Plane wave limits and the string/gauge theory correspondence

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Institute *of* Physics, 12 February 2003

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Which string theory?

A concrete proposal

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- *spectrum* of, say, single trace operators; e.g., chiral primaries (in short multiplets)

$$\mathrm{Tr}'\,\phi \otimes \cdots \otimes \phi$$

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However...recent progress in this direction has been made using a *different* large N limit, the so-called plane wave limit.

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Boost along γ and zoom in on γ while rescaling g.

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$$g = 2dUdV + AdV^2 + B_i dY^i dV + C_{ij} dY^i dY^j$$

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- rescale

$$g_{\text{plane wave}} = \lim_{\Omega \to 0} \Omega^{-2} g(\Omega)$$

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PWL : Solutions → Solutions

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- "lightlike gauge"

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Note: field strength is *null*:

$$F_{\text{plane wave}} = dA_{\text{plane wave}} = dx^+ \wedge \Theta(x^+)$$

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It is useful in classifying the different plane wave limits of a given background.

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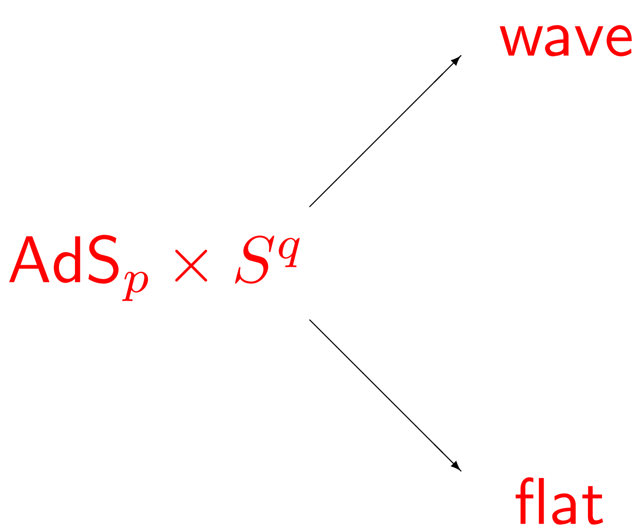

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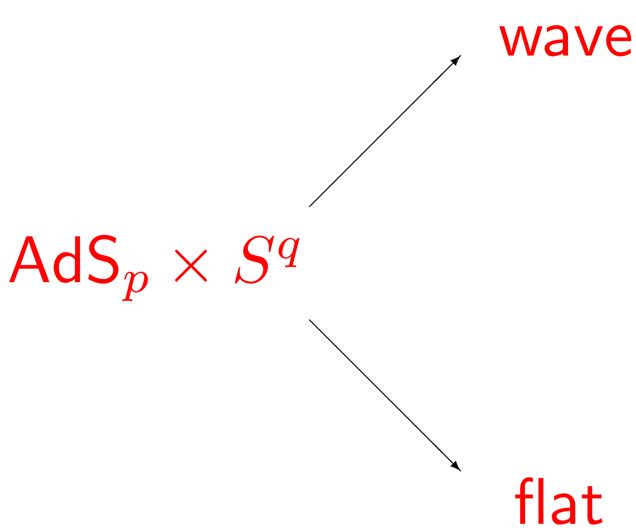

depending on whether or not the component of $\dot{\gamma}$ tangent to S^q vanishes. [Blau et al. (2002)]

Some examples

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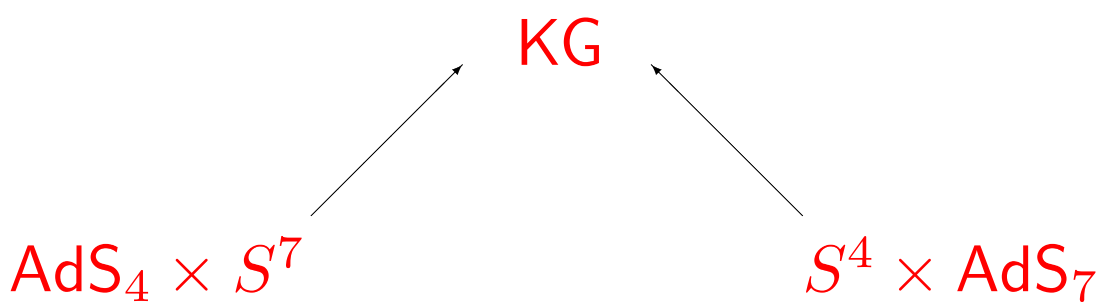

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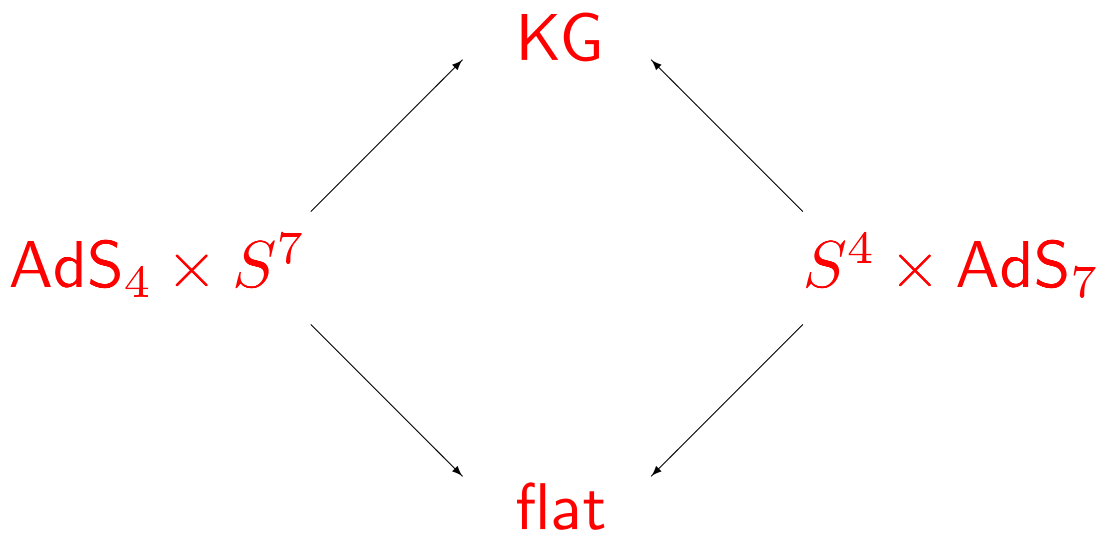

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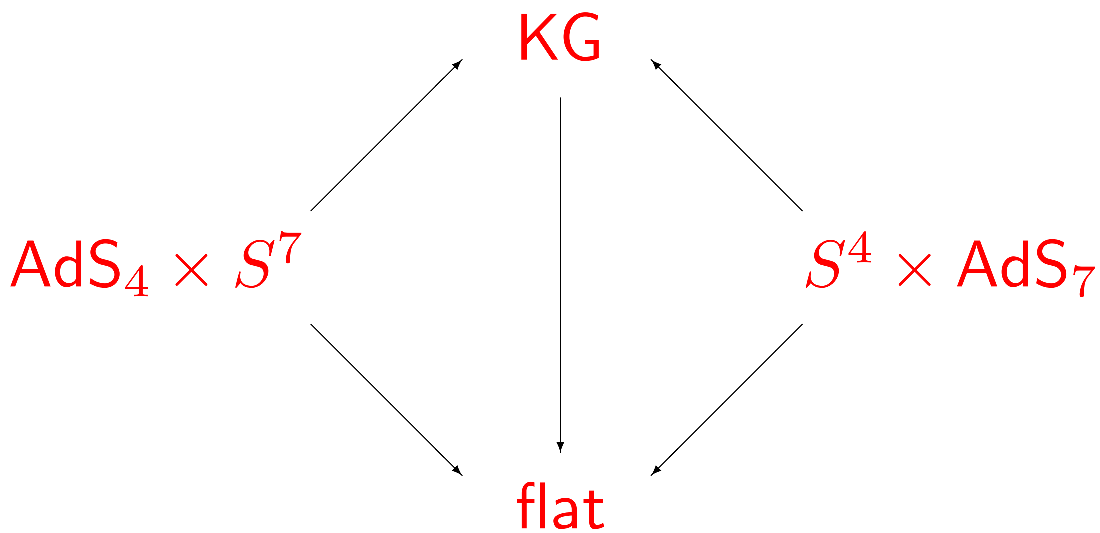

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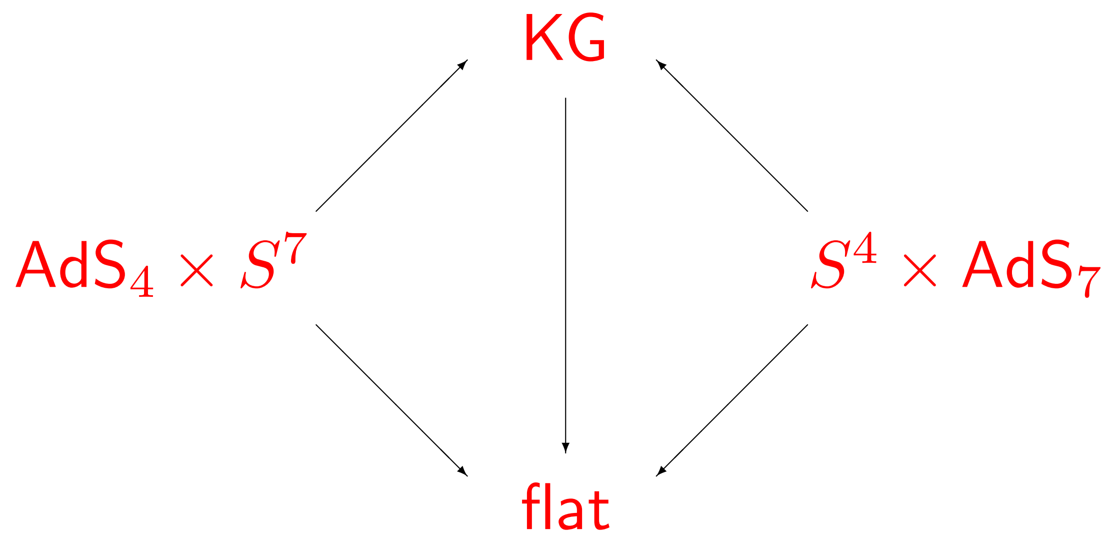

KG: 11-dimensional symmetric plane wave [Kowalski-Glikman (1984)]

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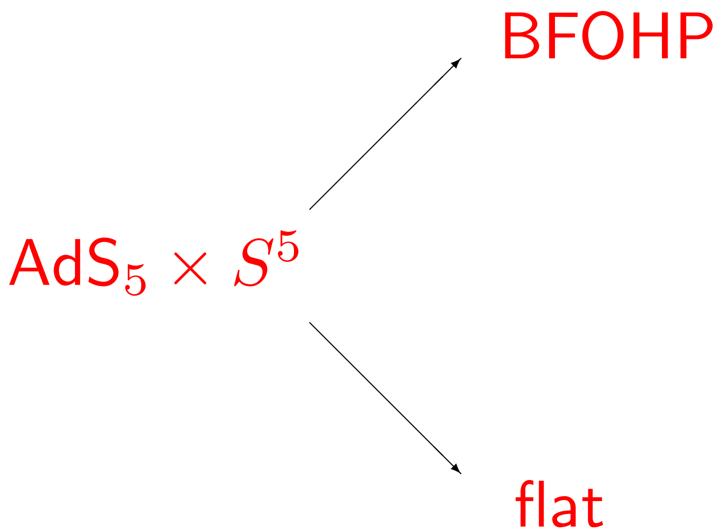

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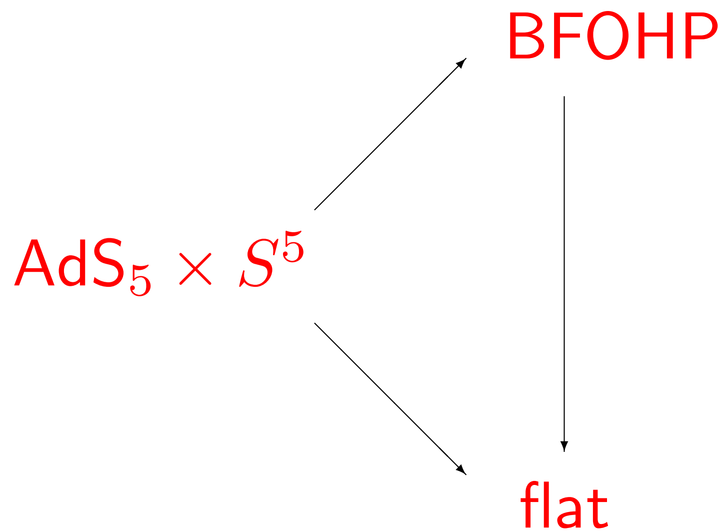

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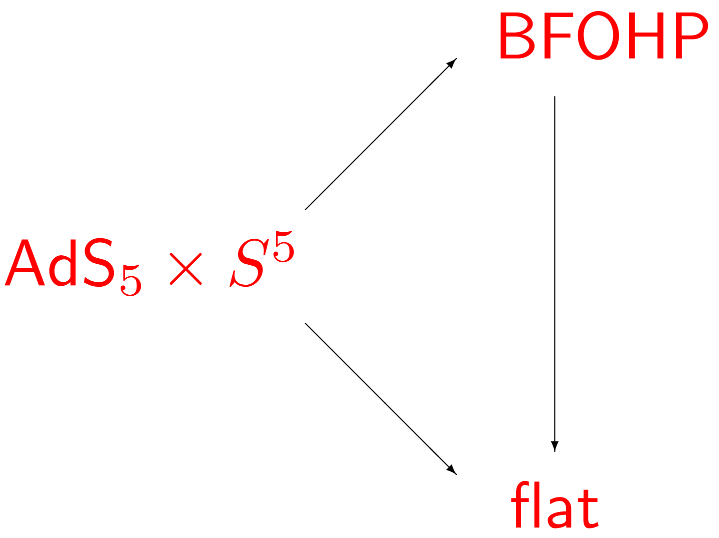

BFOHP: 10-dimensional symmetric plane wave [Blau et al.(2001)]

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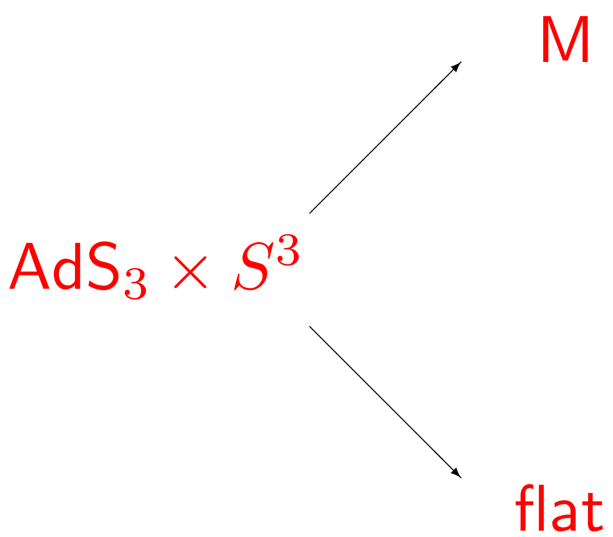

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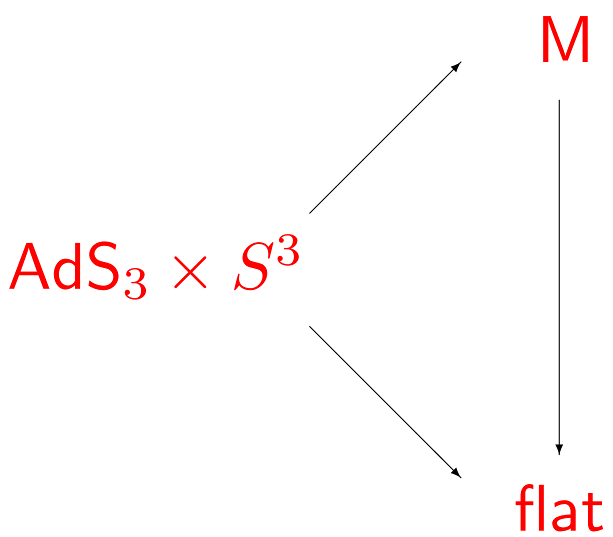

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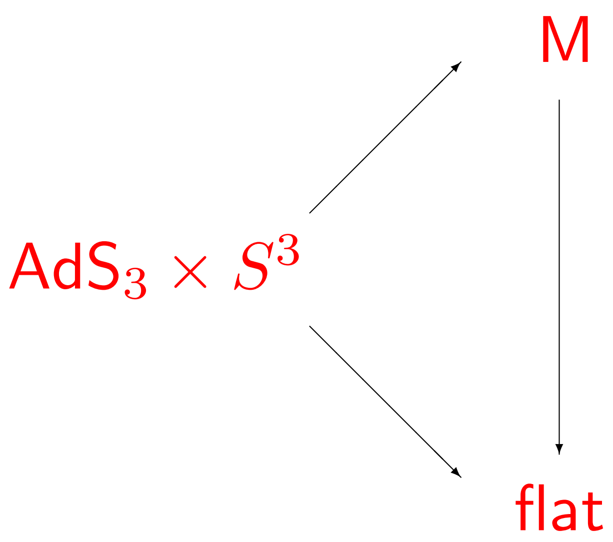

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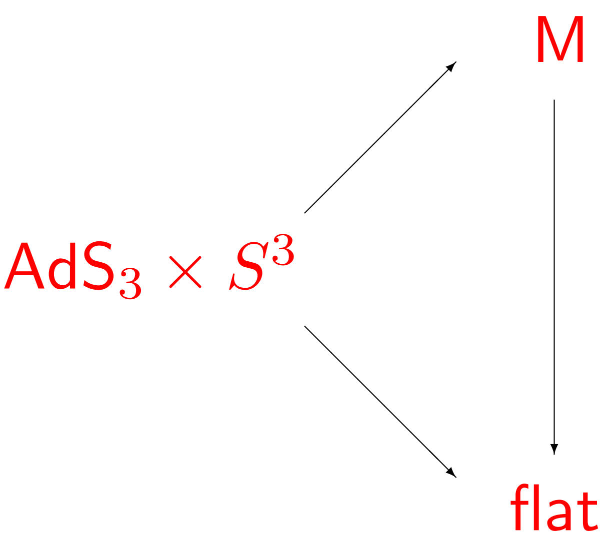

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All solutions are Lie groups and all plane wave limits are group contractions. [FO–Stanciu]

A word on the wave geometry

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Notice: As $N \to \infty$ we focus on states with larger and larger J. Thus observables are not held fixed in this limit.

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- they are dual to the *free* string excitations on the plane wave background

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BMN operators are single trace operators consisting in a string of J Z's and a finite number of *impurities*.

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In other words...

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How about interactions?

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[Kristjansen et al., Gross et al., Constable et al. (2002)]

- theory develops a different effective coupling constant

$$\lambda' = \frac{g_{\mathsf{YM}}^2 N}{J^2} = \frac{1}{(\mu p^+ \alpha')^2}$$

- a different genus-counting parameter

$$g_2^2 = \left(\frac{J^2}{N}\right)^2 = 16\pi^2 g_s^2 (\mu p^+ \alpha')^4$$

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[..., Chu–Khoze, Gomis et al. (2003)]

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- the correspondence has been extended to theories with less supersymmetry; but as usual QCD remains elusive.

Thank you.