Analysis and Stability of Accelerating Solitons and Self-Guiding Beams

David F. Parker

School of Mathematics and Maxwell Institute for Mathematical Sciences,
University of Edinburgh, The King's Buildings, Edinburgh, EH9 3JZ, U.K.

Many physical phenomena (*e.g.* Raman scattering, higher-order dispersion) cause nonlinear optical pulses to have varying speed or cause self-guiding beams to bend (*e.g.* in photo-refractive media). These effects are usually analysed by treating perturbations from a nonlinear Schrödinger equation as small and then using soliton perturbation theory, so yielding evolution equations for the soliton parameters. Many such treatments lead to 'soliton paths' which are parabolae.

While most partial differential equations (pdes) studied in nonlinear optics possess 'travelling wave' solutions, some also allow 'accelerating solutions'. These arise when the governing pde allows a similarity variable which is quadratic in either time or evolution distance. Examples used to illustrate this will include Raman scattering, photo-refractive spatial solitons and sliding frequency filter systems. In each case, the pulse (or beam) profile is governed by an ordinary differential equation which must be solved numerically, taking account of the Airy-function asymptotics in the pulse 'tails'.

Airy-function asymptotics also crucially modify the Evans function method which yields a stability analysis for the accelerating solutions. The Evans function method also, in some cases, predicts the existence of internal modes, consistent with persistent oscillations observed sometimes in numerical simulations of the governing pdes. Finally, some connections between Airy-function tails and emitted radiation will be made.