

 everything

 everything except proofs

 ignore

Stochastic Control and Dynamic Asset Allocation*

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Reading these notes

The reader is expected to know basic stochastic analysis and ideally a little bit of financial mathematics. The notation and basic results used throughout the notes are in Appendix A.