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Hughes, Bruce [Hughes, C. Bruce] (1-VDB);
Ranicki, Andrew (4-EDIN)

★**Ends of complexes. (English. English summary)**

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The topology of high-dimensional manifolds is a subject of substantial complexity and depth which, compared to other parts of mathematics of comparable importance, suffers from a dearth of monographs. The book under review is therefore a welcome addition to the literature. To illustrate its contents we will first give a (non-technical) indication of the theorem around which the book is built. For this we need three definitions. (i) A (connected) "band" is a compact space M together with a map $c\colon M \to S^1$ (the circle) such that the obvious infinite cyclic covering space $\overline{M}$ is finitely dominated. (ii) An end of an open n-manifold W is "tame" if it admits a sequence of finitely dominated open neighborhoods $\{U_i\}$ whose inclusions are π_1-isomorphisms. (iii) An end of W is "collared" if it has an open neighborhood of the form $M \times \mathbf{R} \subset W$ where M is a closed $(n-1)$-manifold.

Let $(W, \partial W)$ be a manifold of dimension at least six, having compact boundary and one end. It is proved in Siebenmann's well-known but unpublished thesis that this end is collared if and only if it is tame and a certain $\tilde{K}_0$-obstruction vanishes. The main theorem of the book under review (Theorem 19 of the Introduction, with proof finally achieved in Section 17 of the text) addresses the question: what geometrical statement about the end is equivalent to tameness even when the obstruction is nontrivial? The answer is that there should be an open neighborhood $\overline{M}$ of the end, on which the group $\mathbf{Z}$ of integers acts as covering translations, so that the associated map $c\colon \overline{M}/\mathbf{Z} \equiv M \to S^1$ is a band. It then follows that these covering transformations are isotopic to the identity, that $M \times S^1 \to M \xrightarrow{c} S^1$ is homotopic to a fiber bundle, and $\overline{M} \times S^1$ is homeomorphic to $M \times \mathbf{R}$, thus producing a collaring of the corresponding end of $W \times S^1$. Moreover, the relevant K-theoretic obstructions (Wall, Siebenmann, Farrell) are nicely and precisely related to one another.

This theorem is also due to Siebenmann (announced but unpublished) and, as the authors point out, a proof can be derived from published work of L. Siebenmann, L. Guillou and H. Hahl [Ann. Sci. Ecole Norm. Sup. (4) 7 (1974), 431–461 (1975); MR 50#14766]. However, the authors take a different approach, providing a proof which constitutes a well-designed advanced course in geometric topology.

The last part of the book generalizes the subject matter of the main theorem to algebra, replacing manifolds by chain complexes, etc.

The authors state their prerequisites thus: "We assume familiarity with the basic language of high-dimensional manifold theory, and the standard applications of algebraic K- and L-theory to manifolds, but otherwise we have tried to be as self-contained as possible." The contents can best be indicated by listing section headings: end spaces, limits, homology at infinity, cellular homology, homology of covers, projective class and torsion, forward tameness, reverse tameness, homotopy at infinity, projective class at infinity, infinite torsion, forward tameness is a homotopy pushout, infinite cyclic covers, the mapping torus, geometric ribbons and bands, approximate fibrations, geometric wrapping up, geometric relaxation, homotopy theoretic twist glueing, homotopy theoretic wrapping up and relaxation, polynomial extensions, algebraic bands, algebraic tameness, relaxation techniques, algebraic ribbons, algebraic twist glueing, and wrapping up in algebraic K- and L-theory. *Ross Geoghegan* (1-SUNY2)