Errata for High-dimensional Knot Theory  
by Andrew Ranicki  

This list contains corrections of misprints/errors in the book. Please let me know of any further misprints/errors by e-mail to a.ranicki@ed.ac.uk


A.A.R. 23.4.2017

p. V  The following statement of Frank Adams makes the dedication of the book to him even more appropriate: Of course, from the point of view of the rest of mathematics, knots in higher-dimensional space deserve just as much attention as knots in 3-space (Article on topology, in 'Use of Mathematical Literature' (Butterworths (1977)).

p. XVIII l. 8 Remove “that of”.

p. XXI l. 9  for a homology framed knot

p. XXIV l. −3  $\pi_1(X)$

p. XXV l. −2  $C_{4+s_1}, C_{4+s_3}$

p. XXVI l. 10  $\pi_1(F) = \{1\}$

p. XXVIII l. 2  chain complex

p. 9 l. 8  i.e.  $f_* : \pi_1(X) \to \pi_1(X)$ is an isomorphism and

\[ \pi_1(T(f)) = \{g z^j \mid j \in \mathbb{Z}\} \]

with $z^{-1} g z = f_*(g)$

p. 10 l. −7  $(f(x), n + 1, 0)$

p. 16 l. −15  If $X$ has a finite 2-skeleton

p. 29 l. −10  for

p. 31 l. 11  Bass [13, XII.7.4]

p. 34 l. 6  4.5, 5.5 (i)

p. 34 l. 19  As for 5.10

p. 35 l. −12  Nil$_0(A)$

p. 47 l. −6  $z^{-N_2^i} b_2$

p. 47 l. −2  $N^+ = \sum_{j=1}^r N_j^+$
for some integers \(N_r^+, N_r^- \geq 0\) (starting with \(N_n^+ = N_n^- = 0\), for example).

Assuming inductively that it is possible to invert \(1 - zb'\) in \(R_{n-1}\) it is now possible to invert \(1 - zb\) in \(R_n\). The inclusion \(B[z] \to B[[z]]\) factors as

\[
B[z] \to \Sigma^{-1}B[z] \to B[[z]]
\]

so that \(B[z] \to \Sigma^{-1}B[z]\) is injective. The morphism \(\Sigma^{-1}B[z] \to B[[z]]\) is an injection for commutative \(B\), but it is not known if it is an injection also in the noncommutative case.

Remove “the \(A[z^{-1}]\)-module subcomplex of”.
p. 83 l. 3  “$A[z]$-module morphisms”.

p. 84 l. 7  Proposition 10.9. For noncommutative $A$ the right hand side of the identity

$$\Omega_+^{-1} A[z] = (1 + zA[z])^{-1} A[z]$$

should be corrected as for Example 9.15 above.

p. 90 l. 12  Replace ”[240, Chap. 8]” by ”[244, Chap. 8]”

p. 92 l. 12  $\tau(1 - f + zf : \Pi^{-1} P[z, z^{-1}] \to \Pi^{-1} P[z, z^{-1}])$

p. 99 l. 1  $\in A[z]$

p. 102 l. 8  [5, 1.10]

p. 102 l. 4  Replace “If the additive group of $A$ is torsion-free . . .” by “If $Q \subseteq A$ . . .”

p. 111 l. 8  Remove “$(P, f)$ is”

p. 120 l. 8  Replace 13.2 by 13.1

p. 121 l. 2  $n - m \geq 1$

p. 121 l. 10  $\Omega^{-1} A[z, z^{-1}]$

p. 121 l. 3  $A[z, z^{-1}]$-module

p. 123 l. 12  $P^{-1} A[z, z^{-1}] = \Pi^{-1} A[z, z^{-1}]$

p. 125 l. 13  Should read ”$(P, h) + (P', h') = (P \oplus P', \begin{pmatrix} g & h \\ 0 & g' \end{pmatrix})$”

p. 135 Chapter 14 The group $\widehat{W}(A)^{ab}$ should be replaced by the image of $\widehat{W}(A)$ in $K_1(A[[z]])$, since the kernel of the morphism

$$\Delta_+ : \widehat{W}(A) \to K_1(A[[z]]) ;$$

$$(a_1, a_2, \ldots) \mapsto \tau(1 + \sum_{j=1}^{\infty} a_j z^j : A[[z]] \to A[[z]])$$

is in general larger than $[\widehat{W}(A), \widehat{W}(A)]$. See the paper

A.Pajitnov and A.Ranicki, The Whitehead group of the Novikov ring


Similarly, $W(A)^{ab}$ should be replaced by the image of $W(A)$ in $K_1(\Omega_+^{-1} A[z])$, since the kernel of the morphism

$$\Delta_+ : W(A) \to K_1(\Omega_+^{-1} A[z]) ;$$

$$(a_1, a_2, \ldots) \mapsto \tau(1 + \sum_{j=1}^{\infty} a_j z^j : \Omega_+^{-1} A[z] \to \Omega_+^{-1} A[z])$$

is in general larger than $[W(A), W(A)]$. 3
Replace by “If \( A \) is a commutative ring such that \( Q \subseteq A \).”

The isomorphism inverse to

\[
\prod_1^\infty A \to \tilde{W}(A) ;
\]

\[
(a_1, a_2, a_3, \ldots) \mapsto \exp \left( \int_0^z (a_1 - a_2s + a_3s^2 - \ldots) ds \right)
\]

\[
= \exp \left( a_1z - \frac{a_2z^2}{2} + \frac{a_3z^3}{3} - \ldots \right)
\]

is given by

\[
\tilde{W}(A) \to \prod_1^\infty A ; q(z) = 1 + b_1z + b_2z^2 + \ldots \mapsto
\]

\[
\frac{q'(z)}{q(z)} = \frac{b_1 + 2b_2z + 3b_3z^2 + \ldots}{1 + b_1z + b_2z^2 + \ldots} = a_1 - a_2z + a_3z^2 - \cdots \to (a_1, a_2, a_3, \ldots)
\]

([5, 6.13]). The reverse characteristic polynomial of an endomorphism \( f : P \to P \) of a f.g. projective \( A \)-module \( P \)

\[
\tilde{c}_z(P, f) = \det(1 - z f : P[z] \to P[z]) = \exp \left( - \sum_{i=1}^\infty \frac{\text{tr}(f^i)}{i} z^i \right)
\]

\[
eq 1 + zA[z] \subset W(A) \subset \tilde{W}(A)
\]

(cf. Example 19.16) has image \((-\text{tr}(f), \text{tr}(f^2), -\text{tr}(f^3), \ldots) \in \prod_1^\infty A\). For any polynomial of the type

\[
p(z) = 1 + \sum_{i=1}^d b_i z^i \in 1 + zA[z] \subset W(A)
\]

the image \((a_1, a_2, a_3, \ldots) \in \prod_1^\infty A\) has components

\[
a_i = (-)^i \text{tr}(f^i) \in A
\]

with

\[
f : P = A[z]/(z^d p(z^{-1})) \to P = A[z]/(z^d p(z^{-1}))
\]

such that \(\tilde{c}_z(P, f) = p(z)\).

For noncommutative \( A \) the right hand side of the identity

\[
\tilde{\Omega}_+^{-1} A[z] = (1 + zA[z])^{-1} A[z]
\]

should be corrected as for Example 9.15 above.
This $\zeta$-function agrees with the $\zeta$-function of Geoghegan and Nicas (Trace and torsion in the theory of flows, Topology 33, 683–719 (1994)).

structure $\phi_B$ on $B \otimes_A C$.

$D[z, z^{-1}] \to D[z, z^{-1}]$

for each $P^{-1}E_r$

the reduced chain complexes

d_{C_{n-r}} = (-)^r(d_C)^*$

A cobordism of $\epsilon$-symmetric Poincaré complexes $(C, \phi), (C', \phi')$ is an $\epsilon$-symmetric Poincaré pair $((f f') : C \oplus C' \to D, (\delta \phi, \phi \oplus -\phi'))$.

$f$ is $i$-connected, $\partial_h f, \partial_t f$ are $(i-1)$-connected

Szczarba

$i$-connected

$i$-connected

g $\times$ 1

$A$-finitely dominated

The stated exact sequence in the $\epsilon$-symmetric case

$$\ldots \to L^n_U(A, \epsilon) \to L^n_{\partial -1U}(\Sigma^{-1}A, \epsilon) \to L^n_D(A, \Sigma, \epsilon) \to L^{n-1}_U(A, \epsilon) \to \ldots$$

should be replaced in general by the exact sequence

$$\ldots \to L^n_U(A, \epsilon) \to \Gamma^\partial_{-1U}(A \to \Sigma^{-1}A, \epsilon) \to L^n_D(A, \Sigma, \epsilon) \to L^{n-1}_U(A, \epsilon) \to \ldots .$$

See the paper Noncommutative localization and chain complexes I. Algebraic K- and L-theory by A.Neeman and A.Ranicki, http://arXiv.org/abs/math.RA.0109118 for the proof that the natural map of $\epsilon$-symmetric groups

$$\Gamma^n_{\partial -1U}(A \to \Sigma^{-1}A, \epsilon) \to L^n_{\partial -1U}(\Sigma^{-1}A)$$

is an isomorphism if Tor$^A_*(\Sigma^{-1}A, \Sigma^{-1}A) = 0$ for $* > 1$ (e.g. if $\Sigma^{-1}A$ is a flat $A$-module, as is the case for a two-sided Ore localization). There is no problem in the $\epsilon$-quadratic case, by virtue of Vogel [296], [297], with the natural maps

$$\Gamma^n_{\partial -1U}(A \to \Sigma^{-1}A, \epsilon) \to L^n_{\partial -1U}(\Sigma^{-1}A)$$

isomorphisms, and with an exact sequence

$$\ldots \to L^n_U(A, \epsilon) \to L^n_{\partial -1U}(\Sigma^{-1}A, \epsilon) \to L^n_D(A, \Sigma, \epsilon) \to L^{n-1}_U(A, \epsilon) \to \ldots .$$
p. 275 l. 13 $1 + T\epsilon : L_n^U(A[s], \epsilon) \to L_n^U(A[s], \epsilon)$

p. 287 l. 3 Replace “will” by “we shall”

p. 290 l. 11,12 Replace conditions (a),(b) by the single condition $\lambda(x, y) = 0$ for all $x, y \in K$.

p. 291 l. 5 Should read “[235, Chap. 9]”

p. 303 l. 4 to describe

p. 312 l. 14 $i$-connected

p. 313 l. 8 $i$-connected

p. 313 l. 10 $(i+1)$-connected

p. 314 l. 5 $(\mathbb{Z}^\ell, \lambda)$

p. 314 l. 16 Cyclic branched covers

p. 340 l. 15 Replace ”28.15” by ”28.17”

p. 344 l. 8 Replace the text of Example 28.31 by

“The 0-dimensional asymmetric $L$-group $L\text{Asy}_{0}^q(A)$ ($q = s, h, p$) is the Witt group of nonsingular asymmetric forms $(L, \lambda)$ over $A$, with $\lambda : L \to L^*$ an isomorphism. Such a form is metabolic if there exists a lagrangian, i.e. a direct summand $K \subset L$ such that $K = K^\perp$, with

$$K^\perp = \{ x \in L \mid \lambda(x)(K) = 0 \} ,$$

in which case

$$\lambda = 0 \in L\text{Asy}_{0}^q(A) .$$

A nonsingular asymmetric form $(L, \lambda)$ is such that $(L, \lambda) = 0 \in L\text{Asy}_{0}^q(A)$ if and only if it is stably metabolic, i.e. there exists an isomorphism

$$(L, \lambda) \oplus (M, \mu) \cong (M', \mu')$$

for some metabolic $(M, \mu), (M', \mu')$. A 0-dimensional asymmetric Poincaré complex $(C, \lambda)$ is the same as a nonsingular asymmetric form $(L, \lambda)$ with $L = C^0$. For a 1-dimensional asymmetric Poincaré pair $(f : C \to D, (\delta\lambda, \lambda))$ with $D_r = 0$ for $r \neq 0$ there is defined an exact sequence

$$0 \to D^0 \xrightarrow{f^*} C^0 \xrightarrow{f\lambda} D_0 \to 0$$

so that $K = \text{im}(f^* : D^0 \to C^0) \subset L = C^0$ is a lagrangian of $(C^0, \lambda)$, and the pair is the same as a nonsingular asymmetric form together with a lagrangian. More generally, suppose given a 1-dimensional asymmetric
Poincaré pair \((f : C \to D, (\delta \lambda, \lambda))\). The mapping cone of the chain equivalence \((\delta \lambda, f \lambda) : C(f)^{1-*} \to D\) is an exact sequence

\[
\begin{array}{c}
0 \to D^0 \xrightarrow{g} C^0 \oplus D^1 \oplus D_1 \xrightarrow{h} D_0 \to 0
\end{array}
\]

with

\[
g = \begin{pmatrix} f^* \\ \delta \lambda \end{pmatrix} : D^0 \to C^0 \oplus D^1 \oplus D_1,
\]

\[
h = \begin{pmatrix} f \lambda & \delta \lambda & d \end{pmatrix} : C^0 \oplus D^1 \oplus D_1 \to D_0.
\]

However (as pointed out by Joerg Sixt), in general

\[
h \neq g^* \begin{pmatrix} \lambda & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} : C^0 \oplus D^1 \oplus D_1 \to D_0
\]

so that \(g\) is not the inclusion of a lagrangian in \((C^0, \lambda) \oplus (D^1 \oplus D_1, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})\).

To repair this, proceed as follows. Use the chain equivalences

\[
\begin{pmatrix} \delta \lambda \\ \lambda f^* \end{pmatrix} : D^{1-*} \to C(f), \quad \begin{pmatrix} \delta \lambda & f \lambda \end{pmatrix} : C(f)^{1-*} \to D
\]

to define a chain equivalence

\[
i = T \begin{pmatrix} \delta \lambda \\ \lambda f^* \end{pmatrix} \begin{pmatrix} \delta \lambda & f \lambda \end{pmatrix}^{-1} : D \to D.
\]

In order to prove that \((C^0, \lambda)\) is stably metabolic, it is convenient to replace \(D\) by a chain equivalent complex for which \(i\) is (chain homotopic to) an isomorphism. The exact sequence

\[
\begin{array}{c}
0 \to D_1 \xrightarrow{(d \atop i_1)} D_0 \oplus D_1 \xrightarrow{(i_0 \atop -d)} D_0 \to 0
\end{array}
\]

splits, so there exists an \(A\)-module morphism \((\alpha \atop \beta) : D_0 \oplus D_1 \to D_1\) such that

\[
(\alpha \atop \beta) \begin{pmatrix} d \\ i_1 \end{pmatrix} = \alpha d + \beta i_1 = 1 : D_1 \to D_1.
\]

The 1-dimensional \(A\)-module chain complex \(D'\) defined by

\[
d' = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix} : D'_1 = D_1 \oplus D_1 \to D'_0 = D_0 \oplus D_1
\]
is such that the inclusion \( D \to D' \) and the projection \( D' \to D \) are inverse chain equivalences. The chain isomorphism \( i' : D' \to D' \) defined by

\[
i'_0 = \begin{pmatrix} i_0 & -d \\ \alpha & \beta \end{pmatrix} : D'_0 = D_0 \oplus D_1 \to D'_1 = D_0 \oplus D_1,
\]

\[
i'_1 = \begin{pmatrix} i_1 & -1 \\ \alpha d & \beta \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & \beta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -i_1 & 1 \end{pmatrix} : D'_1 = D_1 \oplus D_1 \to D'_1 = D_1 \oplus D_1
\]

is such that

\[
i : D \to D' \xrightarrow{i'} D' \to D.
\]

Replacing \( D \) by \( D' \) and reverting to the previous notation, it may thus be assumed that \( i : D \to D \) is an isomorphism. Choose a chain homotopy

\[
(j \ b) : i(\delta \lambda \ f \lambda) \simeq T \begin{pmatrix} \delta \lambda \\ \lambda f \end{pmatrix} : C(f)^{1-*} \to D.
\]

The nonsingular asymmetric form defined by

\[
(M, \mu) = (C^0 \oplus D^1 \oplus D_1, \begin{pmatrix} \lambda & k^* & 0 \\ 0 & j^* & 1 \\ 0 & i_1^* & 0 \end{pmatrix})
\]

is such that

\[
h = g^* \mu : M = C^0 \oplus D^1 \oplus D_1 \to D_0
\]

so that \( g : D^0 \to M \) is the inclusion of a lagrangian and \((M, \mu)\) is metabolic. The \( A \)-module morphism

\[
C^0 \oplus D_1 \to C^0 \oplus M = C^0 \oplus C^0 \oplus D^1 \oplus D_1 ; (x, y) \mapsto (x, x, 0, y)
\]

is the inclusion of a lagrangian in \((C^0, \lambda) \oplus (M, -\mu)\), so that \((C^0, \lambda)\) is stably metabolic.”

p. 346 l. 16 Replace 25.11 by 26.11

pp. 347–348 The construction of \((C', \lambda')\) and \((C'', \lambda'')\) is not correct in general; these complexes should be replaced by the following \((i-1)\)-connected \( n \)-dimensional asymmetric Poincaré complex \((C', \lambda')\) cobordant to the given \( n \)-dimensional asymmetric Poincaré complex \((C, \lambda)\) with \( n = 2i \) or \( 2i + 1 \). Choose a chain homotopy inverse \( \mu : C \to C'' \) for \( \lambda : C'' \to C \) and a chain homotopy
\[ \nu : \mu \lambda \simeq 1 : C^{n-r} \to C^{n-r}, \text{ and set} \]

\[
d_C' = \begin{cases} 
\begin{pmatrix} 
C_r' = C_r \oplus C^{n-r+1} & C_{r-1}' = C_{r-1} \oplus C^{n-r+2} \\
0 & 0
\end{pmatrix} & \text{if } r \leq i-1 \\
\begin{pmatrix} 
C_r' = C_i \oplus C^{i+1} \oplus C_{i+1} & C_{r-1}' = C_{i-1} \oplus C^{i+2} \\
0 & 0
\end{pmatrix} & \text{if } n = 2i \text{ and } r = i \\
\begin{pmatrix} 
C_r' = C_i \oplus C_i+2 & C_{r-1}' = C_i \oplus C^{i+2} \\
0 & 0
\end{pmatrix} & \text{if } n = 2i + 1 \text{ and } r = i + 1 \\
\begin{pmatrix} 
C_r' = C_1 \oplus C_{r+1} & C_{r-1}' = C_{r-1} \oplus C_r \\
(-)^{r-1} \lambda^{*} \mu & d_C
\end{pmatrix} & \text{otherwise,}
\end{cases}
\]

\[
\chi' = \begin{cases} 
\begin{pmatrix} 
C^{n-r} = C^{n-r} \oplus C^{n-r+1} & C_r' = C_r \oplus C^{n-r+1} \\
0 & \mu \lambda^*
\end{pmatrix} & \text{if } r \leq i-1 \\
\begin{pmatrix} 
C^{n-r} = C_i \oplus C_{i+1} \oplus C^{i+1} & C_r' = C_i \oplus C^{i+1} \oplus C_{i+1} \\
0 & 0
\end{pmatrix} & \text{if } n = 2i \text{ and } r = i \\
\begin{pmatrix} 
C^{n-r} = C^{n-r} \oplus C_{r+1} & C_r' = C_r \oplus C_{r+1} \\
\lambda^{*} \nu & 1
\end{pmatrix} & \text{otherwise.}
\end{cases}
\]
p. 369 l. –1 framed codimension 2
p. 372 l. 3 replace "reverse" by "reduced"
p. 374 l. 8 "twisted double bordism groups"
p. 382 l. 17
\[ \beta_s = \left( \begin{array}{cc} \chi_s & (-)^s \phi_s \\ (-)^{n-r-1} \phi_s & (-)^{n-r+s} T \phi_{s-1} \end{array} \right) : \]
\[ B^{n-r+s} = C^{n-r+s} \oplus C^{n-r+s-1} \to B_r = C_r \oplus C_{r-1}. \]
p. 411 l. 10 \[ \Omega_{-1}^{-1} A[s]/A[s] = F(s)/F[s] \]
p. 421 l. 8 Terminology: the covering \( \epsilon \)-symmetric complex in the sense of Definition 32.7 (i) is the \( \epsilon \)-symmetrization of the ultraquadratic complex of Ranicki[237, p.820].
p. 422 l. 9 Proof of 32.8 (ii): Since \( E \) is \( A \)-contractible the \( A[z,z^{-1}] \)-module chain map \( 1 - z : E \to E \) is a chain equivalence. Define a homotopy equivalence \( (E, \theta) \cong U(\Gamma) \) by
\[ (1 + (1 + T \epsilon))(1 - z)^{-1} : E \to C(g - zh), \]
with \( (1 - z)^{-1} : E \to E \) any chain homotopy inverse of \( 1 - z : E \to E \).
p. 437 l. –2 In the proof of (ii) insert:
The natural \( A[s] \)-module morphisms
\[ A[s, s^{-1}, (1 - s)^{-1}] \to Q_{-1}^{-1} A[s], \ Q_{-1}^{-1} A[\min] A[s] \to Q_{-1}^{-1} A[s] \]
are inclusions of submodules. For any elements
\[ \frac{r(s)}{s^j (1 - s)^k} \in A[s, s^{-1}, (1 - s)^{-1}], \quad \frac{p(s)}{q(s)} \in Q_{-1}^{-1} A[\min] A[s] \]
such that
\[ \frac{r(s)}{s^j (1 - s)^k} = \frac{p(s)}{q(s)} \in Q_{-1}^{-1} A[s] \]
it follows from the minimality of \( q(s) \) and the identity
\[ p(s)s^j (1 - s)^k = q(s)r(s) \in A[s] \]
that \( s^j (1 - s)^k \) divides \( r(s) \), and hence that
\[ A[s, s^{-1}, (1 - s)^{-1}] \cap Q_{-1}^{-1} A[\min] A[s] = A[s] \subset Q_{-1}^{-1} A[s]. \]
p. 438 l. –1 –9 Remove. \((\chi_{s, \min} \) is no longer required).
The statement of Proposition 32.45 (i) is false as stated, and should be replaced by:

"The Blanchfield form is such that for any \( x, y \in L \) the composite

\[
P_{\mathcal{L}}^{-1}A[z, z^{-1}] / A[z, z^{-1}] \\
\rightarrow P_{\mathcal{F}}^{-1}[z, z^{-1}] / [F[z, z^{-1}]] = \frac{Q_{F \min}^{-1}[s]}{F[s]} / F[s] = \frac{F[s, 1-s]}{F[s]}
\]

sends \( \mu(i(x), i(y)) \in P_{\mathcal{L}}^{-1}A[z, z^{-1}] / A[z, z^{-1}] \) to

\[
\mu(i(x), i(y)) = \sum_{j=-\infty}^{-1} (\lambda + c\lambda^*)(x, f^{-j-1}(y))s_j \in s^{-1}A[[s^{-1}]] \subseteq s^{-1}F[[s^{-1}]],
\]

where \( i : L \to M \) is the natural \( A \)-module morphism. In particular, \( \mu \) determines \( \chi \) by

\[
\lambda : L \times L \to M \times M \xrightarrow{\mu} P_{\mathcal{L}}^{-1}A[z, z^{-1}] / A[z, z^{-1}] \to F[(s)] / F[s] \xrightarrow{\chi_s} F
\]

with \( s = (1 - z)^{-1} : M \to M \) and \( \chi_s = \text{coefficient of } s^{-1} (31.20) \).

Here is an explicit counterexample to the original statement of 32.45 (i). Let \( A = \mathbb{Z} \), and for any \( m \in \mathbb{Z} \) consider the skew-symmetric Seifert form over \( \mathbb{Z} \) defined in Example 42.2

\[
(L, \lambda) = (\mathbb{Z} \oplus \mathbb{Z}, \begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix})
\]

with

\[
f = (\lambda - \lambda^*)^{-1} = \begin{pmatrix} 1 & -1 \\ m & 0 \end{pmatrix} : L = \mathbb{Z} \oplus \mathbb{Z} \to L = \mathbb{Z} \oplus \mathbb{Z}
\]

and Alexander polynomial

\[
\Delta(z) = \det(1 - f + zf) = m(1 - z)^2 + z.
\]

The corresponding symmetric Blanchfield form \((M, \mu)\) is given by

\[
M = \text{coker}(1 - f + zf) = \mathbb{Z}[z, z^{-1}] / \Delta(z),
\]

\[
\mu(x, y) = \frac{(1-z)^2xz}{\Delta(1-s)} \in P^{-1}\mathbb{Z}[z, z^{-1}] / \mathbb{Z}[z, z^{-1}].
\]

In terms of \( s = (1 - z)^{-1} \)

\[
\mu(x, y) = \frac{xy}{m + s(1-s)} \in Q^{-1}\mathbb{Z}[s] / \mathbb{Z}[s].
\]

If \( m \neq 0 \) then

\[
\mu(1, 1) = \frac{1}{m + s(1-s)} \notin Q^{-1}_{\min}\mathbb{Z}[s] / \mathbb{Z}[s],
\]

since \( s^2\Delta(1-s^{-1}) = m + s(1-s) \in \mathbb{Z}[s] \) is not minimal.
Replace the proof of 32.45 (i) by:

Work in the completion $A[[s^{-1}]]$ to obtain

$$\mu(i(x), i(y)) = (1 - z)(\lambda + \epsilon\lambda^*)(x, (1 - f + zf)^{-1}(y))$$

$$= s^{-1}(\lambda + \epsilon\lambda^*)(x, (1 - s^{-1}f)^{-1}y)$$

$$= \sum_{j=-\infty}^{-1} (\lambda + \epsilon\lambda^*)(x, f^{-j-1}(y))s^j \in A[[s^{-1}]]$$

so that

$$\chi_s(\mu(i(x), i(y))) = (\lambda + \epsilon\lambda^*)(x, y),$$

$$\chi_s(\mu(i(x), s_i(y))) = \lambda(x, y) \in A \subset F.$$
p. 535 l. 14 $\phi_0 = \theta, -zf^*\theta$, $\phi_1 = \theta$.

p. 546 l. –1 identification of 28.33

p. 547 l. 2 from 39.26

p. 547 l. 12 36.3 (i)

p. 547 l. –4 as in 39.20

p. 548 l. –11 Combine 39.20, 39.26

p. 564 l. 17 $\zeta : \overline{X} \rightarrow \overline{X}$ is a generating covering translation.

p. 567 l. –3 $\lambda - \omega \lambda^*$

p. 571 l. 16 Replace [129, 5.6] by [121]

p. 573 l. 10 Replace ‘26.10’ by ‘27.10’.

p. 574 l. –5 Replace ‘$k$ even’ by ‘$j$ even’.

p. 575 l. –12 The exact sequence should read

$$0 \rightarrow \tilde{L} \text{Aut}_p^{2j+1}(A) \rightarrow L_{2j+2}^j(A) \rightarrow \tilde{L} \text{Asy}_h^{2j+2}(A) \rightarrow \tilde{L} \text{Aut}_p(A) \rightarrow L_h^{2j+1}(A) \rightarrow 0$$

p. 575 l. –11 Insert ‘and $L \text{Asy}^{2j+1}(C) = 0$ (Proposition 39.20 (iii))’ after ‘These identifications’

p. 598 l. –1 $n = 2i$ in the braid

p. 616 Replace $V \times 1$ in the figure caption by $V \times I$

p. 617 l. –8 – –5 Replace “Indeed . . . etc.” by

“Indeed, the boundary of a Bing 3-disk $D^3$, which we assume contains the connected binding $N$ in its interior, also bounds a 3-disk in the complement of $D^3$, because $M^3 \setminus N$ is fibered and thus covered by $\mathbb{R}^3$, etc.”

p. 622 Replace $W \times 1$ in the figure caption by $W \times I$

p. 623 l. 11 Replace ”(Jänich, Karras et. al. [117])” by ”(Jänich, Karras et. al. [117], Neumann [211])”

