

Errata for High-dimensional Knot Theory
by Andrew Ranicki
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This list contains corrections of misprints/errors in the book. Please let me know of any further misprints/errors by e-mail to a.ranicki@ed.ac.uk
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<http://www.maths.ed.ac.uk/~aar/books/knot.pdf>

A.A.R. 23.4.2017

- p. V The following statement of Frank Adams makes the dedication of the book to him even more appropriate: *Of course, from the point of view of the rest of mathematics, knots in higher-dimensional space deserve just as much attention as knots in 3-space* (Article on topology, in 'Use of Mathematical Literature' (Butterworths (1977))).
- p. XVIII l. 8 Remove "that of".
- p. XXI l. 9 for a homology framed knot
- p. XXIV l. -3 $\pi_1(X)$
- p. XXV l. -2 C_{4*+1}, C_{4*+3}
- p. XXVI l. 10 $\pi_1(F) = \{1\}$
- p. XXVIII l. 2 chain *complex*
- p. 9 l. 8 i.e. $f_* : \pi_1(X) \rightarrow \pi_1(X)$ is an isomorphism and
- $$\pi_1(T(f)) = \{gz^j \mid j \in \mathbb{Z}\}$$
- with $z^{-1}gz = f_*(g)$
- p. 10 l. -7 $(f(x), n + 1, 0)$
- p. 16 l. -15 If X has a finite 2-skeleton
- p. 29 l. -10 for
- p. 31 l. 11 Bass [13, XII.7.4]
- p. 34 l. 6 4.5, 5.5 (i)
- p. 34 l. 19 As for 5.10
- p. 35 l. -12 $\text{Nil}_0(A)$
- p. 47 l. -6 $z^{-N_2^+} b_2$
- p. 47 l. -2 $N^+ = \sum_{j=1}^r N_j^+$

p. 49 ll. 1,2 Replace by

$$d_E\left(\sum_{j=-N_r^+}^{N_r^-} z^j F_r\right) \subseteq \sum_{j=-N_{r-1}^+}^{N_{r-1}^-} z^j F_{r-1} \quad (r = n, n-1, \dots, 0)$$

for some integers $N_r^+, N_r^- \geq 0$ (starting with $N_n^+ = N_n^- = 0$, for example).

p. 73 l. 17 Replace [297, 9.14] by [297, 7.9]

p. 77 Example 9.15 As before, let $A = B[z]$, and let Σ be the set of B -invertible square matrices in A , with $A \rightarrow B; z \rightarrow 0$. The identity

$$\Sigma^{-1}A = (1 + zB[z])^{-1}B[z]$$

is correct for commutative B . For noncommutative B it should be replaced by the direct limit

$$\Sigma^{-1}A = \lim n R_n$$

with R_0, R_1, R_2, \dots the rings defined inductively by

$$\begin{aligned} R_0 &= B[z] \quad , \quad p_0 : R_0 \rightarrow B ; z \rightarrow 0 \quad , \\ R_n &= (1 + \ker(p_{n-1}))^{-1}R_{n-1} \quad , \quad p_n : R_n \rightarrow B ; z \rightarrow 0 \quad . \end{aligned}$$

(In particular, $R_1 = (1 + zB[z])^{-1}B[z]$.) Given an $n \times n$ matrix $b = (b_{ij})$ in B let $b' = (b'_{ij})$ be the $(n-1) \times (n-1)$ matrix in R_1 defined by the matrix equation

$$\begin{aligned} &\begin{pmatrix} 1 & 0 & \dots & zb_{1n}(1-zb_{nn})^{-1} \\ 0 & 1 & \dots & zb_{2n}(1-zb_{nn})^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} 1-zb_{11} & -zb_{12} & \dots & -zb_{1n} \\ -zb_{21} & 1-zb_{22} & \dots & -zb_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -zb_{n1} & -zb_{n2} & \dots & 1-zb_{nn} \end{pmatrix} \\ &= \begin{pmatrix} 1-zb'_{11} & -zb'_{12} & \dots & 0 \\ -zb'_{21} & 1-zb'_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -zb_{n1} & -zb_{n2} & \dots & 1-zb_{nn} \end{pmatrix} . \end{aligned}$$

Assuming inductively that it is possible to invert $1 - zb'$ in R_{n-1} it is now possible to invert $1 - zb$ in R_n . The inclusion $B[z] \rightarrow B[[z]]$ factors as

$$B[z] \rightarrow \Sigma^{-1}B[z] \rightarrow B[[z]]$$

so that $B[z] \rightarrow \Sigma^{-1}B[z]$ is injective. The morphism $\Sigma^{-1}B[z] \rightarrow B[[z]]$ is an injection for commutative B , but it is not known if it is an injection also in the noncommutative case.

p. 82 l. 4 Remove “the $A[z^{-1}]$ -module subcomplex of”.

p. 83 l. 3 “ $A[z]$ -module morphisms”.

p. 84 l. 7 Proposition 10.9. For noncommutative A the right hand side of the identity

$$\tilde{\Omega}_+^{-1}A[z] = (1 + zA[z])^{-1}A[z]$$

should be corrected as for Example 9.15 above.

p. 90 l. 12 Replace ”[240, Chap. 8]” by ”[244, Chap. 8]”

p. 92 l. 12 $\tau(1 - f + zf : \Pi^{-1}P[z, z^{-1}] \rightarrow \Pi^{-1}P[z, z^{-1}])$

p. 99 l. -1 $\in A[z]$

p. 102 l. -8 [5, 1.10]

p. 102 l. -4 Replace “If the additive group of A is torsion-free ...” by “If $\mathbb{Q} \subseteq A \dots$ ”

p. 111 l. 8 Remove “ (P, f) is”

p. 120 l. -8 Replace 13.2 by 13.1

p. 121 l. 2 $n - m \geq 1$

p. 121 l. 5, l. 10 $\Omega^{-1}A[z, z^{-1}]$

p. 121 l. -3 $A[z, z^{-1}]$ -module

p. 123 l. -12 $P^{-1}A[z, z^{-1}] = \Pi^{-1}A[z, z^{-1}]$

p. 125 l. -13 Should read “ $(P, h) + (P', h') = (P \oplus P', \begin{pmatrix} g & h \\ 0 & g' \end{pmatrix})$ ”

p. 135 Chapter 14 The group $\widehat{W}(A)^{ab}$ should be replaced by the image of $\widehat{W}(A)$ in $K_1(A[[z]])$, since the kernel of the morphism

$$\begin{aligned} \tilde{\Delta}_+ : \widehat{W}(A) &\rightarrow K_1(A[[z]]) ; \\ (a_1, a_2, \dots) &\mapsto \tau(1 + \sum_{j=1}^{\infty} a_j z^j : A[[z]] \rightarrow A[[z]]) \end{aligned}$$

is in general larger than $[\widehat{W}(A), \widehat{W}(A)]$. See the paper

A.Pajitnov and A.Ranicki, *The Whitehead group of the Novikov ring*
<http://arXiv.org/abs/math.AT.0012031>, *K-theory* 21, 325–365 (2000).

Similarly, $W(A)^{ab}$ should be replaced by the image of $W(A)$ in $K_1(\tilde{\Omega}_+^{-1}A[z])$, since the kernel of the morphism

$$\begin{aligned} \tilde{\Delta}_+ : W(A) &\rightarrow K_1(\tilde{\Omega}_+^{-1}A[z]) ; \\ (a_1, a_2, \dots) &\mapsto \tau(1 + \sum_{j=1}^{\infty} a_j z^j : \tilde{\Omega}_+^{-1}A[z] \rightarrow \tilde{\Omega}_+^{-1}A[z]) \end{aligned}$$

is in general larger than $[W(A), W(A)]$.

p. 136 l. -7 Replace by “If A is a commutative ring such that $\mathbb{Q} \subseteq A$ ”.
The isomorphism inverse to

$$\begin{aligned} \prod_1^\infty A &\rightarrow \widehat{W}(A); \\ (a_1, a_2, a_3, \dots) &\mapsto \exp\left(\int_0^z (a_1 - a_2s + a_3s^2 - \dots) ds\right) \\ &= \exp\left(a_1z - \frac{a_2z^2}{2} + \frac{a_3z^3}{3} - \dots\right) \end{aligned}$$

is given by

$$\begin{aligned} \widehat{W}(A) &\rightarrow \prod_1^\infty A; \quad q(z) = 1 + b_1z + b_2z^2 + \dots \mapsto \\ \frac{q'(z)}{q(z)} &= \frac{b_1 + 2b_2z + 3b_3z^2 + \dots}{1 + b_1z + b_2z^2 + \dots} = a_1 - a_2z + a_3z^2 - \dots \rightarrow (a_1, a_2, a_3, \dots) \end{aligned}$$

([5, 6.13]). The reverse characteristic polynomial of an endomorphism $f : P \rightarrow P$ of a f.g. projective A -module P

$$\begin{aligned} \widetilde{\text{ch}}_z(P, f) &= \det(1 - zf : P[z] \rightarrow P[z]) = \exp\left(-\sum_{i=1}^\infty \frac{\text{tr}(f^i)}{i} z^i\right) \\ &\in 1 + zA[z] \subset W(A) \subset \widehat{W}(A) \end{aligned}$$

(cf. Example 19.16) has image $(-\text{tr}(f), \text{tr}(f^2), -\text{tr}(f^3), \dots) \in \prod_1^\infty A$. For any polynomial of the type

$$p(z) = 1 + \sum_{i=1}^d b_i z^i \in 1 + zA[z] \subset W(A)$$

the image $(a_1, a_2, a_3, \dots) \in \prod_1^\infty A$ has components

$$a_i = (-)^i \text{tr}(f^i) \in A$$

with

$$f = z : P = A[z]/(z^d p(z^{-1})) \rightarrow P = A[z]/(z^d p(z^{-1}))$$

such that $\widetilde{\text{ch}}_z(P, f) = p(z)$.

p. 136 l. -1 2.2.5

p. 141 l. -8 For noncommutative A the right hand side of the identity

$$\widetilde{\Omega}_+^{-1} A[z] = (1 + zA[z])^{-1} A[z]$$

should be corrected as for Example 9.15 above.

- p. 142 l. 16 This ζ -function agrees with the ζ -function of Geoghegan and Nicas (*Trace and torsion in the theory of flows*, Topology 33, 683–719 (1994)).
- p. 153 l. –3 [244, Chap.20]
- p. 160 l. 17 *structure ϕ_B on $B \otimes_A C$.*
- p. 172 l. 3 $D[z, z^{-1}] \rightarrow D[z, z^{-1}]$
- p. 173 l. 13 for each $P^{-1}E_r$
- p. 175 l. –5 the reduced chain complexes
- p. 207 l. 5 $d_{C^{n-*}} = (-)^r(d_C)^*$
- p. 211 l. 5 A *cobordism* of ϵ -symmetric Poincaré complexes (C, ϕ) , (C', ϕ') is an ϵ -symmetric Poincaré pair $((f, f') : C \oplus C' \rightarrow D, (\delta\phi, \phi \oplus -\phi'))$.
- p. 223 l. 12,13 f is i -connected, $\partial_0 f, \partial_1 f$ are $(i-1)$ -connected
- p. 241 l. –6 Szczarba
- p. 248 l. –7 i -connected
- p. 249 l. 14 i -connected
- p. 258 l. –6 $g \times 1$
- p. 261 l. 2 A -finitely dominated
- p. 269 Proposition 25.4 The stated exact sequence in the ϵ -symmetric case

$$\dots \rightarrow L_{jU}^n(A, \epsilon) \xrightarrow{i} L_{\partial^{-1}U}^n(\Sigma^{-1}A, \epsilon) \xrightarrow{\partial} L_U^n(A, \Sigma, \epsilon) \xrightarrow{j} L_{jU}^{n-1}(A, \epsilon) \rightarrow \dots$$

should be replaced in general by the exact sequence

$$\dots \rightarrow L_{jU}^n(A, \epsilon) \xrightarrow{i} \Gamma_{\partial^{-1}U}^n(A \rightarrow \Sigma^{-1}A, \epsilon) \xrightarrow{\partial} L_U^n(A, \Sigma, \epsilon) \xrightarrow{j} L_{jU}^{n-1}(A, \epsilon) \rightarrow \dots$$

See the paper

Noncommutative localization and chain complexes I. Algebraic K- and L-theory by A. Neeman and A. Ranicki, <http://arXiv.org/abs/math.RA.0109118> for the proof that the natural map of ϵ -symmetric groups

$$\Gamma_{\partial^{-1}U}^n(A \rightarrow \Sigma^{-1}A, \epsilon) \rightarrow L_{\partial^{-1}U}^n(\Sigma^{-1}A)$$

is an isomorphism if $\text{Tor}_*^A(\Sigma^{-1}A, \Sigma^{-1}A) = 0$ for $* \geq 1$ (e.g. if $\Sigma^{-1}A$ is a flat A -module, as is the case for a two-sided Ore localization). There is no problem in the ϵ -quadratic case, by virtue of Vogel [296], [297], with the natural maps

$$\Gamma_n^{\partial^{-1}U}(A \rightarrow \Sigma^{-1}A, \epsilon) \rightarrow L_n^{\partial^{-1}U}(\Sigma^{-1}A)$$

isomorphisms, and with an exact sequence

$$\dots \rightarrow L_n^{jU}(A, \epsilon) \xrightarrow{i} L_n^{\partial^{-1}U}(\Sigma^{-1}A, \epsilon) \xrightarrow{\partial} L_n^U(A, \Sigma, \epsilon) \xrightarrow{j} L_{n-1}^{jU}(A, \epsilon) \rightarrow \dots$$

- p. 275 l. -13 $1 + T_\epsilon : L_n^U(A[s], \epsilon) \rightarrow L_V^n(A[s], \epsilon)$
- p. 287 l. 3 Replace “will” by “we shall”
- p. 290 l. -11,12 Replace conditions (a),(b) by the single condition ‘ $\lambda(x, y) = 0$ for all $x, y \in K$ ’.
- p. 291 l. -5 Should read ”[235, Chap. 9]”
- p. 303 l. 4 *to* describe
- p. 312 l. 14 *i*-connected
- p. 313 l. 8 *i*-connected
- p. 313 l. 10 $(i + 1)$ -connected
- p. 314 l. 5 $(\mathbb{Z}^\ell, \lambda)$
- p. 314 l. 16 Cyclic branched covers
- p. 340 l. 15 Replace ”28.15” by ”28.17”
- p. 342 l. -6 Replace “band” by “complex”
- p. 344 l. 8 Replace the text of Example 28.31 by
 “The 0-dimensional asymmetric L -group $L\text{Asy}_q^0(A)$ ($q = s, h, p$) is the *Witt group* of nonsingular asymmetric forms (L, λ) over A , with $\lambda : L \rightarrow L^*$ an isomorphism. Such a form is *metabolic* if there exists a *lagrangian*, i.e. a direct summand $K \subset L$ such that $K = K^\perp$, with

$$K^\perp = \{x \in L \mid \lambda(x)(K) = 0\} ,$$

in which case

$$(L, \lambda) = 0 \in L\text{Asy}_q^0(A) .$$

A nonsingular asymmetric form (L, λ) is such that $(L, \lambda) = 0 \in L\text{Asy}_q^0(A)$ if and only if it is stably metabolic, i.e. there exists an isomorphism

$$(L, \lambda) \oplus (M, \mu) \cong (M', \mu')$$

for some metabolic $(M, \mu), (M', \mu')$. A 0-dimensional asymmetric Poincaré complex (C, λ) is the same as a nonsingular asymmetric form (L, λ) with $L = C^0$. For a 1-dimensional asymmetric Poincaré pair $(f : C \rightarrow D, (\delta\lambda, \lambda))$ with $D_r = 0$ for $r \neq 0$ there is defined an exact sequence

$$0 \rightarrow D^0 \xrightarrow{f^*} C^0 \xrightarrow{f\lambda} D_0 \rightarrow 0$$

so that $K = \text{im}(f^* : D^0 \rightarrow C^0) \subset L = C^0$ is a lagrangian of (C^0, λ) , and the pair is the same as a nonsingular asymmetric form together with a lagrangian. More generally, suppose given a 1-dimensional asymmetric

Poincaré pair $(f : C \rightarrow D, (\delta\lambda, \lambda))$. The mapping cone of the chain equivalence $(\delta\lambda \ f\lambda) : \mathcal{C}(f)^{1-*} \rightarrow D$ is an exact sequence

$$0 \rightarrow D^0 \xrightarrow{g} C^0 \oplus D^1 \oplus D_1 \xrightarrow{h} D_0 \rightarrow 0$$

with

$$g = \begin{pmatrix} f^* \\ d^* \\ \delta\lambda \end{pmatrix} : D^0 \rightarrow C^0 \oplus D^1 \oplus D_1 ,$$

$$h = (f\lambda \ \delta\lambda \ d) : C^0 \oplus D^1 \oplus D_1 \rightarrow D_0 .$$

However (as pointed out by Joerg Sixt), in general

$$h \neq g^* \begin{pmatrix} \lambda & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} : C^0 \oplus D^1 \oplus D_1 \rightarrow D_0$$

so that g is not the inclusion of a lagrangian in $(C^0, \lambda) \oplus (D^1 \oplus D_1, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})$.

To repair this, proceed as follows. Use the chain equivalences

$$\begin{pmatrix} \delta\lambda \\ \lambda f^* \end{pmatrix} : D^{1-*} \rightarrow \mathcal{C}(f) , \quad (\delta\lambda \ f\lambda) : \mathcal{C}(f)^{1-*} \rightarrow D$$

to define a chain equivalence

$$i = T \begin{pmatrix} \delta\lambda \\ \lambda f^* \end{pmatrix} (\delta\lambda \ f\lambda)^{-1} : D \rightarrow D .$$

In order to prove that (C^0, λ) is stably metabolic, it is convenient to replace D by a chain equivalent complex for which i is (chain homotopic to) an isomorphism. The exact sequence

$$0 \rightarrow D_1 \xrightarrow{\begin{pmatrix} d \\ i_1 \end{pmatrix}} D_0 \oplus D_1 \xrightarrow{\begin{pmatrix} i_0 & -d \end{pmatrix}} D_0 \rightarrow 0$$

splits, so there exists an A -module morphism $(\alpha \ \beta) : D_0 \oplus D_1 \rightarrow D_1$ such that

$$(\alpha \ \beta) \begin{pmatrix} d \\ i_1 \end{pmatrix} = \alpha d + \beta i_1 = 1 : D_1 \rightarrow D_1 .$$

The 1-dimensional A -module chain complex D' defined by

$$d' = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix} : D'_1 = D_1 \oplus D_1 \rightarrow D'_0 = D_0 \oplus D_1$$

is such that the inclusion $D \rightarrow D'$ and the projection $D' \rightarrow D$ are inverse chain equivalences. The chain isomorphism $i' : D' \rightarrow D'$ defined by

$$\begin{aligned} i'_0 &= \begin{pmatrix} i_0 & -d \\ \alpha & \beta \end{pmatrix} : D'_0 = D_0 \oplus D_1 \rightarrow D'_0 = D_0 \oplus D_1 , \\ i'_1 &= \begin{pmatrix} i_1 & -1 \\ \alpha d & \beta \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & \beta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -i_1 & 1 \end{pmatrix} \\ & : D'_1 = D_1 \oplus D_1 \rightarrow D'_1 = D_1 \oplus D_1 \end{aligned}$$

is such that

$$i : D \rightarrow D' \xrightarrow{i'} D' \rightarrow D .$$

Replacing D by D' and reverting to the previous notation, it may thus be assumed that $i : D \rightarrow D$ is an isomorphism. Choose a chain homotopy

$$(j \ k) : i(\delta\lambda \ f\lambda) \simeq T \begin{pmatrix} \delta\lambda \\ \lambda f^* \end{pmatrix} : \mathcal{C}(f)^{1-*} \rightarrow D .$$

The nonsingular asymmetric form defined by

$$(M, \mu) = (C^0 \oplus D^1 \oplus D_1, \begin{pmatrix} \lambda & k^* & 0 \\ 0 & j^* & 1 \\ 0 & i_1^* & 0 \end{pmatrix})$$

is such that

$$h = g^* \mu : M = C^0 \oplus D^1 \oplus D_1 \rightarrow D_0$$

so that $g : D^0 \rightarrow M$ is the inclusion of a lagrangian and (M, μ) is metabolic. The A -module morphism

$$C^0 \oplus D_1 \rightarrow C^0 \oplus M = C^0 \oplus C^0 \oplus D^1 \oplus D_1 ; (x, y) \mapsto (x, x, 0, y)$$

is the inclusion of a lagrangian in $(C^0, \lambda) \oplus (M, -\mu)$, so that (C^0, λ) is stably metabolic."

p. 346 l. 16 Replace 25.11 by 26.11

pp. 347–348 The construction of (C', λ') and (C'', λ'') is not correct in general; these complexes should be replaced by the following $(i-1)$ -connected n -dimensional asymmetric Poincaré complex (C', λ') cobordant to the given n -dimensional asymmetric Poincaré complex (C, λ) with $n = 2i$ or $2i + 1$. Choose a chain homotopy inverse $\mu : C \rightarrow C^{n-*}$ for $\lambda : C^{n-*} \rightarrow C$ and a chain homotopy

$\nu : \mu\lambda \simeq 1 : C^{n-*} \rightarrow C^{n-*}$, and set

$$d_{C'} = \begin{cases} \begin{pmatrix} d_C & (-)^{r-1}\lambda \\ 0 & d_C^* \end{pmatrix} : C'_r = C_r \oplus C^{n-r+1} \rightarrow C'_{r-1} = C_{r-1} \oplus C^{n-r+2} \\ \text{if } r \leq i-1 \\ \begin{pmatrix} d_C & (-)^{i-1}\lambda & 0 \\ 0 & d_C^* & 0 \end{pmatrix} : C'_r = C_i \oplus C^{i+1} \oplus C_{i+1} \rightarrow C'_{r-1} = C_{i-1} \oplus C^{i+2} \\ \text{if } n = 2i \text{ and } r = i \\ \begin{pmatrix} d_C & 0 \\ 0 & 0 \\ (-)^i \lambda^* \mu & d_C \end{pmatrix} : C'_r = C_{i+1} \oplus C_{i+2} \rightarrow C'_{r-1} = C_i \oplus C^{i+1} \oplus C_{i+1} \\ \text{if } n = 2i \text{ and } r = i+1 \\ \begin{pmatrix} d_C & 0 \\ 0 & 0 \end{pmatrix} : C'_r = C_{i+1} \oplus C_{i+2} \rightarrow C'_{r-1} = C_i \oplus C^{i+2} \\ \text{if } n = 2i+1 \text{ and } r = i+1 \\ \begin{pmatrix} d_C & 0 \\ (-)^{r-1} \lambda^* \mu & d_C \end{pmatrix} : C'_r = C_r \oplus C_{r+1} \rightarrow C'_{r-1} = C_{r-1} \oplus C_r \\ \text{otherwise,} \end{cases}$$

$$\lambda' = \begin{cases} \begin{pmatrix} \lambda & 0 \\ 0 & \mu^* \lambda \end{pmatrix} : C'^{n-r} = C^{n-r} \oplus C^{n-r+1} \rightarrow C'_r = C_r \oplus C^{n-r+1} \\ \text{if } r \leq i-1 \\ \begin{pmatrix} \lambda & 0 & 0 \\ 0 & 0 & \mu^* \lambda \\ \lambda^* \nu & 1 & 0 \end{pmatrix} : \\ C'^{n-r} = C^i \oplus C_{i+1} \oplus C^{i+1} \rightarrow C'_r = C_i \oplus C^{i+1} \oplus C_{i+1} \\ \text{if } n = 2i \text{ and } r = i \\ \begin{pmatrix} \lambda & 0 \\ \lambda^* \nu & 1 \end{pmatrix} : C'^{n-r} = C^{n-r} \oplus C_{r+1} \rightarrow C'_r = C_r \oplus C_{r+1} \\ \text{otherwise.} \end{cases}$$

p. 355 ll. -1,-11,-12 $F \cup_{\partial} -N, T(h) \cup_{\partial} -(N \times S^1)$

p. 357 ll. 4,12 $F \cup -F$

p. 364 l. -13 $(i+1)$ -connected

p. 369 l. 12 $S^3 \times D^4 \cup_{h_1} -(S^3 \times D^4)$

- p. 369 l. -1 framed codimension 2
- p. 372 l. 3 replace "reverse" by "reduced"
- p. 374 l. 8 "twisted double bordism groups"
- p. 382 l. 17

$$\beta_s = \begin{pmatrix} \chi_s & (-)^s \phi_s \\ (-)^{n-r-1} \phi_s & (-)^{n-r+s} T_\epsilon \phi_{s-1} \end{pmatrix} :$$

$$B^{n-r+s} = C^{n-r+s} \oplus C^{n-r+s-1} \rightarrow B_r = C_r \oplus C_{r-1} .$$

- p. 411 l. 10 $\Omega_+^{-1} A[s]/A[s] = F(s)/F[s]$
- p. 421 l. 8 Terminology: the covering ϵ -symmetric complex in the sense of Definition 32.7 (i) is the ϵ -symmetrization of the ultraquadratic complex of Ranicki[237, p.820].
- p. 422 l. 9 Proof of 32.8 (ii): Since E is A -contractible the $A[z, z^{-1}]$ -module chain map $1 - z : E \rightarrow E$ is a chain equivalence. Define a homotopy equivalence $(E, \theta) \simeq U(\Gamma)$ by

$$(1 \oplus (1 + T_\epsilon))(1 - z)^{-1} : E \rightarrow \mathcal{C}(g - zh) ,$$

with $(1 - z)^{-1} : E \rightarrow E$ any chain homotopy inverse of $1 - z : E \rightarrow E$.

- p. 437 l. -2 In the proof of (ii) insert :
The natural $A[s]$ -module morphisms

$$A[s, s^{-1}, (1 - s)^{-1}] \rightarrow Q_A^{-1} A[s] , Q_{A, min}^{-1} A[s] \rightarrow Q_A^{-1} A[s]$$

are inclusions of submodules. For any elements

$$\frac{r(s)}{s^j (1 - s)^k} \in A[s, s^{-1}, (1 - s)^{-1}] , \frac{p(s)}{q(s)} \in Q_{A, min}^{-1} A[s]$$

such that

$$\frac{r(s)}{s^j (1 - s)^k} = \frac{p(s)}{q(s)} \in Q_A^{-1} A[s]$$

it follows from the minimality of $q(s)$ and the identity

$$p(s) s^j (1 - s)^k = q(s) r(s) \in A[s]$$

that $s^j (1 - s)^k$ divides $r(s)$, and hence that

$$A[s, s^{-1}, (1 - s)^{-1}] \cap Q_{A, min}^{-1} A[s] = A[s] \subset Q_A^{-1} A[s] .$$

- p. 438 l. -1 --9 Remove. ($\chi_{s, min}$ is no longer required).

p. 439 l. 12 The statement of Proposition 32.45 (i) is false as stated, and should be replaced by :

“The Blanchfield form is such that for any $x, y \in L$ the composite

$$\begin{aligned} & P_A^{-1}A[z, z^{-1}]/A[z, z^{-1}] \\ & \rightarrow P_F^{-1}[z, z^{-1}]/F[z, z^{-1}] = Q_{F, \min}^{-1}[s]/F[s] = F[s]_{(s, 1-s)}/F[s] \\ & \rightarrow F((s))/F[s] = s^{-1}F[[s^{-1}]] \end{aligned}$$

sends $\mu(i(x), i(y)) \in P_A^{-1}A[z, z^{-1}]/A[z, z^{-1}]$ to

$$\mu(i(x), i(y)) = \sum_{j=-\infty}^{-1} (\lambda + \epsilon \lambda^*)(x, f^{-j-1}(y)) s^j \in s^{-1}A[[s^{-1}]] \subset s^{-1}F[[s^{-1}]],$$

where $i : L \rightarrow M$ is the natural A -module morphism. In particular, μ determines λ by

$$\lambda : L \times L \xrightarrow{i \times si} M \times M \xrightarrow{\mu} P_A^{-1}A[z, z^{-1}]/A[z, z^{-1}] \rightarrow F((s))/F[s] \xrightarrow{\chi_s} F$$

with $s = (1 - z)^{-1} : M \rightarrow M$ and $\chi_s =$ coefficient of s^{-1} (31.20).”

Here is an explicit counterexample to the original statement of 32.45 (i). Let $A = \mathbb{Z}$, and for any $m \in \mathbb{Z}$ consider the skew-symmetric Seifert form over \mathbb{Z} defined in Example 42.2

$$(L, \lambda) = (\mathbb{Z} \oplus \mathbb{Z}, \begin{pmatrix} m & 0 \\ -1 & 1 \end{pmatrix})$$

with

$$f = (\lambda - \lambda^*)^{-1} \lambda = \begin{pmatrix} 1 & -1 \\ m & 0 \end{pmatrix} : L = \mathbb{Z} \oplus \mathbb{Z} \rightarrow L = \mathbb{Z} \oplus \mathbb{Z}$$

and Alexander polynomial

$$\Delta(z) = \det(1 - f + zf) = m(1 - z)^2 + z.$$

The corresponding symmetric Blanchfield form (M, μ) is given by

$$\begin{aligned} M &= \text{coker}(1 - f + zf) = \mathbb{Z}[z, z^{-1}]/\Delta(z), \\ \mu(x, y) &= \frac{(1-z)^2 xy}{\Delta(z)} \in P^{-1}\mathbb{Z}[z, z^{-1}]/\mathbb{Z}[z, z^{-1}]. \end{aligned}$$

In terms of $s = (1 - z)^{-1}$

$$\mu(x, y) = \frac{xy}{m + s(1 - s)} \in Q^{-1}\mathbb{Z}[s]/\mathbb{Z}[s].$$

If $m \neq 0$ then

$$\mu(1, 1) = \frac{1}{m + s(1 - s)} \notin Q_{\min}^{-1}\mathbb{Z}[s]/\mathbb{Z}[s],$$

since $s^2\Delta(1 - s^{-1}) = m + s(1 - s) \in \mathbb{Z}[s]$ is not minimal.

p. 439 l. 12 omit “the natural $A[s]$ -module morphism”

p. 440 Replace the proof of 32.45 (i) by :
 “Work in the completion $A[[s^{-1}]]$ to obtain

$$\begin{aligned}\mu(i(x), i(y)) &= (1-z)(\lambda + \epsilon\lambda^*)(x, (1-f+zf)^{-1}(y)) \\ &= s^{-1}(\lambda + \epsilon\lambda^*)(x, (1-s^{-1}f)^{-1}y) \\ &= \sum_{j=-\infty}^{-1} (\lambda + \epsilon\lambda^*)(x, f^{-j-1}(y))s^j \in A[[s^{-1}]]\end{aligned}$$

so that

$$\begin{aligned}\chi_s(\mu(i(x), i(y))) &= (\lambda + \epsilon\lambda^*)(x, y) , \\ \chi_s(\mu(i(x), si(y))) &= \lambda(x, y) \in A \subset F .”\end{aligned}$$

p. 450 l. -1 $1 \leq r \leq n$

p. 456 l. -2 $\delta\phi \in \text{Hom}_{\mathbb{Z}[\mathbb{Z}_2]}(W, C \otimes_A C)_{n+1}$

p. 457 l. 13 $Q^*(D, -\epsilon) = Q_{\text{end}}^*(D^\dagger, \xi, \epsilon)$

p. 467 l. 11 delete one “will”

p. 475 l. -8 “ $L\text{Aut}_p^n(A, \epsilon) = L_p^n(A, \epsilon) \oplus L\widetilde{\text{Aut}}_p^n(A, \epsilon)$ ”

p. 478 l. 11 26.11 (iii) instead of 25.11 (iii)

p. 479 l. 14 26.11 instead of 25.11

p. 485 l. -12 In 36.3 and 41.19

p. 487 l. 2 By 25.11 and 26.11

p. 488 l. -10 Replace form by $\begin{pmatrix} \phi_0 + \phi_1 d^* & d \\ (-)^i d^* & 0 \end{pmatrix}$

p. 492 ll. 15,16 Replace $\text{char}(F)$ by $|F|$

p. 493 l. -9 Should read ‘Let s_0 be the number of conjugate pairs of non-real roots $\omega \in \mathbb{C}$ of $p(x)$ with $\sigma_\omega(F_0) \subset \mathbb{R}$ and $\sigma_\omega(a) < 0$, so that $\sigma_\omega : F \rightarrow \mathbb{C}^-$ is a morphism of rings with involution.’

p. 495 l. -4 $L^0(\mathbb{Z}_4) = \mathbb{Z}_8$

p. 497 l. 7 Replace quadratic by ϵ -symmetric

p. 508 l. 5 $L^{-1}(A, \epsilon)$

p. 508 l. -11 Should read

$$\text{dimension } L^0(A, \epsilon) = \text{dimension } L^0(F, \epsilon) = r_1 ,$$

with r_1 the number of real roots of $p_0(y)$ such that $\rho_\xi : F_0 = \mathbb{Q}[y]/(p_0(y)) \rightarrow \mathbb{R}; y \rightarrow \xi$ has $\rho_\xi(a) < 0$. Both $L^0(A, \epsilon)$ and $L^0(F, \epsilon)$ are of the form $\mathbb{Z}^{r_1} \oplus 8\text{-torsion}$.

- p. 535 l. 14 $\phi_0 = \theta, -zf^*\theta, \phi_1 = \theta$.
- p. 546 l. -1 identification of 28.33
- p. 547 l. 2 from 39.26
- p. 547 l. 12 36.3 (i)
- p. 547 l. -4 as in 39.20
- p. 548 l. -11 Combine 39.20, 39.26
- p. 564 l. 17 $\zeta : \overline{X} \rightarrow \overline{X}$ is a generating covering translation.
- p. 567 l. -3 $\lambda - \omega\lambda^*$
- p. 571 l. 16 Replace [129, 5.6] by [121]
- p. 573 l. 10 Replace ‘26.10’ by ‘27.10’.
- p. 574 l. -5 Replace ‘ k even’ by ‘ j even’.
- p. 575 l. -12 The exact sequence should read

$$0 \rightarrow L\widetilde{\text{Aut}}_p^{2j+1}(A) \rightarrow L_h^{2j+2}(A) \rightarrow L\text{Asy}_h^{2j+2}(A) \rightarrow L\widetilde{\text{Aut}}_p^{2j}(A) \rightarrow L_h^{2j+1}(A) \rightarrow 0$$

- p. 575 l. -11 Insert ‘and $L\text{Asy}^{2j+1}(\mathbb{C}) = 0$ (Proposition 39.20 (iii))’ after ‘These identifications’
- p. 598 l. -1 $n = 2i$ in the braid
- p. 616 Replace $V \times 1$ in the figure caption by $V \times I$
- p. 617 l. -8 – -5 Replace “Indeed ... etc.” by
“Indeed, the boundary of a Bing 3-disk D^3 , which we assume contains the connected binding N in its interior, also bounds a 3-disk in the complement of D^3 , because $M^3 \setminus N$ is fibered and thus covered by \mathbb{R}^3 , etc.”
- p. 622 Replace $W \times 1$ in the figure caption by $W \times I$
- p. 623 l. 11 Replace ”(Jänich, Karras et. al. [117])” by ”(Jänich, Karras et. al. [117], Neumann [211])”
- p. 629 l. -11 [70] M. Epple, **Die Entstehung der Knotentheorie**, Vieweg (1999)
- p. 633 l. 12 [161] J. Levine and K. Orr, **A survey of surgery and knot theory**, in Surveys on Surgery Theory, Volume 1, Annals of Maths. Studies 145, 345–364 (2000)
- p. 633 l. -17 [174] W. Lück, **The universal functorial Lefschetz invariant**, Fund. Math. 161, 167–215 (1999)