

SUPPLEMENTARY MATERIAL FOR TIME-VARYING EXTREME VALUE DEPENDENCE WITH APPLICATION TO LEADING EUROPEAN STOCK MARKETS

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1. Appendix A: Monte Carlo mean angular surfaces. Figure 1 shows Monte Carlo mean angular surfaces for the three simulation scenarios.

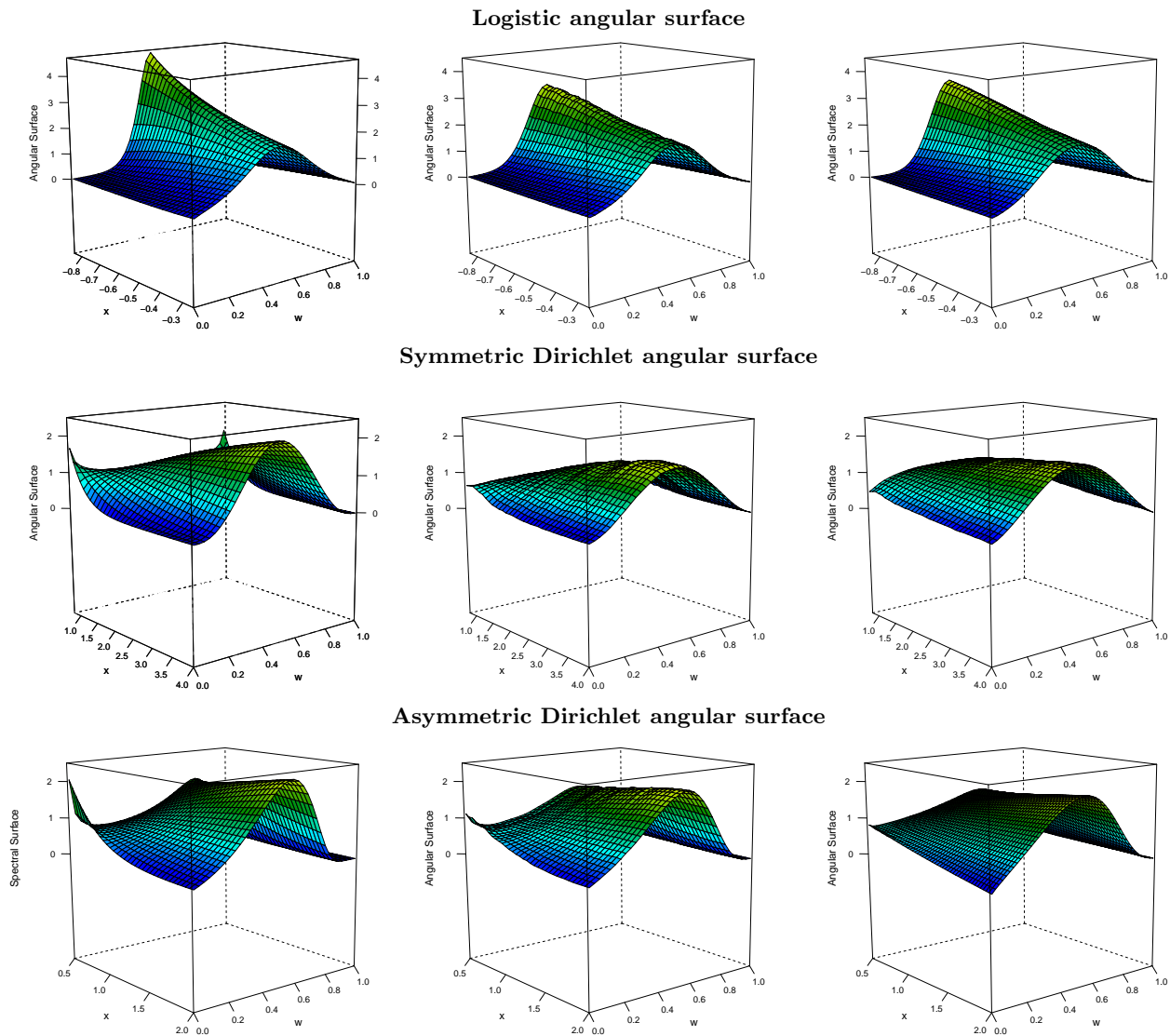


FIG 1. True angular surfaces (left) and corresponding mean estimates using Nadaraya–Watson weights (middle) and local linear weights (right). Top panel: conditional logistic model with $\alpha_x = \Phi(x)$, for $x \in \mathcal{X}_{\text{logistic}} = [\Phi^{-1}(0.2), \Phi^{-1}(0.4)]$. Center panel: conditional Symmetric Dirichlet model with $(a_x, b_x) = (x, x)$, for $x \in \mathcal{X}_{\text{sDir}} = [0.8, 4]$. Bottom panel: conditional Asymmetric Dirichlet model with $(a_x, b_x) = (x, 100)$, for $x \in \mathcal{X}_{\text{aDir}} \in [0.5, 2]$.

2. Appendix B: Supplementary data analysis reports. Recall that our data consists of daily closing stock index levels of three leading European stock markets: CAC 40, DAX 30, and FTSE 100. The sample period spans from January 1, 1988 to January 1, 2014 (6784 observations). Figure 2 shows series of unfiltered daily negative returns and GARCH-filtered residuals, whereas Table 1 presents summary statistics for CAC 40, DAX 30 and FTSE 100.

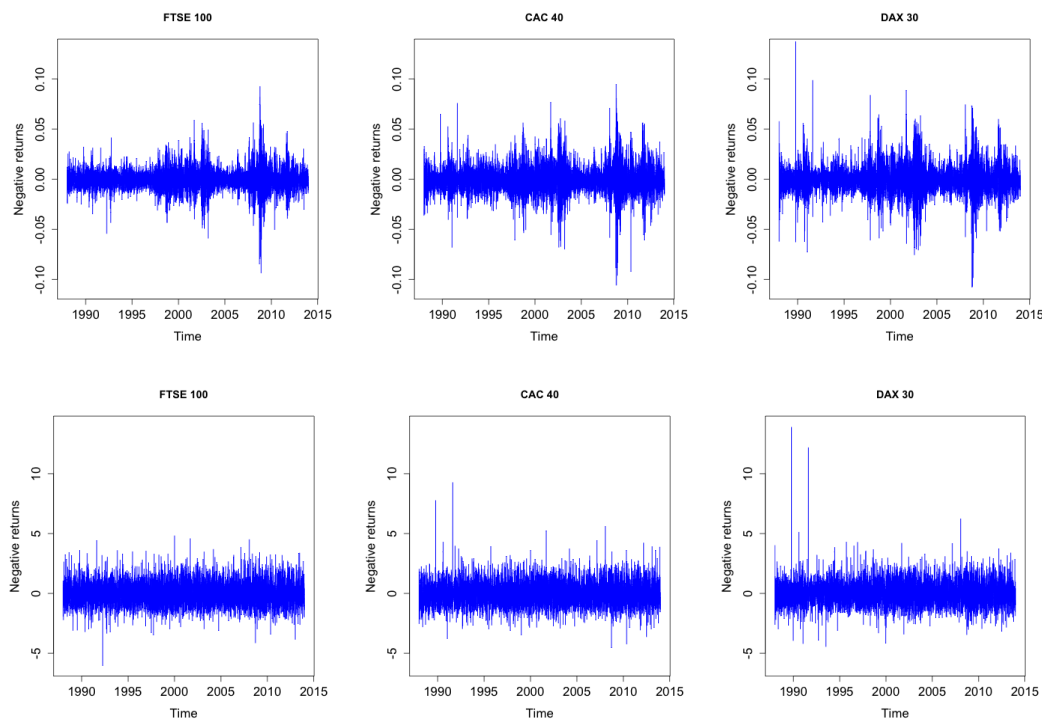


FIG 2. Unfiltered daily negative returns (top) and GARCH-filtered residuals (bottom) for FTSE 100(UK), CAC 40 (FR), and DAX 30 (DE) spanning the period from January 1, 1988 to January 1, 2014.

TABLE 1

Summary statistics of unfiltered daily negative returns and GARCH filtered residual series for the three stock markets indices: FTSE 100 (UK), CAC 40 (FR) and DAX 30 (DE). The data span the period from January 1, 1988 to January 1, 2014.

Statistic	Unfiltered returns			GARCH-filtered residuals		
	UK	FR	DE	UK	FR	DE
Mean	0	0	0	0.027	0.038	0.034
Std Deviation	0.011	0.014	0.015	1.000	1.004	1.001
Minimum	-0.094	-0.106	-0.108	-6.036	-4.552	-4.450
Maximum	0.093	0.095	0.137	4.825	9.268	13.888
Skewness	3.074	1.560	2.953	-2.235	-0.736	6.817
Excess Kurtosis	0.134	0.044	0.261	0.204	0.373	0.919

3. Appendix C: Empirical analysis using nonlinear asymmetric GARCH (NGARCH) filtered residuals. Diagnostic plots (not shown here) suggest that heteroskedasticity is successfully removed with the GARCH and NGARCH filters, and although the GARCH fits are superior than the NGARCH fits for the three stock markets, returns that are filtered using

NGARCH are still suitable for our methods. In the spirit of Poon et al. (2003, 2004) and for the sake of comparison, we conduct an empirical analysis using the NGARCH-filtered residuals, and show the main results here. Before implementing our methods, an assessment on whether NGARCH-filtered residuals are asymptotically dependent was conducted, and the results are in line with those found using GARCH-filtered residuals. Using the same procedure detailed in Section 5.3, we construct the pseudo-angles that give rise to the angular surface estimates shown in Figure 3. All in all, we can clearly see the change from weaker dependence to strong dependence that we have already spotted using GARCH-filtered residuals. Despite some small differences in the edges of $(0, 1)$, the angular surfaces based on both residuals indicate that in recent decades there has been an increase in the extremal dependence in the losses for these leading European stock markets.

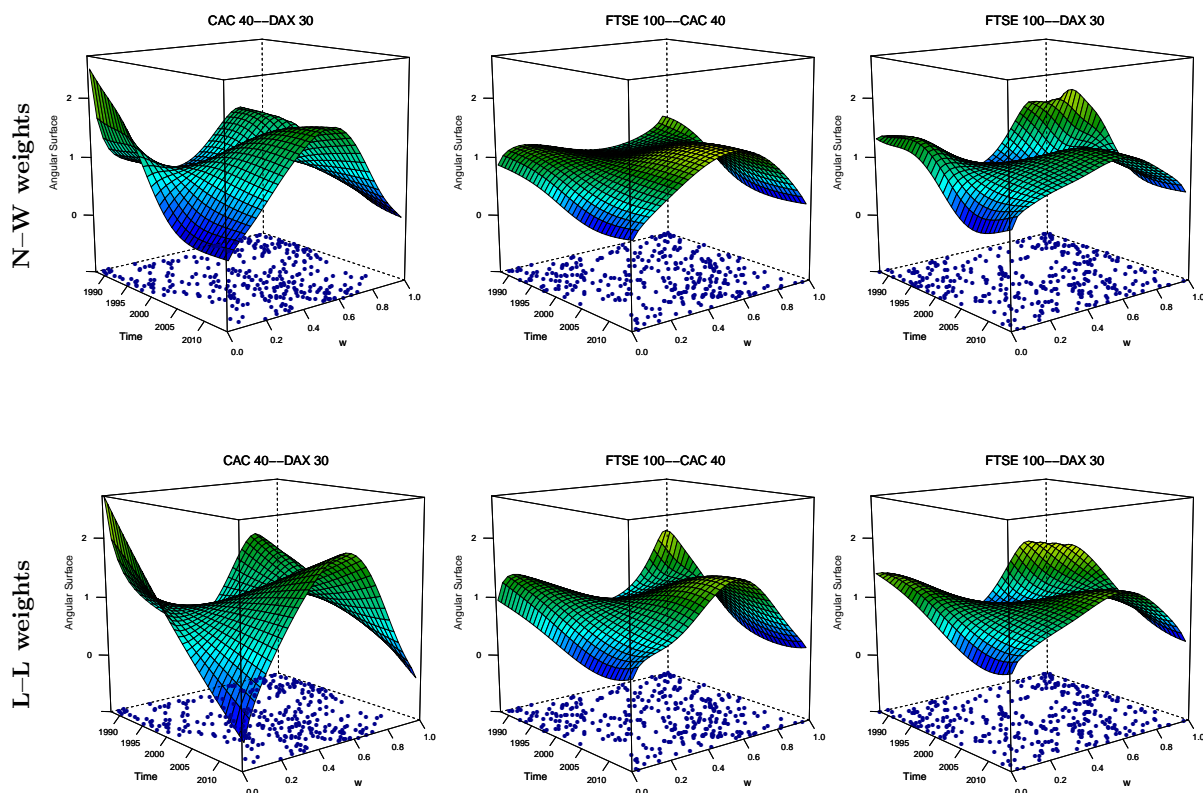


FIG 3. Angular surfaces estimates based on NGARCH-filtered residuals for CAC–DAX, FTSE–CAC and FTSE–DAX using Nadaraya–Watson (top) and local linear (bottom) weights, with pseudo-angles overlaid on the bottom of the box.

4. Appendix D: Smoothing with least-squares cross-validation. Figure 4 is the analog of Figure 7 in the paper, but using least-squares cross-validation (LSCV) instead of maximum likelihood cross-validation (MLCV). Although there are some small differences in the shapes (especially for FTSE–CAC), overall the information in terms of evolution of the extremal dependence structure is similar for both methods.

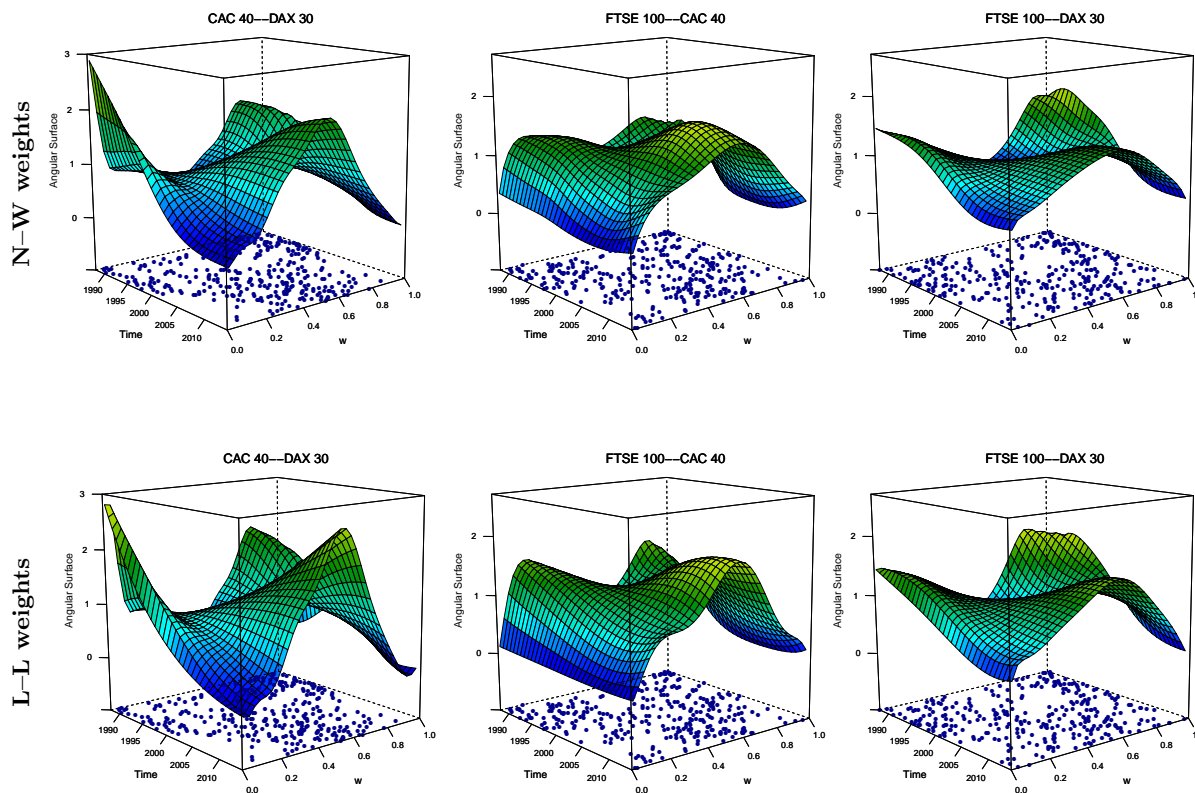


FIG 4. Angular surfaces estimates for CAC-DAX, FTSE-CAC and FTSE-DAX using Nadaraya-Watson (top) and local linear (bottom) weights, with pseudo-angles overlaid on the bottom of the box. This figure compares with Figure 7 in the paper, but using LSCV instead of MLCV.

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