W-SUPERALGEBRAS FROM SUPERSYMMETRIC LAX OPERATORS

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ABSTRACT

We construct an infinite series of classical W-superalgebras via a supersymmetric Miura transformation. These algebras appear as the "second hamiltonian structure" in the space of Lax operators for supersymmetric generalized KdV hierarchies.

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§1 Introduction

W-algebras[1],[2] are playing an increasing rôle in two-dimensional conformal field theory and in string theory. In particular, unexpected relationships have been unveiled between W-algebras and noncritical strings[3] via their matrix model formulation. This seems to lie at the heart of the relationship between string theory and classical integrable systems[4].

Classical W-algebras, in fact, first appeared in the context of integrable systems, where they arise naturally as the "second hamiltonian structure" of the generalized KdV hierarchies[5]. Indeed, the n^{th} order KdV hierarchy (for $n > 2$) is hamiltonian with respect to a classical version of W_n, generalizing the well-known fact that the KdV equation is hamiltonian with respect to a classical version of the Virasoro algebra.

The fact that the Poisson bracket defined by the second hamiltonian structure does indeed obey the Jacobi identity was originally notoriously cumbersome to prove: most proofs being more or less refined versions of a direct computation of the Jacobi identity. In **[6]** , Kupershmidt and Wilson showed that the second hamiltonian structure is induced from a much simpler one—namely that of "free fields"—via a generalization of the celebrated Miura transformation of the KdV theory. But their proof was still rather involved and it was not until Dickey found a short and elementary proof[7] of the Kupershmidt-Wilson theorem that the conceptual and computational advantage (in this context) of the Miura transformation became clear. From the point of view of conformal field theory, the Miura transformation is particularly useful[2], since the free field realizations implicit in this method lend themselves naturally to the quantization of these algebras and to the study of their representations.

It is certainly an interesting question to ask if this method survives supersymmetrization, yielding W-superalgebras[8],[9],[10]. Partial results have already been obtained by the Komaba group[11],[12] who, in the context of super Toda field theory, has explicitly constructed examples of W-superalgebras from supersymmetric Miura transformations. This seems to suggest an affirmative answer to this question.

Indeed in this letter we start such a supersymmetrization program: we show that the space of supersymmetric differential operators of the KdV type is endowed with a bi-hamiltonian structure analogous to the Gel'fand-Dickey brackets of the bosonic case and that the second bracket is induced, via a supersymmetric version of the Miura transformation, by that of free superfields.

§2 Bi-Hamiltonian Structure of Generalized KdV Hierarchies

In this section we briefly review the bi-hamiltonian structure of the generalized KdV hierarchies. We do this to set up the notation and to motivate the objects we will use in the main body of this paper.

The KdV hierarchy is the isospectral problem associated to the differential (Lax) operator $L = \partial^2 + u$. The KdV flows

$$\frac{\partial L}{\partial t_i} = [A_i\, , \, L] \tag{2.1}$$

are hamiltonian with respect to two Poisson brackets. The KdV equation itself $\dot{u} = u''' + 6uu'$ can be written as $\dot{u} = \{H_1\, , \, u\}_1 = \{H_2\, , \, u\}_2$, with

$$H_1 = \int \left(\frac{1}{2}(u')^2 - u^3\right) \quad \text{and} \quad H_2 = -\int \frac{1}{2}u^2 \ , \tag{2.2}$$

and Poisson brackets

$$\{u(x)\, , \, u(y)\}_1 = \partial\, \delta(x-y) \ , \tag{2.3}$$

$$\{u(x)\, , \, u(y)\}_2 = (\partial^3 + 2u\partial + 2\partial u)\, \delta(x-y) \ , \tag{2.4}$$

where, here and in the sequel, the differential operators appearing on the right hand side of the brackets are taken at the point x. Moreover these brackets are coordinated: any linear combination is again a Poisson bracket. This gives the KdV hierarchy its bi-hamiltonian structure.

The generalized $n^{\rm th}$-order ($n > 2$) KdV hierarchy is defined as the isospectral problem of the differential operator $L = \partial^n + u_{n-1}\partial^{n-1} + \cdots + u_0$. Again one can show that its flows are bi-hamiltonian, where the Poisson brackets are now given by expressions of the form

$$\{u_i(x)\,,\,u_j(y)\} = J_{ij}\,\delta(x-y)\ , \tag{2.5}$$

where for one of the structures the J_{ij} are differential operators whose coefficents are nonlinear expressions in the u_i. In particular, the reduction to the case $u_{n-1} = 0$ yields classical versions of the W_n algebras.

It turns out that in terms of L, rather than in terms of its coefficients, these brackets can be written in an elegant and compact form. But for this we need to develop some differential calculus with the L's.

We will define the Poisson brackets on functionals of the form

$$F[L] = \int f(u_i)\ , \tag{2.6}$$

where $f(u_i)$ is a differential polynomial in the u_i (*i.e.* , a polynomial in the u_i and their derivatives). The precise meaning of integration depends on the context: it denotes integration over the real line if we take the u_i to be rapidly decreasing functions; integration over one period if we take the u_i to be periodic functions; or, more abstractly, a linear map annihilating derivatives so that we can "integrate by parts".[1]

It is familiar from classical mechanics that to every function f one can associate a hamiltonian vector field ξ_f in such a way that $\xi_f\, g = \{f\ ,\ g\}$, and where $\xi_f = \Omega\, df$

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[1] Notice that whereas differential polynomials can be multiplied, this does not induce a multiplication on the functionals we are considering. Thus the Poisson brackets will not enjoy the usual derivation property. This, however, does not affect the formalism.

with Ω a linear map from 1-forms to vector fields. In local coordinates, Ω coincides with the fundamental Poisson brackets. This formalism is the most suitable to define Poisson brackets in the infinite-dimensional space of the operators L.

Vector fields are parametrized by infinitesimal deformations $L \mapsto L+\epsilon A$ where $A = \sum a_l \partial^l$ is a differential operator of order at most $n-1$. We denote the space of such operators by R_n. To such an operator $A \in R_n$ we associate a vector field ∂_A as follows. If $F = \int f$ is a functional then

$$\begin{aligned} \partial_A F &= \frac{d}{d\epsilon} F[L+\epsilon A]\bigg|_{\epsilon=0} \\ &= \int \sum_{l=0}^{n-1} \sum_{i=0}^{\infty} a_l^{(i)} \frac{\partial f}{\partial u_l^{(i)}} \ . \end{aligned} \tag{2.7}$$

Integrating by parts we can write this as

$$\partial_A F = \int \sum_{l=0}^{n-1} a_l \frac{\delta f}{\delta u_l} \ , \tag{2.8}$$

where the Euler variational derivative is given by

$$\frac{\delta}{\delta u_l} = \sum_{i=0}^{\infty} (-\partial)^i \frac{\partial}{\partial u_l^{(i)}} \ . \tag{2.9}$$

Since vector fields are parametrized by R_n, it is natural to think of 1-forms as parametrized by the dual space to R_n. This turns out to be given by pseudo-differential operators (ΨDO's) with the dual pairing given by the Adler trace to be defined below. We introduce a formal inverse ∂^{-1} of ∂ and define ΨDO's as formal Laurent series in ∂^{-1} whose coefficients are differential polynomials in the u_i. The multiplication of ΨDO's is given by the following composition law (for any $k \in \mathbb{Z}$)

$$\partial^k f = f\,\partial^k + \sum_{i=1}^{\infty} \binom{k}{i} f^{(i)} \partial^{k-i} \ , \qquad \binom{k}{i} \equiv \frac{k(k-1)\cdots(k-i+1)}{i!} \tag{2.10}$$

extending the usual Leibniz rule for positive k. Given a ΨDO $P = \sum p_i \partial^i$ we define its residue as $\operatorname{res} P = p_{-1}$ and its (Adler) trace as $\operatorname{Tr} P = \int \operatorname{res} P$. One can show that the residue of a commutator is a total derivative so that its trace vanishes. This justifies the name. The Adler trace defines a symmetric bilinear form on the space of ΨDO's given by $\mathrm{Tr}\,(PQ)$, thus allowing us to identify the dual space to R_n as the space R_n^* of ΨDO's of the form $X = \sum_{k=0}^{n-1} \partial^{-k-1} X_k$. In fact, if $A = \sum a_l \partial^l$, then

$$\mathrm{Tr}\,(AX) = \int \sum_{i=0}^{n-1} a_i\, X_i \ , \tag{2.11}$$

which is clearly nondegenerate. We will then think of 1-forms as elements of R_n^* and their pairing with vector fields as given by

$$(\partial_A, X) = \mathrm{Tr}\,(AX) \ . \tag{2.12}$$

In particular, if we define dF, for $F = \int f$, by $(\partial_A, dF) = \partial_A\, F$, it follows from (2.8) that

$$dF = \sum_{k=0}^{n-1} \partial^{-k-1} \frac{\delta f}{\delta u_k} \ . \tag{2.13}$$

Finally we can define the Poisson bracket of two functionals. The map Ω taking 1-forms to vector fields is induced from a linear map $J : R_n^* \to R_n$ by $\Omega(X) = \partial_{J(X)}$. The Poisson bracket is then defined by

$$\{F\,,\, G\} = (\Omega(dF), dG) = \mathrm{Tr}(J(dF)dG) \ . \tag{2.14}$$

Demanding that this be antisymmetric and obey the Jacobi identity, constrains the allowed maps J. Gel'fand and Dickey constructed two such maps:

$$J_1(X) = [X\,,\, L]_+ \ , \tag{2.15}$$

$$J_2(X) = (LX)_+ L - L(XL)_+ \ , \tag{2.16}$$

where for P any ΨDO, P_+ denotes its differential part. The first and second Gel'fand-Dickey brackets are then

$$\{F\,,\, G\}_1 = Tr\,\big([dF\,,\, L]_+\, dG\big) \ , \tag{2.17}$$

$$\{F\,,\,G\}_2 = Tr\ \left((L\,dF)_+ L\,dG - L(dF\,L)_+ dG\right)\ . \tag{2.18}$$

In order to obtain the fundamental Poisson brackets of the u_i it is clearly sufficient to consider linear functionals $F[L] = \int \sum X_i u_i$.

The first bracket has a natural interpretation as the Kirillov-Kostant Poisson bracket on a coadjoint orbit of the Volterra group. A natural interpretation of the second bracket was provided by Kupershmidt and Wilson. Let us factorize the Lax operator L as $L = (\partial - v_n)(\partial - v_{n-1})\cdots(\partial - v_1)$. This allows us to write the u_i as the Miura transforms of the v_j. Kupershmidt and Wilson showed that if the fundamental Poisson brackets of the v_j are given by those of "free fields"

$$\{v_i\,,\,v_j\} = -\delta_{ij}\partial\,\delta(x-y)\ , \tag{2.19}$$

the induced brackets on the u_i coincide with (2.18) . Notice that whereas the fundamental Poisson brackets of the u_i obviously obey the Jacobi identity when defined in terms of the v_j, the fact that they close among the u_i is far from obvious. On the other hand closure is obvious from (2.18) , but the Jacobi identity is not.

We end with an example. Factorizing the KdV operator $L = (\partial + v)(\partial - v) = \partial^2 + u$ yields $u = -v' - v^2$ which is the classical analog of the Coulomb gas realization of the Virasoro algebra. Indeed, computing $\{u(x)\,,\,u(y)\}$ from v reproduces (2.4) .

§3 Supersymmetric Miura Transformation

In this section we exhibit a bi-hamiltonian structure on the space of supersymmetric Lax operators and we prove that the second structure is induced, via a supersymmetric Miura transformation, from free superfields. Our proof follows very closely that of Dickey[7] for the bosonic case.

We will consider differential operators on a $(1|1)$ superspace with coordinates (x, θ). These operators are polynomials in the supercovariant derivative $D = \partial_\theta + \theta\partial$ whose coefficients are superfields. A supersymmetric Lax operator has the form

$L = D^n + U_{n-1}D^{n-1} + \cdots + U_0$ which is homogeneous under the usual $\mathbb{Z}_2$ grading; that is, $|U_i| \equiv n + i \pmod 2$. In analogy to the bosonic case discussed in the previous section, we will define Poisson brackets on functionals of the form:

$$F[L] = \int_B f(U) \ , \tag{3.1}$$

where $f(U)$ is a homogeneous (under the $\mathbb{Z}_2$ grading) differential polynomial of the U and $\int_B$ is defined as follows: if $U_i = u_i + \theta v_i$, and $f(U) = a(u,v) + \theta b(u,v)$, then $\int_B f(U) = \int b(u,v)$.

Vector fields are parametrized by the space S_n of differential operators of order at most $n-1$. We don't demand that A have the same parity as L since we can have either odd or even flows. If $A = \sum A_i D^i$ is one such operator and if $F[L] = \int_B f$ is a functional, then the vector field D_A defined by A is given by

$$D_A F = (-1)^{|A|+n} \int_B \sum_{k=0}^{n-1} A_k \frac{\delta f}{\delta U_k} \ , \tag{3.2}$$

where the variational derivative is defined by

$$\frac{\delta}{\delta U_k} = \sum_{i=0}^{\infty} (-1)^{|U_k|i + i(i+1)/2} D^i \frac{\partial}{\partial U_k^{[i]}} \ , \tag{3.3}$$

with $U_k^{[i]} = D^i U_k$.

As before, to define the 1-forms we introduce super-pseudo-differential operators (SΨDO's)**[13]**. These are defined as formal Laurent series in D^{-1} where the composition law is now given by

$$D^k \Phi = \sum_{i=0}^{\infty} \begin{bmatrix} k \\ k-i \end{bmatrix} (-1)^{|\Phi|(k-i)} \Phi^{[i]} D^{k-i} \ , \tag{3.4}$$

where the superbinomial coefficients are given by

$$\begin{bmatrix} k \\ k-i \end{bmatrix} = \begin{cases} 0 & \text{for } i < 0 \text{ or } (k,i) \equiv (0,1) \pmod 2; \\ \begin{pmatrix} \left[\frac{k}{2}\right] \\ \left[\frac{k-i}{2}\right] \end{pmatrix} & \text{for } i \geq 0 \text{ and } (k,i) \not\equiv (0,1) \pmod 2. \end{cases} \tag{3.5}$$

Given a SΨDO $P = \sum p_i D^i$ we define its super-residue as $\operatorname{sres} P = p_{-1}$ and its (Adler) supertrace as $\operatorname{Str} P = \int_B \operatorname{sres} P$. Again it can be shown that the supertrace vanishes on graded commutators: $\operatorname{Str}[P\,,\,Q] = 0$, for $[P\,,\,Q] = P\,Q - (-1)^{|P||Q|}Q\,P$. This then defines a supersymmetric bilinear form on SΨDO's: $\operatorname{Str}(PQ) = (-1)^{|P||Q|}\operatorname{Str}(QP)$.

We define 1-forms as the space S_n^* of SΨDO's of the form $X = \sum_{k=0}^{n-1} D^{-k-1} X_k$, whose pairing with a vector field D_A, $A = \sum A_k D^k$ is given by

$$(D_A, X) \equiv (-1)^{|A|+|X|+n+1}\operatorname{Str}(AX) = (-1)^{|A|+n}\int_B \sum_{k=0}^{n-1}(-1)^k A_k\, X_k\ , \qquad (3.6)$$

which is again nondegenerate. The choice of signs has been made to avoid undesirable signs later on. Given a functional $F = \int_B f$ we define its gradient dF by $(D_A, dF) = D_A F$ whence, comparing with (3.2) , yields

$$dF = \sum_{k=0}^{n-1}(-1)^k D^{-k-1}\frac{\delta f}{\delta U_k}\ . \qquad (3.7)$$

Finally, to define a Poisson bracket we need a map $J : S_n^* \to S_n$ in such a way that the Poisson bracket of two functionals F and G is given by (*cf.* (2.14))

$$\{F\,,\,G\} = D_{J(dF)}G = (D_{J(dF)}, dG) = (-1)^{|J|+|F|+|G|+n+1}\operatorname{Str}(J(dF)dG)\ . \quad (3.8)$$

Instead of giving the map J directly, we will follow a constructive approach starting from a supersymmetric version of the Miura transformation and, only at the end, write down the resulting map J.

Let us factorize $L = (D - \Phi_n)(D - \Phi_{n-1})\cdots(D - \Phi_1)$. This defines the U_i as differential polynomials in the Φ_j. We define the fundamental Poisson brackets of the Φ_j as follows. If we let $X = (x, \theta)$ and $Y = (y, \omega)$, then

$$\{\Phi_i(X)\,,\,\Phi_j(Y)\} = (-1)^i\delta_{ij}D\delta(X - Y)\ , \qquad (3.9)$$

where $\delta(X-Y) = \delta(x-y)(\theta-\omega)$. Notice that the signs in the Poisson brackets are alternating. This turns out to be crucial: whereas any choice of signs would yield a Poisson bracket, it is only this choice which will give a closed algebra among the U_k. In fact, writing the U_k as the Miura transforms of the Φ_i allows us to write their Poisson brackets from (3.9) . It is clear that these Poisson brackets will obey the Jacobi identity, but it is far from obvious that they will close in terms of the U_k. We will now show, however, that this is indeed the case, yielding a supersymmetric version of the Kupershmidt-Wilson theorem.

Let $F = \int_B f$ and $G = \int_B g$ be two functionals with f and g differential polynomials in the U_k. Via the Miura transformation we can think of them as differential polynomials in the Φ_j. Their Poisson brackets can then be read off from (3.9) :

$$\{F\,,\,G\} = \int_B \sum_{i=1}^{n} (-1)^i \left(D \frac{\delta f}{\delta \Phi_i} \right) \frac{\delta g}{\delta \Phi_i}\ . \tag{3.10}$$

We must first calculate $\frac{\delta f}{\delta \Phi_i}$. The variation of F can be computed in two ways:

$$\delta F = \int_B \sum_{i=1}^{n} \delta \Phi_i \frac{\delta f}{\delta \Phi_i} = \int_B \sum_{k=0}^{n-1} \delta U_k \frac{\delta f}{\delta U_k} = (-1)^{|F|+n+1} \mathrm{Str}\,(\delta L dF)\ . \tag{3.11}$$

Defining $\nabla_i \equiv D - \Phi_i$, one computes

$$\delta L = -\sum_{i=1}^{n} \nabla_n \cdots \nabla_{i+1} \delta \Phi_i \nabla_{i-1} \cdots \nabla_1\ . \tag{3.12}$$

Inserting this into (3.11) one finds after some reordering inside the integrals

$$\frac{\delta f}{\delta \Phi_i} = (-1)^{|F|(n+i+1)+i}\, \mathrm{sres}\, \left(\nabla_{i-1} \cdots \nabla_1 dF \nabla_n \cdots \nabla_{i+1} \right)\ . \tag{3.13}$$

Plugging this into (3.10) , replacing the supercovariant derivative of $\frac{\delta g}{\delta \Phi_i}$ by the graded commutator $[\nabla_i\,,\,\frac{\delta g}{\delta \Phi_i}]$, and using that for any SΨDO P its super-residue can be written as

$$\operatorname{sres} P = (P_- \nabla_i)_+ = (-1)^{|P|+1} (\nabla_i P_-)_+ \ , \tag{3.14}$$

with $P_- \equiv P - P_+$ we obtain

$$\begin{aligned}\{F\,,\,G\} =& (-1)^{(|F|+|G|)(n+1)} \sum_{i=1}^{n} (-1)^{(|F|+|G|+1)i} \times \\ & \operatorname{Str} \left[\nabla_i ((\nabla_{i-1} \cdots dF \cdots \nabla_{i+1})_- \nabla_i)_+ \nabla_{i-1} \cdots dG \cdots \nabla_{i+1} \right. \\ & \left. - \ (\nabla_i (\nabla_{i-1} \cdots dF \cdots \nabla_{i+1})_-)_+ \nabla_i \nabla_{i-1} \cdots dG \cdots \nabla_{i+1} \right] \ .\end{aligned} \tag{3.15}$$

Now notice that we can drop the $-$ subscripts since if we replace them with a $+$ we can then drop the outer $+$'s and the terms cancel pairwise. Using graded cyclicity of the supertrace in the first set of terms we can write

$$\begin{aligned}\{F\,,\,G\} =& (-1)^{(|F|+|G|)(n+1)} \sum_{i=1}^{n} \times \\ & \operatorname{Str} \Big[(-1)^{(|F|+|G|+1)(i+1)} (\nabla_{i-1} \cdots dF \cdots \nabla_i)_+ \nabla_{i-1} \cdots dG \cdots \nabla_i \\ & - \ (-1)^{(|F|+|G|+1)i} (\nabla_i \cdots dF \cdots \nabla_{i+1})_+ \nabla_i \cdots dG \cdots \nabla_{i+1} \Big] \ .\end{aligned} \tag{3.16}$$

Notice that all terms cancel pairwise except for the first term of the first sum and the last term of the second sum, yielding—after some reordering

$$\{F\,,\,G\} = (-1)^{|F|+|G|+n} \operatorname{Str} \left[L (dF\, L)_+ dG - (L dF)_+ L dG \right] \ , \tag{3.17}$$

which is the supersymmetric analog of the second Gel'fand-Dickey bracket (2.18) . If F and G are linear functionals then from this bracket one can recover the fundamental Poisson brackets of the U_i. It is clear from the expression that they close quadratically among themselves giving classical versions of W-superalgebras.

Using (3.8) we can read off the map $J : S_n^* \to S_n$

$$J(X) = (LX)_+ L - L(XL)_+ \ . \tag{3.18}$$

As in the bosonic case the first hamiltonian structure can be obtained by deforming the second structure. If we change $U_0 \mapsto U_0 + \lambda$, where $|\lambda| \equiv n \pmod 2$, then J becomes

$$J_\lambda(X) = J(X) - \lambda \left[(XL)_+ - (-1)^{n+n|X|} (LX)_+ \right] \ . \tag{3.19}$$

The term proportional to λ gives us the first hamiltonian structure. Several remarks are in order. When n is even, the first hamiltonian structure coincides with the Kirillov-Kostant structure on the coadjoint orbit of L under the super-Volterra group and the two brackets are coordinated in the usual way. On the other hand, when n is odd, the bracket has an odd grading (since λ is an anticommuting parameter) and the connection with the Kirillov-Kostant structure seems to be missing. In this case the Poisson brackets can still be considered coordinated if we allow linear combinations with anticommuting parameters.

§4 Conclusion and Outlook

In this letter we have constructed a bi-hamiltonian structure in the space of supersymmetric Lax operators. In the process we have generated an infinite series of classical W-superalgebras generated by superfields of "weights" $\{\frac{1}{2}, 1, \frac{3}{2}, \ldots, \frac{n}{2}\}$ and we have provided free field realizations via a supersymmetric Miura transformation. This series is the supersymmetric analog of the $GL(n)$ series of Drinfel'd and Sokolov. The first nontrivial algebra in the series ($n = 2$) can be checked to be superconformal: although the basic fields do not include the superenergy-momentum tensor, this can be built out of differential polynomials in them.

In analogy with the bosonic case, we also expect that reduction schemes related to simple superalgebras will generalize the results obtained by Drinfel'd and Sokolov. In particular, some of these reductions[14] will yield W-superalgebras extending the super-Virasoro algebra, thus making them more interesting from a physical point of view. For example, one can check that the reduced Lax operator $L = D^3 + \mathbb{T} = (D + \Phi)D(D - \Phi)$ yields the super-Virasoro algebra.

Finally let us comment on the free field realizations associated to the Miura transformations. As we mentioned in section 3, the free fields have to be chosen with alternating signs in their fundamental brackets. This means that in the quantisation of these algebras we will be generically forced to consider indefinite Fock spaces, unless, as in the example commented upon above, reduction leaves only fields with the same sign. The emergence of the indefinite Fock spaces is already present in the Toda approach[11],[12] since Cartan matrices of simple superalgebras are generally indefinite.

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