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BPS algebras and generalised Kac–Moody algebras from 2-Calabi–Yau categories

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The results presented here are selected from the papers [1] and [2], which are joint with Ben Davison and Sebastian Schlegel Mejia.

2-Calabi–Yau (CY) categories feature prominently throughout algebraic geometry and representation theory. Examples of high interest are given by semistable sheaves on K3 or Abelian surfaces, semistable Higgs sheaves on smooth projective curves, representations of preprojective algebras of quivers, and representations of the (possibly twisted) fundamental group algebras of Riemann surfaces. We are interested in the topology and singularities of the moduli stacks and the good moduli spaces of objects in these categories. More precisely, our aim is to understand the Borel–Moore homologies of these geometric objects. We achieve this goal in three steps. First, we define a sheaf-theoretic cohomological Hall algebra for a large class of Abelian categories of homological dimension at most two. Then, we define the BPS Lie algebra, by generators and relations. Last, we relate the BPS Lie algebra to the BPS algebra of the category (the latter is the enveloping algebra of the former), and to the whole cohomological Hall algebra via a PBW-type isomorphism, using a deep understanding of the singularities of the geometric objects involved. Consequences are multiple. We obtain (1) the cohomological integrality of all categories involved and their 3-CY-completions (in particular, semistable sheaves on local K3 surfaces), (2) a stacky nonabelian Hodge isomorphism for smooth projective curves, (3) a proof of the positivity conjecture of cuspidal polynomials of quivers of Bozec–Schiffmann (a strengthening of Kac positivity conjecture) and (4) a lowest weight vector description for the cohomology (in all degrees) of Nakajima quiver varieties.

- [1] Davison B., Hennecart L., Schlegel Mejia S. BPS Lie algebras for totally negative 2-Calabi–Yau categories and nonabelian hodge theory for stacks. 2022, arXiv preprint arXiv:2212.07668
- [2] Davison B., Hennecart L., Schlegel Mejia S. BPS algebras and generalised Kac–Moody algebras from 2-Calabi–Yau categories. 2023, arXiv preprint arXiv:2303.12592

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