C is the MnO_2 equivalent to the available oxygen.

D is the MnO found by weighing as Mn_3O_4.

E is the Fe_2O_3 found by titration with $SnCl_2$.

F is the alumina found by subtracting the Fe_2O_3 found in E from the weight of the precipitate with acetate of soda.

G is the water expelled on ignition; it is obtained by deducting two-thirds of the oxygen found in B from the loss of weight by ignition.

It will be seen from the results given in the above table that the nodules from different localities vary greatly in composition, though in the same locality they have similar composition, irrespectively of the nature of the nodules. The insoluble residue contains, besides silica and clay, sand of the same mineral nature as is found in the bottom at the same locality. The manganese is present wholly as MnO_2, and the iron as Fe_2O_3. In No. 6 there is 3 per cent. of cobalt; this metal, along with copper and a little nickel, is present in all of them. Zinc was not found in any of the above specimens.

2. Note on the Measure of Beknottedness. By Prof. Tait.

In drawing the various closed curves which have a given number of double points, I found it desirable to have some simple mode of ascertaining whether a particular form was a new one, or only a deformation of one of those I had already obtained. Of course the *schemes* (as described in my former paper) contain the desired information, but it may sometimes be difficult to obtain in this way; for, when the number of intersections is large, we may have to change the crossing which is taken as the initial one several times before we hit upon the same notation for like crossings (if such exist) in the two schemes compared. And the methods of deformation already given often present their results in forms so distorted that it is not easy at once to recognise their identity with other drawings of the very same curves.

The method of treatment described in my paper, which depends upon the study of the *plait*, supplies (by the + and − signs *over* the various crossings) exactly the sort of information we require, though it may leave ambiguities. But some simple mode of applying it is requisite.

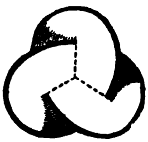
I first tried a modification of the process (formerly described) of going round the curve and pitching a coin into each field or cell as it is reached. To make the required distinction between crossing *over* and crossing *under*, we may suppose the two coins to be of different kinds,—silver and copper for instance. Let the rule be:—silver to the right when crossing over, to the left when crossing under. Then, however the path be arranged, of the four angles at each crossing, one will have no coins, the vertical or opposite corner will have *two* silver or *two* copper coins, the others *one* copper or *one* silver coin each.

It is easily seen that a reversal of the direction of going round leaves the single coins as they were, but shifts the pair of coins into the angle formerly vacant; also that in the deformed figures the circumstances are exactly the same as in the original. Hence we may divide the crossings into silver and copper ones, according as two silver or two copper coins come together. And the excess of the silver over the copper crossings, or *vice versâ*, furnishes an exceedingly simple and readily applied test (not however, as will soon be seen, in itself absolutely conclusive of identity, though absolutely conclusive against it), which is of great value in arranging in family groups (those of each family having the same number of silver crossings), the various knots having a given number of intersections.

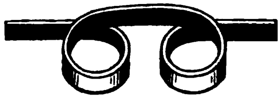
I soon saw that this process, so limited, was intimately connected with that required for the estimation of the work necessary to carry a magnetic pole along the curve, the curve being supposed to be traversed by an electric current, and it occurred to me that we might possibly obtain a definite measurement of beknottedness in terms of such a physical quantity: as it obviously must be always the same for the same knot, and must vanish when there is no beknottedness. The measure may be made more complete by recording the numbers of non-nugatory silver and copper crossings separately, with the number to be deducted as due merely to the *coiling* of the figure. I shall recur to this point later.

When unit current circulates in a circuit, the work required to carry unit pole once 'round any closed curve once linked with the circuit is $\pm 4\pi$. Instead of the current we may substitute a uniformly and normally magnetised surface bounded by the circuit. The potential energy of the pole in any position is always measured by the spherical opening subtended by the circuit; but its sign depends upon whether the north or south polar side is turned to the pole. Hence there is no potential energy when the pole is situated in the plane of the circuit but external to it, and the value is $\pm 2\pi$ when the pole just reaches the plane of the circuit internally. Gauss gave from these results the value of a remarkable double integral extending over each of any two closed curves linked together in space. Clerk-Maxwell (*Electricity*, § 422) has shown that this integral may vanish even for a complex linking of the two circuits; and a similar difficulty is met with in the single circuits with which we are now dealing, so that a special set of rules must be made for determining the beknottedness in terms of the silver and copper junctions. But the difficulty just mentioned leads, as will be seen, to a very curious result.

To construct the magnetised surface which shall exert the same external action on a pole as a current in any given closed circuit does, we may either suppose a surface extending to infinity in one direction (say, for definiteness, upwards from the plane of the paper), and having the circuit for its edge; or we may form, as in the figure, a finite autotomic surface of one sheet, having the circuit for its edge. The only difficulty in estimating the work lies in the definite statement of how the pole is to move along the curve itself. For, if its path screw round the curve, $\pm 4\pi$ must be added to the work for each such complete turn.

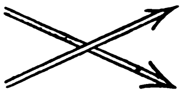


As an illustration, suppose we bend an india-rubber band coloured black on one side, as in the figure, so that the black is always the concave surface, we find on pulling it out straight that it has no twist. If both loops be made by *over*laying, when pulled out it becomes twisted through two whole turns. This is an instance of the kinematic principle that spiral springs act by torsion.



Perhaps the most simple definite condition is that which I first employed, viz., to make the pole move along the curve, keeping always in the osculating plane and on the convex side. But we have then to arrange beforehand what is to be done at a point of inflexion.

A practical rule, however, may easily be given from the consideration of the magnetised surface above mentioned. Go round the curve, marking an arrow head after each crossing to show the direction in which you passed it. Then a junction like the fol-

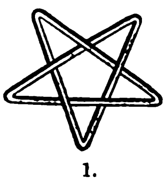


lowing gives + 2π at each time of crossing; or, if we use the infinite surface spoken of above, it gives + 4π for the upper branch, and nothing for the lower (which, on this supposition, does not pass through the magnetic sheet). Change the crossing from *over* to *under*, and these quantities change sign. The junction figured above would, in our first illustration, be a silver one. But a still simpler process is to go round putting a dot to the *right* after each crossing *over*, and *vice versâ*. Silver crossings have two dots in one angle; copper one in each of two vertical angles.

Now, in order that our rule may be such as to give no work where there is no beknottedness, we must make the required expression such as to vanish whenever all the intersections are nugatory. Those which are nugatory only in consequence of their signs are in pairs, silver and copper, and will take care of themselves, as we see by special examples like the following, in which the reversal of one of the directions simply reverses the

signs. Hence the only part to correct for is that depending on the number of whole turns, and the sketch of the india-rubber band above shows that the work at the vertex of each such partial closed circuit is simply not to be counted—*i.e.*, that the $\pm\ 4\pi$, which would be reckoned for each crossing by our rule, is to be considered as made up for by the corresponding screwing of the pole round the curve.

To illustrate the application of this process, let us consider again the two distinct forms with five non-nugatory intersections



1.

2.

(the first being a modified form of the "pentacle," the second, fig. 6

of my paper, which, for the comparison below, must be supposed to have all its signs changed) whose schemes are, respectively—

$$\begin{array}{cccccccccc|l} - & - & - & - & - & - & - & - & - & - & \\ A & D & B & E & C & A & D & B & E & C & A, \\ - & + & - & + & - & + & - & + & - & + & \end{array}$$

$$\begin{array}{cccccccccc|l} - & - & - & - & - & - & - & - & - & - & \\ A & D & B & E & C & A & D & C & E & B & A. \\ - & + & - & + & - & + & - & + & - & + & \end{array}$$

The lower signs refer to over or under, the upper to the electro-magnetic work, or to the silver-copper distinction. These two instances, in which both series of signs are absolutely identical, each with each, show at once that we cannot take the two sets of signs alone as fully descriptive of the knot.

To determine the electromagnetic work for any knot, we must divide the scheme into independent circuits, no one of which includes a less extensive one; and omit from the reckoning the work for the terminal of each such circuit, and for each of the intersections which is not included in any one of the separate circuits. The particular closed circuits chosen do not affect the final result, as is easily seen by thinking of the various deformations of each figure.

In the first of the two schemes above there is but one independent non-autotomic circuit, which may be taken as

$$A\ D\ B\ E\ C\ A.$$

In this all the intersections are included, so that the whole work is to be found by leaving out that for A only; *i.e.*, it is -16π.

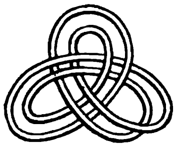
But in the second scheme we may take the two circuits

$$B\ A\ D\ B \text{ and } C\ A\ D\ C,$$

and E is not included in either. Hence we must leave out of count the work for B, C, and E; and thus the whole work is only -8π.

In fact, the figures show that to untie the first knot we must not only have the signs such that we can slip off B and E, but also C and D, *i.e.*, *two* signs must be changed; while the second loses all its beknottedness if A and D could be got rid of, that is if *one* sign only be changed. This is an instance in which the estimate by the electro-magnetic process exactly agrees with the result of simpler considerations. And it is probable after all that the true measure of beknottedness is the smallest number of signs in a scheme which must be altered in order that the wire may cease to be knotted.

It will be found that the alteration of five signs is sufficient to remove the knotting from the annexed figure, and the stages of



operation of the various modes of reduction show that this form can be regarded as made up of simpler knots intersecting one another on the same string. In such a case it is not easy to give a strict definition of the beknottedness in any other way than by defining it as the smallest number of changes of sign which will take off all the knotting. For the separate knots are virtually independent, and to change *all* the signs in any one of them does not in every case necessarily simplify the knot. Uncorrected the work is $-13 \times 4\pi$. Corrected it is $-10 \times 4\pi$, which agrees with the removal of the beknottedness by change of *five* signs only.

If the sign of the one unsymmetrical crossing be altered, four changes of sign will suffice; for the uncorrected work is $-11 \times 4\pi$; corrected it is $-8 \times 4\pi$, corresponding to four changes of sign.

The various modes alluded to in my paper of adjusting the (lower) signs so that there shall be no beknottedness, follow at once from these remarks. For we may make all the free letters in each circuit +, save those which we have taken, *in pairs*, in some previous circuit.

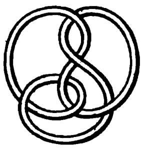
This test, though extremely useful as above explained in classifying knots with the same number of intersections, is not fully descriptive of a knot, being ambiguous whenever there is more than one class of knots with the same number of intersections and with the same excess or defect of silver as regards copper crossings. This consideration, which promises to clear up some obscure and difficult parts of the subject, has led me to some very curious results. The most important of these is when a knot, whatever be its number of intersections, has equal numbers of silver and of copper crossings, or when the uncorrected expression for the electro-magnetic work vanishes. Thinking of this in connection with the fact that a change from right-handed to left-handed in a knot simply changes silver to copper, or *vice versâ*, *i.e.*, reverses the sign of the electro-magnetic work, I was led to see that there is a class of knots which are *capable of being changed from right to left-handed, without change of form*, by the ordinary processes of deformation. Of course this implies that there is a mode of interchanging the letters, two and two, in the scheme, so that their order remains unaltered; or, what comes to the same thing, that we shall get exactly the same scheme (signs not included) by taking either of two different crossings as A, and lettering as usual from it in the same direction round the curve.

It will be readily found by trial that this can be done with the only forms which have four valid intersections—as they are figured in my former paper—if only the wire or cord be so twisted about that, while the *form* is preserved, the junctions B, D be brought into the positions relative to the figure which were formerly occupied by A, C. For the scheme

$$\begin{array}{cccccccccc} + & - & - & + & + & - & - & + & & \\ A & D & B & A & C & B & D & C & | & A \\ - & + & - & + & - & + & - & + & & \end{array}$$

remains the same, so far as the letters are concerned, if we keep the same cyclical order of letters, but write A for B, B for C, &c. In estimating the electromagnetic work, by the rule above, we find we may leave out either A, C, or B, D. So that the work is $\pm 8\pi$, *i.e.*, one degree of beknottedness.

The following case, with six intersections, is very instructive. Either figure is formed from the other by throwing the lower coil over the top of the whole.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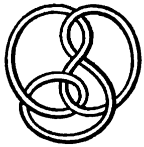
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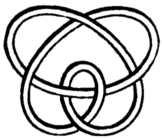
It will be seen that each of these forms may be regarded as a simple loop passed unsymmetrically through a simple knot of three intersections (figures 1 and 3 of my former paper), and that the knot and loop are interchangeable between two groups of three intersections. The knot is right-handed when transferred to the second group; left-handed in the first. But the figures plainly show that they may also be regarded as a right and a left-handed simple knot having a part common to each, so that neither can be pulled tight, subject to our convention that there shall be nothing higher than a double point. And *here* the peculiar difficulty associated with the amphicheiral forms comes in; for, in estimating the electromagnetic work, we find we must leave out one copper and one silver junction—the result being $+8\pi - 8\pi$. This is to be treated as $\pm 16\pi$ (or two degrees of beknottedness) because the portions with different signs belong to what are, virtually at least, two separate knots.*

The possibility of such *amphicheiral* forms is obvious from one of the first illustrations in my former paper; where we have only to suppose the irrelevant crossing removed, and one of the separate simple knots (which are both right-handed in the cut) made left-handed. But I was not at first prepared to find this property in any knot not separable into detached, self-contained portions; so that it is possible that some of my former statements may require modification.

It may be well to notice that when, in a slight variation of the

* *Feb.* 19.—This is not correct. There is but *one* degree of beknottedness, for the two knots are not "virtually separate," as they have a part in common, while one is right-handed and the other left-handed. In fact, the figures above are mere transformations of the last cut in my former paper —which is shown to be capable of being opened up by a single change of sign. This can be done in the figures above, at either end of the lower coil where it forms part of the external boundary. But if, without altering the outline of the figure, we change all the signs in either of the two component knots, so as to make them both right-handed, or both left-handed, the whole acquires the double degree of beknottedness wrongly assigned to it in the text. But it has now continuations of sign, and in virtue of these it happens to be reducible. In fact, when we make it into a clear coil after these changes of sign it becomes the pentacle (fig. 1 above), the only knot with fewer than six crossings which possesses, as we have seen, *two* degrees of beknottedness. I stated in my first paper, that when the signs in any nonnugatory arrangement are alternately + and − the cord "is obviously as completely knotted as its scheme will admit of." This completeness must

arrangement just described, the loop passes symmetrically through the simple knot, we have another six-crossing form, very much resembling the last, but which is essentially not amphicheiral. It is figured below in one of its forms—the others may be got by deformation—and the schemes of the two kinds are appended for comparison.



5

Figs. 3 and 4 A *D* B *E* C *A* D *F* E *C* F *B* | A.
Fig. 5 A *D* B *E* C *F* D *A* E *C* F *B* | A.

It will be seen that the sole difference between the amphicheiral knot and that last figured, lies in the inversion of the positions of A and F in their *even* places in the scheme.

It appears, then, that none of these abbreviated methods, however useful as temporary aids to classification, can take the place of the scheme in fully describing the form of a knot and in measuring the amount of beknottedness in general. Especially is the scheme required in order to calculate the beknottedness in terms of the electromagnetic work. And this conclusion might, I think, have been inferred from the prominent part which the arrangement of the letters in the even places plays in determining the form of a knot; an arrangement of which only traces are left when we substitute the sign of the work at each junction for the letter attached to it, thus losing all control of the amount to be added or subtracted on account of the mere number of coils.

[*Added Jan. 27th.*]—Professor Clerk-Maxwell, to whom I sent some of the above results (and to whom, as well as to Sir W.

be understood of what may be called *Knottiness*, not of *Beknottedness*. For it has just been shown by a particular case that we can occasionally increase the degree of beknottedness, while diminishing knottiness, *i.e.*, losing crossings by so altering their signs as to make some of them nugatory. The point thus raised, *i.e.*, the distinction between Knottiness and Beknottedness, is a very troublesome and delicate one, and is obviously related to several of the difficulties pointed out in the present paper.

Thomson, I am indebted for various hints, usually in the especially valuable form of criticisms and reasons for doubt), has lately called my attention to a paper by Listing, of date 1847, part of which is devoted to the subject of knots. I have this morning obtained it from the Cambridge University Library, but have not yet thoroughly read it. As was to be expected, I find that the author has anticipated some of the contents of my papers; and he mentions at least one very curious fact, which I had thought possible, but had not observed, though it is very directly connected with one of the results of the present note. He virtually shows, by giving a particular case, that the method of deformation which I employ does not always give all possible forms of a completely knotted wire. I believe that this depends on the fact that a part of the scheme is amphicheiral. I propose to give the Society an account of Listing's method and results on the earliest opportunity.

3. Note on the Effect of Heat on Infusible Impalpable Powders. By Professor Tait.

Several years ago Professor Dewar gave me a specimen of silica in a state of exceedingly minute division, which had been produced in Dr Playfair's laboratory in the preparation of fluosilicic acid. I noticed at the time how much its great mobility is increased by heating—so that it behaves almost like a liquid. And I fancied that I observed close to the surface a thin stratum of what might by the same analogy be called a vapour; consisting of particles thrown up and falling back again, like the little drops thrown up at the surface of soda-water. I was inclined to ascribe these phenomena to heat directly—supposing that the particles were fine enough to behave, though in a very imperfect way, as the kinetic theory assumes the particles of a gas to behave. However this may be, the extreme mobility of such powders when heated on a platinum dish; and the fact, noticed by chemists, that a bath of calcined magnesia is capable of propagating waves when heated; seem to show that valuable results might be obtained by seeking for evidence of inter-diffusion as the result of experiments made by very long-continued heating of vessels containing fine silica and mag-