Mathematical Techniques III

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Preface

This the first draft of the Lecture Notes for Mathematical Techniques III (PHY 317), a course offered in the Physics Department of Queen Mary and Westfield College (University of London). These notes are loosely based on pre-existing notes by Professor John Charap. The notes contain all that is said in Lecture and sometimes more. The extra bits are typeset in smaller font and are adorned with one or two "dangerous bend" signs as in the next paragraphs.

Most paragraphs like this fill gaps in the main presentation (e.g., proofs, mathematical remarks,...). They contain material which, although necessary for the logical coherence of the presentation, may be skipped at a first reading or ignored by the less mathematically inclined student who is not interested in proofs,.... They are not an essential part of the course, although I believe they are an essential part of the topic.

Most paragraphs like this contain material which is generally more advanced than the rest of the lectures, but which I personally find interesting and have found useful at one time or other. They are not an essential part of the course, but I have included them in the hope that some of you might find them interesting enough to make the detour.

Some remarks about notation. Terms which are being defined for the first time appear in **bold sans-serif** type. Although the notation will be introduced as we go, here is a summary of the main notational conventions:

- $\mathbb{R}$ and $\mathbb{C}$ stand for the sets of real and complex numbers, respectively;
- vector spaces, subspaces,... are denoted by so-called "blackboard bold" uppercase Latin letters: $\mathbb{V}$, $\mathbb{W}$,...;
- abstract vectors are denoted by bold lowercase Latin letters: $\boldsymbol{v}$, $\boldsymbol{w}$,...;
- linear maps are denoted by uppercase Latin letters A, B,..., except for the identity map which is denoted $\mathbb{1}$.
- column vectors are denoted by sans-serif lowercase Latin letters: v, w,...;

- matrices are denoted by sans-serif uppercase Latin letters: A, B,.... The identity matrix will be denoted I.

The notes are not yet complete: in particular many of the asides are still to be completed, and the introductions have to be rewritten in light of what they are meant to introduce: they were written in advance in most cases. Many diagrams are missing, and many more examples and applications need to be added. The next stage in the development of the notes will consist in some changes in the visual layout, to break the monotony of the present style, and to make the exercises and the problems an integral part of the notes. The solutions, of course, will be available separately.

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