Steering of optical solitary waves by coplanar low power beams in reorientational media

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The interaction of solitary waves in a nematic liquid crystal (NLC) with a coplanar low power optical beam is investigated. The emphasis of the study is on the control of the solitary wave trajectory by the low power beam and the transfer of momentum between the beams. The results of numerical studies are confirmed by a theoretical analysis of this momentum transfer. The implications for all-optical signal control are discussed.

Keywords: Nonlinear optics; optical solitons; all-optical switching; liquid crystals.

1. Introduction

Nonlinear light propagation in nematic liquid crystals (NLCs) has attracted many researchers in recent years, both experimentalists and theorists.\(^1\)\(^-\)\(^6\) The “huge”
optical nonlinearity, as well as the nonlocality which extends the medium response well beyond the size of the beam, render NLCs an ideal material platform for the excitation, propagation and management of optical solitary waves. The non-local response, in particular, prevents the collapse of two-dimensional bulk solitary waves,\textsuperscript{4,7,8} termed “nematicons”, making them extremely robust to perturbations and collisions.\textsuperscript{1,9} Experiments conducted on nematicons have shown that they could be employed in several all-optical devices, including circuits, reconfigurable interconnects, logic and bistable gates, owing to the waveguide character of Kerr-like solitary waves with respect to weaker copolarized signals\textsuperscript{10–13} and provided the response time is not a crucial issue.\textsuperscript{1,14–19} The formation of solitary waves in NLC requires that the Freédericksz threshold is overcome\textsuperscript{3,10,20} and that beam spatial dispersion (diffraction) is balanced out by self-focusing.\textsuperscript{8} To avoid this threshold the organic molecules of NLCs can be pre-tilted by applying an external low frequency electric field across the cell thickness\textsuperscript{1,10} or by anchoring the nematic molecules at a finite angle (typically close to π/4 (see Ref. 21)) with respect to the beam wavevector.\textsuperscript{1,2,22}

The path of nematicons can be altered and controlled by various means, including other solitary waves,\textsuperscript{9,23,24} external beams and refractive index perturbations,\textsuperscript{15,16,18,19,25–29} applied voltage(s)\textsuperscript{22,30,31} and input power.\textsuperscript{32,33} In this paper, we discuss the use of a low power beam launched in the same principal plane as the nematicon in order to modify the latter’s trajectory. The geometry under consideration, since the principal plane is defined by the optic axis (or molecular director) and the beam wavevector, corresponds to propagation either in the plane (x, z) orthogonal to the parallel interfaces of a planar cell or in the plane (y, z) parallel to them, depending on the initial orientation of the optic axis. In both cases the extraordinarily polarized electric field lies in the same principal plane. The copolarized and coplanar control beam is assumed to share the wavelength and be coherent with the nematicon, i.e., to affect its path through both the nonlinear nonlocal induced index perturbation and interference effects.

2. Governing Equations

Two extraordinarily polarized coherent light beams are launched into a planar NLC cell, as sketched in Fig. 1. We take the z-axis as the propagation direction, the x-axis as the direction of the input beam polarisation, with y completing the coordinate system. A solitary wave results owing to the balance between linear diffraction and the self-focusing of the beam via optical reorientation, i.e., the change in molecular director orientation due to the Coulomb torque between the electric field vector and the induced molecular dipoles. In positive uniaxial NLC the optic axis coincides with the molecular director, i.e., the average orientation of the molecular main axes in space. To maximize the nonlinearity (reorientational response) and be able to obtain solitary waves at milliwatt power levels, the molecular director is assumed to be pre-oriented at an angle θ₀ ∼ π/4 with respect to the beam wavevector, the
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![Diagram of NLC cell with principal plane (x, z). Two collinear copolarized (extraordinary wave) beams are launched in the NLC, one is a low power wider beam and the other forms a self-confined nematicon.]

latter taken parallel to \( z \) in Fig. 1. The pre-orientation, obtained by either an applied voltage bias across the cell thickness or suitable anchoring of the molecules at the planar (glass or plastic) interfaces, also eliminates the Freedericksz threshold\(^3,10\) and helps the medium to mimic a Kerr-like (saturable and nonlocal) nonlinear response. Neglecting birefringent walk-off (i.e., the angular offset between Poynting and wavevectors), the nondimensional equations governing the propagation of nonlinear, extraordinarily polarized light beams in the paraxial approximation are\(^34–36\)

\[
\begin{align*}
\frac{i}{\partial z} \frac{\partial E}{\partial z} + \frac{1}{2} \nabla^2 E + 2E\theta &= 0, \\
\nu \nabla^2 \theta - 2q\theta &= -2|E|^2.
\end{align*}
\]

Here the Laplacian \( \nabla^2 \) is in the \((x, y)\) plane. The first of (2.1) is a nonlinear Schrödinger (NLS)-type equation governing the complex electric field envelope \( E \) of the beam, with \( z \) being a “time-like” coordinate. The second equation of (2.1) is the director equation (an elliptic equation) which rules the perturbation \( \theta \) of the molecular director angle from the rest angle \( \theta_0 \). The parameter \( \nu \) relates to the intermolecular elasticity of the liquid crystal, with large values corresponding to a highly nonlocal response. In typical experiments \( \nu = O(100) \).\(^1,10,37\) We take the cell to be infinitely extended as beam sizes are much smaller than any characteristic (thickness, width, length) dimension of a cell. Hereby, we focus on coherent, copolarized, copropagating beams of the same wavelength with their electric field vectors in the principal plane. Therefore, the excitation is chosen as a one colour beam with two components, one representing the nematicon with a Gaussian, propagation invariant, profile and the other representing a low power Gaussian beam. There are no general nematicon solutions. Hence, the electric field of the optical beam will be assumed to have the form of the trial function

\[
E = a_1 e^{-r_1^2/w_1^2} e^{i(\sigma_1 + V_{x1}(x-\xi_1) + V_{y1}(y-\eta_1))} + a_2 e^{-r_2^2/w_2^2} e^{i(\sigma_2 + V_{x2}(x-\xi_2) + V_{y2}(y-\eta_2))},
\]

(2.2)
where $r_{1,2}^2 = (x - \xi_{1,2})^2 + (y - \eta_{1,2})^2$ with the subscripts 1 and 2 referring to the nematicon and low power beams, respectively.

### 3. Discussion and Results

To demonstrate the effects and the diversity of in-plane steering of a nematicon with a low power beam, various numerical experiments were conducted with parameters selected to be consistent with those of typical experiments.

#### 3.1. Change in power via a change in amplitude for the lower power beam

Two coplanar and copolarized beams are input into an NLC cell. One beam is a nematicon and the other is a low power beam with a fixed width. The effect of a change in the power of the low power beam is explored by increasing its amplitude. Due to the nonlocality of the NLC, the low power beam alters the refractive index well beyond its peak intensity, adding to the refractive index change self-induced by the nematicon. The refractive index change due to the presence of the low power beam skews the director’s response, i.e., the nematic molecules reorientate more where the low power beam is present. Figure 2 shows the skewness of the director field at $(\xi_2, \eta_2) = (-20, 0)$ where the peak intensity of the low power beam is located. Hence, by changing the amplitude of the low power beam, a larger reorientation takes place. The corresponding increase in refractive index attracts the nematicon, as shown in Fig. 3(a). Figure 3(a) shows the trajectories of the nematicon as it evolves in the NLC when the low power beam is initially at $(\xi_2, \eta_2) = (-10, 0)$. The trajectory of the nematicon can be steered more by increasing the initial distance between the two beams. Figure 4(a) shows the nematicon trajectory when the low power beam is initially at $(\xi_2, \eta_2) = (-20, 0)$. An increased movement towards the

![Fig. 2. (Color online) The low power beam modifies the refractive index distribution in the NLC. (a) Side-view and (b) top-down view of the same refractive index change due to the control beam. By reducing the maximum value of $\theta$ (in (b)) the change in the refractive index can be seen more clearly. The initial values for the nematicon are $a_1 = 2.5$, $w_1 = 3$, $(\xi_1, \eta_1) = (0, 0)$ and $(V_{x1}, V_{y1}) = (0, 0)$. The initial values for the low power beam are $a_2 = 0.3$, $w_2 = 15$, $(\xi_2, \eta_2) = (-10, 0)$ and $(V_{x2}, V_{y2}) = (0, 0)$, with $\nu = 200$ and $q = 2$.](image-url)
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Fig. 3. (Color online) Nematicon trajectories for different amplitudes of the low power beam, for \( a_2 = 0.0 \) (red solid line), \( a_2 = 0.1 \) (green long dashed line), \( a_2 = 0.15 \) (blue short dashed line), \( a_2 = 0.2 \) (magenta dotted-line) and \( a_2 = 0.25 \) (light blue dot-long dashed line) for (a) the position of low power beam at \((\xi_2, \eta_2) = (-10, 0)\) (region between two black dotted lines). Final (b) nematicon electric field modulus and (c) director field for each of the amplitudes. The initial values for the nematicon are \( a_1 = 2.5 \), \( w_1 = 3 \), \((\xi_1, \eta_1) = (0, 0)\) and \((V_{x1}, V_{y1}) = (0, 0)\). The initial values for the low power beam are \( w_2 = 15 \) and \((V_{x2}, V_{y2}) = (0, 0)\), with \( \nu = 200 \) and \( q = 2 \).

The movement of the nematicon in both cases is towards the refractive index change created by the low power beam if the two are launched in-phase. As the amplitude of the low power beam increases, an in-phase nematicon moves closer towards it. This nonlinear interaction is highly enhanced due to the mutual phase relationship of the two coherent beams.

When the fields of the nematicon and the control (low power) beams are \( \pi \) out of phase, the beams tend to repel in spite of the attraction mediated by the additional refractive index change created by the low power beam. When the latter overcomes this repulsion, then the nematicon “jumps” into the “potential well” associated with the control beam.

Figures 5(a)–5(e) show the evolution of nematicon and low power beams, with the exception of Fig. 5(a) which shows the beams repelling as expected. In all other
Fig. 4. (Color online) Nematicon trajectories for different amplitudes of the low power beam, for $a_2 = 0.0$ (red solid line), $a_2 = 0.1$ (green long dashed line), $a_2 = 0.15$ (blue short dashed line), $a_2 = 0.2$ (magenta dotted-line) and $a_2 = 0.25$ (light blue dot-long dashed line) for (a) the position of the low power beam at $(\xi_2, \eta_2) = (-20, 0)$ (region between two black dotted lines). Comparison of the final (b) nematicon electric field modulus and (c) director field for each of the amplitudes. The initial values for the nematicon are $a_1 = 2.5$, $w_1 = 3$, $(\xi_1, \eta_1) = (0, 0)$ and $(V_{x1}, V_{y1}) = (0, 0)$. The initial values for the low power beam are $w_2 = 15$ and $(V_{x2}, V_{y2}) = (0, 0)$, with $\nu = 200$ and $q = 2$.

cases the beams exchange positions. Figure 6 highlights the positional exchange of the nematicon, showing the abrupt jump occurring within the region between $z = 20$ to $z = 30$ for all the nematicons, except for the nematicon colannanced with the lowest power control beam (Fig. 5(a)), which continues along a quasi-straight trajectory. This jump behaviour occurs when the refractive index of the NLC is altered sufficiently, so that the repulsion initially felt by the two beams is no longer strong enough to maintain their separation. Hence, the director provides a means for the beams to exchange positions. Figure 7(a) shows the electric field at $z = 100$, with the nematicon and low power beam initially coincident. This figure also highlights the diffraction of the low power beam. Figure 7(b) shows the director distribution at $z = 100$, with the low power beam still influencing the refractive index. Finally, Fig. 7(c) illustrates the phase distribution at $z = 100$.
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Fig. 5. (Color online) Evolution of a nematicon and low power beam for different initial amplitudes of the low power beam. (a) \(a_2 = 0.1\), (b) \(a_2 = 0.15\), (c) \(a_2 = 0.2\), (d) \(a_2 = 0.25\) and (e) \(a_2 = 0.3\). The initial values for the nematicon are \(a_1 = 2.5, w_1 = 3, (\xi_1, \eta_1) = (0, 0)\) and \((V_{x1}, V_{y1}) = (0, 0)\). The initial values for the low power beam are \(w_2 = 15, (\xi_2, \eta_2) = (-10, 0)\) and \((V_{x2}, V_{y2}) = (0, 0)\), with \(\nu = 200\) and \(q = 2\).

Fig. 6. (Color online) Comparison of the nematicon trajectory for different amplitudes of a low power beam. \(a_2 = 0.1\) (red solid line), \(a_2 = 0.15\) (green long dashed line), \(a_2 = 0.2\) (blue short dashed line) and \(a_2 = 0.25\) (magenta dotted line) and \(a_2 = 0.3\) (light blue dot-long dashed line). The trajectory of the unimpeded low power beam is the region between two black dotted lines. The initial values for the nematicon are \(a_1 = 2.5, w_1 = 3, (\xi_1, \eta_1) = (0, 0)\) and \((V_{x1}, V_{y1}) = (0, 0)\). The initial values for the low power beam are \(w_2 = 15, (\xi_2, \eta_2) = (-10, 0)\) and \((V_{x2}, V_{y2}) = (0, 0)\), with \(\nu = 200\) and \(q = 2\).
Fig. 7. (Color online) The beam at $z = 100$ showing (a) the modulus of the electric field, (b) the director field and (c) the phase of the nematicon and low power beam. The initial values for the nematicon are $a_1 = 2.5$, $w_1 = 3$, $(\xi_1, \eta_1) = (0, 0)$ and $(V_{x1}, V_{y1}) = (0, 0)$. The initial values for the low power beam are $a_2 = 0.3$, $w_2 = 15$, $(\xi_2, \eta_2) = (-10, 0)$ and $(V_{x2}, V_{y2}) = (0, 0)$, with $\nu = 200$ and $q = 2$.

showing the formation of concentric circles around the position of the nematicon. These concentric circles also occur in the phase plot for the propagation of only one nematicon. However, for the coupled nematicon and low power beam, in the region around $x = 7$, there is a bulge due to the presence of the low power beam which now has the form of diffractive radiation.

3.2. Refraction

To demonstrate nematicon refraction, two coplanar, copolarized beams are input into a NLC cell, with the low power beam fixed at $(\xi_2, \eta_2) = (0, 0)$. The nematicon is given an initial angle and is input at the position $(\xi_1, \eta_1) = (-10, 0)$. The low power beam creates a refractive index change and the nematicon undergoes refraction. Figure 8(a) depicts the evolution of the nematicon without the low power beam, while Fig. 8(b) depicts the evolution of the nematicon with the low power beam perturbing the NLC. At the propagation length $z = 40$ the interaction of the two beams is strongest and refraction of the nematicon can be clearly observed.
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Fig. 8. (Color online) The evolution of a nematicon (a) without the presence a low power beam and (b) with an induced refractive index change due to the presence of a low power beam. The initial values for the nematicon are $a_1 = 2.5$, $w_1 = 3$, $(\xi_1, \eta_1) = (-10, 0)$ and $(V_{x1}, V_{y1}) = (0.2, 0)$. The initial values for the low power beam are $a_2 = 0.25$, $w_2 = 15$, $(\xi_2, \eta_2) = (0, 0)$ and $(V_{x2}, V_{y2}) = (0, 0)$, with $\nu = 200$ and $q = 2$.

Figures 9(a)–9(e) show the interaction of a low power beam of various input powers with a nematicon launched at different angles. Figure 9(a) uses the smallest input angle for the nematicon. Within the propagation length the nematicons do not bend below their paths in the absence of the low power beam. However, the nematicons are attracted to the low power beam, with the largest power control beam creating the strongest attraction of the nematicon, as evidenced by the largest bending. As the nematicon input angle is increased, the bending of its trajectory increases and the nematicon is attracted back towards the low power beam. Thus, the trajectory of the nematicon is bent, as shown in Figs. 9(c)–9(e), where the nematicon deviates from the path of the unperturbed nematicon. Steering of the nematicon can then be achieved by using an initial tilt and controlling the final position by the amplitude (power) of the low power beam.

3.3. Change in width of the low power beam

The effects of increasing the power of the control beam are explored by fixing its amplitude and widening its profile. The tails of the low power beam extend further and affect a larger region of NLC than in the case of a fixed width and increased amplitude (Sec. 3.1). However, due to the small amplitude of the control beam, the rotation induced in the NLC molecules is less than that of Sec. 3.1 as the peak intensity is smaller. Figure 10 displays the director distribution with skewness towards the low power beam at $(\xi_2, \eta_2) = (-20, 0)$. Comparing Fig. 10(b) with Fig. 3(b) a larger NLC region is affected by the wider low power beam (note the scales of the respective plots). Figures 11(a) and 11(b) show the nematicon trajectory after copropagating with a low power beam of different widths and two different initial positions, $(\xi_2, \eta_2) = (-10, 0)$ and $(\xi_2, \eta_2) = (-20, 0)$, respectively. The trajectories of the nematicons are similar to those in Figs. 3(a) and 4(a).
Fig. 9. (Color online) Nematicon trajectories for different initial amplitudes of a low power beam between $a_2 = 0.0$ (red solid line), $a_2 = 0.15$ (green long dashed line), $a_2 = 0.2$ (blue short dashed line) and $a_2 = 0.25$ (magenta dotted line) for different launch angles of the nematicon (a) $(V_{x1}, V_{y1}) = (0.05, 0)$, (b) $(V_{x1}, V_{y1}) = (0.1, 0)$, (c) $(V_{x1}, V_{y1}) = (0.15, 0)$, (d) $(V_{x1}, V_{y1}) = (0.2, 0)$, and (e) $(V_{x1}, V_{y1}) = (0.25, 0)$, for an infinite cell. The trajectory of the unimpeded low power beam (region between two black dotted lines). The initial values for the nematicon are $a_1 = 2.5$, $w_1 = 3$, $(\xi_1, \eta_1) = (-10, 0)$. The initial values for the low power beam are $w_2 = 15$, $(\xi_2, \eta_2) = (0, 0)$ and $(V_{x2}, V_{y2}) = (0, 0)$, with $\nu = 200$, and $q = 2$. 

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3.4. Tilted low power beam

Two coplanar, copolarized beams are launched into a NLC cell, with the low power beam given an initial launch angle. Two different cases are studied, in the first case the low power beam has a fixed width of $w_2 = 15$, while in the second case the width of the low power beam is doubled to $w_2 = 30$. Figures 12(a) and 12(b) show the nematicon trajectory when the control beam has initial angles $(V_{x2}, V_{y2}) = (0, 0)$ and $(V_{x2}, V_{y2}) = (0.4, 0)$, respectively, with initial position at $(\xi_2, \eta_2) = (-10, 0)$.
Fig. 12. (Color online) Nematicon trajectories for different amplitudes with a low power beam with a fixed launch angle for $a_2 = 0.1$ (red solid line), $a_2 = 0.15$ (green long dashed line), $a_2 = 0.2$ (blue short dashed line) and $a_2 = 0.25$ (magenta dotted-line) for (a) $(\xi_2, \eta_2) = (-10, 0)$, $(V_{x2}, V_{y2}) = (0.2, 0)$, (b) $(\xi_2, \eta_2) = (-10, 0)$, $(V_{x2}, V_{y2}) = (0.4, 0)$, (c) $(\xi_2, \eta_2) = (-20, 0)$, $(V_{x2}, V_{y2}) = (0.2, 0)$ and (d) $(\xi_2, \eta_2) = (-20, 0)$, $(V_{x2}, V_{y2}) = (0.4, 0)$ for an infinite cell. Trajectory of the unimpeded low power beam only for $w_2 = 15$: region between two black dotted lines. The initial values for the nematicon are $a_1 = 2.5$, $w_1 = 3$, $(\xi_1, \eta_1) = (0, 0)$ and $(V_{x1}, V_{y1}) = (0, 0)$. The initial values for the low power beam are $w_2 = 15$, with $\nu = 200$ and $q = 2$.

Before the low power beam crosses the path of the nematicon, the latter is attracted towards the low power beam. As the low power beam passes through the nematicon, there is a shift in the position of the nematicon and it tends to follow the low power beam. However, the initial momentum of the nematicon continues to move it in the opposite direction. Figure 12(b) indicates (by the trajectory of the nematicon) that the larger initial angle of the low power beam affects the refractive index only near the nematicon, which results in the nematicon trajectories converging to the same point regardless of the initial power of the control beam. Figures 12(c) and 12(d) show the nematicon trajectory when the control beam has initial angles $(V_{x2}, V_{y2}) = (0.2, 0)$ and $(V_{x2}, V_{y2}) = (0.4, 0)$, respectively, with initial position at $(\xi_2, \eta_2) = (-20, 0)$. Very little movement of the nematicon trajectory is observed, but there is a slight movement of the nematicon towards the control beam at $z = 200$. 
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Fig. 13. (Color online) Nematicon trajectories for different amplitudes with a low power beam with a fixed launch angle for $a_2 = 0.1$ (red solid line), $a_2 = 0.15$ (green long dashed line), $a_2 = 0.2$ (blue short dashed line) and $a_2 = 0.25$ (magenta dotted line). (a) $(\xi_2, \eta_2) = (-10, 0)$, $(V_{x2}, V_{y2}) = (0.2, 0)$, (b) $(\xi_2, \eta_2) = (-10, 0)$, $(V_{x2}, V_{y2}) = (0.4, 0)$, (c) $(\xi_2, \eta_2) = (-20, 0)$, $(V_{x2}, V_{y2}) = (0.2, 0)$ and (d) $(\xi_2, \eta_2) = (-20, 0)$, $(V_{x2}, V_{y2}) = (0.4, 0)$ for an infinite cell. Trajectory of the unimpeded low power beam only for $w_2 = 30$: region between two black dotted lines. The initial values for the nematicon are $a_1 = 2.5$, $w_1 = 3$, $(\xi_1, \eta_1) = (0, 0)$ and $(V_{x1}, V_{y1}) = (0, 0)$. The initial values for the low power beam are $w_2 = 30$ with $\nu = 200$, and $q = 2$.

Figures 13(a) and 13(b) show a different result from the previous figure. The nematicon follows the low power beam after the interaction, rather than continuing towards the initial position of the low power beam. The movement of the nematicon’s trajectory is more significant when the initial angle of the low power beam is small, and its power is large, as the refractive index of the NLC is affected more by both. Shifting the initial position of the low power beam to $(\xi_2, \eta_2) = (-20, 0)$ results in the nematicon trajectory moving more towards the low power beam, as shown in Fig. 13(c). Due to the larger width of the low power beam, the director is perturbed more and as, a result, attracts the nematicon. The wider low power beam acts like a plane wave compared with the narrower low power beam of Fig. 12. The same is true when the initial angle of the low power beam is $(V_{x2}, V_{y2}) = (0.4, 0)$.
as shown in Fig. 13(d), for which the trajectory of the nematicon has been bent dramatically from its original input position.

3.4.1. Asymptotic analysis

An estimate for the position of the deflected nematicon of Sec. 3.4 can be obtained by using an averaged Lagrangian as follows. We assume the impinging wave is a linear wave which, to leading order, is a solution of the Schrödinger equation, that is the first equation of (2.1) with the nonlinear term involving \( \theta \) neglected. This gives the usual plane wave solution

\[
E_{nw} = a_2 e^{i(V_2 \cdot X - \lambda z)}, \tag{3.1}
\]

where \( \lambda \) satisfies the linear dispersion relation for a plane wave in the form

\[
\lambda = \frac{1}{2} V_2 \cdot V_2 - a_2^2. \tag{3.2}
\]

The vector \( V_2 \) gives the direction of the incident beam. We now assume that the total electric field can be taken in the form

\[
E_{nem} = a_1 e^{-r^2/w_1^2} e^{i(V_1 \cdot (X - \zeta(z)) + \sigma z)} + Q, \tag{3.3}
\]

where \( \zeta = (\xi_1, \eta_1), V_1 = (V_{x1}, V_{y1}) \) and \( r = |X - \zeta| \). The nematicon parameters \( a_1, w_1 \) and \( \sigma \) are taken to be constants and to satisfy the approximate amplitude-width relation. Also, the amplitude \( a_2 \) and the wavevector \( V_2 \) are taken to be constants, along with \( \lambda \) satisfying the linear dispersion relation, Eq. (3.2). The only \( z \) dependence is in the position and velocity of the nematicon.

The Lagrangian of the nematicon equations (2.1) is

\[
\int_0^{z_f} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ i(E_z E^* - E_z^* E) - |\nabla E|^2 + 2\theta E \right] dx \, dy \, dz, \tag{3.4}
\]

where \( z_f \) is the final propagation length, \( E = E_{nem} + E_{nw} \) and the director angle \( \theta \) satisfies

\[
\nu \nabla^2 \theta - 2q \theta = -2a_1 e^{-r^2/w_1^2} e^{i(V_1 \cdot (X - \zeta(z)) + \sigma z)} + Q|^2. \tag{3.5}
\]

We now obtain an approximation to the refractive index change induced by the introduction of the linear beam to the nematicon. This is obtained by keeping only the linear terms in \( a_1 \) on the right-hand side of Eq. (3.5). This gives for the total director \( \theta = \theta_{nem} + \theta_{nw} \)

\[
\nu \nabla^2 \theta_{nem} - 2q \theta_{nem} = -2a_1^2 e^{-2r^2/w_1^2}, \tag{3.6}
\]

\[
\nu \nabla^2 \theta_{nw} - 2q \theta_{nw} = -4 \text{Re} \, a_1 a_2 e^{-|X - \zeta(z)|^2/w_1^2} \times \cos (V_2 \cdot X - \lambda z - \sigma z + V_1 \cdot (X - \zeta(z))). \tag{3.7}
\]

The solution for \( \theta_{nem} \) is just the usual term sustaining the nematicon which gives the usual contribution to the nematicon Lagrangian which is independent of \( \zeta(z) \).
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We approximate \( \theta_{nw} \) using the delta function for the nematicon trial function to obtain

\[
\theta_{nw} = 4a_1a_2 G(X - \zeta(z)) \cos(V_2 \cdot \zeta(z) - \lambda z - \sigma z) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-|X - \zeta(z)|^2/w_1^2} dX,
\]

(3.8)

where \( G(X, X') \) is the Green’s function for the director equation, the second of (2.1).

This last expression is substituted in the Lagrangian (3.4) to obtain for the contribution of the director to the Lagrangian

\[
L_{nw} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |E(X)|^2 dX \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(X, X') |E(X')|^2 dX' \\
= 4a_1a_2 \cos(V_2 \cdot \zeta(z) - \lambda z - \sigma z) \\
\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-|X - \zeta(z)|^2/w_1^2} dX \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(X, X') |E(X')|^2 dX'.
\]

(3.9)

The second integral is now approximated by taking \( |E| \) to be the nematicon field. It is then independent of \( \zeta(z) \). The final Lagrangian is therefore

\[
L = \int_{0}^{z_f} \left[ M \dot{\zeta} \cdot \dot{V}_1 - \frac{M}{2} |V_1|^2 - 4\pi a_1^2 a_2 w_1^4 \cos(V_2 \cdot \zeta(z) - \lambda z - \sigma z) \right] dz,
\]

(3.10)

where \( M = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |E|^2 dX \) and we have omitted the \( z \) independent terms.

Taking variations of the Lagrangian (3.10) gives the equations of motion for the centre of the beam

\[
\dot{\zeta} = V_1, \\
\dot{V}_1 = \frac{4\pi a_1^2 a_2 w_1^4}{M} V_2 \sin(V_2 \cdot \zeta(z) - \lambda z - \sigma z).
\]

(3.11)

(3.12)

These equations of motion have a simple solution which gives the nematicon trajectory. This solution must match the phase of the linear wave and the nematicon, which gives

\[
V_2 \cdot \zeta(z) = (\lambda + \sigma) z.
\]

(3.13)

Thus

\[
\zeta(z) = \frac{V_2}{|V_2|} \left( \frac{|V_2|}{2} - \frac{a_2^2}{|V_2|} + \frac{\sigma}{|V_2|} \right) z
\]

(3.14)

is the required solution for the nematicon trajectory.

To compare the modulation solution with numerical solutions, we set \( a_2 = 0.25 \) and \( V_2 = (0.4, 0) \) and the nematicon phase \( \sigma \) is estimated from the numerical evolution of the free nematicon. For the amplitudes and widths in this case we have \( a_1 = 2.5 \) and \( w_1 = 3 \). The frequency of oscillation of the nematicon in the field of the beam is estimated from the period in Fig. 14, which is a plot of the numerical amplitude of the nematicon interactions with the low power beam. This
Fig. 14. (Color online) Amplitude for a nematicon and low power beam interaction. The initial values for the nematicon are \( a_1 = 2.5, w_1 = 3, (\xi_1, \eta_1) = (0, 0) \) and \( (V_{x1}, V_{y1}) = (0, 0) \). The initial values for the low power beam are \( a_2 = 0.25, w_2 = 30, (\xi_2, \eta_2) = (-20, 0) \) and \( (V_{x2}, V_{y2}) = (0.4, 0) \), with \( \nu = 200 \), and \( q = 2 \).

gives the estimate \( \sigma = 2\pi/45 \), which is smaller than the value \( \sigma = \pi/10 \) for a free nematicon.\(^{39} \) With these values we have for the vertical component of the nematicon displacement \( d(z) \) (the only one present in this case)

\[
d(z) = 0.35(z - z_0), \tag{3.15}
\]

where \( z_0 \) is the point where the nematicon starts to move. From Fig. 13(d) we obtain for \( z_0 = 100 \) the displacement of \( d(200) = 35 \), which compares well with the displacement calculated from the full numerical solution, which is \( d(z) = 30 \) (light blue dot-long dashed line in Fig. 13(d)).

The reduction in \( \sigma \) can be explained qualitatively using the modulation equation for \( \sigma \), which is now modified by the interaction between the nematicon and the plane wave. This term decreases the frequency. However, to complete the study it will be necessary to include radiation effects which will further slow the nematicon. This is currently under investigation.

### 4. Conclusion

A NLC has been demonstrated to show numerous different effects when used as a medium to propagate nematicons.\(^1,^2 \) Numerous different techniques for beam steering have been applied experimentally and explored theoretically.\(^{36} \) However, in-plane steering by a low power beam is not one of them. In this article several methods have been employed to explore the potential for steering a nematicon with another beam in the form of a low power beam. It was found that fixing either the amplitude or width of the low power beam results in the deformation of the director which attracts the nematicon, thus altering its trajectory. The nematicon’s trajectory can also be steered using a refractive index change induced by a low power beam, with the inputted nematicon tilted by an angle. The propagation of
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two out of phase beams lead to the interesting result of the nematicon swapping positions with the low power beam, which is diffracted away afterwards. This effect has the potential to be used in all-optical communications. Tilting the initial angle of the low power beam influences the nematicon trajectory which is dependent upon the width of the low power beam. Finally, increasing the width of the low power beam from \( w_2 = 15 \) to \( w_2 = 30 \) created a beam which was similar to a plane wave and “transferred” momentum to the nematicon. The use of low power beams to steer and control nematicons offers a new avenue to explore experimentally and theoretically.

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