Five-minute talk, SLMath, Spring 2024

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Slide 1:

Hi everyone! My name is Lucien Hennecart, and before coming here I was in Edinburgh. My broad area of research is geometric representation theory. I study the symmetries of moduli spaces arising in algebraic geometry. More specifically, I work in cohomological Donaldson–Thomas theory. The idea is that of categorification, that is to find graded vector spaces whose Euler characteristic recovers some well-known enumerative invariants and even more, to find perverse sheaves whose global sections give this graded vector space. The earliest example of categorification goes back to one century ago and is probably the replacement of the Betti numbers of a topological space by its singular cohomology. The richer objects we obtain by the categorification process provide more leverage to understand the enumerative theories.

We start from an Abelian category $\mathcal{A}$ having a stack of objects $\mathfrak{M}_{\mathcal{A}}$. In practice, it is either of representation-theoretic flavour, such as representations of quivers or of topological flavour, such as local systems over a Riemann surface, or of geometric nature, such as coherent sheaves on low-dimensional manifolds.

I seek to understand the cohomological refinements of enumerative theories such as Donaldson–Thomas theory and to understand the symmetries of the cohomology of moduli spaces in algebraic geometry. Donaldson–Thomas theory is originally a theory counting sheaves on Calabi–Yau threefolds. Its scope has since been broaden and applies now to more general categories as above.

The main object we study to achieve this goal is the Borel–Moore homology of the stack $\mathfrak{M}_{\mathcal{A}}$. It encodes information regarding the topology and the singularities of this stack. To understand this object, we define an algebra structure on it, called cohomological Hall algebra. It allows us to study efficiently the purity of the mixed Hodge structure and to find a decomposition of this space in elementary pieces.

Slide 2: My recent results on the subject are the following. First, in a work in collaboration, I proved an isomorphism between the Borel–Moore homology of the stack and the symmetric power of a much smaller and tractable subspace, which is the BPS space.

This smaller subspace has a Lie algebra structure. I determined that it is a generalised Kac–Moody Lie algebra in the sense of Borcherds. This gives a presentation by generators and relations. The generators are given by the intersection cohomology of the coarse moduli space of objects in the category $\mathcal{A}$.

These strong strutural results have several applications in geometry and representation theory. They may be applied to study the positivity of some counting polynomials appearing in the study of constructible Hall algebras.

They can also be used to give a full description of the cohomology of Nakajima quiver varieties as representations over a Lie algebra.

These results and applications give further research directions. First, I aim to understand the full Borel–Moore homology as an algebra and not only as a vector space. Then, I would like to use this finer understanding to study the Borel–Moore homology of more general 0-shifted symplectic stacks. I also would like to go further and investigate the critical cohomology of general (-1)-shifted symplectic stacks. The aim is to give new structural results for the Hitchin fibration for general reductive groups. In these more general settings, the main difficulty is the lack of categorical context.