

Compass and Straightedge Constructions

The Field of Constructible Numbers

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To use one, simply:

- ▶ Place the needle where you want the center of the circle to be.
- ▶ Place the pencil tip on a point you want the circle to pass through.
- ▶ While keeping the needle still and the pencil on the paper, turn the compass (or the paper) to draw your circle.

First Construction

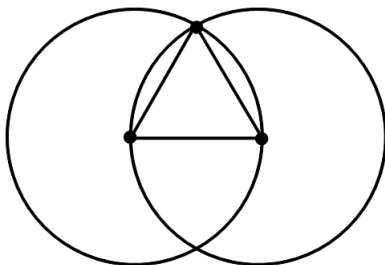
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Given a line segment, construct an equilateral triangle with that segment as one side.



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- ▶ With their limited tools and lack of understanding of modern abstract concepts, compass and straightedge constructions were very important to them.
- ▶ The Greeks successfully constructed many things, but some constructions did stump them.



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- ▶ We'll see today why this proved that the regular 17-gon is constructible.

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Proved impossible in 1882 when Ferdinand von Lindemann proved that π is transcendental.

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- ▶ Add, subtract, multiply, and divide.
- ▶ Take square roots.