his *Compositio Mathematica*. The journal had been discontinued in 1940, when it was confronted with serious difficulties. Freudenthal, who had run the journal almost single-handedly, was the first to bring up the matter of re-issuing Compositio. In a letter to Hopf he gave an account of the situation:[124]

> Concerning Compositio, the matter is that I have officially no business with Compositio. I am simply not a member of the editorial board. Compositio can probably not appear legally with the editorial board as it was on May 14, 1940. For here everything is 'purged', the civil service, the professions, associations, editorial boards, etc. If an editorial board has

[122][Kreisel, G. 1969], p. 46.

[123]In fact Noordhoff merged with Wolters, and it is no longer an independent company. In the transition the relevant material was probably discarded.

[124]Freudenthal to Hopf 9.X.1945.

> not itself been infected, it can of its own proceed to purge itself. How this is done with editorial boards in which also foreigners are present, I do not know. In the case of Compositio the matter is especially unpleasant; Weitzenböck is stepping down anyway. The purging of Brouwer is yet open—I mean his purging as a professor, and the result will have its consequence for his further membership of the editorial board. [...] If Brouwer returns as a professor, he will certainly claim his right to sit on the editorial board. But probably the remaining Dutch mathematicians (apart from Heyting) have no wish to work with him. This can be said with certainty of Van der Corput. [...] I don't see at the moment any possibility but the founding of another journal under a similar name. I will discuss the matter with Van der Corput. Perhaps he can say something. What would be your position with respect to a Compositio without Brouwer or with Brouwer thrown out?

In fact, nothing happened at all. Brouwer did not even consider a quick reanimation of *Compositio*. It would have been rather unlikely that the authorities would have allotted the required amount of paper for the journal.

The activities around *Compositio* during the first years after the war are somewhat obscure. On the one hand, Brouwer started to explore the possibilities of a reissuing of the journal, on the other hand, a number of Brouwer's opponents would rather see a *Compositio* without Brouwer. It seems that Brouwer was approached by Noordhoff with the request to resume the publication of *Compositio*.[125] Apparently the efforts of Noordhoff were not very satisfactory, for Brouwer was in 1947 cautiously shopping around for a new publisher. In January 1947 he inquired with Father Van Breda, professor of philosophy in Leuven, reknown for his founding of the Husserl Archive, if there were printers in Belgium who could handle an international mathematics journal. Van Breda supplied the information.[126] In the same letter he invited Brouwer to Leuven for a series of lectures. In view of the awkward financial situation, he could not promise Brouwer a suitable fee, 3500 Belgian francs was all he could offer, Brouwer would have to pay for his stay in Leuven out of his own pocket, but 'undoubtedly you will repeatedly be invited by various professors for lunch and dinner.' In spite of the scant fee, Brouwer accepted the invitation and lectured 6 times at the end of March.[127]

On February 3, 1948 the difficulties had been in so far overcome that Brouwer informed the Committee of Administration (editorial board) of *Compositio* (de Donder, Hopf, Julia, Whittaker), of his plans to send out a circular letter to all editors.[128] In this letter the editors were asked to stay on and to publish their own papers and 'those originating from your school' in *Compositio*.

[125] Brouwer to Ed. board Comp. Math. 10.VII.1949, 27.I.1950.

[126] Van Breda to Brouwer 25.I.1947.

[127] 17, 19, 21, 24, 26 and 27 March 1947.

[128] Strangely enough 'to the editors belonging to the United Nations.' What had happened to his internationalist convictions of 1919?

Noordhoff set itself to produce a first post-war issue, but it discovered that the printer had lost patience, and re-used the lead of the type of the 1940 issue.[129] Having some doubts as to the wisdom of leaving the daily affairs of *Compositio* to Brouwer, Noordhoff casually asked Freudenthal's opinion on the future of *Compositio* under Brouwer. Freudenthal expressed his willingness to give his opinion, but declined to do so in writing.[130] He urgently counseled Noordhoff to clean up the editorial board—no more than one third of the old board ever took actively part in the editing. One should, in his opinion, attract some 20 young mathematicians 'who are at the peak of their creative power, and who are not yet members of other editorial boards.' Moreover Noordhoff should attract a young, active mathematician with a broad interest and good qualifications for the position of secretary. The person should have relations with the top circles in mathematics, should have enough personal courage to reject mediocre work, etc. He ended with the harsh words: 'if one wishes to salvage anything at all of the goodwill of *Compositio Mathematica*, one should take action promptly and energetically. A journal that keeps plodding on or that degenerates into a rubbish dump would do considerable harm to the international reputation of Dutch mathematics.'

Noordhoff did not act solely on Freudenthal's advice, it even went so far as to poll the mathematics professors in the Netherlands. In January 1949 the publisher sent a letter to the Dutch mathematics professors, asking them for their support, announcing the re-issuing of *Compositio* under the temporary secretarial care of Brouwer, who had taken the initiative. The letter contained the seemingly harmless sentence: 'As we are of the opinion that the journal with its good reputation should appear at the same level as before, we would appreciate if the journal in addition to the support of its foreign contributors, would also receive the total support of the Dutch mathematicians.'

The letter elicited quite a number of reactions, one of which was provided by Van der Corput, who was one of the old editors. Brouwer apparently had not included Van der Corput in his list of recipients of the announcement of the reanimation of *Compositio*. One can easily imagine why; a man who had deftly outmanœuvred Brouwer in the faculty and in the Mathematical Centre affair, was not to be trusted in an editorrial board.

Van der Corput did not accept Brouwer's move without protest, he complained to Noordhoff that he was, to his surprise, unaware of the plans concerning *Compositio*.[131] Noordhoff cleverly made use of Van der Corput's dismay, expressing their surprise that one of the co-founders of *Compositio*, with the same rights as Brouwer, had not been informed by Brouwer.[132] The plan for resuscitating *Compositio* was greeted with applause by most Dutch mathematicians, wrote Noordhoff to Van der Corput; perhaps one should ask Brouwer how (and why) he happened to overlook

[129] Noordhoff to Freudenthal 1.XI.1948.

[130] Freudenthal to Noordhoff 1.XII.1948.

[131] Van der Corput to Noordhoff 26.I.1949.

[132] Noordhoff to Van der Corput 29.I.1949.

Van der Corput. 'Is there any objection on your side, that we show your letter to Professor Brouwer', he subtly inquired. This was not quite what Van der Corput had in mind; he immediately replied that 'Some of the mathematicians consulted by you have expressed themselves very cautiously. It seems to me that my answer should rather not be passed on to Professor L.E.J. Brouwer.'[133]

As one could expect, Brouwer did not react kindly to the Noordhoff circular letter. He interpreted it as an attempt to import more Dutchmen into the editorial board; worse, he viewed it (according to Schouten[134]) as 'an action (by some person or persons unknown) to throw him out, and he took the whole thing as a personal affront.' On these grounds he refused to work any longer with Noordhoff, and the preparations came to a complete halt. That did not mean that Brouwer had put *Compositio* out of his mind altogether. He actively looked for new editors; one of the persons approached was Paul Bernays; in order to get a better representation of the subject mathematical logic in *Compositio*, Brouwer invited him to join the board of editors, asking at the same time his advice as to the another editor from the logical corner of mathematics. Hopf had suggested MacLane, but Brouwer thought hat Kleene might be a good candidate. Bernays apparently advocated Kleene's membership, for Brouwer wrote to Kleene 'I have the pleasure to invite you, *firstly* to enter the editorial staff of *Compositio Mathematica*, *secondly* to favour this periodical with some work of your own.'[135] From the letter it also appears that Brouwer was keenly interested in what was going on in recursion theory. He thanked Kleene for some reprints, and asked for a specific paper. Kleene was at the time occupied finishing his monumental *Introduction to Meta-Mathematics*, so he accepted the invitation provided he could finish the book first. As a matter of fact he spent in 1950 a term in Amsterdam, where he could get first-hand information on intuitionism.[136]

In order to get the journal under way again, Noordhoff and some mathematical colleagues called in the help of Schouten, who was the Dutch mathematician following Brouwer in seniority. By now Schouten was one of the more prominent Dutch mathematicians. He had in 1943 resigned from his Delft chair, and withdrawn himself to a quiet part of the country, but his influence was still considerable and Noordhoff must have seen in him a valuable ally in the attempt to edge out Brouwer. Although Brouwer and Schouten had had their differences in the early twenties—patched up in 1929 after mediation of Weitzenböck (cf. section 8.3)—animosity was certainly not the motivation of Schouten to take Noordhoff's side. Schouten was one of the editors of *Compositio* of the first hour; it was probably a sincere wish to restore *Compositio* to its old glory, that made him an actor in the *Compositio* affair.

Schouten met Brouwer on May 28 and discussed the matter. According to Schouten, Brouwer agreed to enlarge the Committee of Administration with

[133] Van der Corput to Noordhoff 31.I.1949 (draft).

[134] Schouten to Hopf 8.XI.1949.

[135] Brouwer to Kleene 12.IV.1949.

[136] Kleene to Brouwer 19.IV.1949.

Kloosterman, Heyting, and Gerretsen[137] as a secretary.[138] When this agreement was reached Schouten immediately informed Noordhoff and a meeting with Brouwer was scheduled for July 5. To the general disappointment Brouwer asked for postponement of the meeting,[139] and subsequently did not respond to any letters. In all fairness it must be said that no attacks at Brouwer were envisaged, Van der Corput at one point argued forcibly that the combination 'Brouwer–Compositio' was from an international point of view the strongest possible, and that Noordhoff should really try to keep Brouwer in charge. Neither was Schouten out for Brouwer's removal, but he clearly wanted to reduce him to 'one of the editors'. Unfortunately Schouten did not possess the tact needed to handle a mercurial person like Brouwer. His letters, obviously well meant, were of this half patronizing, half schoolmastering kind that goes against the grain. Brouwer in particular had no wish to be lectured. In the end it must have been a mixture of exasperation and genuine worry about the future of Compositio that drove Brouwer to desperate steps.

Brouwer clearly had given up hope of reaching an agreement with the Noordhoff faction. Why is not quite clear. Maybe it was the old story of a personal consultation interpreted differently by the parties. Brouwer had learned a lesson in his relation with Van der Corput: never rely on verbal agreements. Whatever caused the final disruption of connections with Schouten and Noordhoff, Brouwer lost no time in taking counter measures. On July 10 he sent a letter to the members of the Committee of Administration, proposing to sever all ties with Noordhoff.

> Dear Colleagues,
>
> When the House of Noordhoff Groningen, which had functioned from 1934 to 1940 as bookseller-publisher-agent of Compositio Mathematica offered us to resume from 1945 its old function, there was no reason to refuse it the opportunity to prove its claim to be up to that task. However, having taken up this task, it had started by working so miserably, be it through a lack of equipment, be it through a lack of zeal, be it through a lack of good will, and it finally demanded, before continuing its work, a reorganisation of the editorial board which would change completely the character, and in particular the international character, of our journal.

Under the circumstances, he went on, I propose to take our business to another publisher, 'I have good hopes to find for that purpose a house of renown, well directed and equipped, which will serve us better than the one that has deceived us.'

Brouwer must have thought of the North-Holland Publishing Company (which also printed for the Academy, and of which Brouwer was a member of the board of commissioners), for he had already approached that firm early July. Unfortunately

[137] Professor in Groningen, a function theorist.

[138] Ibid. The information is mostly based on Schouten's letter.

[139] Telegram 30.VI.1949.

for Brouwer that particular plan fell flat. When North-Holland was informed by Noordhoff of its purported rights, it lost interest in the acquisition of a journal that might bring a string of lawsuits.

A majority of the Committee of Administration agreed with Brouwer: de Donder, Julia and Saxer sent their approval for further action. Schouten in the meantime tried his best to get Brouwer back to the negotiating table; he asked Van der Corput to talk Brouwer round. Noordhoff wrote a conciliatory letter and Heyting tried to influence Brouwer. None of this was of any avail.[140]

With nothing to lose, Schouten decided to take to the offensive. His first object was to make the members of Committee of Administration see the *Compositio* problem his way. In a long letter to Hopf, Schouten set out to justify his and Noordhoff's cause and to prove Brouwer wrong.[141] Apart from a recount of the events the letter contained a list of refutations of Brouwer's claims or suspicions. Some of Schouten's arguments and claims had a degree of plausibility, but if they contained some truth, certainly not the whole truth. In particular his protestation that he did not attempt to remove Brouwer, seems a bit lame, unless one supplements the claim that the action was not being directed against Brouwer, by the clause 'as long as Brouwer does not interfere with the journal.' As Schouten saw it, the situation held a grave risk for Brouwer:

> Up till now as a mediator I was able to prevent legal action from the side of the publisher. But after the strict refusal of Mr. Brouwer this will not be possible any longer. So if nothing is done, there will be a legal action and the Committee of Administration, especially Mr. Brouwer, will be made responsible for further delay. As I see it now, it would have been better if I had written to the other members of the Committee of Administration at an earlier time. But my intention was to be very careful and to make things for Mr. Brouwer, as little disagreeable as possible, and this held me back till now from this action. Up till now the most influential Dutch mathematicians agreed with me that we must aim at a solution giving Mr. Brouwer the place and the honour that are naturally due to the man who founded the 'Compositio Mathematica.' But with Mr. Brouwer now turning down any compromise, a solution has to be found in whatever way. For Mr. Brouwer this would lead to a very serious defeat and I think we ought to try, if possible, to avoid such a defeat for a man of his age and fame.

On the whole, Schouten's action should be taken at its face value. He was not the evil man Brouwer had thought him to be. It is more likely that he had taken the role of mediator in a sincere wish to solve the problem without hurting Brouwer. He would probably have preferred to solve the *Compositio* conflict without damage to the parties concerned. Nonetheless he had to play the game for Noordhoff, and

[140] Schouten to Hopf 8.XI.1949.

[141] Freudenthal guessed that similar letters went to the other members.

in that role he cleverly bent the facts to his advantage. One should not forget that at the same time Schouten, Van der Corput et al. were the subject of Brouwer's guerilla warfare in the faculty. So, if Schouten showed some exasperation, he was entitled to it.

In order to minimize the damage to all concerned, Schouten launched a proposal for salvaging *Compositio*. A Temporary Committee of Reorganisation should be installed, consisting of four Dutch members (one distinguished man from each of the four Dutch Universities,[142]) with the following task:

1. start the editing of *Compositio*;
2. arrange the election of a new Committee of Administration;
3. draft rules for *Compositio* and a contract with the publisher;
4. submit the rules and contract to the vote of the general committee;
5. dissolve itself.

Schouten did not wish to become a member of the Committee of Adminiatration, 'At my age it is a big mistake to do things or to go on doing things that younger people can do so much better. A wise man has to know the time at which he has to withdraw,' he wrote, but he was willing to act as the 'central man who has to constitute the Temporary Committee and to work as its Chairman, and to mediate the parties concerned. He would make it his special duty to ensure that Mr. Brouwer got the honour and the place due to the founder of the *Compositio Mathematica*.

The secretary designate, Gerretsen, had the task to get legal advise, for it was not unthinkable that Brouwer would take the publisher or the new board to court. After weighing the possible actions, the legal adviser deemed it safe to proceed along the lines indicated by Schouten.[143]

Schouten set to work without delay, and soon he could present the General Committee with his Temporary Committee of Reorganisation, Freudenthal, Gerretsen, Kloosterman, Koksma.[144]

He had ascertained, as he stated that the greatest possible majority of the Committee of Administration had agreed to his proposals. Brouwer was blessedly unaware of all the goings on (which shows how a once central person may get isolated).

Noordhoff had reacted with an invitation for further talks, an unacceptable proposition for Brouwer, who insisted that the work should be resumed first. Van der Corput, in a letter of January 20 to Brouwer, completely ignored the *Compositio* matter. Probably for a good reason, for the Temporary Committee had drafted a rather tactless letter to Brouwer, informing him of the existence of the committee and demanding peremptorily that he hand over of the administration of *Compositio*.

[142]Schouten, for unknown reasons, counted only four universties. There were five at the time.

[143]Kluyver to Gerretsen 11.XII.1949.

[144]Schouten to General Committee 2.I.1950. Cf. Brouwer to eds. 1.I.1950.

J.A. Schouten. (CWI Archive).

This letter reached Brouwer not before February. Brouwer only learned about the activities of the Noordhoff party through a chance remark of a French mathematician, probably P. Levy, who told Brouwer that he was informed by Gerretsen that his paper would soon appear in *Compositio*.

From correspondence of Lize it appears that in the fall of 1949 (and perhaps earlier) Brouwer had been suffering from a mixture of complaints. In particular he feared that his heart and lungs were in bad shape. On 27 November Lize wrote her daughter that Brouwer had been thoroughly been examined by one of his colleagues in the medical faculty, professor Formijne. The examination showed that heart and lungs were in good order, and that the stomach was the problem. Brouwer was immediately put on a diet. In any case he was greatly relieved, he was eying the future with some more optimism, all the more as he had been invited to lecture in Paris in December 1949, January 1950. He had started his lectures on December 13, returned for Christmas to Blaricum, and he taught the second part of his course in January. He had duly reported to the board of the university that his work as a 'professeur d'échange' at the Sorbonne was not yet finished, and he had

asked permission to return to Paris for another month of teaching.[145] His absence suited the *Compositio* conspiritors wonderfully.

On January 9, the day before he returned to Paris in order to resume his series of lectures on intuitionism, he had written to Van der Corput that the latter's intervention with Noordhoff had been a failure. In October Brouwer had asked Van der Corput to inform Noordhoff that he agreed to enlarge the editorial board as soon as Noordhoff resumed the work at the forthcoming issue of *Compositio*.

Brouwer was furious when he found out what was going on, he immediately wrote a long letter to the Committee of Administration.[146] In his function of (temporary) secretary of *Compositio* he was responsible for the manuscripts of the authors, he wrote, and now he found that Noordhoff had betrayed his confidence by giving Gerretsen, the 'self-styled secretary', access to the manuscripts. After a short résumé of the history of *Compositio*, Brouwer concluded that the latest démarches of Noordhoff left him no choice but to go to court. He asked the members of the committee to authorise him to start the legal procedures. 'I don't expect an easy law suit. The manœuvre of Mr. Gerretsen made me think that there is a well meditated machination, by which the House of Noordhoff hopes to get *Compositio* in its possession, and for which purpose it has succeeded to ally itself to some mathematicians in my country who wish to enter into the Committee of Administration of the journal.'

Returning home, he had found the new issue of *Compositio* waiting for him. He again turned to the members of the Committee of Administration.[147] The publication of the issue under the name *Compositio Mathematica* with the traditional cover constituted such a colossal fraud, that the impudence of 'our adversary should hopefully facilitate our law suit.'

The letter had hardly been written when Brouwer received with more than a month's delay, Schouten's letter of January 2, this was the first indication that Schouten had managed to turn the (or at least some) members of the Committee of Administration. Brouwer again addressed the Committee of Administration, explaining them what had been discussed between him and Schouten.[148]

Hopf, who was well aware of the difficult sides of Brouwer's personality, felt that he could not take Schouten's side. The years of friendship with Brouwer could not be erased that easily. In reply to a card from Brouwer with a picture of *Le Penseur*—a reference to the past: 'Does Hopf still remember how he, with Neugebauer, found me here in the Louvre?'[149] the reply was 'Of course I remember very well how Neugebauer and I found you in the summer of 1926, sitting in front of

[145] Brouwer to Mayor and Aldermen, 3.I.1950

[146] Brouwer to Comm. of Adm. 27.I.1950.

[147] Brouwer to Comm. of Adm. 3.II.1950.

[148] Brouwer to Comm. of Adm. 7.II.1950.

[149] Brouwer to Hopf 28.I.1950.

the Mona Lisa'[150]—he lamented the recent developments. In spite of the advanced stage of the fight, he implored Brouwer to accept a compromise. He argued persuasively that one could not blame a publisher for clinging to a journal. Publishers need editors who are prepared for minor compromises, he said, it would not be a shame at all for Brouwer to accept a younger Dutchman at his side in the board. There would be enough foreigners to guarantee the international character.

That same day Hopf sent a letter to Schouten, informing him of his letter to Brouwer. Hopf also castigated Schouten for not informing Brouwer of the setting up of the Temporary Committee. He disapprovingly commented on Freudenthal's membership of the committee; in this way, he wrote, the committee lost its neutrality right at the beginning. Schouten answered with an extensive justification of his policy.[151] Since Brouwer had cut all communication, it had been impossible to consult him. Apparently it had not occurred to Schouten, that in order to inform a person, one does not have to consult him.

The choice of Freudenthal, Schouten argued, was not motivated by anti-Brouwer feelings. One simply had to look for four prominent mathematicians from the four universities. Freudenthal, as a Utrecht professor, was a natural choice; the other professor, Popken, was a young man, who, moreover, was the son-in-law of Van der Corput. In view of the situation in Amsterdam, Schouten did not wish to aggrevate the differences between Brouwer and Van der Corput. Freudenthal's long experience with *Compositio* made him a valuable addition to the committee, and, he added, 'I note with pleasure that Freudenthal is extremely correct, and in no way hostile to Brouwer.' Of course, I was aware of the frictions between Brouwer and Freudenthal, he wrote, 'but as Brouwer permanently gets into a fight, then with one person and then with another, one could not seriously take this into account. Where would the world be if everybody could insist on keeping company only with those they could love, and by whom they are loved? This is ridiculous child's play, and it is most regrettable that an excellent scholar possesses so little wisdom, that he stoops to playing indians and cowboys.'

Schouten left no doubt that he saw no place for Brouwer in the affairs of *Compositio*.

> Personally I hope for officially a reconciliation with Mr. Br. He had 'terminated the friendship' on 28.I.1950 from Paris, apparently in response to some correspondence of Gerretsen with a French mathematician on overdue proofs, without having read the letters of 1 and 2.I.1950 and not being aware of the appearance of the Compositio Mathematica, which by then had already taken place, nor of the real state of affairs. Moreover he had written that he did not wish to receive letters from me and that he would not read any. Those are, however, children's games, which will not

[150] Hopf to Brouwer 12.II.1950.

[151] Schouten to Hopf 14.II.1950.

> induce me to withdraw my friendship from him. From one day to another this could change, as I have experienced before. As you see from the new statutes, the possibility has been created to appoint an especially excellent man, who has been of great importance for the Compositio Mathematica, to honorary president of the editorial board (*Hauptredaktion*). Perhaps the solution is to be found there. At his age something like that would be exactly the right place for Mr. Br., and the honour would be saved!

In spite of Schouten's attempt to treat Brouwer 'en bagatelle', one has no difficulty in seeing through the rather obvious 'propaganda'. Schouten had succeeded in reducing Brouwer's position in the considerations of the committee to that of a capricious child that could be placated with a shining toy. The last sentence, in particular, gives away Schouten's opinion of Brouwer—an elderly dodderer, only good for some honorary position.

The history seemed to repeat itself; exactly as in the case of the *Mathematische Annalen*, Brouwer was completely ignored by the new rulers. The reorganization was carried out according to plan, and on May 5 Schouten could announce to the members of the General Committee of Compositio that the new regulations had been accepted by a majority vote. In a letter to Veblen,[152] Schouten explained the reasons for certain formulations of the rules; he added that 'As matters are now, it is all a bit disagreeable for Mr. Brouwer. It is his own fault, but personally I should like to make things as pleasant as possible for him. I have in mind to propose him for 'Honorary President' of the General Committee. It is quite impossible to make him a member because then the difficulties would begin all over again, he being what he is. Best if he is so wise to accept this honorary Presidentship, the honour is in this way saved and in this position he is quite unable to do any mischief.' It could not be expressed clearer!

On May 31 a letter was sent to the members of the General Committee (formerly the editorial board) containing the list of candidates for the General Committee (Cartan, Gerretsen, Kamke, Kloosterman, Koksma), the Special Committee (formerly the Committee of Adminiatration) (Cartan, Kamke, Kloosterman, Koksma, Saxer and Whittaker) and finally the name of L.E.J. Brouwer for honorary President of the Special Committee.

In fact, when volume 8 of *Compositio* was completed (1951), the cover carried more new members than the above listed: Ancochea, Bompiani, Eilenberg, Freudenthal, Kleene, MacLane and Picone.

Here the *Compositio* affair ended. Brouwer had not only lost a battle, he had lost for the second time a journal—this time it was his own journal, expropriated by his colleagues and supposed friends.

The cover of *Compositio* did not list Brouwer as a honorary president, so either the members did not support Schouten's proposal, or Brouwer refused to accept the

[152] Schouten to Veblen 9.V.1950.

honour. Brouwer refused to resign himself to the inevitable, as late as December 1950 he was still corresponding with his lawyer.[153]

He had not been able to carry on the matter, he wrote, because of two disastrous developments:

1. Shortly after I wrote you for the last time, I had to observe that my foreign confrères in the Comm. of Adm., who were in July 1949 without any reserve on my side, have abandoned me as a consequence of communications and promises of my adversaries, which have remained a secret for me.
2. The physical shock, inflicted on me by this bewildering observation, left me, after a heart attack, weakened in mind and body to such a degree, that I have been put out of action for a longer period, with respect to the mentioned aggression, and that every sojourn in the realm of thought of this conflict was forbidden for some time.

Nonetheless he planned further action against Noordhoff because it 'not only reflected on my honour, but also on the honour of my country.'

Apparently these plans did not materialise, the conclusion to the *Compositio* conflict is lost in vagueness. Brouwer remained very bitter, however, about the whole affair, he used to refer to 'the theft of my journal.' In spite of the insulting treatment, however, he remained a member of the editorial board of *Compositio* until his death.

Looking at the *Compositio* affair from a distance, one can see what the parties wished and feared. Schouten and his followers thought that Brouwer would stand in the way of a recovery of *Compositio*, and they were more incline to be loyal to the publisher than to the founding father. Brouwer, on the other hand, was confident that he could maintain the pre-war quality even without Freudenthal's help. He was firmly convinced that his Dutch colleagues together with the publisher would reduce *Compositio* to a provincial periodical. Justified or not, he feared that the lesser gifted would finally see a possibility to enhance their status by joining an editorial board, rather on the basis of their nationality than on the basis of competence. And Noordhoff had a straightforward commercial interest. The publisher successfully played on the secret dreams of the mathematical community—to be an editor of this prestigious journal!

Probably Brouwer did not fully realise to what extent the success of *Compositio* was due to Freudenthal, but it is not impossible that with a suitable substitute for Freudenthal, he could have made *Compositio* a success. His complaints about Noordhoff were, one would guess, the usual complaints of editors, aggravated by the post-war shortage. In short the conflict seemed to based as much on personalities as on facts.

The standard of conduct in the conflict is maybe best characterised by the motto 'dealing with Brouwer, anything goes.' If one judges by the standards of 'obeying

[153] Brouwer to Baron van Haersolte 22.XII.1950.

the rules', Brouwer definitely cut a better figure than Schouten. The episode does not do credit to the Dutch mathematical community, it is at best an interesting topic for psychologists.

18.5 Rearguard actions

The *Compositio* conflict, for the time being, overshadowed most of the other matters that asked Brouwer's attention. There was little news from the faculty, Brouwer was content to discharge his duties as a Director of the Mathematical Institute; only in March 1949 he suddenly came to life when Bruins was offered a chair at the Agricultural University at Wageningen.[154]

Brouwer happened to be in Switzerland when he alarmed the faculty with the news that it was at risk to loose Bruins. Brouwer was suffering from a bout of asthma, which had already prevented him from attending the inaugural address 'On the interaction between Mathematics and Physics during the last 40 years' of Schouten, which marked his entry in the Amsterdam Faculty.[155]

The faculty had felt that it would profit from the cooperation of the leading Dutch differential geometer, not only because of the prestige that his appointment would lend to Amsterdam, but also in view of his usefulness as an organizer for running the new Mathematical Centre. The Centre indeed benefitted from Schouten's expertise, but his contribution to the mathematics curriculum in Amsterdam was negligible. As Schouten, called back from his self-chosen early retirement (he resigned during the occupation in 1943), was due for mandatory retirement in 1953, the appointment did not carry serious risks for the faculty.

Inaugural addresses traditionally ended by thanking all and sundry, from the Queen down to the assistants; Schouten addressed the (absent) Brouwer with the kind words, 'Dear Brouwer, that serious health problems prevent you to be present here, is a source of serious regret for me. May your cure abroad bring you a complete recovery, so that you may return soon completely recuperated in our midst.' The *Compositio* conflict had at that point not yet reached its climax, and at least superficially everything seemed peaceful.

We read about Brouwer's visit to Switzerland in a letter to Hopf; Brouwer rejoiced that he was finally back in his beloved Tessin, where he hoped to recover from bronchial asthma, which had been troubling him since August 1948.[156] Unfortunately, he wrote, he would have to return soon, since the currency that he had finally—after months of patience—obtained, was disappearing faster than he expected.

From the Kurhaus in Cademario (near Lugano) he called Van der Corput's attention to the possibility of Bruins' exit from the Amsterdam stage, in order to

[154] *Landbouw Hogeschool*.

[155] 21.I.1949.

[156] Brouwer to Hopf 10.III.1949.

accept a chair in Wageningen. Isn't it time, he wondered, that we should do something to keep this mathematician, who had a 'lively and easily roused interest in other sciences than mathematics, which was found before him perhaps only in Professor Korteweg' in Amsterdam. Bruins' flair for tackling problems in chemistry and physics was recognized in Amsterdam; in Brouwer's words these activities 'have to a high degree stimulated in Dr. Bruins the gift to provide scientists in a wide range of non-mathematical areas, who call in his assistance, now and then with simple (but therefore not obvious) mathematical methods, which enable them, to conquer difficulties in their field and to open new vistas.' These talents, in Brouwer's view, could possibly be saved for Amsterdam by offering Bruins an extra-ordinary chair (if necessary without cost to the City of Amsterdam). Bruins did not accept the chair in Wageningen, but he was not promoted in Amsterdam either. Although Freudenthal's appointment in Utrecht had removed the immediate threat to Bruins' Amsterdam career, the position of Bruins had not improved. Hardly anybody in the mathematics department supported him. His role in the treatment of Freudenthal, the paltry handling of the lecturer's position had left an unpleasant taste. And if that was not bad enough, Bruins' mathematical profile did not really fit the Amsterdam department, where, definitely after Van Dantzig, Beth, De Groot, Van der Waerden, De Bruijn had joined, innovation and progress became the order of the day.

In 1949 Brouwer received a letter from a young Swiss mathematician, Ernst Specker, who had just introduced the analogue of Brouwer's methods in recursive mathematics. Specker's letter contained a beautiful and short argument that a monotone, bounded sequence of recursive reals need not have a recursive real as a limit. This was the first step in applying the young theory of recursive functions to the foundations of everyday mathematics, in this case analysis of real functions. No reaction of Brouwer is known—unfortunately.

Specker was personally acquainted with Brouwer, for during one of Brouwer's visits to Zürich Specker was present when Bernays received Brouwer. Brouwer was always interested in young mathematicians and their work, so he soon got into a lively conversation with Specker. In the course of the discussion (or monologue?) Specker inquired if Brouwer had any intuitionistic followers in Holland. The answer was rather revealing, 'Yes, there are some, but they cannot be a lot of good, or they would not run after me.' [157]

Although the situation in Amsterdam was, from Brouwer's viewpoint, far from satisfactory, his old urge for travelling became stronger than his desire for the preservation of his position in the faculty. In April 1949 the authorities give him permission to travel to Spain—and more important, granted him permission to obtain the necessary currency. He was invited to represent the Netherlands at the

[157] Oral communication E. Specker.

Centenary Celebration of the Royal Academy in Madrid. This was his first visit to Spain, and he greatly enjoyed it.[158]

Back in to Amsterdam, Brouwer took a decision that must have been difficult, he handed the supervision of the Mathematics Library over to Schouten. This library had been his pet project since the early days of his career, when the mathematical literature was still a part of the general University Collection. Under Brouwer's management the private mathematics library had slowly expanded into an up to date collection (the reader may recall that one of Brouwer's conditions for staying in Amsterdam was a considerable sum for enlarging the book and journal collection). It was a so-called '*hand bibliotheek*', not a lending library, but a collection to be consulted by staff and students at the premises.

In July 1949 Brouwer again travelled abroad, this time he permitted himself the luxury to fly to Belgium. Evert Beth, the new professor in logic in Amsterdam, picked him up at the airport in Hulsbroeck.

Later that year a series of lectures in Paris at the Sorbonne followed; in the month of December he presented an overview of intuitionism. As we have seen, it was during this second part of his stay in Paris that the Compositio affair exploded.

The return to Paris after years of absence was a memorable occasion for Brouwer, who, after all, started his international career in Paris at the apartment of his brother during the Christmas holiday 1909-10, where the new topology was born.[159] In his opening address of the lecture course he recalled the old days with a certain measure of nostalgia.

> It is for me a great honour and a great satisfaction to have the opportunity to give an exposition of some of my ideas under the aegis of the Sorbonne and in the institute that bears the name of the French scholar who on the one hand was the greatest mathematician of his time, and who on the other hand, gave the philosophy of Science one of the strongest impulses man has known.
>
> Addressing you here, I also feel an emotion of recollection. It is now exactly forty years ago, it was in December 1909 and January 1910, that I inhaled for the first time the scientific air of Paris. And I recall vividly and with a deep gratitude the encouraging benevolence, with which I, a young beginner. was received by the grand old men of that time, whose names had been tied to the grandiose evolution of the mathematical sciences which was taking place, and to whom, through their published courses and their monographs my generation owes the greater part of its knowledge and a considerable part of its inspiration. It were the five classical memoirs on analysis situs of Poincaré and the reflections on dimensionality of Poincaré that were to open the perspectives, in which my thoughts on topology developed themselves and it were the studies of Poincaré and

[158] Brouwer to Hopf 3.V.1949 (from Madrid).

[159] Cf. p. 154.

> Borel, in particular the manner in which the latter had introduced the notion of measure, that made me glimpse the direction in which I had to seek the primordial origin of mathematical thought. It is a very special sensation to find of these great old men of the past, after a so considerable lapse, some who survive full of vigour.

This tribute to Borel and Poincaré was a proper demonstration of his intellectual debt to the two great Frenchmen. It unambiguously mentions the role of Poincaré's topological memoirs; the influence of Poincaré's algebraic approach, or rather the lack of it, seems therefore to have been rather a matter of personal taste (*esprit géometrique*), than of ignorance. To be precise, it was Hadamard who drew Brouwer's attention to Poincaré's topological work. Brouwer. being largely a self-educated man in the more advanced parts of mathematics, had read Cantor and Schoenflies, but missed Poincaré.

During his stay in Paris he wrote two notes for the *Comptes Rendus* of the Académie des Sciences, presented by Emile Borel, both of them dealing with the notion of order. In the last one he published the proof that the continuum can not be 'classically ordered', i.e. that an ordering satisfying the dichotomy principle is not possible. This was a general result; that the usual ordering did not satisfy the trichotomy was not so surprising, but the paper showed that no suitable binary relation of that sort could be found at all. This result had already been presented (as a number of the post-war publications) in the Berlin Lectures of 1927.

Evidently, Brouwer and Borel met during Brouwer's stay in Paris, and it is most likely that they had met in the past. The strange thing is that, although they had strikingly similar ideas on foundational matters, little of mutual influence was acknowledged in their writings. And even at this point Brouwer explicitly mentioned the Borel measure, but not Borel's remarks on choice sequences. Of course, from Brouwer's point of view, Borel contributed no more than heuristic considerations, which in the absence of a systematic revision of the underlying logic, could not compete with Brouwer's approach. Nonetheless Borel quite explicitly spotted the phenomenon, and made appropriate philosophical distinctions. At the same time it is surprising, to say the least, that Borel from his part ignored Brouwer's ideas.

In 1949 Arend Heyting started to make plans for the publication of Brouwer's collected works. He was cut short by Brouwer, who was of the opinion that the time for such an enterprise had not yet come:[160]

> Dear Heyting,
>
> In my opinion, an author who is still alive and in a state of scientific responsibility, and who now brings out again works that have appeared earlier, is obliged to justify for himself for each of those works, both the significance it had at the time of the first publication for the state of science at that time, and the significance it could have for the present

[160] Brouwer to Heyting 28.X.1949.

> state of science if it had appeared now for the first time. And he ought to consider on the basis of this justification, to which comments on the re-printed text the present reader is entitled.
>
> Therefore a re-publication of my collected works would impose on me an amount of work of such a size, that the required time will lack me for several years to come.
>
> To a lesser extent this objection holds also for the planned re-publication of my dissertation with connected publications in English. For this too, I will not have time for the efforts involved for me, as long as not in the first place Compositio Mathematica will permanently be back in operation, subsequently my Cambridge Lectures have appeared, and finally the manuscript of my intuitionistic function theory has been completed. But that time is, I believe (if my working power is at least not completely paralysed by the consolidation of the nazification of Dutch mathematics) in a fairly near future, ...

It is clear that Brouwer did not want a simple reprinting of his papers, he indeed wanted to put all his new insights and a host of corrections and addenda into an edition of his collected works. The ambitious plans that were to be carried out first, all came to naught. The *Compositio* enterprise was successfully taken over by Noordhoff and Schouten, the Cambridge lectures were not finished,[161] and a postwar manuscript of a theory of real functions has not even been found.

The bitter reference to the 'consolidation of the nazification' expresses Brouwer's views on the take-over of mathematics by the new bosses. The clash between Brouwer and the other Dutch mathematicians was the result of a mixture of causes. For one thing, he had dominated Dutch mathematics for a (too) long period; ever since his meteoric rise in international mathematics, none of the Dutch mathematicians matched him in status. The only person who made a comparable rise was Van der Waerden, but he hardly played a role in Dutch mathematics since he left Holland. The older mathematicians, like Barrau, Van der Corput, Weitzenböck, Schouten, Van der Woude, Kloosterman went their separate ways and left Brouwer alone. The younger ones, like Freudenthal, Van Dantzig, Loonstra, De Groot, Beth, played only a minor role before the war, and for them Brouwer was one of those great men of the past—a man to admire, but not to obey, as their predecessors had done. So when the war was over and mathematicians all over the world were looking forward to a period of new life and new ideas, it was natural that the Dutch mathematicians wanted to strike out on their own. That they should find an un-cooperative Brouwer on their path is not surprising. He had long aspired to make Amsterdam a mathematical centre comparable to Göttingen, and when efforts were made after the war to create a mathematical research centre in Holland, Brouwer automatically assumed that it should be situated in Amsterdam, if not actually under his supervision, than at least in harmony with his ideas and goals.

[161] Posthumously published as [Brouwer 1981].

It seems highly unlikely that the post-war Czar of Dutch mathematics, Van der Corput, would have allowed Brouwer much influence anyway, but the political purge was in this case a convenient gift—Brouwer had not been admitted to the inner circle of planners which was going to determine the future Dutch mathematics. Looking back, one gets the impression that Brouwer's political *faux pas* was no serious reason for pulling the rug out from under him. Of course, there were some who considered him guilty of high treason, but that was a small minority, a minority that dwindled after the first excitement was over. The major factor that contributed to Brouwer's downfall was an old-fashioned generation conflict between a new generation and a dominating father. Similar features were certainly also present in the case of the world-famous conductor of the Concertgebouw Orchestra, Willem Mengelberg. Although there were more substantial accusations against Mengelberg, the desire of the younger generation to let fresh air into the repertoire of musical life in Holland cannot be neglected in the history of Mengelberg's downfall.

Brouwer did not view the developments with composure. He saw in the démarches of Van der Corput, Schouten, etc. the continuation of a practice that he had witnessed first in Germany and then in Holland. For him it simply boiled down to a repetition of earlier take-overs: new actors, same play.

The post-war events in Amsterdam had Brouwer firmly convinced that he was surrounded by enemies, and he felt betrayed by colleagues who, generally speaking, were actually kindly disposed to him. His sharp mind analysed all that had happened and came to the conclusion that there was a large scale conspiracy to get him out of the way. The take-over of the faculty and the 'theft' of his journal were two points that rankled so much that he more and more avoided contact with his colleagues. He had gradually cultivated his grieve and anger to such an extent that when his seventieth birthday was coming up, he refused and forbade any attention. In 1951 Holland was still an orderly, neat country, which paid attention to its great scholars, and as a rule important dates of respected scientists were observed in the national newspapers. For example, at the occasion of the fortieth anniversary of Brouwer's doctorate in 1947, the *Algemeen Handelsblad* published a short note of Mannoury. This time Brouwer took no risk, he sent a letter to the national press agency ANP, with the request to inform all newspapers that Brouwer did not wish that any attention should be paid to his seventieth birthday.[162] He expressly notified friends, colleagues and editors of journals that any attention would strike him as painful. When he discovered 'to his embarrassment' that the proceedings of the Academy had published a number of papers dedicated to him on his 70th birthday, he poured out his heart.[163] He did not wish mathematical circles to pay any attention to his seventieth birthday as:

[162] Brouwer to ANP (General Dutch Press Office) 9.II.1951.

[163] Brouwer to Heyting 23.II.1951.

> at all events, which have made me scientifically homeless these last years, and have destroyed my health, working power and joy of life, such as
> – my persecution by the Military Authority (*Militair Gezag*)
> – the deloyally realised founding of Mathematical Centre,
> – the breach of faith of the authorities towards me, which was interwoven with the above,
> – the thus displayed violation of fair play in the arena of Dutch mathematics,
> – the destruction of the mathematical education at the University of Amsterdam which resulted from this,
> – and finally, the theft of my journal effected with the help of deceit, which had almost turned into robbery with murder,
>
> the professional community has, insofar as it did not actively participate, watched doing nothing, and has refrained from any help against the aggressors, whenever and wherever there was an occasion to do so.
>
> If there is thus no occasion for festivity at my 70th birthday, but only for the reflection on a macabre position, this holds in particular with respect to the professional community.

Mannoury, who had submitted a paper to the home journal of the Dutch Mathematical Society, *Het Nieuw Archief voor Wiskunde*—dedicated to Brouwer on his 70th birthday—received a more melancholy letter, begging him to withdraw the paper from this journal, published by the 'deceitful' Noordhoff, and to submit it elsewhere. He closed the letter on a faint note of apology and self-reproach, 'Forgive me my necessity of observing that nowadays I am in a somewhat sorry condition. It is done in a slightly ashamed resignation and in the light of the silent fixed star, in which your person is too.'[164]

The request was easily made, but Mannoury felt that cancelling of this particular dedication, with his own hands, was more than he could reconcile with his conscience and sense of justice, 'For this dedication has a wider intent than the expression of a personal feeling towards you, it is a recognition of your part in the insights that I have acquired, *but which I do not at all consider my property.*'[165] This plea from his former teacher and long-time friend could not fail to impress Brouwer, but he remained nonetheless adamant. A dedication that would even faintly be connected with Noordhoff or his seventieth birthday ('which has been for me in the res publica scientiarum a day of embittered mourning' was too painful to even consider). He even went so far as to offer to bear the cost of having Mannoury's paper printed elsewhere!

In 1950 the appointment Van der Waerden, long thwarted by Brouwer, was finally about to be realized. There are indications that one of the reasons for the

[164] Brouwer to Mannoury 25.III.1951.
[165] Mannoury to Brouwer 28.III.1951.

failing of the attempts to get him a chair (both in Utrecht and Amsterdam) was to be found in the royal palace. Queen Wilhelmina was determined not to accept proposals for appointing individuals who had voluntarily served the enemy. But after her abdication in 1948, her daughter, Queen Juliana, took a more moderate view. Van der Corput had this time taken all precautions, he had asked the Minister of Education in December 1949 if he would object if Van der Waerden would be proposed for a chair in applied mathematics. When the Minister answered that he would not raise any objections,[166] the faculty, now chaired by Professor Cornelia van Arkel, sprung into action. It presented a plan for the number of chairs in mathematics in Amsterdam; it listed four full-time professors, one lecturer and one extra-ordinary professor. The faculty defended the plan as a modest extension of the staff compared to 1937. Van der Waerden was proposed for a new chair in applied mathematics; as usual the obligatory couple of other candidates were added, but the faculty made it abundantly clear that it would not settle for less than Van der Waerden. It pointed out that such an exceptionally gifted man should not be lost for the Netherlands, let alone Amsterdam.

Van der Corput informed Brouwer in Paris about the faculty's plans and told him that Van der Waerden would be put number 1 on the list for the chair that was going to replace Loonstra's lecturership.[167]

Brouwer had already resigned himself to the inevitable; he confined himself to sending of a letter to the Curators of the University drawing their attention to the fact that the tasks of Van der Waerden and Bruins overlapped.[168] He suggested that Van der Waerden could equally well be made a professor in 'Mathematics' or 'Pure and Applied Mathematics.' The letter contained moreover a cautious dose of praise of Dr. Bruins, 'As I have already explained in a letter in 1948, it is difficult to attach a clear meaning to the term 'applied mathematics', but insofar as this might succeed, the gift and the fondness for applied mathematics has not since Korteweg been found so evidently in a Dutch mathematician as in Dr. Bruins.' Before a month had gone by he had changed his mind, he sent after all a letter to the Faculty pointing out that the procedure for the appointment of Van der Waerden had been extremely sloppy, which it indeed had been. And there the matter ended.

Van der Waerden was appointed in the Academic year of 1950.[169] Van der Waerden's activity in Amsterdam did not last long, already in September 1950 the University of Zürich had made him an offer that was hard to resist. And so after one year he left the University that finally had succeeded in creating the chair that both parties wanted. The Rector of the University, in his annual address in 1951 remarked that Van der Waerden had presented the mathematical novelty of coinciding inaugural and farewell addresses.

[166] Secretary General to Van der Corput 31.I.1950.

[167] Van der Corput to Brouwer 20.I.1950.

[168] Brouwer to Curators 5.II.1950.

[169] I count myself lucky to have heard his lectures in that year; Van der Waerden was a wonderful teacher.

No sooner had Van der Waerden's departure been announced, than Brouwer tried to regain the initiative in the shaping of the mathematics department. He suggested that this was the perfect occasion to reduce the mathematical staff to a size fitting the 'reduction of the number of positions at the universities, which took place after the war, and the post-war decline of receptive and investigative intelligence potential of the mathematics students, and the urgency of the discontinuation of any consumptive liberality by the authorities.'[170]

He added that such a 'deflation of the mathematical staff', if combined with (1) a strict specification of the professorial tasks, (2) giving up the practice of leaving the teaching of courses to assistants, (3) an abandoning by the authorities of the 'extra universitary institution which diminishes and tarnishes the mathematical milieu at the university, and at present drains and sequesters the pre-war Amsterdam Mathematical School, which for a time had some importance in the world and drew some attention' could possibly help the Amsterdam School, 'which is down at the moment and only barely breathes, to get up and show itself again.'

The chairman, Professor van Arkel, probably considered this as a letter to her personally, and so no faculty reactions appeared to have been formulated. But a week later Brouwer felt obliged to act again, this time he had found on the agenda of the Faculty Meeting of May 14, an item called 'vacancy mathematics'. It required no superior intelligence to see that this was the vacancy resulting from Brouwer's imminent retirement. In Brouwer's eyes this was the last occasion to undo all the harm that was inflicted upon mathematics in Amsterdam, as he viewed it. He immediately addressed the Faculty,[171] pointing out that it could have to count with the possibility that

1. The Trustees might have understood before September 17, 1951[172] that the manner in which the position of mathematics at the University of Amsterdam gradually was changed after the war, increasingly hampered the discharge of his duties,[173] and had made the discharge of these duties in accordance with the intention, at the time of their establishment, completely impossible.
2. The Trustees might therefore consider to make it possible that I will retire at 17 September, without appointing a successor for me, or that I may be asked to occupy my position a bit longer, with the temporary restoration of the leadership of the mathematical activity at the University of Amsterdam, which was taken away from me after the war, and with the mandate to prepare and initiate the required measures for the re-integration of the pre-war efficiency of the activities.

[170] Brouwer to the Chairman of the Faculty 7.III.1951. The letter consisted of two sentences of 12 and 15 lines!

[171] Brouwer to faculty 14.III.1951.

[172] The official day of retirement of Brouwer.

[173] As a professor and as a director.

When no reaction was coming forth, he again addressed the faculty[174] repeating the arguments, but this time he specifically pointed out that by just adding 'analysis' to the teaching duties of Dr. Bruins—and by promoting him to full professor—the mathematics curriculum would be taken care of.

'My own position', he wrote, 'had been eroded by the founding and operating of the Mathematical Centre since 1946, and it has lost its raison d'être.'

Referring to his letter of October 8, 1946, he remarked that the page of history mentioned there, was almost written, but

> a conciliating last paragraph could still be added, if the City Council recognised at the eleventh hour the hollowness of the argumentation, which had persuaded it to subsidise the Mathematical Centre instead of the University of Amsterdam, and would return to me during a brief extension of my employment, the supervision of the mathematical activities in Amsterdam, with the commission to strip it of all expensive glamour and of all unjustified privileges, and to concentrate again on the discipline that in the past gave the mathematics in Amsterdam some importance in the world, which is still in Amsterdam better represented than elsewhere in the world, and which increasingly attracts the attention of the mathematical world.

The letter contained a number of proposals for the reorganization of the Mathematics Department, including one, that went further than anything that Brouwer had ever proposed: to uniformise the advanced part of the curriculum[175] and to make for all candidates an examination 'of some depth' in intuitionism and symbolic logic mandatory.

This was the first time that Brouwer asked a privileged position for his intuitionistic school. The reason is not easily guessed; did he fear that intuitionism was about to disappear, or had he observed (in Leuven, Cambridge and Paris) signs of a new blossoming of intuitionism and did he want Amsterdam to be in a leading position by the time the world would ask for more constructive mathematics?

The above letter finally drew fire; Schouten wrote an agitated letter to Van der Corput and Van Dantzig, using terms such as 'we cannot leave it at that', 'give me weapons', 'these scandalous, nonsensical scribblings', 'give me the ammunition, the shooting will be taken care of'.[176]

A flurry of letters and memoranda followed. Schouten set out to refute Brouwer's points one by one in a first note (undated)—a draft was prepared by the Faculty Secretary (5.V.1951) Brouwer replied (29.V.1951), Schouten sent a letter to the Faculty (31.V.1951), accompanied by a letter to the Chairman, and it is more than likely that this was only part of the correspondence.

[174] Brouwer to Faculty 1.V.1951.

[175] The first part was already strictly uniform, the second part (after the candidaats exam) was rather flexible.

[176] Schouten to Van der Corput/Van Dantzig 7.V.1951.

Apparently, Schouten c.s. viewed Brouwer's proposals with great apprehension; the arguments were not devastating in themselves, but Brouwer's view that the mathematical institute could be run on a much smaller budget could present authorities with more temptation than they could reasonably be expected to resist.

Schouten's note on the whole was rather the document of a politician than of a scientist, he mostly misrepresented Brouwer's argument and carried out a shadow fight—which he then won on points. For example where Brouwer complained that the raison d'être of his position has disappeared, eroded as it was by the Mathematical Centre, he simply stated that 'It is correct that Professor Brouwer has not given many courses the last few years. The reason is partly the fact that Professor Heyting at the time unofficially has taken over some classes from him, in order to enable him to devote the last years of his professorship to the research and teaching of his special field. By causes beyond the control of the faculty, Professor Brouwer has, however, made only little use of the offered opportunity.' The message is clear enough: Brouwer is himself to blame for his ineffectiveness. While there is a great deal of truth in this, it was not what Brouwer meant by 'erosion of task' and 'loss of raison d'être!'

The draft of the faculty was much more balanced and tempered then Schouten's memorandum. It argued that the present teaching load was excessive—nine hours per week against eight hours per week in Delft.

After a consideration of various factors, the faculty settled on the following main subjects to be taught and studied in the Mathematical Institute:

1. Number Theory (Van der Corput)
2. Philosophy of Mathematics (Heyting)
3. Set theory (Brouwer)
4. Applied mathematics (Van der Waerden)[177]
5. One or more parts of Analysis, Algebra or Geometry.

The Faculty proposed to split the chair of Brouwer (set theory, function theory and axiomatics) into two chairs and to redistribute some topics, so that the following chairs resulted:

I Analysis, algebra and applied mathematics

II Geometry, analysis and set theory.

Running ahead of our story, we can already tell that the two new chairs were approved and that N.G. de Bruijn (I) and J. de Groot (II) were eventually appointed.

In a separate letter Schouten (probably as a Trustee ofthe Mathematical Centre) informed the Faculty of his (or the M.C.'s) views. He persuasively argued that close ties between the University and the Mathematical Centre were to the advantage

[177]The draft mentioned that although Bruins taught applied mathematics, the enormous interest in the courses of Van der Waerden had shown that also advanced courses were in demand.

of both parties, but he firmly denied all claims of Brouwer with respect to possible negative effects of the Mathematical Centre on the Mathematical Institute. He plainly stated that as Brouwer had difficulties in separating personal and objective aspects, he would be an unacceptable risk in a young organisation, where tact was an indispensable commodity. The founders would gladly have given a position in 'accordance with his scientific past', but could not risk doing so.

The accompanying letter to the Chairman was less guarded, Schouten asked her to enclose his letter into the file to be sent to the Trustees, not as he said, to reinforce the arguments of the Faculty, but because also Brouwer had insisted that the faculty should enclose his letters. He bluntly told the chairman that Brouwer's letters bristled with incorrect statements and misconceptions, but he went on to say that he felt himself in a difficult situation; 'I have always maintained that one should not demand from a man like Mr. Brouwer, whose critical sharp mind once managed to revolutionise the foundations of mathematics, that he should on top of that also be moderate, tactful and easy going.' He advocated a liberal attitude towards Brouwer, because he had been in difficulties in 1945, and suffered setbacks and severe disappointments. The state of misery and bitterness of Brouwer, although largely self-inflicted, he added, was deplored by all mathematicians, but when the interests of the mathematical education was at stake, Brouwer should be addressed in the same manner as all other staff members.

In the matter of the destiny of Brouwer's chair, not even Heyting could influence Brouwer. Heyting had not hoped, but at least tried, to change Brouwer's views.[178] Brouwer returned the letter unopened, he kindly but determinedly explained Heyting[179], that 'As I have to make a most economical use of the little time and strength that is probably left to me for my struggle to liberate the Amsterdam Mathematical School from the occupational powers of the Mathematical Centre, I am faced with the necessity to carry on this struggle only in discussion with the authorities involved, and thus with exclusion of any private discussion.' He suggested that Heyting, if he really wanted to ventilate his views, should send them to those same authorities.

Although time was running out fast—only a few months separated Brouwer from the date of retirement—he launched a new campaign. He tried at the last minute to construe a case for a chair for Bruins. He asked Bruins to prepare a list of publications and he collected the opinion of a number of scientists on the work of Bruins, e.g. Rutherford (St. Andrews), Turnbull (London), de Broglie (Paris), Destouches (Paris), Chatelet (Paris).

The mathematicians in Amsterdam were not particularly impressed by Bruins' mathematics, no support for a promotion to full professor could be found. Indeed they had either asked or found reviews and comments on the work of Bruins, which did little to shore up his status. Brouwer sent these comments to Bruins to give him

[178] Heyting to Brouwer 10.II.1951.

[179] Brouwer to Heyting 14.III.1951.

the opportunity to exercise his *droit de réponse*.[180] Apparently all this activity never led to a discussion or proposal in the faculty.

In July 1951 Brouwer acted for the last time as '*promotor*' (i.e. PhD adviser) of Miss J.A. Geldof, the same lady that caused a clash between Brouwer and Freudenthal in 1936. Her thesis, 'On the arithmetisation of axiomatic geometries', was a fairly traditional affair, with little originality. Freudenthal had despaired of her ever getting a doctor's degree and the last assistant of Brouwer, the topologist Daan Kan, spent a great deal of time in straightening out the details.[181] Mannoury, who took part in the opposition (i.e. the traditional public examining of the candidate), said that Brouwer awarded the degree with such an elaborate speech, that it almost had the character of a farewell address.

Kan was Brouwer's assistant after the war and he pursued his career further in Jerusalem, and later at MIT in Cambridge Massachusetts. He recalled that Brouwer warmly recommended him to Fraenkel (who at that time had very mixed feelings about Brouwer, to put it mildly), and it struck him as amusing that those two old gentlemen indulged in an almost ritual criticism of each other.

Shortly before the end of the academic year 1951, Brouwer participated for the last time in a conference as an active professor. It was the St. Andrews Mathematical Colloquium of the Edingburgh Mathematical Society. Brouwer was one of the invited speakers; at the invitation of D.E. Rutherford, the Secretary of the Society, he presented a talk on July 25 from 5.30 to 6.30.

At that time, travelling was still something of an achievement; one had to get a visa and to obtain permission to take money out of the country. The university had no funds to provide its staff members with the necessary cash, and so a prospective traveller had to scrounge money from the scarce sources. Brouwer carried on a lengthy correspondence with the representative of the British Council, Mr. R.P. Hinks, on the topic of a possible financial support. The efforts of Hinks on Brouwer's behalf were finally crowned with success: both the St. Andrews Conference and the stay at Cambridge would be supported.[182] The choice of mode of travel was left to Brouwer; the boat services between Holland and Scotland not having been reopened yet, he travelled by plane from Schiphol to Prestwick and then by train from London to St. Andrews. In view of his age and state of health he asked permission to travel first class, a request that was granted.

The colloquium was, in Brouwer's words, 'on a high level, both successful and delightful.' The colloquium brought him together with the Canadian geometer Coxeter. The basis for a later invitation to the Canadian Mathematical Conference in 1953 was probably laid in St. Andrews.

[180] Brouwer to Bruins 13.VII.1951.

[181] Oral communication D. Kan.

[182] Hinks to Brouwer 4.VII.1951.

Brouwer ended his academic career at the University of Amsterdam, which he had served with an unwavering loyalty for forty-two years, without any official events, no farewell address, no symposium. The Department that he had loved and raised to a status comparable to that of physics in Leiden or Amsterdam let him go without so much as a 'thank you'.

Even the harmless symbolic gesture of asking him to continue some courses until a successor had been appointed, was not made. Heyting, who had succeeded Brouwer as the Director of the Mathematical Institute, wrote a polite letter that barely veiled the beginnings of impatience, which expressed his opinion that Brouwer himself was the cause of the 'misunderstanding'. On Heyting's view, Brouwer had announced that he wished to be put in charge of a reorganisation of the mathematics curriculum in accordance with his views, and he professed not to be aware of Brouwer's willingness to accept the humble role of a substitute. 'After all', he wrote, 'it would not have been possible to make an arrangement so that you could take care of the courses, as you are going in October to England. Moreover, the fundamental conflicts of opinion, which have surfaced during the last months, would doubtlessly have led to problems.'[183] It is one of those quirks of history, that Heyting, who had to thank his career to Brouwer, so soon after Brouwer's retirement showed his former benefactor politely but resolutely the door. It could not have been a pleasant task for Heyting, to disappoint (and actually offend) the man to whom he owed his place in the world of scholars, and who had time and again recommended him. In spite of his indebtedness to Brouwer, Heyting was enough of a realist to have reached the conclusion that the future of the faculty was in better hands with Van der Corput c.s. than with Brouwer. Brouwer had no difficulty to recognise the mood in the Mathematics Department, where nobody (except Bruins) was any longer on his side. He replied coolly, that he could not discuss the matter of the interim arrangements for his courses with Heyting; this only concerned the Chairman of the Faculty and the emeritus professor.[184]

[183] Heyting to Brouwer 25.IX.1951.

[184] Brouwer to Heyting 27.X.1951.

19

THE RESTLESS EMERITUS

19.1 The traveller

Having parted with his creation, the Mathematical Institute, Brouwer had no intention of retiring to the life of a tranquil pensioner. He almost immediately departed for his beloved Cambridge to discuss with Steen the publication of his lecture notes. While in England, he submitted a paper to the Royal Society, of which he was a foreign member since 1948; it was in the vein of his Berlin lectures, *An intuitionist correction of the fixed point theorem on the sphere*.[1] Brouwer had observed long ago that the fixed point theorem in its classical generality failed (already for continuous functions mapping the unit interval into itself). In this paper he gave a counter example to the existence of a fixed point for continuous maps of the sphere that preserved the indicatrix (orientation), and showed that although there is not necessarily a point that remains in its place under such a mapping, there are points that move less than any given distance. This is a basic phenomenon in the constructivisation of classical theorems, precise results are replaced by results 'up to an ε'.

The British Council had generously agreed to pay Brouwer's stay in Cambridge and the return fare; this being negotiated and approved in July, Brouwer planned to visit Cambridge in 1951 from October 10 to November 10. However, a serious attack of bronchitis forced him to postpone his arrival until November 29. The exchange of letters with the British Council official Hinks is of interest as Brouwer explicitly states his grounds for this (and earlier) visit to Cambridge:

> May I take the liberty to recapitulate the object and aim of my planned visit to Cambridge? I am writing an English book on intuitionism, a branch of science initiated by me, but, as I think, rather universally recognised to-day. The book is to be published by the Cambridge University Press.[2] Up to now a textbook on intuitionism does not exist in the English language. There is not even an English vocabulary. Having little training in English expressions, and, moreover, having to coin a lot of English terms for notions introduced in my theories, with the help of my English friends I found the following way to get the book ready: I was given the opportunity to deliver an introductory course in intuitionism in the University of Cambridge. During this course there were regular consultative meetings with some of my hearers, in which intuitionistic notions newly

[1] Cf. section 4.4 and p. 552.

[2] Cf. [Brouwer 1981].

> introduced in the last lectures got their definitive nomenclature, whilst the paragraphs of the book relative to the last lectures were discussed and rewritten to get them as understandable as possible for English readers.[3]

In 1947 and 1948 Brouwer had spent three terms in Cambridge, and five of the six planned chapters were finished in this way. But in 1949 the university ran out of money, so that the project came to a halt. The planned visit in November was in fact part of the publication project. Apparently the effect of this visit was not satisfactory—the book was never finished. After Brouwer's death the 'final' version of the first five chapters was found among the material that was collected by the Dutch Mathematical Society, but chapter six was, as far as we know, never written. The book, eventually published by the Cambridge University Press,[4] indeed shows traces of intense terminological activity; Brouwer introduced a large number of colourful terms for concepts of intuitionistic set theory, but few of those names have survived. One must admit that Brouwer's terminology does justice to the geometrical motivation, but the introduction of a large number of novel names is a rather hazardous didactic policy. Brouwer's paper 'Points and Spaces' uses the same terminology, the foundational literature however, eventually adopted a more conventional vocabulary, mainly under influence of Brouwer's German papers and of Kleene's papers.

Although Brouwer was downright bitter about the treacherous treatment he had suffered at the hands of his mathematical colleagues, one should not get the impression that he sat moping at home. He had made an imaginary partition between his mathematical world and the rest. He continued life much as in the old days. People would drop in, his female admirers would sit at the feet of the great man under the spell of his eloquence. He would gladly set forth on any topic and display an almost unparalleled knowledge of literature, art, politics, medicine, agriculture, history, ... The philosopher Frits Staal,[5] who was in Amsterdam at that time, recalled an instance of Brouwer's extraordinary conversational talent. A visiting mathematician from India had said to him that it was his greatest wish to meet Brouwer. Staal made a telephone call, and Brouwer immediately agreed to receive the visitor. The next day Staal took his colleague to Blaricum. After the usual introductory phrases, the conversation got on its way, and Brouwer soon saw that a mathematical discussion would not be very rewarding He immediately changed the subject, and embarked on a lengthy and precise discussion of Indian politics; he questioned and lectured the visitor on the Hyderabad affair, demonstrating a complete familiarity with all the facts, persons, and backgrounds. The Indian was dumbfounded. This was a demonstration of Brouwer's phenomenal memory and command of facts; a lesser man might be tempted to show off by boning up on the

[3] Brouwer to Hinks 15.III.1951.

[4] [Brouwer 1981].

[5] Oral communication, see also [Noorda-Staal 1999] p. 31.

facts the night before, not so Brouwer, he simply had a natural talent for storing and retrieving information.

In the village Brouwer was well-known for his volubility. Maarten Mauve, the son of Brouwer's old friend, recalled that from time to time, Brouwer would, without a warning, drop in, flatten his foldable hat, and embark on endless stories. Any topic would do.

Scientists from all over the world visited Brouwer in Blaricum and enjoyed his erudite conversation. Mrs. Beth gave an amusing account of one such visit by Tarski. Tarski was a close friend of Beth and whenever he came to Holland the two met. At one of those occasions Tarski expressed the wish to see Brouwer. As Beth was on a friendly footing with Brouwer, an appointment was readily made, and Mrs. Beth drove Tarski and Beth to Blaricum. During the ride Tarski was briefed on the proper conduct during the ceremonies and proceedings which were part of Brouwer's way of life—much as a visiting dignitary was instructed before entering the court of an illustrious monarch. Mrs. Beth judged this a wise precaution in view of the characters of the two men, who each had a strong opinion of their place in science. The party was a great success, Brouwer displayed a magnificent hospitality, and the conversation was spirited. Now it was the case that Brouwer used to invite female company to add lustre to his parties, and the importance of the visitor could be measured by the number of ladies present. Tarski had been informed of this significant detail, and so when the time had come to go home, Tarski and the Beths took leave of Brouwer and the company, and they got into the car. No sooner was the door closed or Tarski asked: 'How many ladies did Brouwer invite for Carnap?'

Mrs. Beth also told that once, when she visited Brouwer she was much impressed by Brouwer's white tropical suit with gilded buttons. She complimented him, how well he looked in it, and she could see that he was very pleased with her remark. Next visit, Mrs. Beth told, he wore the same suit, evidently expecting the same praise, but this time 'I did not pay him a compliment.' In general, Brouwer loved to wear white suits, a reason for the locals to call him jokingly the ice-cream man.

Brouwer's resentment against the post-war bosses of Dutch mathematics, who had 'stolen his journal' and snatched his 'Göttingen at the Amstel', did not stop him from attending the monthly meetings of the mathematical society. He is still remembered by the junior members of that period as the sharp-witted old gentleman, who did not feel too high and mighty to be interested in their ideas and work.

At the mathematics department two important vacancies were to be filled in 1951. They resulted from Brouwer's retirement and van der Waerden's appointment in Zürich. Brouwer had, understandably, showed a keen interest in the matter. As we have seen, Brouwer had suggested that his chair should not be filled for the time being (cf. p. 853). The faculty had wisely ignored Brouwer's desideratum, and submitted a request for the appointment of two new professors, both for Brouwer's chair, N.G. de Bruijn and J. de Groot. The two held chairs in Delft. De Groot was a topologist of the set-theoretic kind, he practiced a mixture of Dutch

and Polish topology. De Bruijn was a versatile mathematician, at that time mostly known for his number theory and algebra. He had started his career as an assistant of Van Dantzig in Delft. When the proposals were submitted to the city council, which had the final say in these matters, a heated discussion followed. Part of the council supported the proposal of the University, but a number of councillors was of the opinion that one chair was enough and that Bruins was the proper person to fill it. It is probably not far-fetched to see the hand of Brouwer in this move. On the other hand, a persistent rumour went around that Bruins had in person been visiting the individual councillors to present his case, and that his efforts had been rather counter-productive. Whatever may have been the case, in the meeting of 10 October 1951 the university proposal got a majority of the votes; De Bruijn later told that it was the support of the communists that made the difference.

Although Brouwer would have preferred another outcome, he did not begrudge the two newcomers their success. When both were about to present their inaugural lecture, he wrote to them that, 'At the time I have hesitated to congratulate you with your appointment in Amsterdam, and in the end I have refrained from doing so, even though, from my side, I could not imagine a more felicitous extension of our academic mathematics staff, than the joining of you both.' The reason being that 'I could not convince myself that there is a sufficiently favourable spiritual climate in the mathematics section of the University of Amsterdam, to warrant a congratulation at your reception by it.'[6] He ended by asking a favour: not to mention his name in their inaugural addresses, for, 'if there were to remain any chance at all to bring my life's work to a satisfactory end, I will not only have to forget my connections in the past with the mathematics section at the University of Amsterdam, but also to make, as far as in my power, others forget it.'

In the last years of his career Brouwer resumed his old travelling habit; he had always been an enthusiastic traveller, be it for pleasure, health or professionally. In his final professional years he had a number of gratifying invitations for mathematical events. So did he revisit his beloved Göttingen in 1951 when he represented the Royal Academy at the bicentenary of the Academy of Sciences in Göttingen.

When Brouwer was making preparations for his visit to the Colloquium in St. Andrews,[7] the next invitation reached him.[8] The Provost of University College, London, R. Pye, invited him to give the Shearman Memorial Lecture, in memory of Dr. A.T. Shearman, who had bequeathed a sum to the College for a course of lectures on Symbolic Logic and Methodology. The status of the lectures may be judged by Brouwer's eminent predecessors, to wit Russell (on 'Scientific Inference'), Schrödinger (on 'The origin and Nature of Scientific Thought'), and Tarski (on 'Fundamental Ideas and Problems in Meta-Mathematics').

[6] Brouwer to De Bruijn and De Groot 22.VI.1952.

[7] See p. 856.

[8] Dr. R. Pye to Brouwer 16.V.1951.

Brouwer was delighted to accept the invitation, provided he could deliver the lectures in the Spring. In the course of the ensuing correspondence, Brouwer, when asked whom of his friends he would like to be invited for the traditional tea party of the Provost in honour of the guest, provided an interesting list: K. Popper, H. Dingle, Mr. and Mrs. Haynal Conyi, Miss Winifred Gordon Fraser, Whitehead, Kneale, Waisman, Steen, Braithwaite, Whitehead, Routledge, Newman, Polanyi and Turing. He added, 'If some of them would have the opportunity to attend my first lecture, I should be happy to see them at the tea party.' It is not known whom of the above actually turned up.

The series of lectures was scheduled for May 12, 16, 19 and 23 of 1952, and the title was 'Outline of Intuitionism'.

During this visit to London[9] Brouwer was briefly confronted with the past. During and after World War I Brouwer had become acquainted with esoteric groups, leftovers from the pre-war designs of intellectual circles. Gutkind was one of those high minded mystical philosophers that had set out to bring a happier world about. After the war Brouwer met Gutkind and his wife, and he was immediately taken with them.[10] After Gutkind found a safe haven in the USA, Brouwer lost contact with him. And that, basically, was the case for all members of the old circles. A particular flamboyant representative of the old generation was Dimitri Mitrinovic. He had been a student in Herzegovina in the old Austro-Hungarian empire; as most of his comrades, he was opposed to the Danube monarchy. After an intense period of political-literary activity, he decided to start a new life of sober cultural reflection. In 1914 he moved to Munich, and after a long odyssey he landed in London. In the meantime he had made friends with the members of the 'blood-brotherhood', the men of a royal spirit, including Gutkind, Van Eeden and Brouwer.[11]

In England Mitronovic made name as a writer for the influential magazine '*New Age*'. Until his death he was the centre of a varying group of thinkers and seekers, idealistic mystics, and political idealists. After World War II the circle had shrunk to a size that did not permit large scale projects and actions. The group met in the 'Renaissance Club', also called the 'Anti-Barbarus Renaissance Club of the New Atlantic'. It became a meeting point for exalted spirits. Martin Buber, one of the great thinkers of the age, dropped in, and Brouwer did so too. He was probably informed about the existence of the New Atlantic group by Winifred Gordon Fraser. She belonged to the group, and she must have recognized a kindred spirit in Brouwer. Doctor Ralph Twentyman described Brouwer's visit, 'I remember vividly one evening when a number of us were at Charlotte Street and Brouwer came in, tall and with the most brilliant eyes I have ever seen. They seemed to light up the whole room. George Adams was present, and he and

[9]The time of the following events is a matter of plausibility, since there are no documents providing the exact information.

[10]Cf. p. 333.

[11]Mitrinovic life is described in [Rigby 1984]. See also p. 247 ff.

Brouwer soon established a warm relationship, and became engrossed in higher mathematical discussions together. George Adams was a fine mathematician and a pupil of Rudolph Steiner. He was engaged in trying to work out indications of Steiner about 'Ethereal' space and projective geometry, and their relevance particularly to plant morphology. He worked with the polarity of point and plane, seeing the planar space as helpful in approaching Steiner's concept of Etheric.'[12] Twentyman never forgot Brouwer's glorious entrance, 'the high spiritedness Brouwer brought into any room is one of my most treasured memories. Everything moved up a floor or two when he appeared.' We may safely assume that Brouwer and Mitrinovic met at the time, although there is no direct information on that point. Twentyman noted that, 'Certainly Mitrinovic had a very high regard for him, both as a man and for his orientation in mathematical higher philosophy.'

That same year Brouwer wandered further afield. He finally fulfilled an old promise, to visit his former PhD student Barend de Loor in South Africa.[13] There are no letters in Brouwer's archive that show all that much contact between the two, be it that in the summer of 1945 Brouwer seriously discussed with De Loor the possibility of (a temporary) emigration to South Africa. Apparently De Loor had managed to arrange a lecture tour for Brouwer in 1952. In July Brouwer addressed the South African Association for the Advancement of Science in Cape Town. The lecture was published as 'Historical Background, Principles and Methods of Intuitionism',[14] and a parallel *Zuid-Afrikaanse* version was also published. The paper is interesting because it signals a retreat from earlier positions on choice sequences; Brouwer asserted that higher-order restrictions on choice sequences were not 'justified by close introspection.' This mental reserve is stronger than an earlier one in his Cambridge lectures: 'But at present the author is inclined to think this admission superfluous and perhaps leading to needless complications.'[15]

In Cape Town he had a minor accident with the result that one of his fingers was incapacitated for some time, Brouwer gratefully thanked the rector of the University of the Witwatersrand for paying the doctor's bills. He also used the occasion to see a bit more of the country, staying with South-African friends and colleagues. Back home, he kept up a correspondence with them. A substantial part of the letters is concerned with the correction of the proofs of his paper for the proceedings; Brouwer was assisted by the South African mathematician Welters.

The next year brought two more major trips. The first one was to Finland, where he was Von Wright's guest in Helsinki. He gave two lectures for the Philosophical Society of Finland, *Changes in the relations between logic and mathematics* (20.V.1953) and *Order on the Continuum* (22.V.1953).[16]

[12] Twentyman to Van Dalen 24.III.1992.

[13] See also p. 778.

[14] [Brouwer 1952b].

[15] How right he was can be seen in e.g. [Atten, M. van and D. van Dalen 2002].

[16] *Wendungen in der Beziehungen zwischen Logik und Mathematik. Die Ordnung im Kontinuum*.

The years had not lessened Brouwer's love of nature, nor his unconventionality. When he stayed with Von Wright, he asked his host one night, 'would you mind if I slept out on the ground tonight.' Von Wright may have had his doubts about his aged guest, but he saw no reason to refuse the request. And so Brouwer once more was one with nature under the sky, as he loved to be.[17]

It is hard to say that Brouwer had mellowed with age, he could fly off the handle like the next man, but generally speaking he was no longer in the line of fire. Old injustices could still provoke him into sharp and sometimes rambling harangues, but the fixation on the misfortunes of his life was slowly giving way to the recognition of the love and appreciation of his friends and students. In particular, Brouwer more and more cherished his love and admiration for his old teacher and friend Mannoury. As it happened, his visit to Helsinki coincided with Mannoury's birthday. As an attentive friend he wrote to Mannoury before leaving. The letter is a touching document, a mixture of friendship and self-pity.

> An early warm congratulation on your eighty sixth birthday, which I will celebrate in Finland in my thoughts. And a belated thank-you for your sign of life at the occasion of my birthday. I really hope that you have now completely recovered. I too have been in bed for months during this winter at a relapse of a heart attack, which I suffered in 1950 being suddenly informed of the theft of my journal and the unbelievable means of deceit and fraud used. It is, I believe, through this event which I on the one hand out of self-preservation must try to forget and to which I can, on the other hand, not resign myself, that I have definitely become misanthropic within our national borders. Remarkable, how little philosophy sometimes protects against psycho-somatic reflexes and reactions.
>
> Have a pleasant day on the seventeenth and do greet the members of your family.

Brouwer's post-war lecturing did not take him to Germany, the country that had shown him hospitality and admiration before the essentially scientific discussion, known as the *Grundlagenstreit*, was ended so abruptly by a non-scientific move. Apart from a few official visits, he had after the war no dealings with German mathematics. This may have been as much his choice as a defensive pro-Hilbert atmosphere that lingered in Germany for years after the *Grundlagenstreit*, and even long after Hilbert's death. His lectures, as well as his publications were now mostly in English; the lectures in Finland were exceptions, and probably a courtesy towards a country where the mathematics was still practised in the German scientific tradition. Nonetheless, even the affronts of the *Mathematische Annalen* affair could not erase from Brouwer's memory the true and sincere friendships of happier days. The man who did most for Brouwer in the dark days of the *Annalen* conflict, Carathéodory, had died in 1950 and the two had not met since the

[17]Personal communication Von Wright.

tragic events of 1928; the loss of such an old friend, even after a long separation, deeply moved Brouwer. In his letter of condolance, he wrote:

> The tiding of the death of your father, which arrived only just now, has deeply shaken me. His friendship and the awareness of his great importance as a thinker and as a humane character, were for me since a number of decades something really essential. By his departure from life, the world has become poorer for me. How I have looked for an occasion to see him again. It was not to be so and hence it only rests me to express my sincerest sympathy to you and the rest of the family and the other relatives at the terrible loss which you suffered, and I assure you that I will keep the memory of Constantin Carathéodory in the highest honour.[18]

Brouwer's last big paper 'Points and Spaces' resulted from his lectures at the *Canadian Mathematical Congress* at Kingston, Ontario in August 1953. Probably, the geometer Coxeter, who had met Brouwer at the St. Andrews conference in 1952, had arranged an invitation for Brouwer as a speaker at this congress. Brouwer immediately accepted the invitation, and proposed a series of talks on intuitionistic mathematics.

Once having accepted the invitation, Brouwer set out to obtain funds for the trip. This was not a simple operation, Coxeter had informed Brouwer in his letter of invitation that the Dutch Government had promised to pay the travelling expenses and that the Congress would take care of the housing plus a fee of $300.-. Brouwer was immensely pleased to get the opportunity to visit Canada; indeed, when he accepted the invitation, he added that he would enjoy a lecture tour after the conference. Coxeter made some inquiries and he soon came up with Queens University at Kingston and McGill in Montreal as possible hosts.

The conference actually consisted of two parts, the conference proper—in conjunction with the Fifty-eighth Summer Meeting of the American Mathematical Society, and the Summer Seminar. The topic of the Summer Seminar was Topology and Geometry. Brouwer's talks were part of the four Lecture Series on Research Topics. The other speakers being Henri Cartan—*Homologie des Groupes discrèts*; Newman—*Theory of fixed points, particularly of periodic functions*, and Beniamino Segre—*Algebraic Geometry*.

Flattering as the invitation might be, it had its problems, mostly of a financial nature; travelling was expensive in those days and a retired professor was certainly not in a position to throw his money around. And so even during his stay in Helsinki Brouwer carried on his correspondence with the Ministry of Education and the Canadian organizers in order to arrange a proper funding. An unfortunate illness had stopped him from answering the mail from the ministry, which had been in his possession since January. The Ministry's letter thoroughly dampened Brouwer's enthusiasm for the trip to Canada. The Minister had politely inquired if

[18] Brouwer to Stephanos Carathéodory 28.II.1950.

Brouwer was planning to accept the Canadian invitation, about which it has been informed by the Foreign Office. If so, it was willing to contribute towards the cost of travelling by boat.[19] In those days flying was still more expensive than sailing, a flight to Canada at that time cost about 2750 guilders. Brouwer expressed his disappointment at this announcement, he pointed out that the invitation was accompanied with a promise that the Dutch Government would furnish the travel expenses. 'In particular', he wrote, 'taking into account the personalities that were invited together with me (Cartan from Paris, Newman from Manchester and Segre from Rome), I would be sorry to withdraw in the end, after I had made my promises.'[20] After a good many letters, the financial problems were solved. Even the excess luggage was the subject of tactful but tenacious negotiating. Brouwer felt he had to remind the Minister that as an emeritus he earned a paltry 5050 guilders a year, and he added bitterly 'which in my case has the consequence that I support my family by labour, which has nothing to do with science and which thousands can do better than I can.'[21] Eventually Brouwer arrived at August 7 at Montreal Airport, from where he travelled on to Kingston. Brouwer not only lectured on intuitionism, but at the request of the organizers, also on 'High School Teaching in Holland.' The lectures on intuitionism, with the title 'Points and Spaces', delivered from 8 to 31 of August, were divided into six sections:

1. History of the gradual disengagement of mathematics from logic.
2. Spreads and fans.
3. Generation of Cartesian and topological spaces.
4. The virtual order of the continuum.
5. Precision analysis of the continuum.
6. Intuitionistic recasting of some classical geometric theorems.

At the request of the organizer, Williams, he had provided a list of reading material; this list by itself showed that intuitionism had lacked resonance in the mathematical community: apart from *Consciousness, Philosophy and Mathematics*, all the material was from the twenties:

Weyl—*Über die neue Grundlagenkrise der Mathematik* 1921;
Dresden—*Brouwer's contributions to the foundations of mathematics* 1924,
Wavre—*Y a-t-il une crise des mathématiques?* 1924, *Sur le principe du tiers exclu, Sur les propositions indémontrables* 1926/28;
Lévy—*Sur le principe du tiers exclu, Critique de la logique empirique, Logique classique, logique Brouwerienne et logique mixte* 1926/27;
Borel—*A propos de la discussion entre M. R. Wavre et M. P. Lévy* 1926/27,
Brouwer—*Intuitionistische Betrachtungen über den Formalismus* 1928.

[19] Minister to Brouwer 16.I.1953.

[20] Brouwer to Minister 27.V.1953.

[21] Brouwer to Minister 27.VII.1953, and to Williams 24.VII.1953.

The conference itself was a joyful experience for Brouwer, he met Coxeter again, talked with Mrs. Coxeter, who was of Dutch descent, and also ran into Paul Gilmore, a young Canadian logician who had studied with Beth in Amsterdam, and who had finished his dissertation on Griss' negationless mathematics in 1953.[22] Jim Lambek, who was also present, told that he had some talks with Brouwer.[23] At one occasion Lambek happened to mention Wittgenstein, whereupon Brouwer politely inquired what Wittgenstein had done for logic.

– He has invented the truth tables.

– And, pray, what are the truth tables?

Lambek proceeded to explain what truth tables were, and how they worked. Only later did he realise that the question had been an ironic one. Lambek also reported how during a conference picnic of the participants; Brouwer who had joined the party, suddenly had disappeared. After some excited searching it turned out that the septuagenarian had not been able to resist the urge to climb one of the trees, and he was discovered somewhere above the heads of the assembled mathematicians.[24]

After the Conference Brouwer set out on a long lecture tour through the United States and Canada. It turned out to be not so trivial to enter the USA, Brouwer had to wait patiently for a visa, but finally he got one. While still in Canada, he had received an invitation of Cairns, the topologist who had proved the triangulation property of differentiable manifolds five years before Brouwer. Brouwer reacted late but enthusiastically from his first stop in the States.[25] He wished, however, first to visit a relative of his wife, Jan de Holl in Birmingham, Alabama. The lecture tour through the USA was largely if not completely organized by Stephen Kleene, the recursion theorist, who knew Brouwer from his stay in Amsterdam in 1950. It was in fact Kleene who had informed Cairns of Brouwer's presence. The precise details of the lecture tour have not been preserved, but we find Brouwer on November 22 in Los Angeles, from where he arranged his lecture in Urbana, the University where Cairns taught. He talked on November 30 on 'The intuitionistic continuum' and moved on to Purdue University, where he gave a talk on December 2. According to Brouwer's family doctor, he also visited the National Laboratory in Los Alamos, for this purpose he obtained a special pass.[26]

The content of Brouwer's lectures is not recorded, but we may safely assume that it closely resembled the Canadian lecture, and perhaps the Finnish one. That is to say, that it contained a good deal of, at that time still rather esoteric, not to say hermetic, material explaining the basics required for a proof of the bar theorem and the fan theorem.

[22] [Gilmore 1953].

[23] Oral communication of J. Lambek.

[24] This fact was also mentioned in a letter of Professor Williams (William to Brouwer 28.VIII.1959) who recalled 'Your prowess in climbing trees.'

[25] Wilton, Connecticut. Brouwer to Cairns 30.IX.1953.

[26] Interview C. Emmer.

For non-experts this certainly was no easy subject. The success of the lectures may thus be questioned. In particular since Brouwer made no concessions to his audience; he lectured much the same way he conducted his classes in Amsterdam, addressing the blackboard rather than the audience, while developing his ideas in a soft voice. When Brouwer had lectured in Berkeley at the mathematical colloquium, Tarski spoke at the end of the lecture, 'It is very good that Brouwer has delivered his lecture, then you mathematicians can see that there are also complicated things in the foundations of mathematics.'[27]

Anil Nerode, who was at that time a young logician in Chicago, had a more positive recollection of Brouwer's visit. He had already privately studied Brouwer's papers beforehand, and so he was, as a student, the only person in the department, who could discuss the content of the talk with the speaker.[28]

The Canadian-American trip gave great pleasure to Brouwer, who enjoyed travelling abroad. And in America and Canada there was quite a number of émigrés from Germany and Austria, many of those were known to Brouwer from the happy years of scientific progress in the years before the *Third Reich*. It had moreover one particular side-benefit that carried for Brouwer probably more weight than all his mathematical lectures together; he hoped and planned to revive the old friendship with the Gutkinds. After their departure from Europe they had not met, and it is even doubtful if there was much contact. The only trace so far is the letter Gutkind wrote to Einstein, relaying Brouwer's wish to emigrate. This trip provided a unique occasion to re-establish the old relations Brouwer eagerly looked forward to a reunion—as a card from Kingston shows:[29]

> Please send me a sign of life by return of post. Badly longing to see you again. Why have we been so silent to each other for so many years, and did I only get your last book from Mitrinovic?[30] Love, love
>
> Bertus (Papagei)[31]

On November 28 he wrote to Lucia Gutkind that he had completed the larger part of his tour; the talks, he said, had taken more time than he expected, but 'between now and my returning home to you, only Urbana, la Fayette, Toronto and Ithaca are left as stages.' The ties of friendship and harmony between Erich and Lucia Gutkind, and Brouwer had been very special indeed; there was that mysterious affinity of souls that never failed to manifest itself with Brouwer. The days which he spent with the Gutkinds in New York were afterwards remembered with deep emotion by Brouwer.

A curious incident took place during Brouwer's stay in New York. Roy Finch, who was in close contact with the Gutkinds, and who witnessed Brouwer's visit to New York, described it in a letter:[32]

[27] Oral communication Dana Scott.

[28] Oral communication Anile Nerode. (One wonders if Menger was present at Brouwer's lecture.)

[29] Brouwer to Gutkind 31.VIII.1953.

[30] Cf. p. 864.

[31] Parrot.

[32] Finch to Van Dalen 11.II.1991.

> Brouwer did not stay with the Gutkinds, but evidently spent most of a day and an evening with them. (I was there for the evening). The Gutkinds told me this story. They lived in the Master Institute and Hotel on Rivers Drive. Downstairs in the same building there were music studios. One of the musicians who had a studio there, in which he taught classes, was Paul Wittgenstein, the brother of Ludwig, who was a pianist who had lost an arm in World War I, but continued to play the piano with his left hand (a number of famous composers wrote compositions for him, including Ravel). Paul had had a serious falling-out with his brother, evidently over the question of inheritance, Ludwig having given his share (the family was very wealthy) to his sister. This is all preliminary to what happened in the elevator at the Masters Institute when Gutkind, Brouwer and Paul Wittgenstein all happened to be in it at the same time. According to Mrs. Gutkind. who was also present and told me, Eric introduced Brouwer to Paul Wittgenstein with the words 'This is a friend of your brother's.' To this Paul Wittgenstein replied: 'I do not wish to have anything to do with anyone who is a friend of my brother's.' He turned his back and left the elevator at the next stop.

Finch had met Brouwer at Gutkind's apartment, after Brouwer had been to Princeton. In Princeton the contact between Brouwer and Einstein, which was broken off after the *Mathematische Annalen* affair, was re-established. He told the company that he had a lengthy conversation with Einstein, and commented 'Einstein's mind never leaves the subject he is interested in, for one minute. He does not want to discuss anything else.'[33] The meeting with Einstein seemed to have healed the old wounds; Casper Emmer was told by Lize that Einstein and Brouwer 'had made peace.'[34]

Finch had asked Brouwer what his present view was of infinity, in a non-technical sense; Brouwer's reply was simply 'It just means that you can go on and on.' When the conversation got round to Wittgenstein, Brouwer told that he and Wittgenstein had met privately for an all-day meeting on an island, during which they discussed Brouwer's Vienna lecture. It is unfortunate that Brouwer did not volunteer any more specific information on this meeting.[35]

Gödel's comments to Brouwer's visit to Princeton can be found in a letter to his mother.[36] After reporting that the faculty now took up more of his time, he went on to discuss Brouwer's visit:

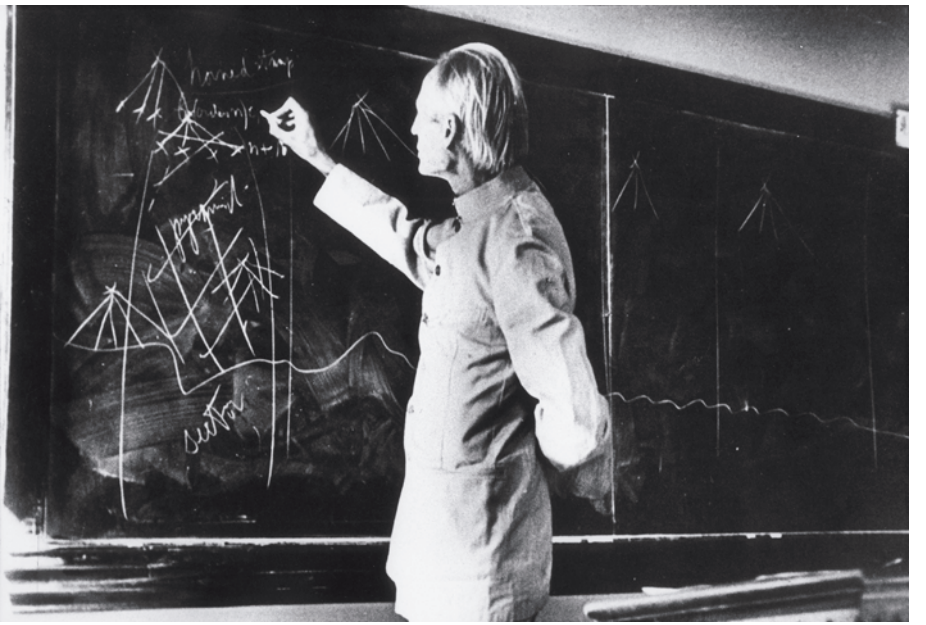
> Moreover a professor from Holland came to visit for 14 days, a famous man who was strongly involved in my field. I had therefore to invite him now and then at our place (once for lunch and once for tea). He is already 72 years old and no longer quite up to date. His lectures have not found

[33] Finch to Van Dalen 10.XII.1990.

[34] Oral communication C. Emmer. Brouwer had resented Einstein's neutrality in the Annalen affair. In his book there was no neutrality in the conflict between good and evil.

[35] R. Finch gathered that the island was somewhere off the coast of Holland.

[36] Gödel to Mrs. Gödel 31.X.1953.



Brouwer lecturing in Princeton (1953). (Brouwer Archive).

> much response (and rightly so). He apparently came only to America to earn money here. There one can see once more the difference between here and over there. The pensions in Holland have generally not been raised (since 1939), although the prices have been tripled, so that this famous man has to work as a bookkeeper in a drugstore, just to survive with his wife and daughter. It is really very sad when something like that happens to distinguished men.

Apparently Brouwer had painted a dramatic picture of his economic position, which was indeed his own perception of the situation. Unfortunately, nothing more is known about the discussions between Brouwer and Gödel.

The lecture tour included more than the above mentioned addresses, e.g. he also visited the University in Boulder Colorado; although it is not confirmed it seems most likely that he met there his old friend Rosenthal. Brouwer moreover lectured at MIT; Hartley Rogers, who attended the talk, recalled its unusual nature.[37] Brouwer's lecture had the appearance of a soliloquy, the audience was evidently of secondary interest. The lecture itself went on longer than usual, lectures traditionally ended punctually after one hour, but Brouwer did not seem aware of the time, he just went on. And so after one and a half hour, a great number of prominent scientists, who otherwise would not tolerate such a breach of decorum, started to sneak out and call their wife that they would be late for dinner.

[37] Personal communication.

Brouwer at the place of Thoreau's cabin at Walden Pond. (Courtesy Dirk Jan Struik).

The visit to Cambridge was a wonderful opportunity to renew contact with Dirk Jan Struik, who had attended Brouwer's lectures in 1916/17. Struik, like Brouwer an inveterate hiker, took Brouwer on a walk in the surroundings of Boston. The reader will not be surprised to learn that Dirk and Ruth Struik took Brouwer also to Walden Pond. There is a picture of Brouwer contemplating the spot of Thoreau's hut. The contact with the Struiks was kept up, and when Struik was a visiting professor in Utrecht in 1962, Brouwer found the couple a nice cottage in walking distance from his own house.

Kleene, who had mostly organized Brouwer's itinerary, hosted Brouwer during his stay in Madison (where he also lectured); he told the following anecdote:

> When Brouwer arrived at one of the places on his lecture tour, his sponsor apologised that there had been a misunderstanding, the funds for his honorarium were not forthcoming. Brouwer told his sponsor not to be concerned about it; he then gave them instead of his one-hour lecture his two-hour lecture.[38]

The tour ended where it had begun; on 31 December Brouwer was back in Montreal, from where he sent a New Year's card to his friend Mannoury. On January 3 he found himself again in Blaricum, full of memories and in particular with

[38] Kleene to Van Dalen 3.II.1977.

a rekindled and loving feeling for Lucia and Eric Gutkind. He immediately sent a card to them, signed by Lize, Tine Vermey, Cor Jongejan and himself, followed by another card, with the words 'In my thoughts I still remain with you.'[39]

Brouwer was enthusiastic about America, in particular where nature and hygiene was concerned. He had since his childhood been sensitive to dust; he in particular suffered from respirational afflictions and inflammation of the nose, and here he had found a country 'where one is not troubled by dust.' For a man like Brouwer, with a strong craving for clean living, the appeal of the Unite States was quite understandable.

Once back in Holland, he submitted in a quick succession the papers '*Addenda and Corrigenda on the role of the principium tertii exclusi in mathematics*', '*Ordnungswechsel in Bezug auf eine coupierbare geschlossene stetige Kurve.*', '*Further addenda and corrigenda on the role of the principium tertii exclusi in mathematics*', *'Intuitionistic Differentiability*', '*An example of Contradictority in classical theory of functions.*'[40] All of those papers consisted of intuitionistic refinements of old results and notions, and of corrections of old papers. None of these papers are notable for new revolutionary ideas, but they show that Brouwer had not lost his mastery of the technical parts of intuitionistic mathematics.

The inevitable loss of old companions and friends that is the fate of the aged, saddened Brouwer's life too; on March 3, 1954 Hendrik de Vries, his fellow mathematician in the faculty, with whom he used to share jokes, and who in the old days came over to play the violin with Brouwer, died in Benjamina (Israel). In December of the same year, Peter van Anrooy, the conductor, passed away. The musician from the Hague had been a true friend, and he was sorely missed by Brouwer. He wrote to the widow,

> Dear Freddy,
>
> Thus Peter has fought the struggle of his life to the end. A life that, directed by a great and indomitable talent, and a tempestuous richness of thoughts and feelings, found the predestined triumphs *and* conflicts on its way. My sincere condolences and my assurance that I, together with the many who are better qualified, will highly cherish his memory.
>
> Your
>
> Bertus Brouwer

A year after Brouwer's American lecture tour the international mathematics conference was held in Amsterdam. It brought the flower of mathematics to Holland. Many of Brouwer's old friends attended, but he apparently stayed away. However, a number of his friends used the occasion to visit him in Blaricum. Alexandrov, who was happy to revisit Holland after all those years, was invited with a number of prominent participants to the royal residence. At the tea party he told Queen

[39] *Fühle mich noch immer dort bei Euch.* Brouwer to Gutkind 12.I.1954.

[40] Submitted resp. 30.1., 27.2, 20.3, 24.4, 24.4 1954.

Queen Juliana receiving a selected group of participants of the International Mathmatics Congress 1954 at her palace in Soestdijk. Left in front: Von Neumann, the Queen is flanked by the Field medal winners, Serre and Kodaira. Behind Serre: Hermann Weyl; behind Weyl: Schouten; to the right behind the Queen: Heinz Hopf; second to the right last row: Veblen; mostly hidden in front of Veblen: Alexandrov. (CWI Archive).

Juliana that in the thirties they were more or less neighbours in Katwijk at the seaside in Holland.

Although he had reached the age of 73 years, Brouwer did not intend to give up his involvement in scientific matters and events. And 1954 indeed offered some occasion for renewing old contacts and making new ones. In March he was invited by Felix Hackett to participate in the Commemoration of Boole's *Laws of thought* of 1854. Brouwer gladly accepted, and on 24 May he found himself in Trinity College Dublin, where he delivered a talk *The effect of intuitionism on classical algebra of logic*. He used the occasion to stress the basic differences between classical and intuitionistic logic. The lecture presented in a nutshell the salient points. 'Classical algebra of logic', he said, 'furnishes a formal image of common-sensical thoughts.'

As such it is based on a threefold belief: (1) the existence of a *truth*, independent of human thought, (ii) obtaining new truths from old ones by logical reasoning, (iii) the principle of the excluded middle. He went on to sketch the status of logic:

Euwe conducting a simultaneous chess session at the Mathematical Conference in 1954. (CWI Archive).

Until not long ago this threefold belief of common-sensical thought was shared by scientific, also by mathematical, thought. As long as mathematics was considered as the science of space and time, it was a beloved field of activity of logical reasoning, not only in the days when space and time were believed to exist independently of human experience, but also after they had been taken for innate forms of conscious exterior human experience. There continued to reign some conviction that a mathematical assertion is either false or true, whether we know it or not, and that after the extinction of humanity mathematical truths, just as laws of nature, will survive.

Only after intuitionism had recognised mathematics as an autonomic interior constructional mental activity, which although it has found extremely useful linguistic expression and can be applied to an exterior world, nevertheless neither in its origin nor in the essence of its method has anything to do with language or an exterior world, on the one hand axioms became illusory, on the other hand the criterion of truth or falsehood of a mathematical assertion was confined to mathematical activity itself, without appeal either to logic or to a hypothetical omniscient being.

> An immediate consequences was that in mathematics no truths could be recognized which had not been experienced, ...

After giving a number of counter-examples to classical theorems, Brouwer concluded on a positive note,

> Fortunately classical algebra of logic has its merits quite apart from the question of its applicability to mathematics. Not only as a formal image of the technique of common sensible thinking has it reached a high degree of perfection, but also in itself, as an edifice of thought, it is a thing of exceptional harmony and beauty. Indeed, its successor, the sumptuous symbolic logic of the twentieth century, which at present is continually raising the most captivating problems and making the most surprising and penetrating discoveries, likewise is for a great part cultivated for its own sake. Don't let us forget that it is Boole who has been the originator of all this.

The lecture appeared in print in 1955, and it was Brouwer's final paper. The paper is worth reading, more because of its reflections on the nature of logic, than because of the logical content. After all those years Brouwer continued to forego the benefits of modern predicate logic; he used the time honoured notion of Boole and the school of algebraic logic. For his purpose that was indeed good enough. In a way the paper was his farewell to mathematics; in a modest way he remained active in the subject, but nothing substantial resulted after 1955. He planned to revise his old papers, but that did not come to more than isolated private notes.

Three months later Brouwer again represented Dutch mathematics. This time he was asked to represent the Academy in Paris at the centenary of Poincaré's birth. A better delegate could not have been found by the Academy; Brouwer was in a double sense a successor of Poincaré. Both in his topological research and in his foundational innovations, he carried Poincaré's torch.

It is good to keep in mind that at almost the same time Heyting was finishing the last parts of his monograph *Intuitionism. An Introduction*. This book finally provided a larger public with an easier access to the mysteries of intuitionism. Its role should not be underestimated, for a large number of logicians it was their first encounter with intuitionism; its readability has always been praised. Brouwer's views on this Introduction are not known, one may safely assume that he approved of its publication, because it appeared in the series *Studies in Logic*, founded by Brouwer himself and of which he was an editor until his death. On the other hand, after his retirement Brouwer got more and more estranged from his colleagues and even Heyting had not escaped the fate of falling into disgrace. The fact that Heyting deserted him in his conflict with the faculty must have hurt Brouwer more than most desertions.

In 1955 Brouwer travelled once more as a delegate of the Royal Academy to Göttingen. This time for the centenary of Gauss' death. It was a memorable occasion to be back in the old capital of the mathematical world. Not only did the town and the university awaken old memories, but some of the participants

were witnesses of the old days; Hopf, Radon, Denjoy, Julia, Perron, Reidemeister, Nöbeling, Süss, A. Schmidt and Courant were present, and at one time or another in the past, they had been more or less close to Brouwer. There may have been some awkward moments when former antagonists came face to face, but by now Brouwer had more or less accepted the past as a fact of life, and, unless provoked, he would behave as a perfect man of the world.

A well-deserved honour befell Brouwer in June of the same year; the University of Cambridge granted him an honorary doctorate. Although he had been rather out of sorts lately, he gathered all his courage and, accompanied by Tine Vermeij,[41] she set out for Cambridge. Lize had not felt like travelling to England, and it was a load of her mind that Tine was prepared to take care of Bertus. Bertus had been in a poor state already for some time, and it was a bit of relief to have a few days for herself. She decided at once to have Louise over in Blaricum.

On the second Thursday of June, Brouwer strode in the colourful procession along King's Parade to the Great Hall. He was in the company of fellow honorary doctors, men of renown, among others the famous architect sir Giles Gilbert Scott, the lawyer sir Charles Bruce, Locker Tennyson, the grandson of the poet, and the Right Honourable David Robert Alexander Earl of Crawford and Balcarres.

The virtues of the aged revolutionary, who even at this occasion managed to look eccentric, were aptly summed up by the Chancellor, lord Tedder, Marshal of the Royal Air Force, in his laudation,

> 'Everyone knows', said Cicero, 'that those who are called mathematicians deal in matters of incomprehensible complexity and subtlety.' Copernicus put it more briefly: 'Mathematics is written for mathematicians.' Hence my quandary. I shall prove a fool if I try to expound the merits of a top-grade mathematician, a coward if I shirk my appointed task. What then? I proceed and beg your indulgence. In two ways Professor Brouwer has earned an outstanding reputation. First, he has so advanced the science of topology that with Henri Poincaré and Georg Cantor he may be reckoned among the founders of this study in modern times. Secondly he has invented a new kind of mathematics, which he calls intuitional and which has revolutionised the whole foundations of the subject. It contains two acts (as he calls them). In the first, mathematics is defined as a languageless activity of the mind, having its origin in the conception of time. From this beginning, he demonstrates the fallibility of those who regard the logical principle of 'tertium non datur' as a reliable instrument for the advancement of knowledge and as for the argument of 'reductio ad absurdum', he exorcises it with a kind of magical incantation: 'The absurdity of the absurdity of the absurdity is absurd.' The second act—but before the second act my courage fails me. Like Socrates on another occasion, I fear

[41] Tine had been married to Willem Langhout but the marriage had ended in divorce.

Brouwer in the procession in Cambridge at the occasion of his honorary doctorate. (Brouwer archive).

> I may fall into a great pit of nonsense. Your applause must make amends for the deficiencies of my speech.[42]

Before leaving the eminent seat of learning that had so graciously recognized his role in science, Brouwer gave a talk on foundational matters. After that he returned to Holland, where he and Cor Jongejan withdrew to Zandvoort for a holiday on the seaside.

Hermann Weyl had in 1951 exchanged the United States for his beloved Switzerland, be it that he spent each year a few months in Princeton. He had renewed his contacts with European colleagues and friends, and attended for example the International mathematics conference in Amsterdam in 1954. He died on 9 December 1955 in Zürich.

Brouwer visited his old friend and fellow revolutionary when Weyl was already seriously ill. Considering how close they had been and how they shared their views on philosophical and foundational matters, there must have been an abundance of material for them to discuss. Brouwer later told that the conversation naturally had led to the foundations of mathematics, to the continuum, choice sequences, laws,

[42]Quoted from *The Times*. Brouwer had himself made a translation from the Latin text into Dutch. A note in his handwriting has been preserved. Cf. [Dalen, D. van 2001] p. 477.

continuity, and of course to the underlying philosophy. No man could cast his spell better than Brouwer if he wished; his softly spoken, beautifully formulated ideas and visions must have evoked in Weyl the memories of the carefree days of their meeting in 1919 in the Engadin.

– '*Ach Brouwer*', Weyl had sighed, '*es ist alles wieder schwankend geworden*.'[43]

And so Weyl ended where he had begun his foundational activities.[44]

In 1955 Brouwer was once more party in a conflict in Blaricum; a small business in his neighbourhood was asking permission to mix explosive fluids, this was for Brouwer and some fellow residents a reason to take legal action to prevent this. When the city council made up its mind to grant the permission, Brouwer went all the way to fight the decision, he even appealed to the Crown. Legal battles like this one took a more and more important place in Brouwer's life; they cost him dearly, both in fees and in emotional stress. Although the arguments were always cool and ingenious, the man behind the arguments was emotionally involved with every fibre of his person. In this particular case, he even travelled to the Hague in order to attend the ruling of the State Court (*Raad van State)* personally (17.XII.1956). The case was eventually won. There were more cases, and some of them had an operetta-like quality.

On January 30, 1956 Gerrit Mannoury died; Brouwer grieved for his long-time companion, in a letter to his colleague Stomps he wrote:[45]

> I feel deeply grieved by the loss of Mannoury. At a very early age I have already learnt from him and many decades I have warmed myself at his personality and received fruitful suggestions for my work from him. His character was of a sincerity and detachedness which commanded respect, and his realm of thought was of an almost infinite wealth [...]. Should at some time his writings be translated into a world language, then I cannot but expect, as far as I can see, that the judgement of the world will grant him a place in the Pantheon of civilisation among the great thinkers.

The treatment at the hands of his former colleagues had filled him with bitterness, which even showed some mild signs of paranoia. He never forgave, e.g., Clay for his role as a chairman of the faculty, and it is said that he refused to sit next to Clay at dinners, fearing that Clay could poison his wine. It is hard to say in how far he really meant this; he was no stranger to a certain provocative eccentricity, and such claims could perfectly serve to produce an effect of shock in an otherwise pleasant conversation. Brouwer's mind remained as brilliant and sharp as ever but his taste for the bizarre may easily have tempted him to shock his visitors.

[43] Everything has become shaky/insecure again.

[44] There is no doubt that, whatever Weyl's technical mathematical practice may have been, philosophically he remained on Brouwer's side. Freudenthal, who knew Weyl very well, and who was certainly qualified in foundational matters, wrote in his obituary [Freudenthal 1955], '*Er entscheidet sich für Brouwers intuitionistische Deutung der Mathematik, aber dem Systembauen abhold, verschmäht er Brouwers aufs Allgemeine zielende Methode. Das ist kein Verrat, denn das Wesen seiner Mathematik war und blieb intuitionistisch*.

[45] Brouwer to Stomps 17.II.1956.

Being well-acquainted with the more cruel sides of history and the practices of (for example) the Borgias, he liked to warn that one had to be careful, because 'many a sugar pot contains ground glass.' In spite of his traumatic experiences, he kept up his habit of visiting the meetings of the Mathematical Society, which still met at that time the last Saturday of the month in the hotel Krasnapolski in Amsterdam; there he mostly talked to the younger visitors. Furthermore he remained a regular visitor of the Royal Academy; at reaching the age of 70, at which an 'active member' becomes a member in retirement,[46] he informed the Academy that he had no wish to have his membership of the Section Physics extended, instead, he wrote, he was willing to join the 'Free Section' if the subject 'exact introspection' was added to its domain.

In due time Brouwer got in mathematical circles the reputation of a quarrelsome, unreasonable man. Heinz Hopf used to say jokingly that he was the only mathematician who never got into a fight with Brouwer, and in particular in the Netherlands, Brouwer's sharp tongue became something of a legend. The following account of De Groot may illustrate this. Once De Groot gave Brouwer a ride home after a meeting of the mathematical society; the conversation was lively, and Brouwer started to list all the fights he had with mathematicians. Suddenly he sat up and said to De Groot, 'but we also have a fight.' De Groot smiled and said, 'well, let us forget about it.' Brouwer possessed enough sense of humour to see that this quarrel at least had lost its grounds for continuation.

It was not only his discontent with the academic world and his colleagues that bothered him. His health, which was, in spite of the above mentioned heart problems, largely in reasonable condition, started to fail him on the smaller points, something that comes with the process of ageing. In 1956 he suffered from complaints of the urinary tract. Far from seeking help from surgical quarters, he stuck to the recipes of his wife: extract of celery and leek. Eventually the pains were alleviated. Some of his afflictions were most likely of a partially nervous origin. In January 1956, Brouwer feared once more for his life. As Lize reported to Louise,[47]

> Dad[48] had in the night of Saturday again a poor night. He thought it was the heart, but that was not the case. It was rather the stomach. He was violently sick during the night, and his heart reacted in a disturbing manner. He has been through this before, e.g. in Almelo in the examination committee; that time he had eaten strawberries, and a herring and a glass of cold water to top it of! He was then taken to the hospital next door.

Lize had hastened to Almelo and with the help of a few bandages cured her husband. A week later she could reassure Louise that,

> Fortunately Dad is well again. It is not the heart, but rather the stomach, which is always fed cookies by Cor. I have cured him with the triangular

[46] Nowadays at 65.

[47] Lize to Louise 15.I.1956.

[48] In her letters to Louise, Lize used the term 'Paps' for Bertus, often 'de Paps'.

compress, and warned him not to eat sweets and to eat and drink all kinds of things between meals. Now everything is well for the time being.

Now and then he asked me if I thought that he would die soon, because it was his heart after all. I told him that it was up to him.

With most of their money tied up in the pharmacy and in real estate, the Brouwers led a sober life. Indeed the only striking extravaganza on Brouwer's side was his love for travelling, in particular abroad. They had to watch their budget very carefully. Under the circumstances, Louise presented a real problem.

Brouwer's financial worries were not wholly imaginary. The pharmacy, in particular, drained his sources. On the other hand he owned large tracts of land and some real estate that could have been turned into money. Brouwer had a naive hope and conviction that he might find a miracle investment; at one time he invested a small sum in an instrument to stretch the hand of a piano player. The gadget did of course not sell at all, and the contraptions laid rusting in his garage for years. His nephew Jan (son of Aldert, named L.E.J. after his famous uncle) was the subject of Brouwer's admiration. Jan, who was like his uncle, a paragon of intelligence, turned his capacities to good use in the service of the Royal Shell. After serving the company in almost every corner of the earth, he ended his career as a very successful president director of Shell.

The relation with Louise had at the best of times been touchy, apart from the personal likes and dislikes there was an ongoing concern for her financial management. The allowance, paid from her father's estate, had not kept pace with the rising cost of living. In the late fifties the burden of a semi-invalid stepdaughter, who had by then already left her 65th birthday behind her, became a serious problem. Replying to Louise's congratulations on the occasion of the fifty-fifth wedding anniversary of her mother and stepfather, Brouwer wrote in September 1959:[49]

Dear Louise,

Many thanks for your sympathy at our 55-th anniversary, and for your fascinating book on Balzac, who appears to have had in his life almost as many difficulties as the undersigned.

As to the financial circumstances these have, alas, deteriorated so much for your mother and me during the last months, as rapidly at the health of both of us, that we can no longer afford you the same standard of living as so far.

In the first place we can no longer allow you to scorn the services of the excellent National Health organization A.Z.A.[50] offers for free both for your teeth and your internal health, and to waste pointlessly money by having you treated privately for the same things.

[49] Brouwer to Louise Peijpers 7.IX.1959.

[50] An Amsterdam organization, which provided medical help for the lower income classes for a moderate fee. A subscription to the National Health Organization was to a certain degree a social stigma. For private patients it was generally considered degrading to join the National Health.

The two L.E.J.'s - Jan Brouwer and his uncle at a meeting of the Shell in London (1960) (Brouwer Archive).

> In the second place we can no longer afford you to continue the payment of a rent which is far above the average, for a length of time …
>
> Should your mother and I have a bit longer to live and should our financial circumstances then take a fortunate turn, then you may rest assured that you will immediately profit from that favourable turn. But you should also reckon with the possibility of a sad event in a very near future.

And indeed that sad event took place sooner than expected.

Lize, more than ever tiny and fragile, had tenaciously run the pharmacies and the Blaricum household with a velvet hand; in spite of the demanding and often grumpy nature of Brouwer, she had managed to protect the interests of Bertus—and also of her daughter Louise. The latter had left the house long ago, but she never managed to earn a living, so that she was an extra financial burden to the somewhat chaotic economy of the Brouwer household. The surviving letters show us Lize perpetually in a hardly enviable position between the Scylla and Charibdis. On the one hand she stuck to her loyalty to her husband, and on the other hand she was visited by feelings of guilt for neglecting her only child.

Louise had in the nineteen twenties become a Roman Catholic and since then the correspondence between mother and daughter often assumed a religious tone. One would expect this from Louise, who had discernible tendency of religious mania, but surprisingly the mother was not outdone by the daughter in pious reflections and remarks. In fact, Mrs. Brouwer had in 1923 gone so far as to join the Roman Catholic Church 'in secret'; she was baptized on 24 December in the St. Dominicus Church in Amsterdam. The secret was well kept, none of the

interviewed persons, with one exception, was aware of the fact. In view of Brouwer's avowed aversion against the Catholic Church, one can understand Lize's reluctance to make her conversion public. It must be added that Brouwer's position with respect to the Church of Rome, did not affect his personal relationships. His friendship with Van Eeden and Father van Ginneken, for example, had been of a personal nature.

In the course of time Lize had grown more and more pious, she sought the assistance of the church in all kinds of matters, e.g. in November 1955 she asked Father Superior Drehmann to pray for a provisor for the pharmacy when it was almost impossible to find one, and when one had turned up, she was sincerely grateful for the prayers of the Father Superior; in view of the success, she did not hesitate to asking for prayers for shop assistants, who were also very scarce at the time.

In her piety she put unlimited trust in the healing powers of faith, later in life she expressed her admiration for the Christian Scientists, who had, in her opinion, taken over the monopoly of 'faith healing' from the church. She urged Louise to look into the Christian Scientist principles of faith, remarking that 'Max Euwe, the genius chess master and his wife had been healed' by the Christian Scientist faith and practice.[51]

Louise had a semi-independent existence in Amsterdam. She lived in one of the apartments over the pharmacy at the Amstelveenseweg. Lize used to praise her for 'taking care of the shop so well,' but that seems rather an attempt to placate her difficult daughter. There was not much sympathy lost between the staff of the pharmacies and Louise, the assistants viewed her, rightly or wrongly, as the lazy, sulking daughter, who reported on the daily affairs when Lize was away.

Lize was not blind to her daughter's shortcomings, but her prevailing attitude was one of protectiveness. Brouwer's dislike for Louise was no secret, and in the interest of domestic peace Lize did her utmost to keep the two apart. Whenever something was going on in Blaricum, a jubilee, a birthday, Christmas, she advised Louise to stay away—'nothing that would interest you.' And when the coast was clear, she would invite Louise, or visit her in Amsterdam.

Louise on her side was perfectly aware of Brouwer's feelings, and those of her mother. Usually she stayed in one of the houses that belonged to the pharmacy and kept up some pretension of taking care of the premises. She had a certain shrewd insight into the feelings of her mother, and she exploited these without much scruples. Some of the surviving letters display that entreating style parents use with difficult or even aggressive children.

Lize had made a will in 1921, which left Louise enough for a comfortable life, the other beneficiaries were Bertus and Cor Jongejan. The proportions were precisely indicated, with—depending at the circumstances—a quarter to Brouwer, a quarter to Cor and a half to Louise. Of course Brouwer had agreed to the will,

[51] Lize Brouwer to Louise Peijpers 11.II.1957. Later Mrs. Euwe became head of the Christian Science in the Netherlands.

Tea on the lawn. Left-to-right Cor Jongejan, Non Emmer, Brouwer, Lize, Tine Vermey. (Courtesy Casper Emmer).

he was one of the witnesses, but neither Lize nor he had foreseen what problems would be in store for the settling of the estate.

In retrospect one easily discerns a definite pattern in the relations of Lize; she had to deal not only with Bertus and Louise, but with the whole entourage of Brouwer and the pharmacies. The impression one gets is that of a resigned and even relaxed attitude towards Brouwer and his numerous girlfriends. None of these ladies seriously threatened her position, she clearly filled the role of the caring and loving mother and spouse, and less that of a passionate, let alone jealous, wife. She once remarked that the sexual aspects of marriage (and by extrapolation, of the relations with the weaker sex) were not prominent in Brouwer's life. Lize had gradually accepted Brouwer's way of forming attachments, in particular she had accepted Cor Jongejan—to whom Brouwer always turned in the long run—much as one accepts a sometimes cheerful, sometimes obnoxious child. Whatever the situation was, there was no sign of jealousy or a hidden struggle for power. With a superior resignation she knew that in the end her views would carry the day.

On the whole their marriage had been a harmonious one, it had started out with the dreams and idealism of the young; Brouwer's meteor-like career had surprised and delighted them both. Brouwer had consistently taken a keen interest in Lize's affairs, in particular he ruled the business matters of the pharmacy as if he were the pharmacist and not his wife. She from her side had guarded his physical and

The Brouwer couple in the late fifties. (Brouwer Archive).

spiritual well-being. Both of them were ardent health adepts, Lize was a master of herb lore and she practised all kinds of cures that avoided the blessings of the pharmaceutical industries. But not only the physical well-being of Bertus claimed her interest, she fervently hoped to bring Brouwer also into the fold of the Catholic Church; the difficulty was that the object of her desires was flatly anti-catholic, so that the only way left open for a conversion was that of constant and intense prayer. Whether Brouwer was aware of the attempts at his soul is unknown. It is a fact that he remained allergic to the catholic church. When a self-styled philosopher, Dr. A.J. Reesink from Laren died in the St. Jans hospital, Casper Emmer told Brouwer that he happened to enter the room where the deceased was lying and saw that he had a rosary in his hands. Brouwer flew up and cried 'Do you hear that Kleine Moek,'[52] if my time has come, you keep them out of the house, those body snatchers.'

Kleene observed during a visit in 1950, how strongly Van Eeden's conversion to Catholicism still rankled after all those years, when Brouwer was still upset and furious about what he called 'Van Eeden's treason.'

In 1959 Brouwer severed the connections with Bruins. The cause was a paper of Bruins, that he had asked Brouwer submit to the Proceedings of the Academy. The paper dealt with invariant theory. Brouwer had checked the contents and sent a number of comments and corrections, e.g. he suggested that a suitable title would

[52]The Brouwer couple used to refer to each other as '*klein Moek*' (little Mom), and to '*Lief man*' (sweet husband), in letters abbreviated as kM and lM.

be *on pathological metrisation elucidating theory of invariants*. He also pointed out that the references were in need of correcting.[53] Somehow Bruins was dragging his feet, and when he returned the revision,Brouwer was not satisfied, so that he waited for further corrections. This led to an exchange of letters, and when Brouwer had still not submitted the paper in August, Bruins made the mistake to hint that he wished to take his paper elsewhere. When that did not work, he took a high tone and demanded his manuscript back. As we know from past incidents, this was exactly the kind of thing Brouwer could not tolerate. For all practical purposes that was the end of the relationship between Brouwer and Bruins. The impression left by this episode is that Bruins had come to take Brouwer for granted. Brouwer's repeated attempts to get Bruins promoted, combined with Brouwer's isolation in Dutch mathematics, probably gave Bruins ideas.

19.2 The pharmacy

Among all the changing projects and affairs, there was one element that remained constant: the pharmacy. It is not easy to guess what its attraction was for Brouwer; was it a material tribute to his wife, or was it a palpable testimony to his importance and success? A man like Brouwer could not have overlooked the fact that, say, the fixed point theorem was a much better investment in eternal fame than the ownership of a pharmacy. Yet his attachment to the pharmacies was genuine and no doubt dictated by emotion rather than by calculation. Probably his fascination with the shops was the result of a sincere devotion to his wife, mixed with a feeling of family obligation, and the conviction that a clever man like himself could make a success of this commercial activity. Whatever the motivation for Brouwer was, it had started innocuous enough with the old shop at the Overtoom. This was the place where Bertus and Lize started their married life, that is to say, the Amsterdam part of it. Here Brouwer did part of his research, took care of his correspondence and met with his friends of the signific circle. If the city had not chosen to redevelop that part of the Overtoom, it is likely that the pharmacy had lasted as a calm centre for dispensing medicine and for homeopathic specialities. Fate, however, decided otherwise. In 1929 the shop had to be moved elsewhere. Brouwer found a suitable place, roughly a mile to the south at the Amstelveenseweg (cf. 572). Somewhat later Brouwer acquired two more locations, not far from the shop at the Amstelveenseweg, he bought a location at the Surinameplein and rented a wooden temporary building at Surinamestraat number 12.[54] The shops at the Amstelveenseweg and those at the Surinamestraat and -plein (square, place) were separated by a canal; after a bridge was built, it took no more than 10 minutes to get from one place to the other.

In the middle of the war Brouwer started to make plans for moving the pharmacy of the Surinamestraat (which was only a temporary location) to the shop at

[53] Brouwer to Bruins 4.VII.1959.

[54] The transaction was concluded on 2 April 1930.

the Surinameplein. Since the house at Surinameplein did not satisfy the requirements for a pharmacy, a great deal of reconstructing was required. Plans were made by a local contractor, but understandably, no actual work was done before the end of the war. The situation during the war was, as a matter of fact, far from pleasant, as the German army had constructed a concrete platform in the back garden, right behind the pharmacy, and installed an anti-aircraft battery.[55] After the war Brouwer managed to get permission to make large scale alterations to the house at the Surinameplein. In fact, he planned to have a basement added to the pharmacy, necessary for safe storage of pharmaceutical material. This was an unconventional plan; the newer houses in Amsterdam had as a rule no cellars or basements, the soft underground and the technique of building houses on piles made such extras far too expensive. Finally in 1955 the new basement was added. As a kind of constructional tour de force, the building was jacked up, so that work under the house could be carried out. As it was a four story brick building, this was at the time a major operation. The building did survive it rather well, although some unavoidable cracks showed up. The tenants, understandably, blamed the construction activity for it. Brouwer, on the other hand, blamed it on shortcomings of the original building (1936). We must keep in mind that Brouwer was already 74 years old when he undertook the basement construction, an age at which most people prefer a life of leisure and reflection. His wife, for whom he was creating the perfect pharmacy, had passed her eighty-fifth birthday! It seems clear that retirement was not an option for her, but considering her age, it was unlikely that she could enjoy the new shop for long. Financially speaking, the extension was also an irrational project. It was highly unlikely that a sale would make up for the investments. Perhaps one should admire the tenacity with which Brouwer extended his pharmacy, but his business instincts seem to have played a secondary role.

Brouwer carried on lengthy and cunning negotiations with surrounding tenants and owners to obtain a larger share of the central court. Surprisingly enough, he was successful; he indeed acquired enough space to connect the pharmacies of the Surinamestraat and the Surinameplein by way of his own backyard. And, to put the finishing touch on the new pharmacy, he commissioned the building of a laboratory for Lize on this ground behind the pharmacies. This was a simple, one story structure, which had all the facilities one could wish for a laboratory. Like the pharmacy at the Amstelveenseweg, the laboratory was more than a utilitarian construction, its interior was drawn up in style, with nice panelling and stained glass, decorated windows.

The shop itself was furnished in the traditional style of a chemist's shop. Of course in the process of perfecting the building, misfortunes occurred, for example, most of the year 1951 was taken up by a dispute with the firm that installed the central heating. The heating did not work at all, and finally Brouwer had to call in another firm for the reparations, which cost him an extra 1384 guilders.

[55]The pre-war building style in Amsterdam was that of (mostly) four story housing blocks with an enclosed court, split up among the tenants of the ground floor apartments.

Brouwer's relation with the pharmacy was somewhat ambiguous. On the one hand he could paint it as a cruel master, for which he had to slave as a freelance administrator. There are a number of private notes in Brouwer's archive on various topics; the following refers to the pharmacy: 'For several years I have been forced to earn my living by doing unskilled labour of the lowest level uninterruptedly, all day long, Sundays and days of illness not excepted.' Gödel's letter to his mother in which he reported on the visit of a great man from the past (i.e. Brouwer) also mentions Brouwer's complaints, see p. 871. There is a letter from Brouwer to his solicitor, Hardenberg, in which he expresses similar grieves; he wrote that he 'has lived in poverty ever since, to save the estate. Works 10 hours a day at it, must count with a sudden death.'[56] It was not just a matter of keeping the books in order and of keeping an eye on the staff. Brouwer spent a considerable amount of time and energy on negotiations with the city office for building and planning, with contractors, with third parties who made all kinds of absurd demands. In Lize's letters there is often mention of the state of exhaustion Brouwer was in, after spending whole days on business. True, the bleakest period in Brouwer's relation to the pharmacy came after his wife's death, when his stepdaughter enlisted legal assistance to get the most out of the inheritance, but it is equally true that Brouwer spent much time at all kinds of chores in the pharmacy.

On the other hand he considered the pharmacy his private pet. He devoted much time and thought to the well-being of the pharmacy and its staff. His relation to the assistants was that of a benevolent feudal lord, he cared about the 'girls' and he kept them on until a ripe old age. One of the assistants had joined the pharmacy as a very young girl, she worked up her way to the managing of the shop at the Amstelveenseweg, and when she retired at the age of eighty, it turned out that many customers had taken her for Mrs. Brouwer. Brouwer had a liking for all the assistants, but some more than others, he was for example very much impressed by a blond beauty, Greetje. None of this worried Cor Jongejan, she just made fun of it, as she made fun of almost everything, including herself. This was possibly her way of protecting herself.

When Brouwer bought the building (a temporary wooden construction) in the Surinamestraat, he obtained a long lease on the ground (a standard procedure in Amsterdam) with the condition that the city could at any time give notice when it needed the ground for development. In 1953 the city plans, which had been postponed on account of the war, had reached a phase in which the ground of the Surinamestraat was required. On March 13, 1953 the official notice of the city, announcing the termination of the lease, dropped in Brouwer's letter box. In 1954 the City of Amsterdam officially cancelled the rent for the lot of the premises at the Surinamestraat, in view of plans for developing the street. And so Brouwer had to move out of the temporary pharmacy.

[56] Brouwer to Hardenberg 11.II.1963. He is referring to Lize's estate and the lawsuits of Louise.

The Brouwer-de Holl pharmacy at Surinameplein. (Photo Dokie van Dalen).

The actual moving of the pharmacy to the Surinameplein presented a serious problem, as the pharmacy, including its laboratory and office did not fit into the ground floor of the premises at Surinameplein 8. An obvious solution to the problem was to give notice to the tenant of the first floor, and to add that part of the house to the pharmacy. However, the general housing shortage, caused by the demolitions during the war and the post-war baby boom did not allow for such steps. There was a strict rent protection. By regulation, no space should be converted from living- to office- or business space. Unless, of course, one could get permission from the official housing office to do so. Brouwer set immediately to work on the problem. He engaged his solicitor and spent his own time to convince the present tenant that she should find alternative quarters, while fighting the City of Amsterdam at the same time. The tenant was not inclined to cooperate, and the City of Amsterdam was adamant. In November, 1955, Brouwer lost the lawsuit and was ordered to evacuate the temporary shop before December 1. He appealed, and managed to win time. In June 1956 the shop in the Surinamestraat was finally and definitely pulled down.

From then on the business was conducted at the Amstelveenseweg and the Surinameplein, be it that the Surinameplein shop became the more important of the two. The following example may illustrate how seriously Brouwer took the commercial aspects of the pharmacies. When the City, after pulling down the house in the Surinamestraat, reconstructed the Surinameplein, it designed a small public garden in front of the shops. The plan also implied the moving of the big red

letterbox from the front of the pharmacy to this public garden. Immediately Brouwer started a protest: it would reduce the number of customers, he argued. And when the City did not give up its plan, he took it to court and won.

Since there were two chemist shops to be manned, again a 'provisor' was required. This provisor was to take care of the shop at the Amstelveenseweg, while Lize supervised the shop at the Surinameplein. The engagement of new provisors was always left to Brouwer. One of the later provisors, mrs. Paulssen remembered clearly how one day in 1952, Brouwer dropped in at her place in the Jan van der Heydestraat. This was in the older part of Amsterdam, notorious (or famous) for its almost infinite stairs that led from the street all the way to the top floor. Mrs. Paulssen was pregnant and her husband was away working for his Ph.D. One day the bell rang, and when she had pulled the rope that opened the street door, a tall, lean man with long white hair stormed up the stairs, taking two steps at the time. He had reached her floor before she could take of her apron, he looked round and said 'not very suggestive, the way you live here.' This vital elderly gentleman turned out to be professor Brouwer, the owner of a pharmacy; he invited her to become the provisor in his pharmacy, and she gladly accepted the offer. To her astonishment there were virtually no customers at the pharmacy that was assigned to her, yet the shop was open all day. Thus she simply went over to the shop at the Surinameplein, where she helped out. There she found a free-and-easy group; the customers bossed over the staff, and it took her some time to straighten things out. Mrs. Brouwer was completely out of touch with professional matters. Brouwer was very tender and patient with her, he took his time, kindly explaining things to her when she happened to be puzzled. Lize was equally concerned about Bertus and his health. At one occasion Brouwer was asked to go out and do some shoppings. It was very cold, and Lize insisted that Bertus could not go out without a hat. Unfortunately he had forgotten his own hat. After a long exchange of arguments, Brouwer finally left the shop with a hat of Lize on his head, blessfully unaware of his rather unusual look.

Mrs. Paulssen's impression of Brouwer was that of a remarkable person, a bit unworldly, and sometimes inclined to talk about mysticism. When her child was born, she got a bed jacket with thirty moth holes. After four years she moved to Rotterdam, remembering the pharmacy Brouwer-de Holl as quite an exceptional place.

On 11 October, 1959 Lize died at the age of 89 years in the hospital in Bussum. In spite of her advanced years, she had insisted on regularly visiting the pharmacy in Amsterdam and on carrying out the small duties that she considered hers. In the autumn of 1959 she fell down in the shop and broke her hip, after being laid up for several weeks in her apartment over the pharmacy, she was transported to the hospital in Bussum.[57]

[57] She was burried on 13 October in Blaricum in a private grave, from where her body was moved on 21 October to a double grave. Brouwer insisted on two separate graves for his wife and him.

Lize's death was a bitter blow to Brouwer, life without her would never be the same. Although Cor Jongejan from now on ran the household and took charge of the pharmacy, one cannot say that she filled the place Lize had left. With the death of Lize the duties of the executor of the will fell upon Brouwer. In itself the handling of the estate should not have been an overly taxing job, but there were aggravating circumstances. In the first place, the estate was rather more complicated than the ordinary 'family with some savings' estate. Not only did the couple, which was married without a settlement, own land and real estate, but the pharmacy posed real problems. The value of a pharmacy depended on a lot of factors, such as its house, the circle of clients, the management, etc. Nonetheless Brouwer succeeded in getting all the required figures right, but that was not enough; his stepdaughter, who heartily returned the dislike that Brouwer felt for her, finally saw her opportunity to get even with her stepfather. She got herself a lawyer, who like Louise was an ultra catholic, bordering on bigotry.[58] With his help she managed to delay the settling of the estate for some five years. This plunged Brouwer once more into a quack mire of legal and financial problems.

On August 24 1960, Brouwer had completed the registration of the credit and debit of the joint possessions of him and his deceased wife. The document drawn up by the notary, lists an impressive array of real estate: two houses in the Amstelveenseweg (Amsterdam), two houses on the Surinameplein (one of which housed the pharmacy) (Amsterdam), two villas and a wooden cottage (the hut) in Blaricum and Laren, furthermore an assorted collection of holdings in 't Gooi, mostly heather and wooded land, but also agricultural land. The estate also comprised a fair (but not large) collection of shares, (roughly 86,000 guilders), the capital of the pharmacy (28,497 guilders) and some fifteen hundred guilders at various banks. On the debit side there was something like D.fl. 80,000.-. (We note that Brouwer was so conscientious as to even list a claim of D.fl. 15,000.- on the Sodalitas Bath Company; a dubious claim, as he expressed it).

On 5 July 1961, Brouwer, Louise, and Cor reached under the watchful eye of the notary a settlement, it was agreed that the joint property would not be split up for the next five years. Louise received immediately an amount of ten thousand guilders (to be deducted from her share). Brouwer was put in charge of the administration of the estate, from which he had to pay out 400 guilders monthly (and 600 in December) to Louise—again to be deducted from her share. As Brouwer had to manage the pharmacies, he had to be able to raise the required funds for running the shops. For this purpose he was allowed to mortgage the property for no more than fifty thousand guilders. Once an agreement was reached, Louise's solicitor started to raise problems, after long negotiations he was prepared to concur with a mortgage of thirty thousand guilders on Brouwer's property Torenlaan 68, on the condition that Brouwer surrender his mandate for a mortgage of twenty

[58]Much later in life Louise had acquired enough critical sense to separate her worldly interests from her religious convictions—her adviser at the end of her life was a shrewd and impartial Jewish lawyer.

five thousand guilders at the 'Nationale'.[59] 'Miss A.L.E. Peijpers acquiesces—for the time being—in the situation, now that her stepfather is prepared to allow her five thousand guilders, to be deducted from her legitimate inheritance, where he wants to draw 30,000 guilders from the estate,' as the solicitor of Louise wrote to the notary.

In spite of the formal arrangement, a substantial residue of distrust remained on both sides. Louise was in spite of everything a shrewd operator. She was determined not only to get the maximum out of the estate, but also to put her stepfather a spoke in the wheel wherever she could. Her lawyer apparently had no compunctions carrying out her wishes, at the same time serving his own interest. There is an extremely bitter letter from Brouwer to his notary, Mr. W. van der Ploeg, which shows that Brouwer was at the end of his tether.[60]

> Now that the position Mr. Groos has taken with respect to the settlement of the estate Brouwer-de Holl seems to be beyond the power of the judiciary, on the basis of this position mr. Groos assigns himself in the matter of the settling and its duration the exclusive authority, to charge me with the complete management of the estate, which demands from me complete workdays of at least 12 to 14 hours, under refusal of remuneration, to forbid me to have the mentally deranged co-heiress miss Peijpers placed under legal constraint, to forbid me to charge the cost of the administration to this administration, and to use the revenues of the estate, which are by far exceeded by the expenditures, exclusively for fees, determined by himself, for himself;
>
> now that this attitude of Mr. Groos has finally, after almost four years, caused me bitter poverty, serious illness, and a harmed international reputation;
>
> now that any prospect of rescue from this situation is lacking,
>
> logically speaking the consequence of this situation would for me be suicide, were it not that the consequences for others which follow from it, prevent me from it.

The letter is no doubt a dramatic protest against mostly real injustice; this time Brouwer was up against a formidable opponent, a slick lawyer with a keen eye for weak spots. In cases like this logic and common sense are no match for the legal mind, sharpened by ages of casuistry and sophistry. Although Brouwer could not produce hard evidence, he was convinced that even his stepdaughter was in the end the victim of her councillor; there is a private note of Brouwer in the archive in which he mentions that he was told, that Louise had said to the informant, 'How did dad find out that Mr. Groos had given me no more than one half of the ten thousand guilders?' Not that this information was very useful to Brouwer, but it did little to make him more accommodating towards his stepdaughter.

[59] Insurance company.

[60] Brouwer to van der Ploeg 12.VIII.1963.

Under the management of Cor Jongejan the shop went gradually further down the road to friendly anarchy. She was easygoing to the extreme; at the end of the day she would often, without even counting, empty the cash register into her shopping bag or a plastic bag. When, as happened from time to time, she forgot her bag, one of the assistants would find it the next day. The stock was not systematically checked, old and obsolete medicines packed the cupboards and the replenishing of the stocks was left to the wisdom of the visiting salesman. When the pharmacy had night duty, instead of the single assistant required, there were often four assistants on duty. It was not unusual that cash was taken from the till and used to buy delicacies at the night shop round the corner, to pass the night. By and large Brouwer lost a good deal of money on the pharmacy. He often dropped in at the Surinameplein, carrying out his administrative duties, surrounded by the small army of assistants (all female). Brouwer enjoyed these visits, it was a special treat for him to retire to the bathroom at the first floor and to conduct his business from the bath. Covered by a towel he would sometimes call in an assistant to give instructions; lying in the bathtub he read books, recited poems, did the administration and felt happy.

In 1965 Brouwer finally sold the pharmacy; this was not a simple business transaction, dealing with Brouwer required patience and subtlety. The new owner, Mrs. Hanny van Lakwijk-Naojan, negotiated for a full year before the sale was concluded. Brouwer expected the new owner to come up to his expectations, just any buyer would not do. Hanny visited Brouwer repeatedly in Blaricum to discuss business, and to be inspected. The negotiations were more similar to a courtship than to a business transaction—the pharmacy had to be won. Hanny was thoroughly familiar with the strong and weak points of the pharmacy, as she had served as a 'provisor', so she could not be fooled about the business details. But dealing with Brouwer was not just a matter of looking at figures and real estate. She would find Brouwer in Blaricum reclining on his old wicker chair, in his white tropical suit, under a straw hat with holes. Brouwer opened the discussions innocently enough and acted as though he was a stranger to business, leaving the floor to the other party. Then suddenly he would draw a sheaf of paper with all the relevant data from under his chair, and argue sharp and to the point. Brouwer, moreover, had simple but non-negotionable conditions, e.g. Cor Jongejan should stay on as an assistant, and the pharmacy should continue to operate under the name 'Brouwer-de Holl.'

The negotiations were finally successful, partly because Brouwer had developed a strong sympathy for Hanny, without however allowing her to get too close to him. He kept a certain distance by corresponding with her in English. Cor said to her 'the professor is quite fond of you, but he wants to keep his distance.' He viewed his relation to the prospective owner more or less as a grandfather would look at a promising grandchild. He could in a paternal manner be quite strict with Hanny's correspondence. It happened, for instance, that Cor Jongejan handed her back a letter of hers with the words, 'The Professor has corrected your letter, he wants you to rewrite it.' He had said to Cor, 'It looks like the letter of a little girl; she has studied, she must be able to do better.'

Brouwer's reputation as a difficult man was made clear to Mrs. van Lakwijk by her former professors; when it became known that she was interested in buying the pharmacy, she suddenly got a telephone call from Prof. Cornelia van Arkel, 'would she immediately come and see her.' Van Arkel warned her in no uncertain words, that Brouwer could not be trusted, illustrating this with many examples. If there was any person that could mislead you, it was Brouwer! A couple of days later the warning was repeated by Professor Kok, he summoned her to his house—what was she thinking of? Kok lectured her in even stronger terms on Brouwer's evil reputation, his ruining the pharmacy, etc.

In spite of this undoubtedly well-meant advice, Hanny carried on the negotiations, and finally in 1965 she could call the pharmacy her own. Brouwer was proud of her because she had conquered the pharmacy, as he put it, and left him the honour of graciously handing over the property.

An amusing detail of the transaction was that Brouwer insisted on payment in cash; she travelled to Blaricum at the appointed day with a suitcase full of small denominations! At the opening of the the pharmacy under the new owner, Brouwer presented Hanny with a beautiful antique mortar filled with flowers.

Although the pharmacy had changed hands, Brouwer kept a vivid interest in it; he often visited the Surinameplein, where the assistants saw him, watching the shop from a distance.

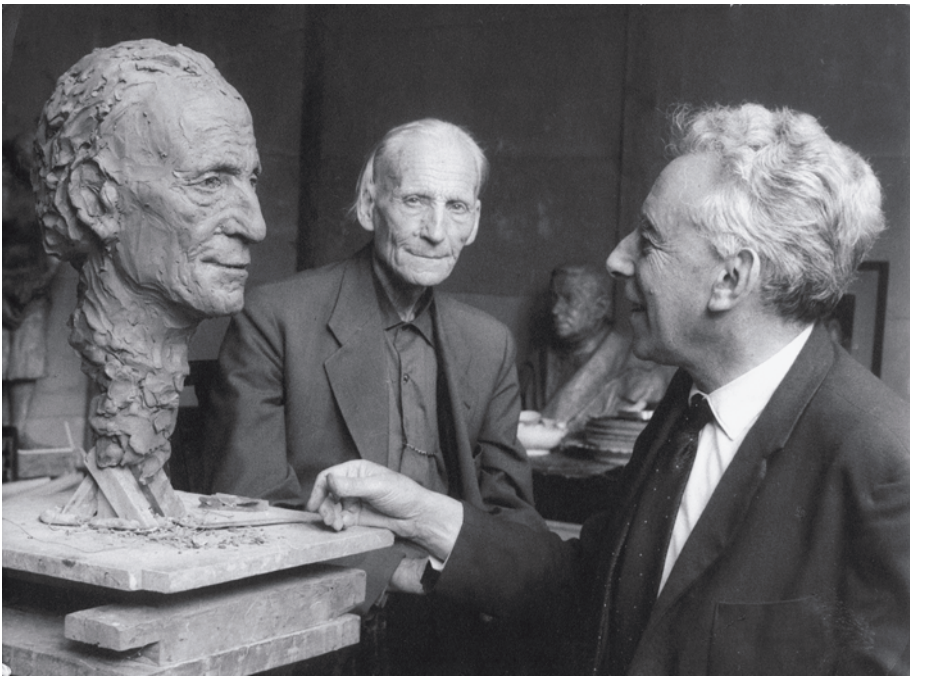
—Oh, Mrs. van Lakwijk, there is Professor Brouwer watching us.
—Well, then he will shortly be in.

After some time, Brouwer would then move off in the direction of the shop, crossing the heavy traffic and walking along the track of the trams. The traffic then miraculously parted, like the waters the Red Sea. On this point all witnesses agree, that Brouwer managed to ignore the traffic in his path. Students and colleagues who saw him making his way to the mathematical institute, or some other place, were horrified to see him plunge absentmindedly into the flow of cars and cyclists.

One thing Brouwer was loath to give up; owner or not; he would insist to take his customary bath in the bathroom belonging to the pharmacy, if Hanny happened to be absent. The collected assistants tried to stop him—'Oh no professor, you can't do that, you are no longer the owner.' and when they failed to do so, they started to talk him out of the bath. Finally he would give in and say, 'All right, then give me a glass of warm milk.'

19.3 The last years

The accumulated pressure of the legal problems, the loss of his wife and the never forgotten or forgiven injuries of the post-war years increased his tendency to see enemies all around him. His last years were not free from forms of paranoia, he felt threatened by criminal conspiracies. In the most literal sense he was convinced that 'they' were after him. He was under the impression that criminal parties broke into his house and spirited away incriminating material. Often the police was called in



Brouwer posing for the sculptor Wertheim. (Brouwer Archive).

after a reported theft; usually the missing files or briefcase were discovered somewhere else in the house. The legal battles, of which he fought a number simultaneously, gradually transformed the living room. The floor was occupied with stacks of documents of case 1, case 2, … Like the commander of a beleaguered fortress Brouwer would, before going to bed, make the rounds, and bolt all the doors and windows, only to have them reopened by Cor Jongejan, who quietly let in a friend when Brouwer had gone to bed. After the ladies had spent the rest of the evening with animated discussions, she would re-lock the doors and Brouwer was none the wiser.

His paranoia did not improve over the years. There is a (unconfirmed) story that he wrote to Dag Hammarskjöld, the Secretary General of the United Nations, telling him about his situation, and the criminal designs at his life and property. According to the story, Hammarskjöld send a kind reply guaranteeing him protection within 30 kilometres of his home.

Although Brouwer had taken his leave from the Science section of the Academy, he kept an interest in the matters of the Academy. He visited the meetings of the Science section when he felt like it. In the early sixties he was asked to present a revised set of voting rules. A considerable amount of time and correspondence was taken up by that project in 1961, and well into 1962.

The fact that Brouwer considered himself the victim of machinations of some of his colleagues, and deplored the lack of support from the remaining mathematicians, did not stop him from attending the Academy meetings. He was a frequent and outspoken visitor of the meetings of the Physics section (which he had left to join the Free Section). He could comment on the past with a cynical detachment. One such case was recalled by N.G. de Bruijn: after a meeting of the Academy Schouten came upon Brouwer, who was talking with a small group of members. Schouten, in whose opinion the conflict with Brouwer had lost its edge long ago, judged this a good opportunity to restore peace.

Schouten—let us forget about the old differences.
Brouwer—do I then get my journal back?

In 1960 Brouwer suddenly became active in the Academy; at the extra-ordinary meeting of the section of the sciences the appointment of a new foreign member had been tabled. A number of members had put forward Kuratowski as a candidate, but Brouwer had proposed his own candidate, Sierpinski. Brouwer conducted his campaign with great display of words. Apparently one of the members had remarked that if Sierpinski—a man well advanced in years—were appointed, it might be for only a year or so. This provoked a spirited rejoinder of Brouwer, who was actually only one year older. He praised Sierpinski's qualities with gusto,

> During his whole life so far, Sierpinski has continuously broadened, deepened and rejuvenated his thoughts; recently his researches took him into number theory, and he has started with fervour a seminar. His leadership on the areas which he opened up, has remained undisputed—no matter how famous, astute and original some of his collaborators may have been. Without claiming completeness, I mention among them Borel, Baire, Lebesgue, Hausdorff, Young, Hobson, Alexandroff, Kuratowski, Tarski and Carnap. Among them Sierpinski shines with undiminished luminosity in the epistemological sky as a star of the first magnitude.

The nomination of Sierpinski was put into a flowery language that clearly betrayed Brouwer's hand,

> When round the latest turn of the century the physiology of the real functions had drawn the general attention, and thereby a vast domain of new problems had emerged, the treatment of which demanded a drastically deeper foundation of epistemology, and questions such as the justification of existence and the scope of the axiom of choice and the notion of continuity could no longer be ignored, the international mass of researchers, which was thereby inspired, has right from the beginning been led by Sierpinski, who, through his mental power and originality, obtained results that mark him as a grandmaster, and by means of his inspiring suggestiveness shaped the Polish mathematical school which finds its expression in the renown journal Fundamenta Mathematicae. [...]

> With Chopin, Paderewski and Mrs. Curie, Sierpinski belongs to the group of through the ages admirable figures, which Poland has given to the world.[61]

Freudenthal had been one of the supporters of Kuratowski's appointment, he sharply criticized the decision of the Academy to reject Kuratowski and to appoint Sierpinski instead. He remarked that it would be hard to substantiate the mere claim of Sierpinski's fame as an equal of the above mentioned Poles. 'It would not have been an easy task to unearth from the possibly thousand mathematical papers of Sierpinski (mostly of 1 or 2 pages) enough material to justify this comparison.' Brouwer reacted with more praise for the old master—and he poured his indignation out over 'a member' who had remarked that on account of Sierpinski's advanced years, he probably would serve probably for no more than a year. In a first indignant reaction Brouwer quoted a member[62] describing Sierpinski as a senile old gentleman, well in his eighties. In the end Brouwer won the battle and Sierpinski became a foreign member of the Academy.

In spite of his advancing age Brouwer still considered emigration as a way to start a new life abroad, leaving his beloved, but, in his eyes, corrupt and degraded native country behind. In a letter to Marston Morse, he fondly recalled his stay in Princeton, and added that he was 'toying with the idea of being called back to the Institute some day for a longer stay. Which might also be to the profit of science, my circumstances in the Netherlands being absolutely prohibitive for scientific research.'[63]

In 1959 a correspondence with B.N. Moyls (acting head of the Mathematics Department of the University of British Columbia) was conducted. Moyls asked Brouwer whether it was true that he was interested in a position,[64]and Brouwer eagerly replied that 'As a matter of fact I want to emigrate from the Netherlands, find a field of activity on your continent, and change my nationality as quickly as possible.'[65] When this did not work out, he asked in desperation if the 'joint universities could rescue me?' Apparently there was little that tied him to Holland after the death of Lize, nonetheless he remained active in his own characteristic way.

In spite of his advanced age he kept up his travels. At the third centenary of the Royal Society in 1960, he together with the astronomer Oort represented the Netherlands.

Brouwer had lost none of his fierceness and his sympathy for the underdog. His last action in the Academy is a worthy illustration of the spirit that never left him. In 1962 Brouwer put A.D. de Groot up as a candidate for a vacancy in the 'Free Section' of the Academy. De Groot was a psychologist who had as a student

[61] There are a number of versions of this text in the archive, which differ in formulation.

[62] Not Freudenthal, probably Schouten.

[63] Brouwer to Morse 4.I.1955.

[64] Moyls to Brouwer 7.VIII.1959.

[65] Brouwer to Moyls 5.IX.1959.

The Three brothers Brouwer. (Brouwer Archive).

enrolled in the mathematics faculty and subsequently had switched to psychology. He was a student of Révész, the father of the Amsterdam school of psychology. He wrote a dissertation on 'The thinking of the chess player', a work that more or less established his reputation (and which has been rediscovered now by the cognitive artificial intelligence people). Brouwer devoted much time and ingenuity to De Groot's candidacy, but the opposing forces this time got the better of him.

Among the smaller actions in which Brouwer took part one must mention his support of the candidacy of the Dutch author Simon Vestdijk for the Nobel prize for literature (1964).[66] This action was not successful either.

Brouwer still regularly visited the meetings of the Academy, where he had a fierce reputation. It was, for example, not unusual to see him quarrel there with his brother Aldert, the geologist.

By now his old friends had died or were no longer active. His brother Lex had died in November 1963, and Brouwer spoke at his grave:

> In Izaak Alexander Brouwer there was an experience of beauty and a capacity for expressing beauty, in painting, music and literature, which seemed boundless.
>
> Further a rare erudition in the area of technology, critical, inventive and constructive.
>
> Next to this talent, which was also visible in his personal appearance, there was no place for a social fighting spirit, not for a regular and hygienic way of life, not for finishing, displaying in public and delivering own creations, not for efficient measures against setbacks.

[66] Vestdijk was not only an outstanding , but also an extremely prolific author, one of his fellow authors wrote 'he who writes faster than the gods can read'.

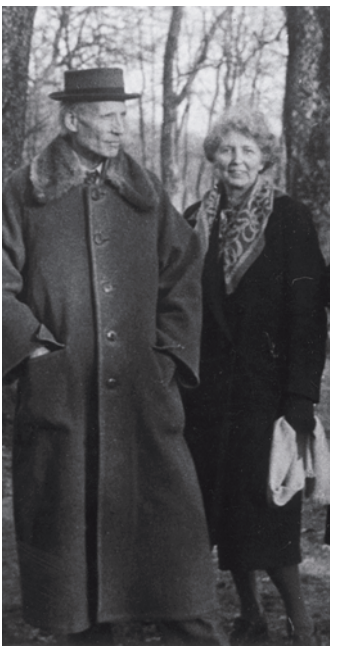
> Izaak Alexander Brouwer has been surrounded in his inexorable isolation by the admiration, love and friendship of many. They will cherish an unforgettable memory of him.

True and compassionate words for a brother who had great dreams, but did not know how to realize them. Lex Brouwer lives on in the memory of his pupils of the Baarn's Lyceum, where he taught French. The eccentric, scholarly man with his motor bike was one of the more striking personalities at school.

Where the oldest and the youngest Brouwer brothers had making a lightning career, the one in the middle had not chosen to follow the wishes of his parent. Lex had started to study chemistry (and according to Aldert's son Aldie, he was very successful in the subject), but he soon gave up a prospective academic career, in order to become a painter. He ran away to Paris, and started to learn the basics of the art. He also found an attractive girl friend, Winnie, whom he married. His parents were not happy at all with this unexpected demonstration of independence, they hastened to Paris, and carried the young man and his wife back to Holland. There he, rather reluctantly, enrolled in the study of French to become a teacher. In the end he lived between the two worlds of Art and Teaching. Life had not brought him what he dearly wished, and he remained a rather disenchanted man to his last days. He settled in Blaricum, where he became the owner of a charming little house with a studio. There is a legend that he was so gifted in the French language that he wrote a brilliant dissertation, but never defended or published it; instead, it is told, he buried the manuscript in his backyard (it has never been found).

In the same year Tine Vermeij, one of his intimate friends closed her eyes; at the funeral Brouwer could not master his emotions. Ru Mauve, the last of the small band of Brouwer's fellow students, also passed away in 1964. When his son Maarten went to Brouwer's house to bring the sad news, Brouwer was silent for a while, and turned to Cor, sighing 'Ru was a good man, he learned easily, but he never in his whole life had an original idea. However, if you talked to him, he was so sharp, that he could always catch you at smaller points.' Thus, as it is with true friends, the memories of the weak and strong points had blended into a tender remembrance. Maarten recalled vividly how Brouwer came to the house to pay his respect. Ru was lying in state at home, as was the custom. There, in a coffin lined with satin, he was in a suit with silver buttons. When Brouwer arrived, he went straight up to the room where the coffin was placed and kneeled before the coffin. There he remained in thoughts in total silence for at least twenty minutes. Returning to the family gathered in the sitting room, he immediately embarked on a lively conversation. His disregard for the conventional atmosphere of mournfulness did by no means reflect on the sincerity of his grief.

The aged Brouwer had not lost his eagerness for human company. He regularly entertained a small circle of female admirers, who loved to follow his lengthy and often fantastic tales. He had, ever since he shed off the reserved habits of his student years, been an almost compulsive talker. He could expand any subject, his



Brouwer and Cor Jongejan. (Brouwer Archive).

phenomenal memory allowed him to use details that he had picked up, nobody knew how long ago. In addition to the, say, rational part of his conversation, he loved to add little mystifications and fantasies.

Colleagues from abroad, who visited him after his retirement, recall his conversation, from which one had to pick up little bits of historical information, as one picks gold nuggets from a mass of mud and stones.

When Kleene visited him, he learned the story of Brouwer's walking tour to Italy in 1902; Brouwer illustrated how he wrapped himself in his cape for sleeping in the open air.

Although he avoided contact with his Dutch mathematical colleagues, he was hospitable to visitors, and there often were old friends or students who dropped in. A regular visitor was the chess master Euwe, who could get along very well with Brouwer. Euwe, after his chess career (which coincided with his teaching of mathematics at a girl's school in Amsterdam), had become a computer scientist. He got a chair in Rotterdam, and in addition he was an advisor to Remmington. One might wonder how Brouwer viewed the advent of the computer and computer mathematics, which was already taking place during his life time. Not much is known beyond the little that Euwe told. Brouwer attended in 1962 Euwe's congress 'Man and Robot', and Euwe related how Brouwer had shown a clear grasp of the principles behind the (then) modern computers, when he was given a guided tour by Euwe along the computing machines which were under his care. The attending

staff at the occasion showed surprise at the understanding 'of the old gentleman.' He had, however, enough self-knowledge to take a realistic view of the offer of an adviser's position with the firm, which would have brought him 10,000 guilders. Just his name was worth the money, Euwe told him. Brouwer declined the offer, he had to finish more urgent things first.

After the death of Lize, Cor Jongejan and Tine Vermeij had become, if not 'next of kin', at least 'next of relationship'. They played a major role in the remaining years of Brouwer's life The relationship with Louise was a disaster, to put it mildly, a situation of a mutual dislike and distrust, interrupted by moments of open warfare. With Cor and Tine Brouwer could get along well enough; where love had been a feeling of loyalty and admiration remained on the side of the ladies. In the past they had fought fierce battles for Brouwer's favour, but at a riper age—Cor was 66 and Tine 72 when Lize died, 89 years of age—the competition had petered out. At many occasions Cor or Tine accompanied Brouwer on his trips abroad. Brouwer felt that his obligations were really to Cor, who had sacrificed her personal future to that of Brouwer. Consequently, he made her a marriage proposal, which she turned down. She remained his universal support, managed the pharmacy, ran the house, did the garden, etc. Hanny van Lakwijk described the quiet leisurely atmosphere in the garden. Brouwer was lying in a hammock, and from above instructed Cor how to plant what where. An old-fashioned scene of a man with a straw hat and a pretty middle aged woman in the mixture of light and shadow, among the luxurious trees and shrubs.

The housework was done by a local help, Ali. One would have thought that by now, Brouwer turning eighty, it was time to settle into a comfortable routine. One of slippers, good books, a warm fire, music.

Not so Brouwer, he kept a vivid interest in the university; he became a familiar figure at Ph.D. examinations, where he took part in the questioning of a rich variety of candidates, not only in the mathematics and physics sections, but also in the literary faculty. He displayed a remarkable mastery of the classic languages, examining candidates in Latin or Greek if the occasion arose. As his knowledge of a considerable part of the subjects in mathematics and in physics was often based on first hand information from the grand old men of the subjects, he liked to interlace his questions with long and often intriguing references to his experiences in the far past. When Hans Mooij defended his dissertation on the work of Poincaré, Brouwer could not let the occasion slip by without presenting an extensive exposition of the views of Poincaré, as observed by him first hand. The excursions in the literary faculty may have had something to do with a new development: he fell once more violently in love with a most remarkable lady, a colleague in the literary faculty. The object of his love was Emily Haspels. She was a professor in archaeology, with a great expertise in Greek vases. Emily, who was thirteen years Brouwer's junior, had entered Brouwer's life through his interest in languages. His love for the classic languages had been the factor that brought them together. They had regular sessions, during which they read together the ancient-literature. And soon this

Brouwer in his favourite wicker chair in the lawn in front of the Pimpernel. (Brouwer Archive).

mutual interest turned into something more personal. This new person in his life was a paragon of intelligence, determination and tenacity. She had in the thirties travelled in China and worked in Turkey. It was in Turkey, where she carried out her research on vases, that the Second World War surprised her. When she learned about the fate of Holland, it was too late to return, the Turkish government gave her no permission to leave the country. After working as a cleaning lady in schools, she eventually got a teaching job at the university, as the authorities had discovered her expertise. After the war she got in 1946 a chair at the University of Amsterdam.

The new relationship differed in an important respect from the loves of his younger years. This time Brouwer's relationship was of a different nature. He was the party who stayed behind while Emily travelled around the world and carried out her research. The letters that Emily wrote to Brouwer were eagerly read, inscribed with the dates of reception and other information. There is no doubt

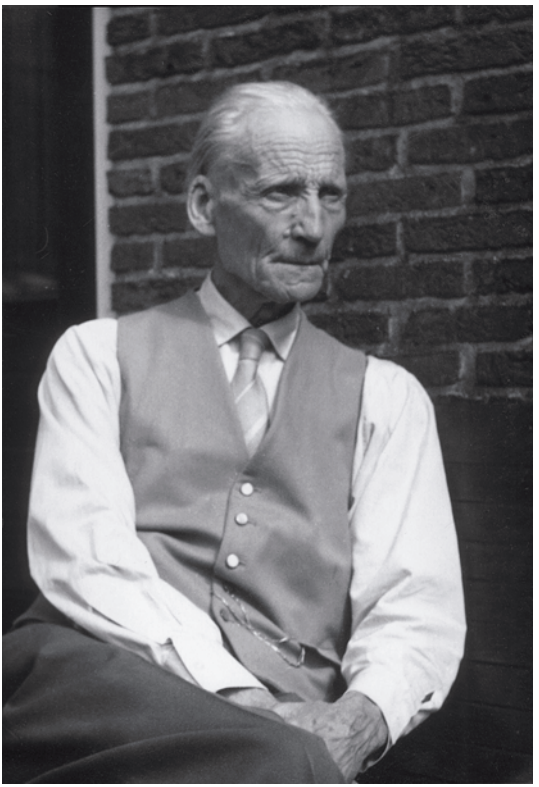
Brouwer taking part in the examination at the Ph.D. ceremony of Jan Hilgevoort (1960). (Courtesy Jan Hilgevoort).

that the affection was mutual, this was a case of two strong characters, who met in a mixture of love and cherishment. Both were undoubtedly serious about their affections; there is a large number of letters and jottings, probably given to Emily by Cor Jongejan after Brouwer's death, and they show a picture of sincere interest and affection. Emily travelled all over the world for her research, there are letters from Lyons, Venice, Oxford, Stanford, ... In Oxford she met the ladies Deneke, who were descendents of Maurice Philip Deneke, a musicologist, who left money for a series of lectures, called the Deneke lectures. Staying with Margaret Deneke, she found out that Brouwer had in 1949 given a lecture in the series, with the title: 'The influence of mathematics on logic'. A lecture that was not only appreciated by the ladies Deneke, but also by the scholarly community. Emily wrote that she had been in Lady Margaret Hall (in Oxford), where she was told of the impression Brouwer made, 'all the mathematicians' faces in the audience lighted up, because they got something worth their while.'[67] Suddenly, in the middle of the correspondence, there is a letter from Brouwer, mentioning an infarct, 'My infarct a few times very dangerous but already the awareness of a wide variety of indispensability keeps me alive.'[68]

In 1966 Emily visited the United States for a lecture tour, combined with negotiations with various publishers concerning her new book. The tour, from May till

[67] Haspels to Brouwer 25.VI.1964.

[68] Brouwer to Haspels 30.VIII.1965.



Brouwer dressed up for a photo session. (Courtesy G.F. Viets).

August, took her also to Princeton, where, clearly on Brouwer's request, she tried to meet Oppenheim and Marston Morse. Both appeared rather elusive, Oppenheim because of his terminal illness, and Morse because he made few appearances. Finally, in July, she happened to run into him. After Morse introduced himself, she immediately asked him if he knew Brouwer. 'Yes, he has been here for a few weeks once; he has a very famous name, internationally, at home he has a rather stormy career, I gather.' Yes', she replied, 'apparently others cannot look ahead the way he does.'[69]

The fact that Brouwer's travelling range had become smaller did not mean that he had lost his appetite for company and for social and scientific meetings. One of those meetings took him in 1962 to Middelburg in Zeeland. Brouwer was attending some meeting in Middelburg, when he decided to look up Karen, the daughter of his friend Ru Mauve, who lived in Veere, in a roughly three hundred and fifty years old warehouse. Since the busses were running only once every one and a half hour, the eighty-one year old Brouwer set out on foot for a walk of some ten kilometres.

[69]Haspels to Brouwer 31.VII.1966.

Agnes van den Noort-van Gelder, Karen's daughter, described her meeting with Brouwer:[70]

> I was 16 at the time, and I was with Karen in the kitchen (on the first floor, we lived upstairs, downstairs was too cold). Then there was a ring at the door, and Karen looked out of the window—'Say, there is Brouwer!', she said astonished and surprised, and hastened down the stairs.
>
> Curiously I also looked from the window and saw a piece of what appeared to be a black cape. Down there the greeting took time, but finally this Brouwer mounted the steep stairs, and there he appeared from the curtain of the stair well—with his sharp pale face and the white spiky hairs of such a length, that they curled into his collar. I did not know how to react—what a shoes. And the cape. And how funny he smelt, dusty, not very pleasant. [...]
>
> Pretty soon I was sent to the green grocer for mushrooms. Mushrooms! Not so long ago these could only be bought to order, and then only on Saturdays! Well, Brouwer stayed for dinner: mushrooms and rice and lettuce. [...]
>
> Karen told me that Brouwer was ve-ge-ta-rian. A new word! Hence the fancy mushrooms. It was with reluctance that I sat next to him at the table—he sat at the place of my sister, who had already 'left the house.' I remember that the conversation was difficult to follow. Brouwer's use of language was curious for me, and his sentences were rather complicated. But he taught me that one should *always* eat with little bites, and then *always* chew 32 times. I did not dare to think this nonsense (but I found out later that 32 times is not so much after all)
>
> After my parents had taken him to the bus, my father said: 'Now you have seen a genius!'

A severe loss was the death of Erich Gutkind in September 1965. The Gutkinds and Brouwer had renewed their friendship in 1953, and Brouwer immediately had resumed the old relationship. The death of Erich left him 'crushed like you, and with you,' as he wrote to Lucia[71], and a couple of weeks later he begged her to join him,

> How Eka continually floats around me and looks at me! With respect to travel abroad I am for the time being a prisoner. But couldn't you come to me, for a short time or as long as you like? Eye in eye, Bertus[72]

[70] Agnes van den Noort-van Gelder to Van Dalen 14.III.2002. In the same letter Agnes provided some extra information on Brouwer's famous walking tour to Italy in 1901 (Cf. p. 27, 34). Old family documents that had been found showed that Brouwer had stayed with Ru and his wife in Florence. The young couple had rented the Villa Belvedere at the Via Dante da Castiglione.

[71] Brouwer to Lucia Gutkind 6.IX.1965.

[72] Brouwer to Lucia Gutkind 17.IX.1965.

In March 1966 his old friend Zernike died, he attended the funeral in Groningen, but when he was begged not to speak at the grave, he angrily marched out of the room.[73]

The estate of Lize was not yet settled and Brouwer (one would guess, correctly) surmised ill-will or worse on the side of Louise and her representative, Mr. Groos; on April 19, 1966 he filed a formal complaint against Groos. 'Criminal actions of private persons and official authorities,' he said, 'led to his bankruptcy.' The battle with the tax collector also went on, on October 31, 1966 he got a distress warrant ordering him to pay the overdue taxes within two days, on penalty of seizure of Brouwer's possessions by the bailiff.

The year 1966 was almost over when Brouwer finally fell victim to motorized traffic, as so many of his friends had feared and predicted. Cor Jongejan was celebrating with the staff of the pharmacy Saint Nicholas' eve (Sinterklaas) on the second of December (three days early), and in order not to be interrupted, she had proposed to turn off the ring of the telephone. Hanny van Lakwijk then observed that the light of the telephone was continually flashing, finally she picked up the telephone and received the stunning news that Brouwer had died as the result of an accident. It appeared that he had been asked by Cor to deliver some presents for St. Nicolas eve to friends across the road; he had picked up a grey horse blanket and wrapped around him. When crossing the Torenlaan in front of his house, almost invisible in the ill lit lane, he was hit by a car, which knocked him into the other lane. There, two more cars ran over him, and he died almost immediately. Cor was heartbroken, she felt that she had been responsible. If only ....

A few days later Brouwer was buried in the local cemetery, next to the grave of Lize. He had always expressed his objections to a shared grave. The funeral was, at Brouwer's request, attended by a handful of close friends; Max Euwe held the funeral oration.

Emily was in Oxford when all this took place. There was a small scrap of paper among her notes and letters, that said 'and then when you call his house, suddenly—"he is dead—will be burried in one and a half hour." You do not survive something like that—you live on, but you don't get back to normal.'

19.4 Epilogue

Brouwer's will appointed Cor Jongejan as the sole beneficiary. Cor thus came into the possession of a substantial fortune, at least on paper—most of it was tied up in real estate and land. The negative side of the inheritance was the debt to the tax collector. No love was lost between Brouwer and the Internal Revenue Service, his relation with the tax collector consisted of a long series of requests for postponement of payment, appeals, arguments, personal interviews, etc. Later in life, in particular after his retirement, he regularly got into serious problems with the internal revenue service. In 1964 the inspector turned down Brouwer's claim for a

[73] Oral communication N. van Kampen.

reduction in tax, and in 1965 the bailiff was at his door, giving Brouwer ten days time to pay his taxes, under threat of a considerable penalty. Brouwer did not cheat on his taxes, but he certainly looked for legal means to minimize the tax assessment. He had quite a reputation with the tax inspectors, he could talk until the inspector's head was spinning. Louise told that, speaking about taxes, Brouwer had said, 'When I die, I will leave a mess that nobody will be able to sort out.' Poor Cor, as the sole heiress, had to take the rap; three days after Brouwer's death she was summoned to the police station to surrender her passport. No foreign travel until the tax debt was cleared. Through Brouwer's, accidental or intended, mismanagement of his tax returns, a considerable debt had accumulated. Moreover, being no kin of the deceased, she had to pay the maximal death duty of 67%.

Brouwer's estate contained a large amount of documents, of a diverse nature. There were legal documents, scientific ones, and of course personal correspondence. Cor conscientiously sifted the whole mass of paper and put the scientific *Nachlass* into the hands of the Dutch Mathematical Society. The chairman, Professor Dijkman (an intuitionist and a former student of Brouwer), collected the material and stored it at the Technical University of Delft. Freudenthal and Heyting had access to the material, which they used for their edition of the Collected Works.[74] Before the transfer of the material, Alexandrov visited Cor Jongejan, when he was in Holland to give a lecture at the occasion of the opening of Freudenthal's new mathematical institute in Utrecht (1967). He asked for, and obtained, the remaining documents that related to Urysohn's and his own connections with Brouwer. As an editor of Urysohn's collected works he had an understandable interest in the documents.[75] The material was not returned, but fortunately it was preserved in Alexandrov's own archive, and Professor Shiryaev made it kindly available for the present biography. Alexandrov's talk in Utrecht was an interesting historical survey of topology in Holland, in particular in Brouwer's school, it has been published in the *Nieuw Archief voor Wiskunde*.[76] Brouwer's private library was sold to a well-known Amsterdam bookseller-antiquarian. It would have been a valuable source of historical and biographical information, as many books and monographs carried Brouwer's notes and comments in the margin. Alas, the sale of the books could not be traced in the files of the firm.[77]

Brouwer had prudently protected his correspondence with Lize from curious eyes; he had instructed the notary to burn the whole bundle. In fact, much more correspondence was destroyed at various occasions. For example, the correspondence of Tine Vermeij was turned over to Brouwer by her niece, mrs. Versteegh-Vermeij, and subsequently destroyed by Brouwer. Cor Jongejan sifted the correspondence and notes of Brouwer, and the more intimate items were burned. After the death of Brouwer's brother Aldert, the son Jan made a huge bonfire of his papers

[74] In 1976 Dijkman handed the material over to the Brouwer archive in Utrecht.

[75] [Urysohn1951].

[76] [Alexandrov 1969].

[77] It seems that the library was sold in its entirety to some university library in Canada or the United States.

Alexandrov and Freudenthal at the International Congress in Amsterdam 1954. (Freudenthal Archive).

and letters, remarking that the world would better be spared the uncountable fights and conflicts for which his father had a reputation. In the process many letters of Bertus were lost too.

Brouwer's brother Aldert almost immediately laid a claim on Cor. He was in poor health, and he avoided spending his money if he could get someone else to pay for him. Together with one of her friends, mrs. Volmer, Cor went three times a week to Aldert's house with homemade vegetarian meals. She drove him around in her little car; accompanied by Mrs. Volmer, she drove him, for example, several times to the Spa in Bad Pyrmont. Cor did not survive Brouwer for long; she died two years after Brouwer, leaving the estate to a son of Aldert, the teacher Aldie, stipulating that Aldert had the usefruct during his life. This transfer of the estate once more was taxed at the maximum rate.

Cor was a generous person, who did not begrudge others a possible intimate friendship with Brouwer. Where female 'admirers' were concerned, she philosophically shrugged her shoulders, 'when I was young I did the same thing', she said. Her relationship with Emily Haspels was friendly and uncomplicated. After Brouwer's death she made her a present of Brouwer's own copy of '*Life, Art and Mysticism*'; a copy with Brouwer's notes in the margin.[78]

[78]The copy came after her death into the hands of a nephew, and after his death it could not be traced.

The Brouwer-de Holl pharmacy was successfully run by the new owner, Hanny van Lakwijk. But when in 1995 the time had come for her to retire, the pharmacy fell victim to the reorganization of the system of pharmacies in Amsterdam. The pharmacy was taken over and subsequently closed down, so there is nothing left that reminds of the joint enterprise of Lize and Bertus.

After Brouwer's retirement, intuitionistic mathematics was taught by his student Heyting. It is hard to think of a greater contrast between teacher and student; Brouwer had the flamboyance of the artist-scholar, the man of the world, whereas Heyting was the cautious, precise scholar, who avoided the theatrical gestures and bold formulations, who felt safety in symbols. Brouwer felt no scruples about being right and telling people so, Heyting on the other hand was a man of compromise, who recognised other foundational views as legitimate possibilities, even though he did not share them. The only condition for him was exactness. For him the *Grundlagenstreit* was a historical accident, a blatant failure of common sense and of communication. As Heyting wrote to Bernays after Brouwer's death, 'Brouwer's expectation that intuitionistic mathematics would take the place of classical mathematics will remain an illusion (fortunately), but I am pleased to say that intuitionism nowadays gets more recognition and acceptance, then was the case only a few years ago.'[79]

Heyting was successful in making intuitionism respectable, he wrote and lectured in a style that was better adapted to the taste and customs of the newer generations. He had little or no affinity with Brouwer's more extravagant views and aims, a work like *Life, Art and Mysticism* was viewed by him as something that might jeopardise the recognition of intuitionistic mathematics, and its content was alien to him. Indeed the spirit of the times had changed, the fighting mood had become a thing of the past. A man like Paul Lorenzen, who did not hide his views on right and wrong in science, strongly disapproved of Heyting's lack of fighting spirit, but he was one of the last of the foundational scholars with a moral conviction where science was concerned.

Under Heyting's supervision a steady supply of dissertations (almost all on intuitionistic topics) appeared. These were basically contributions to the programme of rebuilding mathematics on intuitionistic principles. The foundational research into intuitionistic mathematics and logic was at the same time being picked up in the United States. The two leading scholars in this area were doubtlessly Stephen Kleene and George Kreisel. They, and Myhill and Kripke, gave a powerful boost to the study of systems of intuitionistic analysis. This short look at Brouwer's scientific legacy would not be complete without mentioning the man who took Brouwer's pragmatic programme really serious: Erret Bishop. Bishop launched a rebuilding of mathematics on a constructive basis, this time using all the experience gained in the past century, without being hampered by philosophical hangups, likes or dislikes. Bishop, withhis considerable experience in modern analysis, set the standard

[79]Heyting to Bernays 25.V.1968. Indeed, at the Buffalo congress of 1968, Intuitionism and Proof Theory were given equal time in the programme, cf. [Myhill. J. 1970].

for a new practice of constructive mathematics; his book *Foundations of Constructive Analysis* became the starting point for a wealth of interesting and successful research.[80]

Those who had played a role in Brouwer's life, were getting on in years; Van der Corput and Schouten, the two bugbears of Brouwer's later years, had left the stage soon after the retirement of Brouwer. Van der Corput had spent only four years in Amsterdam before he accepted a visitor's chair in Berkeley. When he returned two years later to Amsterdam, he had lost his enthusiasm for the local activities. He soon returned to California, where he spent another twelve years.[81]

Schouten had retired in 1953; he took an active part in the management of Dutch mathematics, in particular he remained on the board of the Mathematical Centre until 1968. He died in 1971 in Epe, a place in the middle of the Netherlands, where he had settled in 1943.[82]

The younger generation, Beth, Freudenthal and Heyting, took over the foundations of mathematics. Beth, a man with a precarious health, had died in 1964, leaving a rich inheritance of logical ideas and techniques. Freudenthal, the man who had suffered most under Brouwer's rejection, had acknowledged without a grudge the greatness of the man who had been his inspiration at the turning point of his career.

Van der Waerden had left Amsterdam for Zürich. To his many interests in mathematics, he had in the meantime added the history of mathematics; this earned him the everlasting enmity and fierce attacks of Bruins, who used every opportunity to spell out the low opinion he had of Van der Waerden's historical expertise. Bruins' wrath was in particular reserved for Freudenthal. It may not be uncommon to see this inversion of guilt feeling; but when one realises that Bruins hated Freudenthal because he himself failed to do justice to the man whose position he had appropriated, one cannot suppress a feeling of astonishment. Indeed, Bruins went so far as to send Freudenthal a copy of his farewell lecture, with the message 'this is the first attack' on the cover. Bruins never left the University of Amsterdam; over the years he had become a specialist in Babylonian mathematics. In the fifties he had been a visiting professor in Baghdad. There he had the opportunity to study historic material and places. Returning to Amsterdam he resumed his lecturer's position. In the following years Bruins' career was plagued by a long string of conflicts with his colleagues and the university. Eventually he was made extraordinary professor in the history of mathematics. Looking back at his career, one can only feel sad that such a gifted man generated so many conflicts—making enemies came natural to him. It is idle speculation to ask what would have become of him if there had not been a war. His role in the matter of Freudenthal's position in Amsterdam was never forgotten nor forgiven. The fact that Brouwer used him can only

[80] See [Bishop, E. 1967], [Bishop, E. 1985].

[81] See [Korevaar 1975], [Duparc, H.J.A. and J. Korevaar 1982].

[82] [Nijenhuis 1972], [Struik 1971].

be seen as a partial excuse. In all likelihood he would have been a difficult man at all places and at all times.

Freudenthal, the Cinderella of the Amsterdam institute, was after all the most successful of Brouwer's entourage. He created a strong mathematical department in Utrecht where he was for years the unchallenged master of. With a firm hand he established a powerful tradition in the subjects of his expertise: geometry, algebra, Lie groups, topology; he left at his retirement a department behind that had earned a prominent place in the international community. He never felt tempted to take his revenge on Brouwer for the dubious treatment during and after the war. On the contrary, he honoured him in word and writing. Freudenthal's obituaries fully did justice to the greatness of the man who revolutionised topology, and mathematics as a whole. Freudenthal's edition of Brouwer's topological work stands out as a precise analysis of Brouwer's ideas and as an homage to the man who surpassed all his contemporaries in the extraordinary force of his geometrical intuition. When Freudenthal retired he had become a major force in the didactics of mathematics.

Louise's history is easily told. Financial problems were no longer her worry. She was already, partially under her mother's exhortations, an example of piety, and when her mother and Brouwer had gone, she became even more deeply religious. In due time she choose to follow the anti-pope. In her house she had a rich variety of statues and icons. In her simple apartment she celebrated the mass. She gradually developed a measure of religious mania. Her mother, Cor Jongejan, and her stepfather peopled her dreams. Brouwer would rudely keep her awake at night. Only after Brouwer's centenary, when she got to read a brief story of his life,[83] she suddenly realised that the man was not crazy after all. The nightly disturbances stopped immediately, 'Brouwer feels that now justice is done' she remarked. She was bedridden for many years, but that did not stop her from ruling the world her way. She died at the ripe old age of eighty seven, a woman with a strong will, who had not found a world adjusted to her views.

Of Brouwer's property, the villa 'de Pimpernel' still exists, the hut and the other small houses and buildings have been replaced by run-of-the-mill estate housings for the affluent. Fortunately the town of Blaricum has generously taken over the care for the graves of Bertus and Lize; in 2003 the *Koninlijk Wiskundig Genootschap*,[84] the University of Amsterdam, and a generous private sponsor placed a tasteful commemoration glass monument at the grave.

[83] [Dalen, D. van 1981].

[84] The Queen had in 2002 granted the predicate 'Royal' to the Dutch Mathematical Society.

20

APPENDIX

20.1 Dissertations under supervision of Brouwer

Bernardus Petrus Haalmeijer, (28.11.1917) *Bijdragen tot de theorie der elementairoppervlakken*.
Maurits Joost Belinfante, (12.12.1923) *Over Oneindige Reeksen*.
Barend de Loor, (23.2.1925) *Die Hoofstelling van die Algebra van Intuïtionistiese Standpunt*.
Arend Heyting, (25.5.1925) *Intuïtionistische Axiomatiek der Projektieve Meetkunde*.
Wilfrid Wilson, (10.2.1928) *Afbeeldingen van Ruimten*.
Frans Loonstra, (11.7.1941) *Analytische Untersuchungen über bewertete Körper*.
Johanna Adriana Geldof, (11.7.1951) *Over de arithmetisering van axiomatische meetkunden*.

20.2 Correspondence and Archives

Below we have listed the correspondence from both volumes. The documents are in chronological order. Each date is followed by the archive it is in. There is a variety of archives; some of them are open to the public, some are just private collections. The abbreviations used are:

Name	Description	Place
Alexandrov	private collection	Moscow
Brouwer	RANH	Haarlem
Freudenthal	RANH	Haarlem
Boerhaave	Museum Boerhaave	Leiden
Göttingen	Niedersächsische Staats- und Universitätsbibliothek Göttingen	Göttingen
NLM	Nederlands letterkundig museum	Den Haag
CWI	Centrum voor Wiskunde en Informatica	Amsterdam
GAA	Gemeente Archief	Amsterdam
UB-UVA	Universiteits Bibliotheek,	Amsterdam
UB-Leiden	Universiteits bibliotheek	Leiden
ETH	Eidgenössische Technische Hochschule	Zürich

(Continues)

Continued.

Name	Description	Place
Einstein	Einstein Archive, Hebrew University	Jerusalem
Fraenkel	Hebrew University	Jerusalem
Courant	NY University Archives (Bobst Library)	New York
Menger	Menger Archive, Special Collections Library Duke University	Durham, NC
van Heemert	private collection	Leersum
Springer	Springer Verlag	Heidelberg
Arch-UU	Utrecht University	Utrecht
Veblen	Veblen collection, Library of Congress	Washington
Gödel	Gödel archive, Institute for Advanced Study	Princeton
RANH	Rijksarchief Noord-Holland	Haarlem

1898
08.14 - NLM
09.20 - NLM
1900
01.12 - Brouwer
1901
05.04 - NLM
07.05 - NLM
12.05 - NLM
1902
06.11 - NLM
06.12 - NLM
07.29 - NLM
08.23 - NLM
09.04 - NLM
12.29 - NLM
1903
01.07 - NLM
05.02 - NLM
05.23 - NLM
07.21 - NLM
08.09 - NLM
08.26 - NLM
11.15 - NLM
1904
01.18 - NLM
03.05 - UB-Leiden
03.20 - Brouwer
05.14 - UB-UVA
07.04 - NLM
10.08 - NLM
1905
04.07 - NLM
05.13 - Brouwer
05.16 - NLM
1906
09.07 - UB-UVA
01.11 - GA-Leeuwarden
09.07 - NLM
10.18 - UB-UVA
11.05 - UB-UVA
11.06 - UB-UVA
11.11 - UB-UVA
11.13 - UB-UVA
1907
01.11 - UB-UVA
01.18 - UB-UVA
01.23 - UB-UVA
02.13 - Brouwer
02.16 - UB-UVA
07.08 - NLM
08.01 - UB-UVA
08.06 - NLM
1908
01.03 - Brouwer
05.00 - UB-UVA
08.11 - NLM
09.04 - NLM
10.11 - NLM
11.08 - Brouwer
1909
02.16 - Brouwer
03.01 - NLM
05.12 - NLM
05.14 - Göttingen
05.22 - Göttingen
05.22a - UB-UVA
05.27 - Brouwer
06.02 - NLM
06.08 - Brouwer
06.17 - Brouwer
06.18 - UB-UVA
06.22 - UB-UVA
06.24 - Göttingen
07.07 - NLM
07.26 - Göttingen
08.14 - Brouwer
09.12 - Brouwer
09.22 - UB-UVA
10.15 - Göttingen
10.28 - Göttingen
11.09 - NLM
12.03 - NLM
12.13 - Brouwer
12.19 - Brouwer
12.24 - UB-UVA
24.12 - Brouwer
1910
01.01 - Brouwer
01.04 - Brouwer
06.11 - NLM
07.29 - Brouwer
10.06 - UB-UVA
10.07 - UB-UVA
10.27 - Brouwer
11.16 - UB-UVA
24.07 - Brouwer
1911
01.01 - Brouwer
02.06 - UB-UVA
06.16 - Brouwer
06.19 - Brouwer
08.26 - Brouwer
09.10 - UB-UVA
11.05 - Brouwer
11.07 - NLM
12.05 - Brouwer
1912
01.00 - UB-UVA
01.21 - Brouwer
01.28 - Brouwer
02.02 - Brouwer
02.12 - Brouwer
02.24 - Göttingen
02.29 - Göttingen
03.02 - Brouwer
03.06 - Brouwer
03.06a - Brouwer
03.09 - Brouwer
03.12 - Brouwer
03.24 - UB-UVA
03.30 - Göttingen
05.05 - Hilbert
05.06 - Göttingen
05.31 - Göttingen
06.21 - Göttingen
07.01 - Brouwer
09.03 - Brouwer
09.30 - UB-UVA

12.02 - Brouwer
21.11 - Brouwer

1913

04.16 - Göttingen
05.04 - Brouwer
06.16 - Göttingen
07.08 - Göttingen
09.11 - Brouwer
12.31 - UB-UVA

1914

05.25 - Göttingen
06.04 - Brouwer
07.10 - Brouwer
07.13 - Brouwer
08.15 - NLM

1915

03.28 - Brouwer
06.10 - Brouwer
06.11 - Brouwer
06.19 - RANH
09.18 - Brouwer
10.12 - Brouwer
10.15 - UB-UVA
10.17 - Brouwer
10.18 - Brouwer
10.19 - UB-UVA
11.04 - Brouwer
11.19 - Brouwer
11.29 - UB-UVA
12.01 - UB-UVA

1916

02.25 - NLM
08.27 - Brouwer
12.07 - Brouwer

1917

01.09 - GAA
03.17 - Brouwer
09.27 - NLM
12.17 - RANH

1918

00.00 - Brouwer
01.09 - Brouwer
01.16 - RANH
02.04 - Brouwer
02.16 - RANH
04.19 - ETH
05.02 - RANH
07.12 - RANH
09.08 - RANH
09.08 - NLM
10.21 - Brouwer
11.25 - Brouwer

1919

03.18 - Wolters
03.24 - Brouwer
07.09 - Brouwer
08.28 - Göttingen
09.08 - Brouwer
09.12 - Brouwer
09.19 - Göttingen
10.14 - Brouwer
10.18 - Brouwer
10.21 - Brouwer
12.18 - Brouwer
12.20 - Brouwer
12.29 - Brouwer

1920

01.15 - UB-UVA
01.25 - Brouwer
01.28 - Brouwer
02.04 - GAA
02.12 - Brouwer
02.22 - Brouwer
03.25 - Brouwer
04.29 - Brouwer
05.06 - Brouwer
05.16 - ETH
09.07 - ETH
09.27 - ETH
09.28 - Göttingen
10.04 - Brouwer
10.06 - ETH
10.17 - Brouwer
10.20 - Brouwer
10.27 - Brouwer
10.29 - Brouwer

1921

01.11 - ETH
01.17 - Brouwer
05.14 - Brouwer
09.20 - Brouwer
09.23 - Brouwer
11.13 - Brouwer
11.22 - Brouwer
16.02 - Brouwer

1922

02.18 - Brouwer
04.26 - Brouwer
09.27 - Brouwer
11.24 - UB-UVA
15.02 - Menger

1923

02.12 - Brouwer
04.18 - Brouwer
06.28 - Brouwer
09.01 - Brouwer
12.15 - Fraenkel
16.04 - ETH
24.10 - Brouwer
27.12 - Alexandrov
29.09 - Alexandrov

1924

00.00 - Brouwer
01.02 - Fraenkel
01.22 - Alexandrov
02.12 - Brouwer
02.19 - Alexandrov
03.20 - Alexandrov
03.22 - Menger
03.25 - Alexandrov
03.28 - Alexandrov
04.09 - Alexandrov
06.06 - Menger
06.13 - Alexandrov
06.14 - Göttingen
06.14a - Brouwer
06.24 - Alexandrov
06.26 - RANH
06.27 - Alexandrov
07.29 - Alexandrov
08.01 - Alexandrov
08.04 - Alexandrov
08.31 - Alexandrov
09.11 - Alexandrov
10.13 - Alexandrov
10.20 - Brouwer
10.20a - Alexandrov
10.20 - Alexandrov
10.24 - Alexandrov
11.01 - Einstein
11.04 - Alexandrov
11.04a - Alexandrov
11.06 - Einstein
11.13 - Brouwer
11.17 - Menger
11.24 - Menger
11.25 - Alexandrov
12.03 - Alexandrov
12.15 - Einstein
12.16 - Einstein
12.21 - Alexandrov
12.24 - Alexandrov
26.12 - UB-UVA

1925

01.12 - Einstein
01.12a - Einstein
01.13 - Einstein
01.15 - Alexandrov
01.20 - Einstein
01.20 - Einstein
01.23 - Einstein
01.26 - Sommerfeld
02.17 - Alexandrov
02.25 - Göttingen
03.15 - Alexandrov
05.04 - Brouwer
05.11 - Brouwer
05.14 - Brouwer
05.24 - Menger
07.03 - Brouwer
09.10 - Hilbert
10.25 - Hilbert
11.07 - Alexandrov
11.11 - Alexandrov
11.14 - Brouwer
11.18 - Hilbert
11.21 - Brouwer
12.21 - ETH

1926

02.21 - Brouwer
04.07 - Menger
04.10 - Brouwer
04.10a - Brouwer
08.12 - ETH
08.19 - Brouwer
08.22 - Brouwer
12.21 - Fraenkel
12.23 - ETH

1927

01.10 - GAA
01.16 - Brouwer
01.26 - Brouwer
01.31 - Fréchet
01.9 - Brouwer
02.03 - Alexandrov
02.15 - ETH
03.03 - Brouwer
03.05 - Fraenkel
03.08 - ETH
03.15 - Brouwer
03.26 - Brouwer
04.08 - Brouwer
04.10 - ETH
07.03 - Brouwer
10.20 - Fraenkel
12.01 - Fraenkel
12.22 - Brouwer
12.22a - Brouwer
18.01 - Fraenkel
28.01 - Fraenkel

1928

01.17 - Brouwer
01.23 - Brouwer
01.24 - Brouwer
01.24a - Brouwer
02.16 - ETH
03.24 - Brouwer
04.30 - Brouwer
05.06 - Bieberbach
06.08 - Brouwer
06.18 - UBGtt
06.29 - Hilbert
07.02 - Brouwer
07.03 - Brouwer
07.07 - Brouwer

07.10 - Bieberbach
07.15 - Brouwer
07.17 - RANH
08.02 - Springer
09.06 - Brouwer
09.09 - Mises
10.03 - Springer
10.04 - Springer
10.08 - Boerhaave
10.10 - Menger
10.15 - Einstein
10.19 - Einstein
10.19a - Einstein
10.20 - Einstein
10.23 - Einstein
10.25 - Brouwer
10.27 - Brouwer
11.02 - Brouwer
11.02a - Brouwer
11.02b - Brouwer
11.03 - Brouwer
11.04 - Brouwer
11.05 - Brouwer
11.05a - Brouwer
11.06 - Brouwer
11.06a - Brouwer
11.10 - Brouwer
11.13 - Brouwer
11.14 - Brouwer
11.14a - Brouwer
11.16a - Einstein
11.17 - Brouwer
11.18 - Brouwer
11.19 - Fraenkel
11.24 - Brouwer
11.27 - Brouwer
11.28 - Brouwer
11.30 - Brouwer
11.30a - Brouwer
12.00 - Brouwer
12.06 - ETH
12.12 - Einstein
12.15 - Brouwer
12.16 - Brouwer
12.16a - Einstein
12.17 - Brouwer
12.19 - Brouwer
12.19a - Einstein
12.23 - Brouwer
12.23a - Brouwer
12.31 - Brouwer

1929

01.14 - ETH
01.23 - Brouwer
02.24 - Brouwer
04.09 - Brouwer
07.11 - Brouwer
07.15 - Brouwer
07.30 - Brouwer
08.04 - Brouwer
08.09 - Brouwer
08.21 - Brouwer
08.27 - Brouwer
08.27a - Brouwer
08.29 - Brouwer
09.07 - Brouwer
09.08 - Brouwer
09.8 - RANH
10.10 - Brouwer
10.12 - Menger
10.15 - Brouwer
10.22 - Brouwer
10.26 - Brouwer
11.01 - Brouwer
11.14 - Brouwer
11.26 - Brouwer
11.26a - Brouwer
12.14 - Brouwer
12.14a - Brouwer

1930

00.00 - Boerhaave
01.04 - Brouwer
01.10 - Brouwer
01.11 - Brouwer
01.12 - Brouwer
01.21 - Brouwer
02.06 - Boerhaave
03.17 - Menger
04.19 - Brouwer
06.13 - Brouwer
10.11 - Veblen papers
10.16 - Einstein
10.28 - Brouwer
11.15 - Einstein
12.10 - Gödel
12.22 - ETH

1931

03.06 - Brouwer
09.02 - ETH

1932

.0..0 - Brouwer
01.22 - ETH
04.02 - Veblen
04.22 - CWI
10.20 - Alexandrov

1933

11.11 - Göttingen
06.17 - Brouwer
11.23 - Brouwer

1934

06.19 - Brouwer
06.21 - Brouwer
07.16 - RANH
10.12 - Brouwer
10.20 - Library of Congress

1935

01.08 - Brouwer
01.15 - Brouwer
01.19 - Univ.Arch. Freiburg
03.20 - Univ.Arch. Freiburg
05.17 - Brouwer
08.03 - Brouwer
08.03 - Brouwer
08.20 - Freudenthal
10.05 - CWI

1936

08.12 - Brouwer
08.17 - Brouwer

1939

07.08 - Freudenthal
10.04 - Freudenthal
10.04a - Brouwer
10.25 - Freudenthal
10.27 - Freudenthal
12.21 - Freudenthal
12.29 - Library of Congress

1940

01.14 - Brouwer
03.02 - Freudenthal
03.18 - Brouwer
04.30 - Freudenthal
06.06 - Freudenthal
06.15 - Brouwer
06.26 - Brouwer
07.27 - Freudenthal
08.09 - Freudenthal
10.17 - Brouwer
11.10 - ETH
11.30 - Freudenthal
12.27 - Freudenthal

1941

04.28 - Freudenthal@
07.26 - Freudenthal
12.18 - Brouwer

1942

05.24 - Freudenthal
05.26 - Brouwer
05.29 - Freudenthal
06.26 - van Heemert
06.29 - Brouwer
06.30 - Brouwer
08.03 - van Heemert
08.20 - Brouwer
09.23 - Arch-UU
24.05 - Brouwer

1943

.0..2 - Brouwer
07.27 - Arch-UU

1945

12.03 - Brouwer
01.06 - Freudenthal
06.11 - Freudenthal
06.20 - Freudenthal
06.20a - Brouwer
06.22 - Freudenthal
07.01 - Freudenthal
07.09 - Freudenthal
07.17 - Brouwer
08.18 - GAA
08.20 - Brouwer
08.23 - Freudenthal
08.25 - Freudenthal
08.30 - Brouwer
08.30a - Brouwer
09.05 - Courant
09.07 - Freudenthal
09.12 - Brouwer
09.17 - Freudenthal
09.17a - Freudenthal
09.21 - Freudenthal
10.09 - ETH
10.19 - Fraenkel
10.22 - GAA
11.13 - GAA
11.15 - Fraenkel
12.09 - Brouwer
12.11 - GAA
12.13 - Arch-UU
15.10 - GAA

1946

00.05 - Brouwer
01.07 - Brouwer
01.10 - Brouwer
01.12 - Brouwer
01.23 - CWI
05.06 - Freudenthal
05.06a - Brouwer
05.08 - Freudenthal
05.15 - Freudenthal
09.27 - CWI
10.08 - CWI
10.14 - CWI
10.31 - CWI
11.02 - CWI
12.14 - CWI

1946

12.21 - CWI
01.25 - Brouwer
27.09 - Brouwer

1948

02.27 - GAA
02.27a - Brouwer
03.13 - GAA

04.02 - Brouwer
04.09 - CWI
04.14 - CWI
04.15 - Brouwer
11.01 - Freudenthal
12.01 - Freudenthal
1949
.0..1 - Brouwer
11.08 - ETH
01.26 - CWI
01.29 - CWI
03.10 - Brouwer
04.19 - Brouwer
05.03 - ETH
08.11 - ETH
10.28 - Brouwer
11.08 - Brouwer
11.27 - Brouwer
12.11 - Brouwer
31.01 - CWI
1950
01.02 - CWI
01.03 - Brouwer
01.20 - CWI
01.27 - Brouwer
01.28 - ETH
01.31 - CWI
02.03 - Brouwer
02.05 - Brouwer
02.07 - Brouwer
02.12 - Brouwer
02.14 - ETH
02.28 - Brouwer
05.09 - Veblen papers
12.22 - Brouwer
1951
02.09 - Brouwer
02.10 - Brouwer
02.23 - Heyting
03.07 - Brouwer
03.14 - Brouwer
03.14a - Brouwer
03.15 - Brouwer
03.25 - Brouwer
03.28 - Brouwer
05.01 - Brouwer
05.07 - CWI
05.16 - Brouwer
07.04 - Brouwer
09.25 - Brouwer
10.27 - Heyting
1952
06.22 - De Bruijn
1953
01.16 - Brouwer
05.27 - Brouwer
07.27 - Brouwer
08.31 - Brouwer
09.30 - Brouwer
10.31 - Gödel
1955
01.04 - Brouwer
1956
01.15 - Brouwer
02.17 - Brouwer
1957
02.11 - Brouwer
1959
07.04 - Brouwer
08.07 - Brouwer
09.05 - Brouwer
09.07 - Brouwer
1963
02.11 - Brouwer
08.12 - Brouwer
1964
06.25 - Brouwer
1965
08.30 - Brouwer
09.06 - Brouwer
09.17 - Brouwer
1966
07.31 - Brouwer
1967
01.19 - Gödel
1968
05.25 - ETH
1972
04.21 - Gödel
1976
11.27 - van Dalen
1977
02.03 - van Dalen
1990
10.10 - van Dalen
12.10 - van Dalen
1991
02.11 - van Dalen
1992
03.24 - van Dalen
2000
14.03 - van Dalen
2004
01.39 - Dyckhoff

20.3 Chronology

Birth L.E.J. Brouwer	1881	
Entrance High school	1890	
Entrance University of Amsterdam	1897	
	1900	Schoenflies Bericht I
	1903	Mannoury privaat docent
Doctoral exam (masters), papers on four-dimensional rotations. Marriage with Lize de Holl; cottage in Blaricum.	1904	
Art, Life, and Mysticism, aquisition pharmacy	1905	
Ph.D. (advisor D.J. Korteweg)	1907	
Rejection Princ. Excl. Middle.	1908	Schoenflies Bericht II. Intern. Congr. Rome
Cantor–Schoenflies topology; fixed points on spheres, vector distributions on spheres. Indecomposable continua, Lie groups. *Privaat docent*, 'The essence of geometry'.	1909	

The new topology. Invariance of dimension, and of domain. Mapping degree, homotopy, *n*-dimensional Jordan thm., translation thm., uniformization, etc.	1911	
Professor Amsterdam, member Royal Ac. Sc. Amsterdam, 'Intuitionism and Formalism'.	1912	Karlsruhe Conf.
Dimension definition and correctness proof	1913	Schoenflies revised Bericht
Ed. Math. Ann.	1914	World War I. Hausdorff "Grundzüge der Mengenlehre".
Chair in Leiden offered.	1915	Cor Jongejan in Blaricum
Introduction choice sequences	1916	
Member Königliche Gesellschaft der Wissenschaften zu Göttingen	1917	
First Begründungs paper	1918	Hermann Weyl Das Kontinuum, Hilbert Axiomatisches Denken. Mannoury full professor in Amsterdam
Meets Weyl in the Engadin Weyl joins intuitionism. Chairs in Göttingen and in Berlin offered.	1919	Conseil intern. de recherch, Union intern. de math. Significa.
On decimal expansions of reals, Denjoy conflict.	1920	Nauheim Conf.
	1921	Weyl New Crisis paper, beginning Grundlagenstreit.
	1922	Hilbert Neubegründung. (Hamburg lecture)
Brouwerian counterexamples	1923	Urysohn & Alexandrov in Göttingen, Marburg conference
Bar induction, fan theorem., continuity thm. Member Leopoldinische-Carolingische Deutsche Akademie (Halle). Lecture intuitionism in Göttingen	1924	Urysohn & Alexandrov in Laren, death Urysohn death Adama van Scheltema
Amsterdam topological school Alexandrov, Menger, Newman Vietoris, Wilson. Visit Emmy Noether	1925	Hilberts Münster lecture - on Infinity, Hilbert pernicious anaemia.
Brouwers talk in Göttingen, reconciliation with Hilbert	1926	Hilberts second Hamburg lecture. Hurewicz assistant in Amsterdam
Brouwers Berlin lectures. Death Brouwers mother.	1927	

Brouwers Vienna lectures, Menger conflict, Hilbert dismisses Brouwer from *Math. Ann.*, War of the frogs and mice.	1928	Menger publishes "Dimensionstheorie" Bologna conflict. Hilberts Bologna lecture.
Corr. Member Prussian Ac. Sc.	1929	
	1930	Königsberg conf., Gödels incompleteness thm., Heytings formalization of intuitionistic logic, Freudenthal comes to Amsterdam..
Brouwer Knight in the Order of the Lion of the Netherlands	1932	
	1934	Hilbert-Bernays I
Hon. Doctorate in Oslo	1936	Hurewicz emigrates to USA Alexandroff-Hopf
	1937	Heyting lecturer, Freudenthal conservator.
Brouwers Triangulation lecture	1939	Hilbert-Bernays II , Blumenthal forced to leave Germany
	1940	Germany occupies the Netherlands, Freudenthal dismissed. Bruins appointed.
	1941	Death Korteweg
	1943	Declaration of loyalty for students, death Hilbert.
	1945	End of World War II. Brouwer, Bruins, Heyting suspended. Van der Waerden back in Holland.
Brouwer resumes publications. Introduction "creating subject", Course Intuitionism in Cambridge. Death of Brouwer's father.	1946	Freudenthal appointed in Utrecht
Consciousness, Phil., and Math.	1948	
Compositio Mathematica conflict	1949	
Foreign fellow Royal Soc. London	1950	
Shearman lecture, London	1951	
Lecture tour South Africa	1952	
Lecture tour in USA	1953	
Honorary Doctorate Cambridge; Honorary Fellow of Royal Society of Edinburgh	1955	Death Weyl
	1956	Death Mannoury
Death Lize Brouwer-de Holl	1959	
Death Brouwer	1966	

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INDEX