.

Detection of Higher Dimensional Topological Manifolds Among Topological Spaces [1]

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Abstract. In this survey, based on my 1990 lecture at the University of Bologna I describe the history and the present status of one of the most interesting problems of modern geometric (i.e. Bing) topology – the problem of detection of topological manifolds among topological spaces, also known as the Recognition problem. I begin by a primary on the topology of generalized manifolds. In the sequel I concentrate on the higher dimensional case. The Recognition problem naturally splits into two parts – the Resolution problem and the Disjoint Disks Property problem. The former is still unsolved whereas the solution to the latter is due to R. D. Edwards. I describe the attempts at the Resolution problem – from J. L. Bryant and J. G. Hollingsworth via J. W. Cannon, J. L. Bryant and R. C. Lacher, to F. S. Quinn – and analyze the problem of Quinn's local surgery obstruction. As for the Disjoint Disks Property problem I merely give an outline of the Edwards' Shrinking theorem – as a prime example of Bing topology. I discuss the low dimensional case (dimensions 3 and 4) only briefly – in the Epilogue – as it will be the subject of another survey. In the References I have collected an extensive bibliography on the subject of recognizing topological manifolds (of all dimensions).

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Contents:

1. Introduction

One of the most interesting and at the same time most challenging problems of modern geometric topology is to determine whether a given topological space is a topological manifold or not. (For some history see e.g. [13], [28], [74], [76], [89], [91], [98], [108], [122], [151], [182], [186], [187], [221], [232], [238], [290].) In 1977, J.W. Cannon [74] explicitly formulated this, so-called *Recognition problem*: find a short list of simple topological properties which characterize topological manifolds. For example, S^1 is known to be the only compact, connected metric space containing at least 2 points, which is separated by every pair of its points [224], whereas S^2 is the only nondegenerate locally connected, connected, compact metric space which is separated by no pair of its points but is separated by each of its simple closed curves [22].

The purpose of this survey article is to give an up-to-date review of the current status of the recognition problem for higher dimensional manifolds, i.e. of dimension ≥ 5. In 1984 I wrote a similar survey for dimension 3 [238]. In [238] the higher dimensional case was dealt with only briefly since at that time it seemed that it had been successfully settled through the work of R. D. Edwards [118] and F. S. Quinn [232]. However, the unexpected discovery [233] [234] in 1985 – by S. Cappell and S. Weinberger – of a fundamental gap in [232] suddenly reopened this problem, in particular the status of the *Resolution problem* which asks whether every generalized n–manifold admits a resolution. In the following years no success was reported on this subject. Therefore, I have decided to prepare a survey of the work done so far in this field. Perhaps a description of different methods of approach various authors have used in the past may inspire and stimulate future attempts at the Resolution Problem.

This paper doesn't treat the other two dimensions which remain of interest, namely 3 and 4 (in dimensions ≤ 2 everything is classics [22] [224]). In the Epilogue we give some references concerning the lower dimensional case, in particular the surveys [238], [98], and [221].

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2. Preliminaries

Throughout this paper, we shall be working in the category of locally compact Hausdorff spaces and continuous maps throughout this paper. Manifolds (TOP, PL or DIFF) will be assumed to have no boundary unless specified. Homology (resp. homotopy) equivalence will be denoted by $\sim$ (resp. $\simeq$). Isomorphisms (resp. TOP, PL, DIFF homeomorphisms) will be denoted by $\cong$ (resp. $\approx$, $\underset{\text{PL}}{\approx}$. $\underset{\text{DIFF}}{\approx}$). The singular (resp. Čech, Borel-Moore, sheaf) (co)homology over a principal ideal domain (PID) R will be denoted by $H(\_;R)$ (resp. $\check{H}(\_;R)$, $\mathbb{H}(\_;R)$, $\mathcal{H}(\_;R)$). Wherever $R = \mathbb{Z}$ we shall not write the coefficients.

The euclidean n-space (resp. the closed n-ball, the standard n-sphere, the n-cube $= [0,1]^n$ will be denoted by $\mathbb{R}^n$ (resp. B^n, S^n, I^n). A *homotopy* (resp. *R-homology*) *n-cell* is a compact n-manifold with boundary M such that $M \simeq B^n$ (resp. $M \sim B^n$ over R). The definition of a *homotopy* (*R-homology*) *n-sphere* is analogous.

A compact subset K of an n-manifold M is *cellular* in M if K is the intersection of a *properly nested* decreasing sequence of n-cells in M, $K = \cap_{i=1}^{\infty} B_i^n$ (i.e. for every i, $B_{i+1}^n \subset \text{Int} B_i^n$). A space X is *cell-like* if there exist a manifold N and an embedding $f : X \to N$ such that $f(X)$ is cellular in N. A map defined on a space (resp. an ANR, a manifold) X is *monotone* (resp. *cell-like*, *cellular*) if its point-inverses are continua (resp. cell-like sets, cellular sets) in X. A closed map is *proper* if its point-inverses are compact. A map $f : X \to Y$ is *one-to-one over* $Z \subset Y$ if for every $z \in Z$, $f^{-1}(z)$ is a point.

Let F be a covariant (resp. contravariant) functor defined on some topological category $\mathcal{C}$ and let $\Phi : F(X) \to F(Y)$ (resp. $F(Y) \to F(X)$) be a morphism, where $X \subset Y$ are any two objects of $\mathcal{C}$. Then it will always be assume that $\Phi = F$ (incl.) unless otherwise specified.

A compactum K in a manifold is *point-like* if $M - K \approx M - \{pt\}$. A subset Z of a space X is π_1-*negligible* if for each open set $U \subset X$ the homomorphism $\pi_1(U - Z) \to \pi_1(U)$ is one-to-one. A space X is *k-lc(R)* (resp. $lc^k(R)$, $lc^\infty(R)$) *at* $x \in X$ ($k \in \mathbb{Z}_+$, R a PID) if for every neighborhood $U \subset X$ of x there is a neighborhood $V \subset U$ of x such that $H_k(V;R) \to H_k(U;R)$ is trivial (resp. $H_j(V;R) \to H_j(U;R)$ is trivial for every $0 \leq j \leq k$, $H_j(V;R) \to H_j(U;R)$ is trivial for all $j \geq 0$). A compactum K in an ANR X has the $k - uv(R)$ (resp. $uv^k(R)$, $uv^\infty(R)$) *property* ($k \in \mathbb{Z}_+$, R a PID) if for each neighborhood $U \subset X$ of K there is a neighborhood $V \subset U$ of K such that $H_k(V;R) \to H_k(U;R)$ is trivial (resp. $H_j(V;R) \to H_j(U;R)$ is trivial for every $0 \leq j \leq k$, $H_j(V;R) \to H_j(U;R)$ is trivial for all $j \geq 0$). The uv properties are related to the Čech cohomology: if a compactum K has the properties $j - uv(R)$ ($j = k-1, k$) then $\check{H}^k(K;R) \cong 0$ and conversely, if $\check{H}^j(K;R) \cong 0$ ($j = k, k+1$) then K has the property $k - uv(R)$ [185]. If instead of homology R-modules one uses homotopy groups one gets the corresponding definitions of the $k - LC$, LC^k, LC^∞ and $k - UV$, UV^k, UV^∞ properties [185]. A map defined on an ANR is $uv^k(R)$ (resp. UV^k) ($k \in \mathbb{Z}_+$, R a PID) if its point-inverses have the $uv^k(R)$ (resp. UV^k) property.

A countable collection of pairwise disjoint compacta $\{C_i\}$ in a metric space X is a *null-sequence* if for every $\varepsilon > 0$ all but finitely many among the $C_i's$ have diameter less than ε. A compactum $K \subset \mathbb{R}^m$ has *embedding dimension* $\leq n$, $\text{dem}\, K \leq n$, if for every closed subpolyhedron $L \subset \mathbb{R}^m$ with $\dim L \leq m - n - 1$, there exists an arbitrarily small ambient isotopy of $\mathbb{R}^m$, with support arbitrarily close to $K \cap L$ which moves L off K. This concept is due to M.A. Štan'ko [265] - for more see [117].

A *crumpled cube* is the complementary domain of an open n-cell in S^n. A *fake* cube is a homotopy 3-cell which is not homeomorphic to B^3. The classical Poincaré conjecture asserts that there are no fake cubes [153]. A space X is said to have the *Kneser finiteness* (KF) if no compact subset of X contains more than finitely many pairwise disjoint fake cubes. A *homotopy handlebody* is a regular neighborhood of a wedge of finitely many circles in some 3-manifold.

Let X be a σ-compact space and present it as the union $X = \cup_{i=1}^{\infty} K_i$ of a properly nested increasing sequence of compact subsets $K_i \subset X$. An *end* of X is sequence $e = \{U_i\}$ of properly nested decreasing sequence of components of $X - K_i$. The *Freudenthal compactification* $\hat{X}$ of X is $X \cup \{e\}$ with $\{U_i\}$ as the basis of topology at the end e [137] [138] [256]. For example, if X is generalized n-manifolds with 0-dimensional singular set $S(X)$ (see Chapter 3 for definitions) then X is the Freudenthal compactification of the open n-manifold $X - S(X)$.

A space X is 1-*acyclic at* ∞ if for every compact set $K \subset X$ there exists a compact set $K' \supset K$ such that $H_1(X - K') \to H_1(X - K)$ is trivial.

A subset $Z \subset X$ is *locally simply connected* (1-LCC) if for every $x \in X$ and every neighborhood $U \subset X$ of x there is a neighborhood $V \subset U$ of x such that $\pi_1(V - Z) \to \pi_1(U - Z)$ is trivial. A metric space X is *uniformly locally simply connected* (1-ULC) f for every $\varepsilon > 0$ there exists a $\delta > 0$ such each loop in X of diameter less than δ bounds a disk in X of diameter less than ε.

Let G be a decomposition of a space X into compact and connected subsets and let $\pi : X \to X/G$ be the corresponding quotient map, H_G the collection of all *nondegenerate* (i.e. $\neq$ pt) elements of G, and N_G their union. A set $U \subset X$ is G-*saturated* if $U = \pi^{-1}\pi(U)$. A decomposition G is *upper semicontinuous* if for each $g \in G$ and for each open neighborhood $U \subset X$ of g there exists a G-saturated open neighborhood $V \subset U$ of g. Equivalently, π is a closed map. A decomposition G of a separable metric space X is k-*dimensional* (resp. *closed* k-*dimensional*), $k = -1, 0, 1, \ldots$, if $\dim \pi(N_G) = k$ (resp. $\dim \pi(\overline{N_G}) = k$). A decomposition G of a metric space X is *weakly shrinkable* if for each $\varepsilon > 0$ and each neighborhood $U \subset X$ of N_G there is a homeomorphism $h : X \to X$ such that $h \mid X - U = id$ and for each $g \in G$, $\operatorname{diam} h(g) < \varepsilon$. A decomposition G of a space X is *shrinkable* if for every G-saturated open cover $\mathcal{U}$ of N_G and every open cover $\mathcal{V}$ of X there is a homeomorphism $h : X \to X$ such that:

(i) $h \mid X - \mathcal{U}^* = id$ where $\mathcal{U}^* = \cup\{U \in \mathcal{U}\}$;

(ii) for each $x \in X$ there exists $U \in \mathcal{U}$ such that $\{x, h(x)\} \subset U$; and

(iii) for each $g \in G$ there exists $V \in \mathcal{V}$ such that $h(g) \subset V$.

Let $f : X \to Y$ be a map. The *nondegeneracy set* of f is defined by $N(f) = \{x \in X \mid f^{-1}f(x) \neq x\}$ and its image $S(f) = f(N(f))$ is called the *singular set* of f. Let $f : M \to X$ be a proper, cell-like map from manifold onto an ENR. Then the associated decomposition $G(f) = \{f^{-1}(x) \mid x \in X\}$ of M is upper semicontinuous and cell-like. Moreover, $H_{G(f)} = f^{-1}(S(f))$ and $N_{G(f)} = N(f)$. For more on decompositions see [91].

A general reference for algebraic topology will be [263], for PL topology [250], for surgery [57] and [283], for 3-manifolds [31], [153], and [160], for ANR's [39] and [156], for cell-like maps and related topics [108], [185], and [221], and for decompositions [91].

3. An introduction to generalized manifolds

The concept of a generalized manifold goes back to the 1930's. The first results concerning this class of spaces were obtained during 1930-1945 by P. S. Aleksandrov, E. G. Begle, E. Čech, S. Lefschetz, L. S. Pontrjagin, P.A. Smith, R.L. Wilder, and some others, each approaching the subject with a different motivation. For example, R.L. Wilder discovered that generalized manifolds were the proper framework in which some fundamental results about 2–manifolds (e.g. the Jordan curve theorem and the Schoenflies theorem) generalized to higher dimensions [291]. From another direction, P.A. Smith entered this subject in the course of his investigations of group actions on topological manifolds [260]. During this period the foundations of the algebraic topology of (generalized) manifolds were developed.

The second period of increased activity in this area was during 1950–1965 and it was led by A. Borel's seminar group in Princeton [37]. They were mostly interested in transformation groups and they exploited heavily the sheaf theory, which was developed around that time by J. Leray, J.P. Serre and some others [37] [43] [45] [235]. A. Borel, G.E. Bredon, P.E. Conner, E.E. Floyd, D. Montgomery, J.C. Moore, R.S. Palais, F.A. Raymond, C.T. Yang, and others did some most important work in this period.

The third period of interest in generalized manifolds started sometime in the early seventies - among geometric topologists, in at least 3 areas: taming theory, the double suspension problem and the desingularization problem. It has since been dominated by some important results of M.G. Brin, J.L. Bryant, J.W. Cannon, R.J. Daverman, W. T. Eaton, R. D. Edwards, J. G. Hollingsworth, R.C. Lacher, D.R. McMillan,Jr., F.S. Quinn, M. Starbird, L. C. Siebenmann, T.L. Thickstun, J.J. Walsh, and some others. As it is often the case in geometric topology much of the foundations of their work can be traced to the pioneering work of R.L. Wilder [290],[291] and R.H. Bing [28], [31].

The following is a modern, more geometric definition of a generalized manifold [76]: A space X is a (geometric) *generalized n-manifold* ($n \in \mathbb{N}$) if:

(i) X is *euclidean neighborhood retract* (ENR), i.e. for some integer m, X embeds in $\mathbb{R}^m$ as a retract of an open subset of $\mathbb{R}^m$; and

(ii) X is a *homology n-manifold*, i.e. for every $x \in X$, $H_*(X, X - \{x\}; \mathbb{Z}) \cong H_*(\mathbb{R}^n, \mathbb{R}^n - \{0\}; \mathbb{Z})$.

Note that condition (i) is equivalent to: X is a locally compact, finite dimensional separable metrizable ANR [156].

Classical definitions were much more general: condition (i) above was usually weakened to some (co)homological local connectivity requirement and finite (co)homological dimensionality was also assumed in most cases. On the other hand the singular homology in condition (ii) above was replaced mostly by the Borel-Moore homology [38], with coefficients in an arbitrary principal ideal domain. The following (classical) definition seems to have been most commonly used among algebraic topologists since 1950's [43] [235]. A locally compact Hausdorff space X is a (classical) *generalized n-manifold* ($n \in \mathbb{N}$) *over a principal ideal domain* R if:

(i) X is *cohomologically locally connected over* R, clc_R, i.e. for every $x \in X$ and every neighborhood $U \subset X$ of x there is a neighborhood $V \subset U$ of x such that the restriction $\tilde{H}^*(U; R) \to \tilde{H}^*(V; R)$ of reduced (Čech or sheaf) cohomologies is trivial;

(ii) X has finite cohomological dimension over R, $\dim_R X < \infty$, i.e. there exists an integer m such that $H^j_c(U; R) \cong 0$ for every open subset $U \subset X$ and every integer $j \geq m$;

(iii) The homology sheaf, generated by the presheaf $U \to H_q(X, X-U; R)$, $\mathcal{H}_q(X; R)$, has each stalk a free R-module of rank 1 if $q = n$ and is trivial otherwise.

Condition (iii) can be rephrased in the language of the Borel-Moore homology: for every $x \in X$, $\mathbb{H}_q(X, X - \{x\} : R) \cong R$ if $q = n$ and $\cong 0$ otherwise. The main reason for using the Borel-Moore homology above is that it doesn't have the well-known deficiencies of the classical homology theories, e.g. the Čech homology is not exact [202], the singular homology doesn't respect the dimensions properly [16], etc. However, on the class of $lc^\infty(\mathbb{Z})$ spaces, the Borel-Moore homology coincides with the singular homology [43] [195]. Therefore, this classical definition includes our first one - the geometric definition above. In particular, all fundamental results above classical generalized manifolds also hold for the geometric ones.

Although generalized manifolds are defined by a set of local properties of topological manifolds, they nevertheless satisfy most of the basic global properties of manifolds, e.g. the invariance of domain, standard separation properties, linking theory, and intersection theory. By far the most significant fact is that the Poincaré duality in its most general form) holds in generalized manifolds [38]:

Theorem 3.1 (A. Borel and J.C. Moore [38]) *Let $\mathcal{A}$ be a coefficient sheaf of R-modules and φ any paracompactifying family of suppotrs on a generalized n-manifold X ($n \in \mathbb{N}$) over a principal ideal domain R (i.e. φ is a collection of closed, paracompact subspaces of X such that* (i) *a closed subset of a member of φ belongs to φ,* (ii) *φ is closed under finite unions, and* (iii) *every element of φ has a (closed) neighborhood in X which is in φ). Then there exists a natural isomorphism $\Delta : H^p_\varphi(X; \mathcal{O} \otimes \mathcal{A}) \to H^\varphi_{n-p}(X; \mathcal{A})$, induced by a cap product and natural with respect to inclusion maps and boundary-coboundary homomorphisms.*

All standard duality theorems (Alexander's, Lefschetz's, Poincaré's) can essentially be deduced from Theorem (3.1). In fact, one could consider generalized manifolds as the class of finite dimensionl, locally compact Hausdorff spaces in which the Poincaré duality holds *both* locally and globally [43], [215], [235], [264].

One of the important applications of Theorem (3.1) is the following result which is indispensable for almost all applications of (classical) generalized manifolds (in order to derive global facts from locall hypotheses):

Theorem 3.2 (G.E. Bredon [44]) *Every generalized n-manifold X (over any PID R) is locally orientable, i.e. the orientation sheaf $\mathcal{H}_n(X, R)$ is locally constant.*

Local orientability means, roughly speaking, that the local homology modules at neighboring points have certain nice relationship to one another. Although this fact may today seem only too natural it was for many years one of the outstanding conjectures [291] until Bredon put it to rest in 1969 (see also [215]).

We conclude our discussion concerning the classical generalized manifolds and hereafter we shall only consider the geometric ones, in the narrow sense of the definition at the begining of this chapter. So let X a (geometric) generalized n-manifold. If $n \leq 2$ then X is a genuine n-manifold because in these lowest dimensions algebraic properties are strong enough to imply the geometric ones. In higher dimensions however, X may not be locally euclidean at some (or perhaps at all) points. Such exceptions are called *singularities* of X and they form the *singular set* of X, $S(X) = \{x \in X \mid x$ *doesn't have a neighborhood in* X *homeomorphic to an open subset of* $\mathbb{R}^n\}$. The complement, $M(X) = X - S(X)$ is the *manifold set* of X.

At the beginning of this century, in their search for an appropriate definition of a PL n-manifold, early topologists came upon a concept we would today recognize as a PL generalized n-manifold. It is nowadays easy to see that in the lowest dimensions, $n \leq 3$, this class of spaces coincides with the class of n-manifolds (which they were trying to capture) - so they were completely successful in this range. In higher dimensions, $n \geq 4$, it has been observed by R.D. Edwards that, as a consequence of J.W. Cannon's work on the double suspension problem [75], such spaces may fail to be n-manifolds only at the vertices of some triangulation [74]. Consequently, the classical topologists may have missed the essential properties of (higher dimensional) manifolds only by a finite collection of singularities (see the discussion on pp. 835-838 in [74].

Generalized manifolds arise in many situations:

(1) As cell-like, upper semicontinuous *decomposition of manifolds*: every proper, cell-like surjection from a (generalized) n-manifold onto a finite dimensional metric space yields a generalized n-manifold [185] (for examples see [238]).

(2) As *manifold factors*: using the Künneth formula one can show that given locally compact Hausdorff spaces X_1 and X_2, their product $X_1 \times X_2$ is a generalized $(n_1 + n_2)$-manifold if and only if each X_i is a generalized n_i-manifold [37].

(3) As orbit spaces of the action of *transformation groups*, e.g. P.E. Conner and E.E. Floyd proved in 1959 that the Smith manifolds [260] are (classical) generalized manifolds [86]: the fixed point set of a toral group action (resp. a $\mathbb{Z}_p$ action with p any prime) on a manifold is a generalized manifold

(4) As *suspensions of homology spheres*: the k-fold suspensions of a generalized n-manifold with the singular homology of S^n is always a generalized $(n+k)$-manifold. J.W. Cannon [75] and, independently, R.D. Edwards [118] have proved that the k-fold suspension ($k \geq 2$) of a topological n-manifold ($n \in \mathbb{N}$) with the homology of S^n is homeomorphic to S^{n+k}. (Note that for $k = 1$ this need not be true in general).

(5) As those ENR's which admit maps onto closed manifolds with arbitrarily small point-inverses [196].

(6) The Freudenthal compactifications of certain open manifolds [46]–[48], [52]–[55], [51], [114], [281], [289], etc. Note that these constructions may, in general, produce quite exotic spaces, e.g. the endpoint compactification of an infinite connected sum of Poincaré homology 3-sphere [168] yields a homology 3-manifold with an uncountably generated fundamental group.

4. The Resolution Problem

The problem of recognizing (detecting) topological manifolds is usually restricted to the class of generalized manifolds and it splits naturally into two problems: Given a generalized n–manifolds X, one must (1) find a resolution $f : M \to X$ and (2) check if the associated cell–like decomposition G_f of M is shrinkable, if we are given that X has a sufficient amount of general position properties, e.g. the DDP.

A *resolution* of an n-dimensional ANR X is a proper, cell–like map $f : M \to X$ from a topological n-manifold M onto X. If X has a resolution then X is a generalized n-manifold [185]. A resolution $f : M \to X$ of X is called *conservative* if f is one-to-one over the manifold set $M(X)$ of X.

The following is the best known result so far. Note that, in particular, it implies that if X is a generalized n–manifold, $n \geq 4$, and X is not *totally singular*, i.e. $S(X) \neq X$, then X always has a (conservative) resolution.

Theorem 4.1 (F. S. Quinn [232]–[234]) *Let X be a generalized n–manifold.*

(a) If $n \geq 4$, then X admits a conservative resolution if $X \times \mathbb{R}$ resolves.

(b) If $n \geq 5$ and if a certain local surgery obstruction $\sigma(X) \in 8\mathbb{Z} \oplus 1$ vanishes, then X admits a conservative resolution.

Moreover, if (M_i, f_i) are any two conservative resolutions of X, $n \geq 4$, and $U \subset X$ is a neighborhood of $S(X)$, then there is a homeomorphism $h : M_1 \to M_2$ such that $f_1(x) = f_2 h(x)$, for every $x \in X - U$.

We now review the history of attempts at the Resolution problem. The first resolution theorem is due to J.L. Bryant and J.G. Hollingsworth [64]: they found a (conservative) resolution $f : M \to X$ with M a smooth (resp. PL) n-manifold, for an arbitrary generalized n-manifold X ($n \geq 5$), provided $X \times \mathbb{R}^k$ was smooth (resp. PL) $(n + k)$-manifold, for some integer k, and that $\dim S(X) = 0$. Their argument went as follows: using the Product Structure Theorem of R.C. Kirby and L.C. Siebenmann [170], a smooth structure can be imposed upon $M(X)$, compatible with the smooth structure on $M(X) \times \mathbb{R}^k$ (which is inherited from $X \times \mathbb{R}^k$). Take now a closed neighborhood $N' \subset X$ of $S(X)$ such that each component $N \subset N'$ is compact and has a closed smooth $(n-1)$-manifold as boundary. Then N has the homotopy type of a finite cell-complex since Wall's obstruction to finiteness of N [282] can be shown to vanish. The hard part is then to construct (using the main results from [256]) a smooth n-manifold with boundary M and a map $g : (M, \partial M) \to (N, \partial N)$ such that g is cell-like over $N \cap S(X)$ and a diffeomorphism over the complement.

It is a consequence of the double suspension theorem [75] that the result of Bryant and Hollingsworth doesn't extend to dimension 4: let X^4 be the open cone over the dodecahedral space H^3 [168]. Then X^4 satisfies the hypotheses of the theorem (for $n = 4$ and $k \geq 1$). However, if the conclusion were valid for X^4 then H^3 would bound a contractible 4-manifold which is known to be impossible [166]. (Since, by Theorem (4.1), X^4 has a conservative resolution $f : M^4 \to X^4$, we see that such M^4 cannot admit a smooth structure - although it is smoothable away from any of its points [231]. In dimension 3, the analogue of the Bryant - Hollingsworth theorem is equivalent to the Poincaré conjecture (see [238]).

Few years later the resolution problem was attacked by J.L. Bryant and R.C. Lacher [65]. Rather than trying to resolve a wider class of generalized manifolds (than those covered in [64])they weakened the requirement on the resolution $f : M \to X$. Instead, they proved that every generalized n-manifold is the base of an approximate fibration with the total space a topological m-manifold, $m \geq n+2$. (Subsequently, R.J. Daverman and L.S. Husch [91] have verified that a (proper) approximate fibration from a topological manifold onto an ANR always yields a generalized manifold. Thus the class of generalized manifolds is essentially the same as the class of those ANR's which are the base spaces of some (proper) approximate fibrations on topological manifolds.)

Since approximate fibrations didn't seem to offer any applications in the characterization of manifolds, Bryant and Lacher continued their investigations of resolvability of higher dimensional generalized manifolds. In their next paper [66] they proved Theorem (4.1) for the case $n \geq 5$ and $\dim S(X) = 0$. Here's an outline of their argument: Suppose first, that $n \geq 6$ and that $S(X) = \{p\}$. Because of the local contractibility of X, the Kirby-Siebenmann obstruction to triangulating $U - \{p\}$ [170] vanishes for a sufficiently small neighborhood $U \subset X$ of p. We can therefore find a smaller neighborhood $V \subset U$ of p such that V is contractible in U and $V - \{p\}$ is a PL n-manifold with a compact and connected boundary. Then V has the homotopy type of a finite complex and a well-defined simple homotopy type [214] [287]. Since $(V, \partial V)$ is a simple Poincaré pair [283] we can choose V small enough to make the restriction of the (stable) normal bundle η of $X - \{p\}$ to $V - \{p\}$, $\eta \mid V - \{p\}$, to be a trivial PL bundle [264] [283]. Using the Thom-Pontrjagin construction [283] one can then obtain a PL n-manifold M with a normal bundle ν and a degree one normal map $f : (M, \partial M) \to (V, \partial V)$ such that $f \mid \partial M$ is a PL isomorphism and ν is PL trivial. Using simply connected surgery [57] one can transform f into a simple proper homotopy equivalence, such that f is a PL isomorphism over the complement of Int$V$. Finally, extend $f : M \to X$ over the rest of X to get a homotopy resolution of X over V which, by the s-cobordism theorem [250], are unique up to a PL homeomorphism. The desired cell-like resolution is then obtained as the limit of such homotopy resolutions, much like in [64]. The general case is then deduced from the "one singularity" case by an inverse limit argument (see [66]).

In the case when $n = 5$ (and $S(X) = \{p\}$) we need some extra work since the deleted neighborhoods of p do not necessarily have a vanishing Kirby-Siebenmann obstruction. However, one can get around this problem by geometric methods (compare [6] [120] [141] [170]).

At this point we wish to mention two papers of J.W. Cannon: [72] and [73]. For, [72] was the first paper to advertise that generalized manifolds can be treated not only algebraically but also geometrically much like topological manifolds. Cannon was the first to sistematically study geometric properties of generalized manifolds. Using taming arguments, he proved in [72] that all generalized codimension one submanifolds of S^n ($n \geq 5$) arise, at least stably, as cell-like decompositions of topological manifolds. His other paper [73] played a crucial role in his solution of the double suspension problem few years later [75]: in [73] he proved that the double suspension $\sum^2 H^3$ of a homology 3-sphere H^3 admits a resolution $f : S^5 \to \sum^2 H^3$. (As R.D. Edwards later pointed out this proof can easily be generalized to all dimensions $n \geq 5$). In [75], Cannon then used the DDP to shrink the associated cell-like, upper semicontinuous decomposition $G(f)$. (See [238] for more details.)

New ideas and insights from [72] [73] [74] enabled Cannon to improve the results of

[66]: in 1977, J.W. Cannon and, independently, J.L. Bryant and R.C. Lacher proved Theorem (4.1), for $n \geq 5$, in the trivial range, i.e. for the case $2 + 2 \dim S(X) \leq n \geq 5$ [77] (see also [74] and [186]).

Their proof splits naturally into two parts. First, given a generalized n-manifold X such that $2 + 2 \dim S(X) \leq n \geq 5$, they detect homologically a sequence of (pinched) crumpled n-cells in X which capture the "homotopic" nontriviality of the embedding of $S(X)$ in X. By replacing these crupled cubes by pinched real n-cells, they produce a better generalized n-manifold Y in that, now, $S(Y) \subset Y$ is 1-LCC. Furthermore, they obtain a proper, cell-like map $g : Y \to X$. In the second step they show that Y must, in fact, be a manifold since $S(Y)$ cannot be tamely embedded if it is in the trivial range.

They use geometric techniques which are different from the surgery arguments of [66] described earlier. They invoke the 1-ULC taming theorems of A.V. Černavskii [82] and C.L. Seebeck [255] among others (see also [131] which improved upon both [82] and [255]) in their fairly technical proof. We suggest Cannon's outline of the proof in the simplest cases [74] as a preliminary reading.

It is interesting to observe that, modulo the Poincaré conjecture, analogous results (for the trivial range) were later shown to hold in dimension three, although in the reverse historical order. First, J.L. Bryant and R.C. Lacher proved the 1-LCC taming theorem [67] and few years later T.L. Thickstun found a blow up $g : Y \to X$ with the properties described above [271]. We have discussed [67] and [271] in some detail in [238].

We should also mention another interesting resolution theorem, from 1982: using the more basic techniques of [64] and the Waldhausen (analogue of) simple homotopy type and projective class group for infinite complexes [280] (see an exposition in [227]), M.Y. Kutter applied an obstruction theory due to J.L. Bryant and M.E. Petty [69] to splitting $X \times \mathbb{R}$ as a manifold, to resolve generalized n-manifolds X $(n \geq 5)$ with $S(X)$ a polyhedron and such that $X \times \mathbb{R}$ is a PL $(n+1)$-manifold [179].

Around the time of Cannon's solution of the double suspension problem [75] and subsequent Edwards' definitive higher dimensional shrinking theorem [118], F.Quinn began to present his program on *ends of maps* which in the years to follow materialized in a sequence of papers [229] - [232]. The starting idea was to develop an analogue of the completion theory, done for manifolds by L.C. Siebenmann [256], for functions: given an n-manifold M and a continuous map $e : M \to X$ onto, say, an ENR, the question was - when does there exist a *completion*, i.e. a compact n-manifold with boundary M' such that $M' - M \subset \partial M'$ and an extension of e to a proper onto map $e' : M' \to X$? Quinn obtained several important results on this subject with many applications (e.g. to constructions of mapping cylinder neighborhoods, to resolutions of manifold factors, to block bundle approximations of approximate fibrations, to local flatness of embeddings, to locally flat approximations of wild embeddings, and finally - some starting results in dimension 4 (see [229] - [232]). The methods employed in the proofs of his End theorem and the corollaries were ε-versions of algebraic topology, homotopy theory, algebraic K-theory, and surgery which Quinn had been working out since the early 1970's.

One of the main problems Quinn had been attacking since mid 1977 was the problem of the existence of resolutions for higher dimensional generalized manifolds. The shrinking theorem of Edwards [118] was "almost one half" of the proof of the Cannon manifold characterization conjecture - the missing half was Theorem (4.1) $(n \geq 5)$. Quinn announced a proof in 1978 and a full version of it appeared few years later [232]. The case $n = 4$ of (4.1) followed soon after M.H. Freedman's [134] and S.K. Donaldson's [104] fundamental contributions to the topology of 4-manifolds [231]. We remark here that before [231] appeared, M. Ue [275] [276] had resolved the class of generalized 4-manifolds with isolated singularities.

For the general proof of Theorem (4.1) one needs the following result which is a consequence of Edwards' shrinking theorem [118], Quinn's end theorems [229]–[231], and R.J. Daverman's observation [89] that all generalized manifolds adopt the DDP after having been crossed by $\mathbb{R}^2$. Note that this result also shows that the two, in the past most useful methods of desingularizing a generalized n-manifold ($n \geq 4$), resolving and stabilizing, are equivalent.

Theorem 4.2 *Let X be a generalized n-manifold, $n \geq 4$. Then the following statements are equivalent:*
(i) X has a resolution;
(ii) $X \times \mathbb{R}^k$ has a resolution, for some $k \in \mathbb{N}$; and
(iii) $X \times \mathbb{R}^2$ is a manifold.

For the proofs of (4.2) and using that, (4.1) we refer to the survey [238]. In conclusion, we consider resolutions of *PL homology manifolds* (see [203] [165] for their basic properties). M.M. Cohen and D. Sullivan [83] [84] [85] [270] see also [200] [254]) have completely solved the Resolution problem for this class of spaces (they might have been inspired by H. Hironaka's resolution theorem for algebraic varieties [155]: the obstruction to finding a PL acyclic resolution $f : M \to X$ for a PL homology n-manifold X (i.e. M is a PL n-manifold and f is a PL, strongly acyclic surjection [209]) lies in $H^4(X;\Theta)$ where Θ is the abelian group of oriented PL h-cobordism classes of oriented homology 3-spheres, modulo those which bound acyclic PL 4-manifolds, and the group operation in Θ is induced by taking connected sums. Furthermore, if X admits a PL acyclic resolution then there is a one-to-one correspondence between $H^3(X;\Theta)$ and the set of the concordance classes of PL acyclic resolutions of X. (One can show that $\Theta \cong \Pi_3(H/\mathrm{PL})$, where H/PL is a homotopy fiber of the natural map $\mathrm{BPL} \xrightarrow{j} \mathrm{BH}$ between the classifying spaces for stable PL block bundles and stable homology cobordism bundles [165] [199]. For an interesting relationship between the Cohen-Sullivan obstruction and the Kirby-Siebenmann obstruction to putting a PL manifold structure on a topological manifold [169] see [2].)

These existence and classification theorems have later been recast in the language of classifying spaces by A.L. Edmonds and R.J. Stern: a PL homology n-manifold X admits a PL acyclic resolution if and only if the classifying map τ of the homology tangent bundle of X lifts to BPL. Furthermore, if X admits a PL resolution then there is a one-to-one correspondence between the set of homotopy classes of lifts of τ to BPL and the set of the concordance classes of PL acyclic resolutions of X [113]. Both properties follows from the Product Structure Theorem in [113], analogous to our Theorem (4.2): a PL homology manifold X admits a PL acyclic resolution if and only if $X \times I^k$ admits a PL acyclic resolution for some $k \in \mathbb{N}$.

Note that by Theorem (4.1), every generalized n-manifold, $n \geq 4$, is simple homotopy equivalent to a topological n-manifold [185]. Analogous result for PL homology n-manifolds, $n \geq 5$, was proved earlier by D.E. Galewski and R.J. Stern [140].

5. The DDP Problem

Topological n manifolds ($n \geq 5$) the *disjoint disks property* (DDP) and Theorem(5.1) shows that the DDP is also their characteristic property. A metric space X has the *DDP* if for every pair of maps $f, g : B^2 \to X$ and every $\varepsilon > 0$ there exist maps $f', g' : B^2 \to X$ such that $d(f, f') < \varepsilon > D(g, g')$ and $f'(B^2) \cap g'(B^2) = \emptyset$ [75]. In this chapter we shall give a brief account of the results which culminated in the following definite higher dimensional shrinking theorem:

Theorem 5.1 (R.D. Edwards [118]) *Let G be a cell-like, upper semicontinuous decomposition of an n-manifold M ($n \geq 5$) such that* $\dim M/G < \infty$. *Then G is shrinkable if and only if M/G has the DDP.*

In an arbitrary generalized n-manifold ($n \geq 5$) the DDP can fail badly [100] (see also [238]). But if it is valid it *detects* topological manifolds:

Corollary 5.1 F.S. Quinn [232]–[234]) *A space X is a topological n-manifold ($n \geq 5$) if and only if X is a generalized n-manifold with the vanishing obstruction $\sigma(X)$ and possessing the DDP.*

Corollary 5.2 (J.W. Cannon [75]) *The double suspension of every homology n-sphere is homeomorphic to S^{n+2}.*

Edwards' proof of Theorem (5.1) is one of the nicest achievements of modern geometric topology. It generalizes many earlier related results, e.g. [75], [175], [180], [257], and [273]. The proof's ingredients are classical shrinking techniques of R.H. Bing, radial engulfing and some fundamental taming results of R.H. Bing and J.M. Kister [34] and J.L. Bryant and C.L. Seebeck [70].

We shall present an outline for the following, by [198], equivalent formulation of Theorem (5.1).

Theorem 5.2 (R.D.Edwards [118]) *A proper, cell-like map $f : M \to X$ from an n-manifold ($n \geq 5$) onto an ANR can be approximated by homeomorphisms if and only if X has the DDP.*

Corollary 5.3 (L.C. Siebenmann [257]) *A proper, cell-like map between topological n-manifolds ($n \geq 5$) can be approximated by homeomorphisms.*

This corollary is also known in lower dimensions: $n = 2$ [249] [279] (for S^2 already [223], $n = 3$ [12], [180], [257] and for $n = 4$ [231].

The main idea of Edwards' proof of Theorem (5.1) is as follows: Using the DDP, he embeds the (infinite) 2-skeleton S of M into X. Then the applies the 1-ULC taming theory for decompositions of manifolds to make f one-to-one over $f(S)$. After this process the remaining nondegenerate point-inverses of f have 1-ULC complements in M, actually they have low embedding dimension. Consequently, they are essentially tame. Using an engulfing type induction he then untangles them and shrinks them to points.

The proof of (5.2) is based on three key proposition [191]: the 0-dimensional shrinking theorem (5.3), the $(n-3)$-dimensional shrinking theorem (5.4), and the 1-LCC shrinking theorem (5.5).

Theorem 5.3 (R.D. Edwards [191]) *Let $f : M \to X$ be a proper, cell-like map from an n-manifold onto an ANR such that* $\dim S(f) \leq 0$ *end* $\operatorname{dem} N(f) \leq n-3$. *Then f can be approximated by homeomorphisms.*

Note that Theorem (5.3) is false if the condition $\operatorname{dem} N(f) \leq n-3$ is replaced by $\leq n-2$, even if f is cellular - as a counterexample for $n = 3$ one can take the Bing countable planar Knaster continua decomposition [29] and for $n \geq 4$, Eaton's generalized dogbone space decomposition [110].

The idea is to first prove Theorem (5.3) for a special case: when the components of $N(f)$ form a null-sequence and $\operatorname{dem} f^{-1}(x) \leq n-3$ for every $x \in X$. The general case then follows by a standard amalgamation technique. The proof of the special case is a (quite technical) bare-handed shrinking, done in M - a generalization of a technique used in dimension 3 by Bing [26] for the case when $f^{-1}(x)$ was a (geometric) cone, lying in some coordinate patch in M. The crux is that our point-inverses are cell-like hence almost contractible, so with some effort one can find a sufficiently good conelike structure for them. Because we are dealing with only a null sequence of nondegenerate point-inverses we only need to shrink finitely many of them (others are already small enough). However, in doing this we must use the conelike structure on the chosen point-inverse (which we want to shrink), to prevent others from being inadvertly stretched larger.

Theorem 5.4 (R.D. Edwards [191]) *Let $f : M \to X$ be a proper, cell-like map from an n-manifold onto an ENR such that* $\operatorname{dem} N(f) \leq n-3$. *Then f can be approximated by homeomorphisms.*

Proof: Find a *filtration* of X with σ-compact subsets $p_k \subset X$

$$\emptyset = p^{-1} \subset p^0 \subset p^1 \subset \ldots \subset p^n = X$$

such that for every k, $\dim p^k \leq k$ and $\dim(p^k - p^{k-1}) \leq 0$. (This can easily be done with a (downward) induction: construct p_{k-1} by taking the fronties of a countable base of open neighborhoods of p^k.) Choose an open neighborhood $W \subset M$ of $N(f)$. Without losing generality we may assume that M is compact. Choose an $\varepsilon > 0$ and construct inductively, using at each step Theorem (5.3), for each $k = 1, 2, \ldots, n+1$, cell-like maps $f_k : M \to X$ with the following properties:

(i) $d(f_k, f) < \frac{k\varepsilon}{n+1}$;

(ii) f_k is one-to-one over p^{k-1};

(iii) $\operatorname{dem} N(f_k) \leq n-3$; and

(iv) $f_k|M - W = f|M - W$.

Clearly, $f_{n+1} : M \to X$ is then a homeomorphism and $d(f_{n+1}, f) < \varepsilon$ as required.

Theorem 5.5 (R.D. Edwards [191]) *Let $f : M \to X$ be a proper, cell-like map from an n-manifold ($n \geq 4$) onto an ENR, such that* $\dim \overline{S(f)} \leq n-3$ *and* $\overline{S(f)}$ *is 1-LCC in X. Then f can be approximated by homeomorphisms.*

Proof: ($n \geq 5$) The idea is to approximate f by a cell-like map $f' : M \to X$ such that $\operatorname{dem} N(f') \leq n-3$ and then apply Theorem (5.4) to approximate f' by a homeomorphism. Such an f' can be constructed as the limit of cell-like maps $f_k : M \to X$

such that for every k, the nondegeneracy set $N(f_k)$ lies in the complement of the 2-skeleton $T_k^{(2)}$ of some triangulation T_k of M, where mesh $T_k \to 0$ as $k \to \infty$. The main ingredient of the construction of $\{f_k\}$ is radial engulfing.

Proof of (5.2): (M PL and compact, $n \geq 6$). There are three main steps. Choose an $\varepsilon > 0$.

Step 1. Let $\{P_s\}$ be the (countable) set of all finite 2-complexes in M. Use the DDP in X to get a countable collection $\{C_t\}$ of compact sets in X such that:

(i) C_t is 1-LCC in X;

(ii) for every C_t there is some P_s such that $C_t \approx P_s$; and

(iii) for every map $g_s : P_s \to X$ there exists a homeomorphism $h_{st} : P_s \to C_t$ such that h_{st} approximates g_s.

Apply Theorem (5.5) to show that for every t, f can be approximated by homeomorphisms over C_t. So there is a cell-like map $f_1 : M \to X$ such that $d(f_1, f) < \frac{\varepsilon}{3}$ and f_1 is one-to-one over $\cup_{t=1}^{\infty} C_t$.

Step 2. We want to lower the embedding dimension of the nondegeneracy set of f_1. Therefore we shall approximate f_1 by a cell-like map $f_2 : M \to X$ such that $d(f_2, f_1) < \frac{\varepsilon}{3}$ and $\operatorname{dem} N(f_2) \leq n - 3$. In order to find such an f_2 construct triangulations $\{T_k\}$ of M with pairwise disjoint 2-skeleta (use general position) and such that mesh $T_k \to 0$ as $k \to \infty$. We then get f_2 as the limit of a sequence of cell-like maps $g_k : M \to X$ with the following properties:

(i) $g_0 = f_1$;

(ii) for every k, $d(g_{k+1}, g_k) < \frac{\varepsilon}{2^{k+1}}$; and

(iii) for every $k > 0$, g_k is one-to-one over $g_k(T_k^{(2)}$.

We must make certain that $f_2 = \lim_{k\to\infty} g_k$ agrees with each g_k over $T_k^{(2)}$ hence

$$N(f_2) \subset M - \cup_{k=1}^{\infty} T_k^{(2)}$$

so $\operatorname{dem} N(f_2) \leq n - 3$. The main ingredients of the proof are the Bing-Kister [34] and the Bryant-Seebeck [70] taming theorems combined with [145].

Step 3. Use Theorem (5.4) to find a homeomorphism $f_3 : M \to X$ such that $d(f_3, f_2) < \frac{\varepsilon}{3}$. Consequently, $d(f_3, f) < \varepsilon$ and we are done.

In conclusion, we mention a generalization of the disjoint disks property - the so-called *disjoint k-cells property* (DD^kP): A space X is said to have the DD^kP if every pair of maps $f, g : B^k \to X$ can be arbitrarily closely approximated by maps with disjoint images. H. Toruńczyk proved in 1977 a remarkably simple characterization of manifolds modelled on the Hilbert cube [274] (for an excellent exposition see [121] or [212]): a compact AR X is homeomorphic to the Hilbert cube if and only if X satisfies the DD^kP for all $k \geq 0$. M. Bestvina has used the DD^kP to characterize universal Menger compacta [19] [20]: if X is a k-dimensional, $(k-1)$-connected, LC^{k-1}, compact metric space, satisfying the DD^kP then X is homeomorphic to the k-dimensional universal Menger space μ^k. (Some results along these lines were at the same time obtained by A.N. Dranišnikov [106]). As a corollary, various constructions of μ^k that have appeared in the past (e.g. in [192] [211] [226]) all yield the same space. Another important result in [20] is a topological characterization of manifolds modelled on μ^k: a locally compact, LC^{k-1} k–dimensional metric space locally looks like μ^k if and only if it satisfies the DD^kP. As it is demonstrated in [20] μ^k turns out to be the k-dimensional analogue of the

Hilbert cube. For another variation of the DDP and its applications for decompositions of manifolds see the work of D.J. Garity [142] [143]. For more history of DDP see [238].

6. Epilogue

In this survey only the case of higher dimensions (≥ 5) was discussed. In dimensions ≤ 2 all generalized n–manifolds are genuine n–manifolds so the Recognition problem in the sense of Cannon doesn't concern this case. In dimension 4 we still don't know enough about the geometric properties of 4–manifolds to successfully attack this problem – in spite of the recent breakthrough in this area by M. H. Freedman [134] and subsequent developments [105], [136], [167], [194]. For example, we still have no good analogue of the DDP for this dimension (for partial results see [21] and [96]).

In dimension 3, however, there has been a considerable amount of work done in the past 3 years. Anyone interested in this dimensions can consult the surveys [238] (which describes the results up to 1984) and [98] (which is about the work done after 1984, e.g. [21], [40]–[42], [52]–[55], [56], [59], [92], [95]–[97], [102], [103], [132], [142]–[144], [146]–[148], [161]–[164], [220]–[222], [237]–[246], [261], [266], [267], [272], [293]) as well as a related survey [221] on topology of cell–like maps.

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