# Henri Poincaré. A Life in the Service of Science

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In 1954 the scientific community celebrated the 100th anniversary of Henri Poincaré's birth. At that time, Poincaré's fame was not at its highest point among mathematicians, and the spirit of Hilbert dominated most mathematical minds. His fame was also not at its highest point among physicists, as physics was then essentially concerned with quantum theory.

Nevertheless, the celebration was important in the many places where Poincaré's presence or name had been significant. The proceedings of this celebration were published as a memorial book and were reproduced in the last volume of Poincaré's *Scientific Work*.

This year, as we celebrate the 150th anniversary of Poincaré's birth, his popularity has reached new summits in the scientific world and even among laymen. Chaos theory and the origins of special relativity have brought Poincaré's name and picture to most popular science journals. But although several new books about Einstein have been added this year to an already long list, we are still waiting for a detailed biography of Poincaré. Presented here is an impressionistic introduction to Henri Poincaré, the man and the scientist.

## Family, Childhood, Studies

Poincaré was born on April 29, 1854, in Nancy, in the Hôtel Martigny, a town mansion that had been transformed into a drugstore and that still exists at the corner of Grande-Rue and rue de Guise. Poincaré's family was well known in Lorraine. His grandfather on his father's side, Jacques-Nicolas, was a pharmacist; his father, Léon, a neurologist, was a professor in the Faculty of Medicine; and his uncle Antoni (the father of Raymond, future president of the French Republic), a graduate of the École Polytechnique, was the general inspector of roads and bridges. Henri's mother, born Eugénie Launois, came from a family of gentlemen farmers in Arrancy. Henri's younger sister, Aline, married the famous philosopher Emile Boutroux, and their son Pierre was a talented mathematician and philosopher.

With the exception of a dangerous bout with diphtheria at age five, Poincaré's childhood resembles those described in old-fashioned storybooks. The games he invented with his sister and cousins revealed Henri's unlimited imagination, and a clever private tutor nurtured his fantastic memory. At the high school in Nancy (the future Lycée Henri-Poincaré), he was soon noticed as a first-class student, showing himself to be a "mathematical monster" in his last few years. After earning his *baccalauréat* in letters and science, he became well known during the two years he spent preparing in mathematics for the entrance examination for the "Grandes Écoles".

Ranked as the fifth-best student admitted to the École Normale Supérieure and as the best one admitted to the École Polytechnique, Poincaré chose to attend the latter and graduated second in his class. He then went to the École des Mines, where crystallography appealed to his mathematical taste and may have inspired his abiding interest in group theory. After he was refused permission to attend lectures at the Sorbonne, Poincaré received his diploma in mathematics from the Faculty of Science of Paris in August 1876.

During his last two years at the École des Mines, Poincaré prepared his Ph.D. thesis in mathematics. He defended on August 1, 1879, at the Faculty of Science in front of a jury consisting of Bonnet, Bouquet, and Darboux. The thesis extends to partial

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differential equations some classical results of Briot and Bouquet on singular ordinary differential equations. Darboux's report, which was very positive about the results and the methods, was far less enthusiastic about the clarity of the style.

## Career and Personality

Poincaré started working as a mining engineer in Vésoul in April 1879. During his several months there he paid a dangerous visit to the Magny pit, where a firedamp explosion had killed sixteen workers. Poincaré remained all his life on leave (and was also promoted!) as a member of the Corps des Mines.

His academic career started at the Faculty of Science of Caen, where he taught analysis starting in 1879. Two years later he moved to Paris as lecturer of analysis in the Faculty of Science. He was successively appointed lecturer in physical mechanics and experimental physics in 1885, professor of mathematical physics and probability in 1886, and professor of mathematical astronomy and celestial mechanics in 1896. He also taught astronomy at the École Polytechnique and theoretical electricity at the École des Postes et Télégraphes. He was a member of the Bureau des Longitudes.

His former students described Poincaré as a more devoted than brilliant teacher. According to Robert d'Adhémar:

> From the beginning, the blackboard was covered with formulas, and one had an extraordinary feeling of power; the words came fast and without hesitation. The lectures were infinitely austere.

According to Maurice d'Ocagne:

> One could not say that Poincaré was a marvelous professor. He did not have the oratorical gifts required for excellence in teaching.

According to Léon Brillouin:

> I have seen him many times leaving his notes, announcing he would try another method and improvising in front of us at the blackboard.

According to Louis Bourgoin:

> Poincaré was, in 1910 and 1911, a fashionable scientist attracting the mundane Paris crowd to listen to him. During the first lectures, the room was more than full, but, rapidly and happily, the crowd soon decreased. From the third lecture on, only a few students and a few of the avid remained. Poincaré always ended with simple formulas, translated into a language full of imagery, that we were forced to understand.

A detailed analysis by Toulouse gives interesting information about the man Henri Poincaré at age forty-three:

> Poincaré is 1 m. 65 cm. in height, weighs 70 kilos, is bent, a little big-bellied. His face is florid, his nose big and red. His hair is chestnut color and his moustache blond.
>
> He does not smoke and never tried.
>
> He seems neither timid nor much more sensitive to the cold than others. Nevertheless he is subject to frequent colds. He does not sleep with an open window.
>
> What dominates his physical appearance is an expression of constant distraction. When speaking to him, one has the feeling that he has not followed or understood what was said, although he answers or thinks about the question.
>
> He believes he has a quiet, sweet, and even-tempered character. But he has no patience in his actions, and even in his work.
>
> He is passionate neither about his feelings nor about his ideas, and he is neither sociable nor someone to confide in.
>
> In practical life, he is disciplined. He is not orderly but appreciates the value of this quality.
>
> He speaks correctly, but with some shyness he is aware of. Hence he avoids speaking publicly without preparation, except in scientific meetings. Before giving a speech, he prepares a number of sentences but does not learn them by heart.
>
> He does not play chess and believes he could not be a good player. He does not hunt.

The popular magazine *L'Illustration* confirmed this picture in 1912:

> Very simple, very affable, looking somewhat like his head is in the clouds, he gave the impression of being a quite unusual balance between the mathematician of the new school, looking if

not like an artist at least very much like a Parisian and the classical mathematician, forbidding and absorbed in his equations.

## The Annus Mirabilis

The stay in Caen was certainly Poincaré's double annus mirabilis. Between August 1879 and October 1881, Poincaré not only married Louise Poullain d'Andecy (they had three daughters—Jeanne, Yvonne, Henriette—followed by one son, Léon) but also sent more than twenty notes to the *Comptes Rendus de l'Académie des Sciences de Paris* dealing with three completely different topics: the arithmetic of forms, the qualitative theory of differential equations, and automorphic functions.

The study of quadratic and ternary forms was inspired by the work of Charles Hermite, who at the time reigned over French mathematics. Hermite taught analysis to Poincaré at the École Polytechnique and owes his reputation to, among other things, his proof of the transcendental character of the number *e*. He reacted very enthusiastically to Poincaré's work, even though Poincaré's introduction of non-Euclidean geometry in the study of ternary forms completely disgusted the old analyst, who had always hated geometry. Hermite suggested that Poincaré read Kronecker's work ("without omitting anything") and made suggestions, which Poincaré ignored, for improving his writing style.

Poincaré himself has told the story of how he discovered automorphic functions, and I will not recall the famous tale of the omnibus. Automorphic functions, which extend the (periodic) trigonometric functions and the (doubly periodic) elliptic functions, recover their values under the action of a discrete group of homographic substitutions. The corresponding tessellation of the complex plane, made by rectangles for elliptic functions, is replaced by curvilinear figures bounded by curves that Poincaré identified with the "straight lines" in a new model of Lobatchevsky's geometry. Striking illustrations can be found in some paintings of Escher. In Göttingen, Felix Klein, trying to follow Poincaré's pace, ended up having a nervous breakdown, which ruined his career as a researcher. When Klein reproached Poincaré for having named some of his newly discovered functions "Fuchsian" to acknowledge the inspiration found in a memoir of Fuchs, the French mathematician reacted somewhat ironically by calling the next class of functions he discovered Kleinian!

Poincaré's motivation in this area comes from a problem raised by Hermite for the Grand Prix des Sciences Mathématiques de l'Académie des Sciences of 1880: To improve in some important way the theory of linear ordinary differential equations.

Poincaré's response to this problem was a disordered sequence of submitted, withdrawn, and supplemented memoirs, following the impetuous evolution of his thinking. This chaotic strategy disturbed the venerable Académie, and the Grand Prix was awarded to Georges Halphen for a more carefully written but less revolutionary memoir, with only an honorable mention to Poincaré. Besides non-Euclidean geometry, another basic ingredient of this first stream of research of Poincaré was Kronecker's index, inaugurating his use of topological tools in the study of singular points and limit cycles of differential equations, periodic solutions of the three-body problem, and bifurcation of the equilibrium shapes of a rotating fluid when the speed of rotation increases.

## The Three Stars of Hermite and King Oscar Prize

Besides Poincaré, two other rising stars revolved around Hermite for what could be described as family-mathematical reasons. The first one was Paul Appell, who married a niece of Joseph Bertrand, brother-in-law of Hermite and among the most influential mathematicians and academics. The second was Emile Picard, already famous in 1879 for his theorem on entire functions and son-in-law of Hermite. The poor Hermite was subject to pressure from his wife, who supported Picard, and from his authoritarian brother-in-law, who defended Appell. He wrote to Mittag-Leffler, former student of Weierstrass and husband of the wealthy daughter of Finland's "king of tobacco": "In a low voice and in confidence, afraid of being heard by Mrs. Hermite, I tell you that, from our three mathematical stars, Poincaré seems to me the most brilliant one. Furthermore he is a charming young man, from Lorraine like me, who knows my family very well."

Skillfully orchestrating appointments for vacant positions at the Sorbonne, Hermite succeeded in appointing almost simultaneously Appell in mechanics, Picard in calculus, and Poincaré in mathematical physics and probability. A similar game started a few years later for their election to the Académie des Sciences. Poincaré was elected in 1887, Picard in 1889, and Appell in 1892.

In 1885, following a suggestion of Mittag-Leffler, King Oscar II of Sweden decided to celebrate his sixtieth birthday by giving a prize crowning an important discovery in mathematical analysis, an example unfortunately followed by few other monarchs. The award consisted of a gold medal and 2,500 golden crowns. Any memoir submitted had to deal with one of the following topics:

1. The *n*-body problem in celestial mechanics.
2. Fuchs's generalization of ultraelliptic functions.
3. The functions defined by a first-order differential equation.
4. The algebraic relations between two Fuchsian functions having a common group.

The competition fit perfectly with Poincaré's mathematical interests, and he decided to work on the first question. He sent in May 1888 a 160-page memoir entitled "Sur le problème des trois corps et les équations de la dynamique". Although the work did not completely answer the question, the committee, made up of Weierstrass, Hermite, and Mittag-Leffler, gave the award to Poincaré, adding that: "It is the deep and original work of a mathematical genius whose position is among the greatest mathematicians of the century. The most important and difficult questions, like the stability of the world system, are treated using methods which open a new era in celestial mechanics."

The French newspapers commented widely on the event, and Poincaré was made Chevalier de la Légion d'Honneur.

During the printing of Poincaré's memoir, from July to November 1889, Phragmén, a young collaborator of Mittag-Leffler in charge of the editorial work, found some parts mathematically unclear. The first explanations of Poincaré, concretized by nine added notes, were followed by a long silence. In a moving letter of December 1, 1889, Poincaré admitted an error having important consequences: the conclusion of stability of the solar system is in fact invalid! When the letter reached Stockholm, the optimistic Mittag-Leffler had already started the distribution of the issue of *Acta Mathematica* containing the memoir, and he had to use all his diplomacy and influence to get the issues back to Sweden. One of them was rediscovered in Stockholm in the last decade of the twentieth century, in contradiction to a hand-written mention: "The whole edition has been destroyed."

Poincaré finally sent a new version of the memoir in June 1890—270 pages long—and had to pay for its printing, which was more than the 2,500 crowns of the prize! But the curse was not extinguished: King Oscar's medal itself was stolen a few years ago from Poincaré's grandson's apartment!

In correcting his mistake, Poincaré discovered a gold mine for mathematics and science by paving the way to chaos theory. In his own words:

> When one tries to depict the figure formed by these two curves and their infinity of intersections, each of which corresponds to a doubly asymptotic solution, these intersections form a kind of net, web, or infinitely tight mesh. One is struck by the complexity of this figure that I am not even attempting to draw.

In one of his more popular writings, he later explained in a prophetic way the possible consequences of this discovery:

> It may happen that small differences in the initial conditions produce great ones in the final phenomena.

The butterfly effect was born, but hunting this butterfly was for Poincaré quite a painful experience!

## Mathematical Physics

The extraordinary and turbulent period of research related to King Oscar's Prize did not prevent Poincaré from taking very seriously his teaching position in mathematical physics. If not a great lecturer, he was a very conscientious one. Each semester he chose new topics, and he prefaced and edited the notes taken by his best students. All were published in more than a dozen volumes, covering the whole of classical physics (hydrodynamics, elasticity, potential theory, capillarity, thermodynamics, heat theory, optics, electromagnetism) and probability, where Poincaré revealed his inventiveness and mathematical virtuosity.

Among other things, he carefully discussed Hertz's experiments on the propagation of electromagnetic waves and the beginnings of wireless telegraphy. His books on Maxwell theory contain the germs of special relativity and led him to analyze, correct, and name the Lorentz transformations. Poincaré published in 1905 a note (followed by an extended memoir) on the dynamics of the electron, containing the whole mathematics of special relativity. Historians of science still passionately discuss the priority between Einstein and Poincaré, and if one follows some recent publications, one might conclude that Hercule Poireau might be the only one able to uncover the whole story. Curiously, the mathematician Poincaré reached relativistic kinematics via Maxwell's electromagnetic theory, while the physicist Einstein used an axiomatic method. But it is unquestionable that Poincaré anticipated the so-called Minkowski space-time.

Poincaré also devoted three long memoirs between 1890 and 1895 to the partial differential equations of classical mathematical physics. He invented the sweeping method to solve the Dirichlet problem, proved for the first time the existence of infinitely many eigenvalues for this problem, and introduced some inequalities that are still the cornerstones of the modern theory of partial differential equations.

One of the last scientific conferences attended by Poincaré was the first Conseil Solvay, in Brussels, from October 30 to November 3, 1911, held at the Hôtel Métropole. Lorentz, Poincaré, Planck, Marie Curie, Einstein, Perrin, Langevin, Rutherford, and others were there to discuss the latest developments in quantum theory. During this conference Poincaré insisted on the main challenge of physics at that time: the construction of a coherent quantum theory:

> What struck me in the discussions we have just heard is to see the same theory depend sometimes on the principles of the old mechanics, and sometimes on new hypotheses that are its negation; one should not forget that any proposition can be proved, as soon as one uses in the proof two contradictory statements.

Back in Paris, he published on this hot topic, in February 1912, one of his last papers, which shows the necessity of quantum jumps in interpreting experimental data.

With forty-nine proposals between 1901 and 1912, Poincaré is the most-nominated scientist for the Nobel Prize in physics. The priority given to experimental physicists, Mittag-Leffler's enemies at the Swedish Academy of Science, and the untimely death of Poincaré prevented him from adding the Nobel Prize to his amazing list of scientific awards.

## Celestial Mechanics and Topology

After Tisserand's sudden death in 1896, Poincaré accepted, at the request of his dean, Darboux, the chair of theoretical astronomy and celestial mechanics. In academic affairs, Poincaré never behaved like a prima donna, and he gave priority to the benefit of the institutions. Again, his lectures were published: one volume on the shapes of equilibrium of rotating fluids; a three-volume set on celestial mechanics developing perturbation methods, lunar theory, and a study of tides based on Fredholm integral equations; and a last volume on cosmogonic hypotheses. But his most famous publication in this area is the immortal *Méthodes nouvelles de la mécanique céleste*, published between 1892 and 1899, a widely expanded version of the memoir recognized by King Oscar's Prize.

The turn of the century also saw the publication of a series of six long memoirs on *Analysis situs*, i.e., on algebraic topology, where geometrical properties in arbitrary dimensions are deduced from those of associated algebraic structures. The motivation came from the study of nonlinear differential equations and of the three-body problem, but the theory is developed for its own sake with applications to algebraic geometry. Between 1892 and 1901, Poincaré created almost ex nihilo the basic tools of algebraic topology: the fundamental group, simplicial homology, the Euler-Poincaré formula, and the duality principle. He even sketched de Rham cohomology, and after proving that

> any compact and simply connected 2-dimensional manifold is homeomorphic to the usual sphere,

he stated a famous conjecture:

> any compact and simply connected 3-dimensional manifold is homeomorphic to the 3-sphere.

This is today one of the seven famous million-dollar problems of the Clay Mathematics Institute and may have been proved very recently by the Russian mathematician Perelman. In another direction, Poincaré also initiated the modern theory of dimension, and in a moving paper in the year of his death, he described a fixed-point theorem for continuous area-preserving mappings of the annulus that rotate the two boundaries in opposite directions. He knew his proof was incomplete but feared he had not enough time left to fix it. It was fixed by George D. Birkhoff in 1913, and generalizing Poincaré-Birkhoff's theory is nowadays a very active area in Hamiltonian dynamics and symplectic geometry.

## Scientific Philosophy and the Académie Française

Besides technical works, Poincaré regularly published papers in popular science and philosophy journals. He discussed the role of logic in mathematics, the birth of set theory, the foundations of arithmetic, geometry and mechanics, and recent developments in physics. In 1902 the editor Flammarion convinced Poincaré to collect and edit this material for his famous series Bibliothèque de Philosophie Scientifique. The first volume, *La Science et l'Hypothèse*, was published in 1902, followed by *La Valeur de la Science* in 1905, and *Science et Méthode* in 1908. A posthumous volume, *Dernières Pensées*, came out in 1913, and an aborted fifth volume was published in 2002.

The four orange-covered books are often reprinted and have been translated into many languages. Written in a witty style, they differ from books written in the usual philosophical language by frequently showing a sharp irony and a definite taste for paradox. Many philosophers have had difficulty understanding the books' ever-changing and self-criticizing mode of thought, which refuses to encapsulate the world in a single idea. Scientism is criticized by a first-class scientist, who defends the idea of a "convenient" model and can be truly understood only by readers fully aware of his scientific contributions.

Those popular books gave Poincaré an unwanted celebrity. In France the secularization of primary and secondary schools generated a strong tension between Catholics and radicals. While discussing the relativity of motion in mechanics, Poincaré wrote:

> Absolute space, that is to say, the point to which it would be necessary to refer the Earth to know whether it really moves, has no objective existence. The two propositions: "The Earth turns round" and "It is more convenient to suppose the Earth turns round" have the same meaning; there is nothing more in the one than in the other.

In Catholic circles this justified Galileo's condemnation by the church! Needless to say, Poincaré lost time and energy in refuting such tendentious interpretations.

Following a long tradition, the Académie Française elects a few scientists who have added style to their scientific discoveries. These scientists are helpful in writing the definitions of scientific words for the dictionary that the learned assembly publishes. d'Alembert, Condorcet, Laplace, Fourier, Bertrand, Poincaré, and Picard were members of the Académie. Since Picard's death in 1941, no mathematician has received this honor, showing that our community should care more about style.

Henri Poincaré was elected in 1908, at seat number 24, formerly occupied by Sully-Prudhomme. Following tradition, at his induction into the Académie on January 28, 1909, Poincaré had to present a eulogy for the poet. The story is pleasantly told in a chronicle of André Beaunier:

> He had, like a socialite writer, like a playwright, attracted the crowd. Algebra may become the new fashion this winter. The greatest living mathematician did not read his speech badly. From time to time, he gave the impression that he was thinking of something else, but when remembering his present adventure, he took a clearer and more colloquial tone. When a page was read, he looked quite happy and threw it rapidly behind him. At the end of his speech, he sat on the pile with great satisfaction.

The eulogy for Sully-Prudhomme is reproduced in the exquisite but obscure little volume *Savants et écrivains*, whose introduction remains the best description of Poincaré's idea of the activity of a scientist. Sully-Prudhomme was born in Paris in 1839. First attracted by science but prevented from entering the École Polytechnique due to an eye illness, he received a bachelor's degree in letters. After translating Lucrece, he dreamed of unifying poetry and science and wrote long philosophical poems that brought him in 1901 the first Nobel Prize in literature. In a sense Poincaré was lucky, as he himself observed:

> One will perhaps be surprised to learn that Sully-Prudhomme has left a long manuscript on the philosophy of mathematics; it looks like he was trying from the beginning to justify as much as possible my presence here.

Instead of taking advantage of this unexpected help, Poincaré carefully analyzed Sully-Prudhomme's poetry and philosophy, concluding in a way that underlined his own conception of philosophy:

> But I must stop, for there are in the philosophical vocabulary too many words ending in "iste", and this infinite multitude scares me. The soul of a true philosopher is a battlefield; this is not a peaceful monarchy where there is room for one master only.

## Poincaré and Public Affairs

The famous Dreyfus affair gave Poincaré another opportunity to leave his ivory tower. In 1894 the French Intelligence Service found a memorandum sent to the German military attaché in Paris announcing the sending of confidential information. An apparent similarity in handwriting led to the arrest of a French officer of Jewish origin, Alfred Dreyfus. He was declared guilty by a court-martial in 1895 and deported to the Ile du Diable in Guyane. France was soon divided into supporters and opponents of Dreyfus. After a long and passionate fight, a new trial took place in Rennes in 1899. The famous police expert Bertillon used pseudoscientific techniques and probability theory in his graphological analysis of the memorandum. He concluded his testimony as follows:

> In the collection of observations and agreements that constitute my demonstration, there is no place for doubt; and this is with not only theoretical but material certitude, that with the feeling of responsibility following from such an absolute certitude, in all honesty, I affirm, today as in 1894, under oath,

> that the memorandum is the work of the defendant.

Such a statement was philosophically unbearable to Poincaré. In a letter written at the request of Painlevé and read to the court, he strongly reacted against the use of probability theory in Bertillon's conclusions:

> Nothing in it has any scientific character. I do not know if the defendant will be sentenced, but if he is, it must be on other evidence. It is impossible that such an argument makes any impression on free-minded people who have received a solid scientific education.

But the court-martial again declared Dreyfus guilty, this time with mitigating circumstances. Dreyfus got a presidential pardon, but his supporters again obtained an appeal in 1904. Poincaré concluded a long report, jointly written with Appell and Darboux, as follows:

> All those systems are absolutely deprived of any scientific value:
>
> 1. because the application of probability theory to those questions is not legitimate;
>
> 2. because the reconstruction of the memorandum is false;
>
> 3. because the rules of probability theory have not been correctly applied.
>
> In a word, because the authors have wrongly argued on the basis of false documents.

The conclusions of this report took a full page out of the twenty-one presenting reasons adducing the court's decision, which declared Dreyfus innocent and restored his honor and his rights.

In contrast to other contemporary scientists—such as Painlevé, Hadamard, Borel, Perrin, or Langevin—Poincaré always refused any political engagement or duty. In 1904, in response to an inquiry of the *Revue bleue*, he expressed his feelings about politics in his usual ironic style:

> Politics is nowadays a profession that entirely absorbs a man; any scientist who wants to dedicate himself to politics must give up his vocation; if he really wants to be useful to the country, he must give half of his time to the affairs of the Republic; if he wants to keep his seat, he must give the other half to the affairs of his electors; nothing will be left for science. It would therefore be inopportune if all scientists would aim at the Parliament, because then there would be no scientists anymore. One can be resigned to, or even rejoice not only for the country, but for science itself, the sacrificing from time to time of one of us, who is more able to be understood by the multitude or the assemblies. After all science needs somebody to defend its interests.

But Poincaré never refused any engagement or duty in the administration and organization of science. The list of his responsibilities in this respect covers two long pages. The astronomer J. Levy expressed his regrets about the time lost by Poincaré in those activities, especially in the last ten years of his life:

> Maybe one should regret that, from this time, the increasingly heavy duties he conscientiously accepts no longer give him time to polish his works. He gives a part of the best of himself to the numerous academies, associations, councils, and committees that have solicited the favor of receiving him. He consumes himself in tasks that do not fit his measure. For example, as chairman of the Commission devoted to organizing the revision of the arc of meridian in Quito, he writes himself, from 1900 to 1905, all the corresponding reports. In 1900, he discusses the savings that can be had when buying the she-mules; in 1902, the measures to be taken to remedy the destruction of the geodesic signals by the Indians; in 1905, the reproduction in colored pictures of the insects found by the expedition.

His repute and the attraction that events in the heavens have for the multitude constantly attracted to him journalists short of copy. When the exceptionally rainy year 1910 was linked in the media to the passage of a comet, Poincaré reacted with humor, referring to the tradition of connecting the production of good wine, not water, to the presence of a comet!

## Poincaré and Science

Some superficial readers confuse Poincaré's philosophical ideas about science, his so-called conventionalism, with a skepticism about science. All we have said about his activity contradicts such an assertion and proves the French mathematician's deep feelings about and complete devotion to scientific activity. As Émile Borel wrote in 1954:

> Some have considered Poincaré a skeptic, while others have considered him the forerunner of axiomatic methods; but he would have refused to be put in the service of any sect, even if this sect follows his thought. For him, the moral of the scientist can be summarized in a law that the usual ethics condemns: the aim justifies the means. The aim is the knowledge of the Universe, it is the agreement between the numerical results deduced from the formulas and the numbers written by the physicists and the astronomers in their observation books. The means, for the mathematician, are the formulas and a language he has the right to create at his own convenience.

Poincaré expressed strong feelings about the freedom of science. In 1909, when he was promoted to doctor honoris causa at the 75th anniversary of the Université Libre de Bruxelles, he said:

> Freedom is for Science what air is for animal; deprived of this freedom, it dies from suffocation, like a bird deprived of oxygen. And this freedom must be without limits, because, if one wants to impose limits, one gets a half-science only, and half-science is no longer science, because it can be, it necessarily is, a false science. Thought must never be subordinated to any dogma, political party, passion, interest, preconceived idea, to anything indeed, except the facts themselves, because, for science, to be subordinated means to die.

The last sentence is reproduced on the walls of the main building of the University of Brussels.

Poincaré often insisted on the aesthetic motivations of scientific activity:

> The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living. Of course I do not speak here of that beauty that strikes the senses, the beauty of quality and appearances; not that I undervalue such beauty, far from it, but it has nothing to do with science; I mean that deeper beauty coming from the harmonious order of the parts, and that a pure intelligence can grasp.

Some of the mathematical considerations of Poincaré have inspired various trends in modern art.

Poincaré, the graduate of the Polytechnique who contributed so much to the applications of science, unambiguously insists on the essential long-term investment of fundamental research:

> The scientist must not dally in realizing practical aims. He no doubt will obtain them, but must obtain them in addition. He never must forget that the special object he is studying is only a part of this big whole, which must be the sole motive of his activity. Science has had marvelous applications, but a science that would only have applications in mind would not be science anymore, it would be only cookery.

Those words remain more than ever of crucial importance. A victim of the increasing resignation of the public authority, science is increasingly threatened in its freedom by economic forces dominated by immediate profit, and fundamental research is constantly identified, in the minds of decision makers and in public agency budgets, with development.

## Conclusion

When Poincaré suddenly died July 17, 1912, from an embolism following surgery, the scientific world was far from ready to benefit from his legacy. According to the great French mathematician Jean Leray:

> Very few men were able to follow his thoughts; he had no students. After one century of mathematical work, we can understand his thoughts more easily, speak about them in a more familiar way; but the more we approach them, the more we admire and respect them.

Another great French mathematician, André Weil, insists on the modern aspect of Poincaré's work:

> On this point like on many other ones, I hope to have shown you that Poincaré's work belongs not only to the history of our science; it also belongs to the most vibrant mathematics of today.

I leave the last words to the famous mathematical physicist David Ruelle:

> Mathematical physics tries to understand a world of unknown complexity with tools of known limitation. This requires boldness, and modesty. Obviously Henri Poincaré lacked neither of these two qualities.

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