Nonlocally Linear Manifolds and Orbifolds

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A *topological manifold* is, by definition, a Hausdorff topological space where each point has a neighborhood homeomorphic to Euclidean space. The geometrical topology of manifolds is a beautiful chapter in mathematics, and a great deal is now known about both the internal structure of manifolds (transversality, isotopy theorems, local contractibility of homeomorphism groups, etc.) and their classification (cobordism theory, surgery theory, etc.). The subject that I would like to explore is the extension of this picture to a larger class of intrinsically interesting spaces (finite-dimensional ANR homology manifolds). Part of our exploration is motivated by an analogy between homology manifolds and orbifolds, that is, spaces that are modeled not on Euclidean space, but rather on the quotients of representation spaces by their finite linear actions.

1 The Topological Characterization of Manifolds

There are several different ways that one can be lead to the nexus of problems considered here. One useful way is to ask: How can one tell whether or not a space that arises in some natural fashion is a manifold?

In low dimensions, there are some classical criteria. A connected space is a circle if no point separates it, but each pair of points separates it. There is a similar characterization of the 2-sphere in terms of nonseparating points and separating circles, due to R. L. Moore. However, in dimension 3 and higher this is not possible because of the existence of homology manifolds: A *homology manifold* will be, until the very last section, a finite-dimensional ANR X with $H_*(X, X - x) = H_*(\mathbb{R}^n, \mathbb{R}^n - 0)$ for every point x in X. Such spaces have many of the properties of manifolds. They satisfy Poincaré duality, and therefore will be separated by exactly the same spaces that would separate a manifold.

The simplest example of a homology manifold that is not a manifold is the cone on a nonsimply connected (manifold) homology sphere. All deleted neighborhoods of the cone point are nonsimply connected, so this space is not a manifold, but it is a trivial calculation to see that the local homology is as required.

Another way to obtain many more and much wilder examples is that of *decomposition spaces*, pioneered by Bing.[2] One starts with a manifold M and

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2) The earliest striking application was the construction of a nonmanifold whose product with $\mathbb{R}$ is a manifold.

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describes a (suitably semicontinuous) collection of subsets that are in some weak sense contractible (technically, cell-like), and identifies each of these subsets to a point. This identification space X is the image of a natural *CE map*[3] $M \to X$.

Because every homology sphere bounds a contractible manifold (see [K] for high dimensions and [Fr] for dimension 3) the first example is a special case of the second. In fact, for quite some time it seemed as if every homology manifold could be obtained in this fashion:

RESOLUTION CONJECTURE (Cannon): Every homology manifold (of dimension at least five) is the decomposition space of some cellular decomposition of a manifold.

This conjecture was attractive in light of an amazing theorem of Edwards (see [Dav, E]).

THEOREM (Edwards). *A CE map $f : M \to X^n$, $n \geq 5$, can be approximated by homeomorphisms iff X satisfies the disjoint disks property (DDP), that is iff any pair of continuous maps $D^2 \to X$ can be ε-approximated by maps with disjoint images.*

COROLLARY: A resolvable homology manifold is a manifold iff it has the DDP.[4]

COROLLARY: A resolvable homology manifold $\times\mathbb{R}^2$ is a manifold.

With some diffidence, I would like to suggest calling homology manifolds with the DDP *nonlocally linear manifolds.* The conjectures made in [BFMW1] suggest that these will be locally modeled on some new (topologically homogenous) spaces and that they will share many of the geometric properties of manifolds. For instance, in [BFMW2] the resolution conjecture is verified with nonlocally linear manifolds replacing manifolds. However, alone, this conjecture does not give us any insight into what the local geometry of such spaces can be.

An important rigidification of the situation was made by Quinn [Q1]. He showed:

THEOREM (Quinn). *There is a locally defined $i(X) \in H^0(X;\mathbb{Z})$ valued invariant of homology manifolds. Thought of as a function on components, it assumes values in $1 + 8\mathbb{Z}$, and equals 1 (on every component) iff X is resolvable.*

This integer is a signature, and it would be appropriate to think of it as the 0th Pontryagin class of (the "tangent bundle" of) X. We call it the *local index* of X.

Its definition is about the same level of depth as the topological invariance of Pontryagin classes (Novikov's theorem) as it requires defining L-classes in a topological fashion for ANR homology manifolds (and in particular for topological manifolds). L-classes for homology manifolds are constructed in [FP] and [CSW], and we will return to this in Section 3.

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3) A map is CE if, when restricted to the preimage of any open subset of the range, the map is a homotopy equivalence.

4) For an example of how dramatically the DDP can fail, see [DW].

The locality of Quinn's obstruction implies that for connected X if some open subset of X is resolvable, then X is. In particular. any "manifold with singularities" is resolvable by a manifold. Thus, constructing nonresolvable homology manifolds involves building the whole space simultaneously.

THEOREM ([BFMW]). *For every number $i \in 1 + 8\mathbb{Z}$ there is a homotopy sphere*[5] *that is a DDP homology manifold with local index i.*

We conjecture these spaces to be uniquely determined by i and dimension, at least if the dimension is ≥ 4.

2 Lacunae in the Theory of Topological Manifolds

In this section, I would like to show how the theory of manifolds itself cries out for some missing spaces. The spaces turn out to be supplied by the nonlocally linear manifolds (DDP homology manifolds).

DEFINITION: For M a manifold of dimension ≥ 5, let $S(M)$ denote the set of homotopy equivalences modulo homeomorphisms. That is,

$$\begin{aligned} S(M) \cong \{&h : M' \to M \text{ a simple homotopy equivalence} \\ &\text{with } h : \partial M' \to \partial M \text{ a homeomorphism}\} \\ &/\text{homeomorphism (rel } \partial). \end{aligned}$$

THEOREM (Siebenmann, as corrected by Nicas, [KS, Ni]). *Let M be a manifold of dimension ≥ 5. If ∂M is nonempty, one has*

$$S(M) \cong S(M \times D^4).$$

In general, there is an exact sequence

$$0 \to S(M) \to S(M \times D^4) \to \mathbb{Z}.$$

This means that $S(M \times D^4)$ can be as much as a $\mathbb{Z}$ larger than $S(M)$. The simplest manifold, the sphere, gives an example of this. $S(S^n) \cong 0$, but $S(S^n \times D^4) \cong \mathbb{Z}$. From the point of view of periodicity there should be a $\mathbb{Z}$s worth of homotopy spheres. These are filled in by the homology manifolds. (For manifolds with boundary, the boundary condition forces the domain homology manifold mapping to M to be a homology manifold — essentially because of locality, so that using homology manifolds would not increase the size of S.)

THEOREM ([BFMW1]). *Let X be a homology manifold of dimension ≥ 5, and let $S^H(X)$ denote $\{h : X' \to X$ a simple homotopy equivalence with $h : \partial X' \to \partial X$ a homeomorphism$\}/s$-cobordism* (rel ∂)*. Then if ∂M is nonempty, one has*

$$S^H(X) \cong S^H(X \times D^4) \quad (\cong S(X \times D^4), \text{ if } X \text{ is a manifold}).$$

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5) In fact every simply connected manifold has an "evil twin" with given local index, but typically aspherical manifolds do not.

Thus, periodicity is true in the category of homology manifolds. The periodicity map interchanges manifolds and homology manifolds. We have to use s-cobordism rather than homeomorphism as our equivalence relation because we cannot yet prove an s-cobordism theorem for DDP ANR homology manifolds. Nonetheless, this enables us to ignore any DDP conditions in the definitions of our structure sets. (There can be no s-cobordism theorem without assuming DDP because a CE map has as mapping cylinder an s-cobordism, which cannot be a product unless the quotient space is a manifold!)

THEOREM ([BFMW1]). *$S^H(X)$ can be computed as the fiber of the assembly map (see [R]).*[6] *Consequently it is an abelian group, which is functorial for orientation true*[7] *maps between manifolds of dimension that differ by multiples of 4.*[8]

Using $S(M)$ one loses functoriality for maps where the dimension of the target is smaller than that of the domain. So the theory of homology manifolds has better formal properties than the theory of manifolds. In particular, pushing manifolds forward leads to (nonresolvable) homology manifolds.

These theorems imply that the rigidity theory of high-dimensional topology adapts gracefully to include homology manifolds. For instance any DDP homology manifold homotopy equivalent to a nonpositively curved manifold is homeomorphic to it (see [FJ]).[9] Moreover, existence theorems in our larger category work out a little more nicely:

THEOREM ([BFMW]). *Any Poincaré space $\mathbb{Z}$ homology equivalent to a nonpositively curved manifold. is homotopy equivalent to a closed DDP homology manifold, which is unique up to s-cobordism.*

However, such a Poincaré space is not necessarily homotopy equivalent to a closed manifold.[10] Another example where such spaces occur naturally is the following result of Smith theory:

THEOREM ([CW]). *Tame semifree circle actions on a manifold have ANR homology manifolds as fixed sets. If the fixed set has codimension* 2 *mod* 4,[11] *any equivariant homotopy equivalent free circle action on (a manifold homotopy equivalent to) the complement of the fixed set of this action extends to a unique concordance class of circle actions.*

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6) This is a more modern formulation of surgery theory in the topological category than one finds in Wall's book.

7) That is, a map that preserves the orientation character of curves.

8) There are reindexing tricks that allow one to define functoriality for a related theory for all orientation true maps.

9) A simpler approach would be to show that the local index is necessarily 1, which is of the depth of the Novikov conjecture: it is a kind of tangentiality statement (see [FW] for this point of view, taken in a different direction). Then resolution implies it is a manifold, and the usual rigidity takes over.

10) There is a nonresolvable homology manifold proper homotopy equivalent to a symmetric space of noncompact type iff it has q-rank greater than 2. ([BW])

11) There is a rather different analysis for the case of codimension 0 mod 4.

If one doesn't allow homology manifolds to arise there is a $\mathbb{Z}$ obstruction to the existence of a semifree manifold completion for the free action.

3 Properties of Homology Manifolds

In the previous section we described some systematic features of the class of homology manifolds. In particular, we described the result of surgical classification for these spaces. Next, we describe the cobordism classification (suggested by David Segal).

THEOREM ([BFMW]). *For $n \geq 6$, the formula $\Omega_*^H \cong \Omega_*^{\mathrm{Top}}[1+8\mathbb{Z}]$ (monoid ring) is correct additively. At least rationally, this formula gives the correct multiplicative structure.*

Note that unlike the classical manifold case, not all bordism classes are represented by connected spaces. Locality implies that there is no analogue of connected sum. I conjecture that the bordism calculation is multiplicatively correct even integrally.

Unlike the usual proofs of bordism results, this is not achieved by direct analyses of transversality, but rather by using the fact that all topological bordism classes have simply connected representatives, and applying the "evil twin" theorem of Section 1.

There are obstructions to transversality, as one can see by the following reasoning. Consider a homotopy equivalence (say) from a manifold to a homology manifold, and assume that it could be made transverse to a point; then the "funny local type" would be present in a manifold giving a contradiction. One can cross this example with a manifold to see that this is not an oddity because of low-dimensional preimages.

Similarly, if one embeds a nonresolvable homology manifold in high-dimensional Euclidean space, one cannot stably hope for any "normal bundle" structure because of multiplicativity properties of the local index.[12]

The following calculation demonstrates the systematic failure of transversality:

THEOREM (Stabilized structure calculation). *One has an isomorphism:*

$$\lim S^H(M \times K \downarrow K) \cong \lim S^H(M \times \mathbb{R}^n \downarrow \mathbb{R}^n)_{(2)}.$$

Here the $\downarrow$ denotes controlled structures, and the K's run over the DDP ANR homology manifolds proper homotopy equivalent to Euclidean spaces ("fake Euclidean spaces"). The right-hand side is a convenient stabilization of S, and only differs from the usual S when there is some algebraic K-theory present (see e.g. [R], [We]) — in particular, for M simply connected it is the structure set $S^H(M)$.

12) There is another kind of structure present: a teardrop neighborhood (see [HTWW]). It is to a bundle what a CE map is to a homeomorphism in the sense that the same homotopical data as is present for the classical notion arises here but with respect to open subsets of the range.

If there were transversality, then the part of $S(M)$ caught by characteristic classes could not die on crossing with any manifold: by projecting down to that manifold, the transverse inverse image of any point would recover this data. However, the theorem asserts that all odd torsion is lost.

Another version of this theorem describes when a map between manifolds is stably homotopic to an s-cobordism: iff it is stably tangential at 2 (it is a classical theorem of Mazur that one has the same condition for stable homeomorphism if stabilization is with Euclidean spaces, except that one does not localize.)

A good analysis of transversality obstructions could lead to the calculation of the multiplicative structure of bordism suggested above. A final ramification of transversality, classically, is Sullivan's $KO[1/2]$ orientation for manifolds (see [Su]). Here we have a variant:

THEOREM (See [CSW]). *For every homology manifold X there is a canonical $KO[1/2]$ class, the signature class, that is an orientation iff X is resolvable.*

Thus, the orientability is closely related to transversality, but independently of transversality, homology characteristic classes can in any case be defined, and used. Sullivan's formulation of surgery in terms of these classes is implicit in the theory adumbrated in Section 2. Away from 2 the homology normal invariant is just the difference of these signature classes.

REMARK: There is a refinement at 2 related to the Morgan-Sullivan class [MS]. The method of proof is to understand the chain complex of X as a self-dual sheaf and recognize the Witt group of such (away from 2) as KO-homology. (This is dependent on the work of Quinn and Yamasaki — or alternatively, Ferry and Pederson — for nonconstructible sheaves, as arise here.) A little thought checks that the local calculation one would do of this invariant agrees with the local index (up to powers of 2).

REMARK: The assignment of characteristic classes to self-dual sheaves has a number of other applications. For instance, one has a close parallel to classical surgery theory for spaces with even codimensional strata and simply connected links by making use of the intersection sheaves (see [GM II]), see [CSW], and also [Sh1] for other applications of cobordism of self-dual sheaves.

4 Conjectured Properties of Homology Manifolds

We have already alluded several times to a conjectured view of homology manifolds.

VAGUE CONJECTURE: High-dimensional homology manifolds with the disjoint disk property (nonlocally linear manifolds) share all the geometrical properties of manifolds.

Obviously one cannot include transversality among the geometrical properties we have in mind! On the other hand, general position holds. We will be guided somewhat by the analogy presented in the next section. Until then, let us be more precise and list the following package of conjectures [BFMW]:

CONJECTURE (Homogeneity): Nonlocally linear manifolds are homogenous.

For manifolds this is a triviality. If the signature class is an orientation then, of course, it is true because of the work of Edwards and Quinn cited above: the space is a manifold. Because the deviation from "manifoldness" is entirely measured by a conventional top locally finite homology class (or 0-dimensional cohomology class), an algebraic sort of homogeneity is guaranteed. Still, the conjecture has eluded us. The next conjectures are somewhat stronger, and we will discuss some progress by analogy in the next section:

CONJECTURE (S-bordism): For nonlocally linear manifolds of dimension ≥ 6, the s-cobordism theorem holds.

In the smooth category this is Smale's theorem (or rather its generalization by Barden-Mazur-Stallings) and is true in the topological category by Kirby-Siebenmann's reinstitution of handlebody theory.

CONJECTURE (CE-approximation): If dim ≥ 5, any CE map from a nonlocally linear manifold to another is approximable by homeomorphisms.

If one assumes the domain and range are conventional manifolds, then this is a theorem of Siebenmann, if just the domain is, then it is Edwards' result, and if just the range is, then one obtains this as a consequence of the work of Edwards and Quinn.

CONJECTURE (Local contractibility of homeomorphism groups): Every homeomorphism sufficiently close to the identity can be canonically isotoped to the identity.

For manifolds, this is due to Cernavski, and Edwards-Kirby.

5 The Analogy to Orbifolds

It is somewhat reassuring that there are other settings in which one can both define objects in terms of explicit models, or alternatively in terms of local homotopy properties, and the latter not only fill in lacunae in the theory of the former, but they themselves possess many nice geometric properties, and homogeneity, in particular.

One such setting is that of orbifolds (although much of what follows is a special case of a general theory of stratified spaces).

DEFINITION: A locally linear orbifold is a space that is locally modeled on the orbit space of an orthogonal representation of a finite group.

DEFINITION: A nonlocally linear orbifold is a space that is locally the quotient of a disjoint disk homology manifold under a finite group, where the fixed sets of all subgroups are (not necessarily locally linear) homology manifolds that are embedded in one another in a "locally homotopically trivial fashion". This local condition (aside from codimension 2) asserts that in the local chart, small 2-disks in one fixed set can be homotoped (in an arbitrarily small way) disjoint from a smaller fixed point set. (See [Q2], [We].)

REMARK: In the second definition, if one is trying to imagine phenomena not stemming directly from the existence of nonresolvable homology manifolds, not much is lost in assuming that one is locally the quotient of an action on a manifold where all of the fixed point sets are submanifolds. The local homotopy condition (by work of Bryant, Chapman, and Quinn) boils down to the assertion that these fixed sets are locally flat.

Now for the analogous theorems to what we discussed above.

(CE APPROXIMATION) There is an equivariant CE approximation theorem[13] (see [StW] for the locally linear case, and [Hu], [We] in general). The analogue of the resolution conjecture would be the coincidence of the two definitions of orbifolds. However, it is quite simple to see that the cone on a fake real projective space (which are produced in profusion by surgery theory, but are not hard to come by explicitly, using linear involutions on Brieskorn spheres, for instance) is never resolvable by a linear orbifold!

(SURGERY) The analogue of the surgery exact sequence was established for odd order locally linear group actions by [MR] by a complex induction depending on transversality methods. Simultaneously with establishing transversality, they proved that there is an equivariant Sullivan orientation $\in KO_*^G[1/2]$ for the actions they considered. Unfortunately for nonlocally linear actions, and for even order groups, equivariant transversality fails, and although subsequently [RtW] (see also [RsW]) a signature class[14] was constructed for more general actions, it was not an orientation. This necessitates a deviation from the Sullivan-Wall exact sequence of classical surgery theory, and is given in [We].

The new sequence boils down to an equivariant extension of the homological form of surgery theory due classically to Quinn and Ranicki (see [R]). Indeed, that theory naturally has Siebenmann's periodicity built into it, and is the one alluded to in Section 2. Moreover, in this formulation, all the theories (including the topological theory for all statified spaces) take a beautiful "local-global" form:

$$\cdots \to L(X \times I) \to S(X) \to H_0(X; L(\mathrm{loc})) \to L(X) \to$$

where L denotes a surgery spectrum (a generalization of the notion of surgery groups, and adapted to stratified spaces), which when applied to open subsets gives a cosheaf of spectra. The difference between the L-cosheaf homology and a global L-spectrum gives rise to the spaces stratified homotopy equivalent but not homeomorphic to X.[15]

As for the conjectures.

(HOMOGENEITY) The version of homogeneity is due to Quinn [Q2]. It asserts that generally a "manifold stratified space" will have all of its connected strata homogenous. Quinn has also established an h-cobordism theorem for these. (Steinberger had independently done the locally linear case.)

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13) And even an α-approximation theorem [CF].

14) The original method for doing this was analytic, but the paper [CSW] referred to above sketches a topological approach.

15) There is a deviation at the prime 2 that we are ignoring here.

(Local Contractibility of the Homeomorphism group) Local contractibility is also true, according to [Si2] for locally linear orbifolds, and [Hu] for general ones.

To sum up, the definitions of orbifolds suggested parallel the possibilities of definitions of manifold and DDP homology manifold in the unequivariant setting. (With a little twist: even order locally linear orbifolds are not oriented by their signature classes, and correspond to a "manifold type category in which transversality fails".[16].) In both settings the analogues of simple homotopy theory and surgery are understood.

Regarding the local structure, one is in much better shape in the orbifold setting, assuming the strata are manifolds, ultimately because inductive arguments are possible. Interestingly enough, it is the same basic ideas that are responsible for our advances in both of these directions: the methods of controlled topology. However, as of yet, it does not seem natural to combine the detailed arguments of these two situations. It seems to me that the explicit nature of the stratification in the orbifold setting suggests methods for exploring the problems of homology manifolds at least at the level of conjecture.

On the other hand, in the case of homology manifolds, we have a very good feel of what the aggregate of local structures should be: there are a $\mathbb{Z}$'s worth of them. For the orbifold case, the algebraic problems are much more subtle: see [tD] for the underlying local homotopy theory, and, e.g. [Sh2] for some of the geometry: the part of identifying what role the linear examples play. The signature class can fail, even rationally, to be an orientation, which should mean that the failure of transversality is more striking in the orbifold setting than in the homology manifold case. Consequently, problems in group actions should be addressable by finding their concomitants in the theory of homology manifolds, and working out the easier algebra there. For instance, the stable structure calculation above, in contrast to Mazur's theorem, should lead, by analogy, to interesting phenomena in the cancellation problem for nonlinear similarity.

Finally, and most speculatively, the first method for obtaining signature classes for orbifold was by doing Lipschitz or quasiconformal analysis of signature operators. One would hope that homology manifolds, which have more algebraic signature classes, also support a suitable type of analysis: one that must be based on something other than calculus and linear approximation.

6 Some Remarks on Infinite Dimensions

One can also inquire regarding the nature of infinite-dimensional homology manifolds. These are d-dimensional homology manifolds by the homological definition, but which have infinite covering dimension.

They exist as a consequence of work of Edwards (see [Wl]) and the construction of an infinite-dimensional space of finite homological dimension by Dranishnikov [Dr]. The following begins a study of their geometrical topology

16) In the orbifold case, there isn't the same close connection between the signature class being an orientation and local linearity. However, in light of homogeneity, there are locally defined obstructions that tell you when you're locally linear: examine the local structures at a few strategic points!

THEOREM ([DF]). *There are infinite dimensional homology manifolds that do not have any finite-dimensional resolution. When a resolution exists, it need not be unique. However, according to a theorem of Ferry, the number of s-cobordism classes of resolutions is finite.*

This has had applications [DF] to constructing pairs of manifolds that converge to each other in Gromov-Hausdorff space, through metrics with some fixed local contractibility function.

It has also been applied to large-scale geometry in the construction of a uniformly contractible manifold with no degree one Lipschitz map to Euclidean space [DFW] and the failure of a bounded analogue of the rigidity conjecture for aspherical manifolds. Their further study promises to contain many more surprises.

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