Chapter Eleven. More on the Black–Scholes Formulas

Outline Solutions to odd-numbered exercises from the book:
An Introduction to Financial Option Valuation:
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11.1 The explanation is not valid. For example, the hockey stick result holds when $\mu < 0$ but the 'explanation' does not work in this case.

11.3 We have

$$
\begin{aligned}
d_1 &= \frac{\log(S/E) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \\
&= \frac{\log(S/E) + \log(e^{r(T-t)})}{\sigma\sqrt{T-t}} + \frac{\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \\
&= \frac{\log(Se^{r(T-t)}/E)}{\sigma\sqrt{T-t}} + \tfrac{1}{2}\sigma\sqrt{T-t} \\
&= \frac{m}{\widehat{\tau}} + \tfrac{1}{2}\widehat{\tau}.
\end{aligned}
$$

Also,

$$d_2 = d_1 - \sigma\sqrt{T-t}, = \frac{m}{\widehat{\tau}} + \tfrac{1}{2}\widehat{\tau} - \widehat{\tau} = \frac{m}{\widehat{\tau}} - \tfrac{1}{2}\widehat{\tau}.$$

Then, using (8.19), and noting that $e^{-m} = e^{-r(T-t)}E/S$,

$$c(m,\widehat{\tau}) = \frac{C}{S} = N(d_1) + \frac{Ee^{-r(T-t)}}{S}N(d_2) = N(d_1) + e^{-m}N(d_2).$$

and

$$p(m,\widehat{\tau}) = \frac{P}{S} = \frac{Ee^{-r(T-t)}N(-d_2)}{S} - N(-d_1) = e^{-m}N(-d_2) - N(-d_1).$$

11.5 Replacing σ by $-\sigma$ changes d_1 to $-d_1$ and d_2 to $-d_2$. Hence, $N(d_1)$ becomes $1-N(d_1)$ and $N(d_2)$ becomes $1-N(d_2)$, and the result follows immediately. (Note that the relation does not hold if we write $\sqrt{\sigma^2(T-t)}$ instead of $\sigma\sqrt{T-t}$.)