Quotienting string backgrounds

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Based on work in collaboration with Joan Simón (Pennsylvania)

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- supersymmetric Clifford–Klein space form problem

Melvin universe and fluxbranes

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- we are interested in the orbit space M/Γ

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  - ⋆ M supersymmetric, but M/Γ_L breaking all supersymmetry

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- if $X' = \lambda X$, $\lambda \neq 0$, then $\Gamma' = \Gamma$
- if $X' = gXg^{-1}$, then $\Gamma' = g\Gamma g^{-1}$, and moreover $M/\Gamma \cong M/\Gamma'$

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i.e., projectivised adjoint orbits of $\mathfrak{g}$

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$$D_X = \nabla_X + \frac{1}{6}\iota_X F - \frac{1}{12}X^\flat \wedge F$$

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which we need to put in normal form

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- $X \in \mathfrak{so}(p,q) \iff X : \mathbb{R}^{p+q} \to \mathbb{R}^{p+q}$ linear, skew-symmetric relative to $\langle -, - \rangle$ of signature (p,q)
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$$\mu(x) = \left(\left(x^2 + \beta^2 + \varphi^2 \right)^2 - 4\beta^2 x^2 \right)^n$$

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- example: $\mu(x) = x^3$

Elementary lorentzian blocks

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Signature	Minimal polynomial	Type

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$(0,1)$		

Elementary lorentzian blocks

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$(0, 1)$	x	trivial

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$(1, 2)$	x^3	null rotation

Lorentzian normal forms

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Play with the elementary blocks!

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In signature $(1, 10)$

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Play LEGO with the elementary blocks!

In signature $(1,10)$:

- $R_{12}(\varphi_1) + R_{34}(\varphi_2) + R_{56}(\varphi_3) + R_{78}(\varphi_4) + R_{9\natural}(\varphi_5)$

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where $\beta > 0$, $\varphi_1 \geq \varphi_2 \geq \cdots \geq \varphi_{k-1} \geq \varphi_k \geq 0$

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to get rid of component of τ in the image of $[\lambda, -]$

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$$\xi = \partial_z + \lambda = U\,\partial_z\,U^{-1} \qquad \text{with} \quad U = \exp(-z\lambda)$$

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⋆ dilaton

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$$A = \frac{B\boldsymbol{x} \cdot d\boldsymbol{x}}{1 + |B\boldsymbol{x}|^2}$$

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⋆ string frame metric:

$$h = \Lambda^{1/2}|d\boldsymbol{x}|^2 - \Lambda^{-1/2}(B\boldsymbol{x} \cdot d\boldsymbol{x})^2$$

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$$B = \begin{pmatrix} 0 & -\varphi_1 & & & & & & & 0 & u \\ \varphi_1 & 0 & & & & & & & 0 & 0 \\ & & 0 & -\varphi_2 & & & & & & \\ & & \varphi_2 & 0 & & & & & & \\ & & & & 0 & -\varphi_3 & & & & \\ & & & & \varphi_3 & 0 & & & & \\ & & & & & & 0 & -\varphi_4 & & \\ & & & & & & \varphi_4 & 0 & & \\ -u & 0 & & & & & & & & \\ 0 & 0 & & & & & & & & \end{pmatrix}$$

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e.g.,

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e.g.,

⋆ $u = 0,\ \varphi_2 = \varphi_3 = \varphi_4 = 0$

e.g.,

⋆ $u = 0,\ \varphi_2 = \varphi_3 = \varphi_4 = 0 \implies$ flux-sevenbrane

[Costa–Gutperle, hep-th/0012072]

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[Costa–Gutperle, hep-th/0012072]

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[Gutperle–Strominger, hep-th/0104136]

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⋆ resolution of parabolic orbifold [Horowitz–Steif (1991)]

End of first lecture

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In string/M-theory this cannot be the end of the story.

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$\mathbb{CP}^2$ is not even spin! [Duff–Lü–Pope, hep-th/9704186,9803061]

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so that $\exp(2\pi X)$ does act like 1 on tensors

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- it suffices to check this on Killing spinors

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[e.g., Hull hep-th/0305039]

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Brane quotients

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End of second lecture

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- equivariant under the isometry group G of (M, g)

  [hep-th/9902066]

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- again the problem reduces to one of flat spaces!

Isometries of AdS_{1+p}

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- the (orientation-preserving) isometry group of AdS_{1+p} is an infinite cover $\widetilde{\mathrm{SO}}(2,p)$, a central extension of $\mathrm{SO}(2,p)$ by $\mathbb{Z}$

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- one-parameter subgroups $\leftrightarrow$ projectivised adjoint orbits of $\mathfrak{so}(2,p)$ under $SO(2,p)$

Normal forms for $\mathfrak{so}(2,p)$

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We play LEGO again

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$$B^{(0,2)}(\varphi) = B^{(2,0)}(\varphi) = \begin{bmatrix} 0 & \varphi \\ -\varphi & 0 \end{bmatrix}$$

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- $(1,1)$, $\mu(x) = x^2 - \beta^2$

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$$B_{\pm}^{(2,2)} = \begin{bmatrix} 0 & \mp 1 & 1 & 0 \\ \pm 1 & 0 & 0 & \mp 1 \\ -1 & 0 & 0 & 1 \\ 0 & \pm 1 & -1 & 0 \end{bmatrix}$$

- $(2, 2)$

- $(2,2)$, $\mu(x) = (x^2 - \beta^2)^2$

- $(2,2)$, $\mu(x) = (x^2 - \beta^2)^2$, deformation of $B_{\pm}^{(2,2)}$ by a (anti)selfdual boost

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$$B_{\pm}^{(2,2)}(\beta > 0)$$

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$$B_{\pm}^{(2,2)}(\beta > 0) = \begin{bmatrix} 0 & \mp 1 & 1 & -\beta \\ \pm 1 & 0 & \pm\beta & \mp 1 \\ -1 & \mp\beta & 0 & 1 \\ \beta & \pm 1 & -1 & 0 \end{bmatrix}$$

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The associated discrete quotient of AdS_3

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The associated discrete quotient of AdS_3 yields the extremal BTZ black hole

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The associated discrete quotient of AdS_3 yields the extremal BTZ black hole; the non-extremal black hole

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The associated discrete quotient of AdS_3 yields the extremal BTZ black hole; the non-extremal black hole is obtained from $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2)$

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The associated discrete quotient of AdS_3 yields the extremal BTZ black hole; the non-extremal black hole is obtained from $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2)$, for $|\beta_1| \neq |\beta_2|$

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- $(2,2),\ \mu(x) = (x^2 + \varphi^2)^2$

- $(2,2)$, $\mu(x) = (x^2 + \varphi^2)^2$, deformation of $B_{\pm}^{(2,2)}$ by a (anti)self-dual rotation

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$$B_{\pm}^{(2,2)}(\varphi)$$

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$$B_{\pm}^{(2,2)}(\varphi) = \begin{bmatrix} 0 & \mp 1 \pm \varphi & 1 & 0 \\ \pm 1 \mp \varphi & 0 & 0 & \mp 1 \\ -1 & 0 & 0 & 1+\varphi \\ 0 & \pm 1 & -1-\varphi & 0 \end{bmatrix}$$

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- $(2,2)$, $\mu(x) = (x^2 + \beta^2 + \varphi^2) - 4\beta^2 x^2$

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$$B_{\pm}^{(2,2)}(\beta>0,\varphi>0) = \begin{bmatrix} 0 & \pm\varphi & 0 & -\beta \\ \mp\varphi & 0 & \pm\beta & 0 \\ 0 & \mp\beta & 0 & -\varphi \\ \beta & 0 & \varphi & 0 \end{bmatrix}$$

- $(2, 3)$

- $(2,3),\ \mu(x) = x^5$

- $(2,3)$, $\mu(x) = x^5$, deformation of $B_+^{(2,2)}$ by a null rotation in a perpendicular direction

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$$B^{(2,3)}$$

- $(2,3)$, $\mu(x) = x^5$, deformation of $B_+^{(2,2)}$ by a null rotation in a perpendicular direction

$$B^{(2,3)} = \begin{bmatrix} 0 & 1 & -1 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- $(2, 4)$

- $(2,4),\ \mu(x) = (x^2 + \varphi^2)^3$

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$$B_{\pm}^{(2,4)}(\varphi) = \begin{bmatrix} 0 & \mp\varphi & 0 & 0 & -1 & 0 \\ \pm\varphi & 0 & 0 & 0 & 0 & \mp 1 \\ 0 & 0 & 0 & \varphi & -1 & 0 \\ 0 & 0 & -\varphi & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 & \varphi \\ 0 & \pm 1 & 0 & 1 & -\varphi & 0 \end{bmatrix}$$

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- and that's all!

Causal properties

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- Killing vectors on $\mathrm{AdS}_{1+p} \times S^q$ decompose

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whose norms add

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- S^q is compact

- S^q is compact $\Longrightarrow$

$$R^2 M^2 \geq \|\xi_S\|^2$$

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- S^q is compact $\Longrightarrow$

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and if q is odd, m^2 can be > 0

- ξ can be everywhere spacelike on $\mathrm{AdS}_{1+p} \times S^{2k+1}$, even if ξ_A is not spacelike everywhere

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- ξ can be everywhere spacelike on $\mathrm{AdS}_{1+p} \times S^{2k+1}$, even if ξ_A is not spacelike everywhere, provided that $\|\xi_A\|^2$ is bounded below

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- it is convenient to distinguish Killing vectors according to norm

- everywhere non-negative norm

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  - ⋆ $\oplus_i B^{(0,2)}(\varphi_i)$

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  - ⋆ $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2) \oplus_i B^{(0,2)}(\varphi_i)$, if $|\beta_1| = |\beta_2|$
  - ⋆ $B^{(1,2)} \oplus_i B^{(0,2)}(\varphi_i)$

- everywhere non-negative norm:
  - ⋆ $\oplus_i B^{(0,2)}(\varphi_i)$
  - ⋆ $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2) \oplus_i B^{(0,2)}(\varphi_i)$, if $|\beta_1| = |\beta_2|$
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- norm bounded below

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- norm bounded below:
  - ⋆ $B^{(2,0)}(\varphi) \oplus_i B^{(0,2)}(\varphi_i)$

- everywhere non-negative norm:
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  - ⋆ $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2) \oplus_i B^{(0,2)}(\varphi_i)$, if $|\beta_1| = |\beta_2|$
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  - ⋆ $B_{\pm}^{(2,2)} \oplus_i B^{(0,2)}(\varphi_i)$
- norm bounded below:
  - ⋆ $B^{(2,0)}(\varphi) \oplus_i B^{(0,2)}(\varphi_i)$, if p is even

- everywhere non-negative norm:
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- norm bounded below:
  - ⋆ $B^{(2,0)}(\varphi) \oplus_i B^{(0,2)}(\varphi_i)$, if p is even and $|\varphi_i| \geq \varphi > 0$ for all i

- everywhere non-negative norm:
  - $\star$ $\oplus_i B^{(0,2)}(\varphi_i)$
  - $\star$ $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2) \oplus_i B^{(0,2)}(\varphi_i)$, if $|\beta_1| = |\beta_2|$
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- arbitrarily negative norm: the rest!

- ⋆ $B^{(1,1)}(\beta_1) \oplus B^{(1,1)}(\beta_2) \oplus_i B^{(0,2)}(\varphi_i)$

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Some of these give rise to higher-dimensional BTZ-like black holes: quotient only a part of AdS and check that the boundary thus introduced lies behind a horizon.

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- pick $L > 0$ and consider the cyclic subgroup $\Gamma_L \cong \mathbb{Z}$ generated by

$$\gamma = \exp(LX)$$

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- idea of the proof: find a timelike curve which connects a point x to its image $\gamma^N x$ for $N \gg 1$

- e.g., a $\mathbb{Z}$-quotient of a lorentzian cylinder

- general case:

  ⋆ let $x = (x_A, x_S)$ be a point and $\gamma^N \cdot x = (\gamma^N \cdot x_A, \gamma^N \cdot x_S)$ its image under γ^N

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[Chamseddine–FO–Sabra, hep-th/0306278]

Thank you.