Entry-exit functions: beyond eigenvalue separation
Mattia Sensi, INRIA Sophia Antipolis–Méditerranée,
11 March 2022

Abstract
We study delayed loss of stability in a class of fast-slow systems with one slow variable and two fast ones, in which the linearization of the vector field along the one-dimensional critical manifold has two real negative eigenvalues which intersect before at least one of them becomes positive. This lack of separation between eigenvalues and eigendirections renders the use of known entry-exit relation functions unsuitable. We expose additional hidden structures of these systems, illustrating the different possible qualitative scenarios, and we propose novel formulae for the entry-exit functions for the phenomenon of delayed loss of stability in this setting.

Geometric analysis of fast-slow PDEs with fold type singularity
Thomas Zacharis, University of Edinburgh
1 April 2022

Abstract
We study a fast-slow, reaction-diffusion PDE system, consisting of two equations on a bounded domain, with a fold singularity in the reaction term of the fast variable, while the slow variable assumes the role of a dynamic bifurcation parameter. This extends the classical fast-slow dynamic fold bifurcation problem to an infinite-dimensional setting. Our approach is to use a spectral Galerkin discretization and use the techniques of Geometric Singular Perturbation Theory (GSPT) on the resulting high-dimensional ordinary differential equation systems. In particular, away from the fold singularity we obtain the existence of invariant manifolds, while in a neighbourhood of the singularity we use geometric desingularization via a blow-up method. A key ingredient is the inclusion of the domain length of the PDEs as a variable. Finally, we connect the invariant manifolds obtained through this discretization process with invariant manifolds that exists in the original phase space of the PDE system.
Discrete Geometric Singular Perturbation Theory
- joint event with TUM Applied Mathematics group meetings -

Sam Jelbart, Technical University of Munich
29 April 2022

Abstract

Geometric singular perturbation theory (GSPT) is a powerful and established mathematical framework for the analysis of fast-slow systems of ordinary differential equations (ODEs). In this talk we review the development of discrete GSPT or DGSPT, i.e. a corresponding theory for discrete dynamical systems induced by fast-slow maps. The theory provides (i) a means for identifying and analysing more tractable limiting problems associated to each scale (fast and slow), and (ii) a set of perturbation results theorems allowing one to infer dynamical information about perturbed (multi-scale) dynamics based on the dynamics of the limiting problems in (i). Similarly to the established GSPT for ODEs, perturbation theorems in the sense of (ii) are derived from established results in invariant manifold theory under normally hyperbolic conditions. We outline and sketch the proof of these results and, if time permits, consider the extension of DGSPT to the non-normally hyperbolic regime.

Geometric singular perturbation analysis of the multiple-timescale
Hodgkin-Huxley equations
- joint event with INRIA MathNeuro Seminars -

Panagiotis Kaklamanos, University of Edinburgh
13 May 2022

Abstract

The Hodgkin-Huxley (HH) equations [A. L. Hodgkin and A. F. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve, The Journal of Physiology, 117 (1952), pp. 500?544] are one of the most successful models to describe the propagation of action potentials in neurons. For their work, Hodgkin and Huxley received the 1963 Nobel Prize in Physiology and Medicine. The original HH system is four-dimensional, with dynamics evolving on at least three distinct timescales. In this talk, we consider a non-dimensionalised version of the four-dimensional Hodgkin-Huxley equations [J. Rubin and M. Wechselberger, Giant squid-hidden canard: the 3D geometry of the Hodgkin-Huxley model, Biological Cybernetics, 97 (2007), pp. 5-32], and we present a novel and global three-dimensional reduction that is based on geometric singular perturbation theory (GSPT). We investigate the dynamics of the resulting reduced system in regimes in which the flow evolves on three distinct timescales. Specifically, we demonstrate that the system exhibits bifurcations of oscillatory dynamics and complex mixed-mode oscillations (MMOs), in accordance with the geometric mechanisms introduced in [P. K., N. Popovic?, and K. U. Kristiansen, Bifurcations of mixed-mode oscillations in three-timescale systems: An extended prototypical example, Chaos: An Interdisciplinary Journal of Nonlinear Science, 32 (2022), p. 013108], and we classify the various firing patterns in dependence of the external applied current [P. K., Nikola Popovic, K. U. Kristiansen, Geometric singular perturbation analysis of the multiple-timescale Hodgkin-Huxley equations, arXiv preprint]. Time permitting, we will demonstrate how this methodology can be applied to other systems that are expressed in similar formalisms, such as models from cardiac dynamics.