MR620795 (82h:57027) 57R67 (18F25 57-02)
Ranicki, Andrew

★**Exact sequences in the algebraic theory of surgery.**

Mathematical Notes, 26.
Princeton University Press, Princeton, N.J.; *University of Tokyo Press, Tokyo*, 1981. xvii+864 *pp*. $16.50 paperbound. *ISBN* 0-691-08276-6

Surgery is concerned with two problems: When is a space homotopically equivalent to a manifold, and when are homotopically equivalent manifolds homeomorphic, diffeomorphic or PL equivalent? This theory has been vigorously developed during the last 20 years. Unfortunately, there are few books containing an exposition of this topic, and those are basically research monographs and hence make for rather heavy reading. In particular, the reviewer is familiar with three books, including the one here reviewed. The other two are *Surgery on simply-connected manifolds* by W. Browder [Springer, New York, 1972; MR0358813 (50 #11272)] and *Surgery on compact manifolds* by C. T. C. Wall [Academic Press, London, 1970; MR0431216 (55 #4217)]. These two books together with the one under review should be owned by everyone interested in surgery.

For someone who wants to learn this subject, the reviewer would suggest the following approach. First, read J. W. Milnor's paper [*Proceedings of the Symposia in Pure Mathematics, Vol. III*, pp. 39–55, Amer. Math. Soc., Providence, R.I., 1961; MR0130696 (24 #A556)]. It introduces the basic geometric construction of surgery. Then, read M. A. Kervaire and Milnor's paper [Ann. of Math. (2) **77** (1963), 504–537; MR0148075 (26 #5584)], which thoroughly examines the surgery obstruction in the simply connected case with the object of classifying the possible differential structures on the n-sphere, $n \neq 4$. After this, read Browder's paper [Proc. Cambridge Philos. Soc. **61** (1965), 337–345; MR0175136 (30 #5321)], which introduces codimension-one splitting problems and leads to S. P. Novikov's paper [*International congress of mathematicians* (Moscow, 1966), pp. 172–179, Amer. Math. Soc., Providence, R.I., 1968; MR0231401 (37 #6956)], which contains an important application of surgery, namely, the topological invariance of the rational Pontrjagin classes. This last paper also motivates the extension of the theory to non-simply connected manifolds, which is the principal focus of both Wall's and the present author's books. Next, read Browder's book, which develops surgery theory for simply connected manifolds and contains a very good discussion of the Kervaire-Arf invariant. Browder's book (together with Sullivan's lecture notes ["Triangulating and smoothing homotopy equivalences and homeomorphisms", Geometric Topology Seminar Notes, Princeton Univ., Princeton, N.J., 1967], which expounds the homotopy structure of $G/$PL) gives one a good picture of surgery theory for simply connected manifolds—at least, modulo some unsolved problems about the Kervaire-Arf invariant.

Next, one should read Wall's book. It extends the theory to non-simply connected manifolds and discusses many interesting applications, such as space-form problems. The crucial thing is to analyze the surgery map $\sigma\colon [M^n, G/\mathrm{Top}] \to L_n(\pi_1 M)$, where $[M^n, G/\mathrm{Top}]$ is the group of homotopy classes of maps of the manifold M^n to $G/$Top and $L_n(\pi_1 M)$ is the surgery obstruction group. Very important but only partially solved problems are to calculate $L_n(\pi_1 M)$ and σ.

The monograph under review recasts the foundational material in Wall's book in a more algebraic and functorial setting and answers many questions posed by Wall. For instance, it allows for a better understanding of the surgery map σ by taking care of the anomaly that $L_n(\ )$ is covariant while $[\ ,G/\mathrm{Top}]$ is contravariant. It also gives more insights into calculating $L_n(\Gamma)$ via localization theorems and by giving algebraic proofs of most of the splitting theorems, thus allowing them to be applied inductively to larger classes of groups. (The details of some of these splitting results will be given in a later paper by the author ["Splitting theorems in the algebraic theory of surgery", to appear].) Also, the author has asked the reviewer to mention that the asserted "mild generalization of the splitting theorem of J. Shaneson [Ann. of Math. (2) **90** (1969), 296–334; MR0246310 (39 #7614)]" on p. 813 is wrong, and that "the discussion on pp. 812–814 should therefore be restricted to the case $\omega = +1$ only".

For the experts, a panorama of this book is best provided by a glance at its table of contents. Chapter 1. Absolute L-theory: 1.1 **Q**-groups; 1.2 L-groups; 1.3 Triad **Q**-groups; 1.4 Algebraic Wu classes; 1.5 Algebraic surgery; 1.6 Forms and formations; 1.7 Algebraic glueing; 1.8 Unified L-theory; 1.9 Products; 1.10 Change of K-theory. Chapter 2. Relative L-theory: 2.1 Algebraic Poincaré triads; 2.2 Change of rings; 2.3 Change of categories; 2.4 Γ-groups; 2.5 Change of K-theory. Chapter 3. Localization: 3.1 Localization and completion; 3.2 The localization exact sequence $(n \geq 0)$; 3.3 Linking Wu classes; 3.4 Linking forms; 3.5 Linking formations; 3.6 The localization exact sequence $(n \in \mathbf{Z})$; 3.7 Change of K-theory. Chapter 4. Arithmetic L-theory: 4.1 Dedekind algebra; 4.2 Dedekind rings; 4.3 Integral and rational L-theory. Chapter 5. Polynomial extensions $(\overline{x} = x)$: 5.1 L-theory of polynomial extensions; 5.2 Change of K-theory. Chapter 6. Mayer-Vietoris sequences: 6.1 Triad L-groups; 6.2 Change of K-theory; 6.3 Cartesian L-theory; 6.4 Ideal L-theory. Chapter 7. The algebraic theory of codimension q surgery: 7.1 The total surgery obstruction; 7.2 The geometric theory of codimension q surgery; 7.3 The spectral quadratic construction; 7.4 Geometric Poincaré splitting; 7.5 Algebraic Poincaré splitting; 7.6 The algebraic theory of codimension 1 surgery; 7.7 Surgery with coefficients; 7.8 The algebraic theory of codimension 2 surgery; 7.9 The algebraic theory of knot cobordism.

To summarize, this is a carefully written and lucid (but lengthy) account of an important topic in topology which the reviewer strongly recommends to anyone interested in the structure of manifolds.

Reviewed by *F. T. Farrell*