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Implementing STACK questions concerning integration techniques combined with the usage of STACK in ILIAS

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30.04.2019

1 Examples from Integration

What do I do?

Question
What should be trained in using integration techniques in times of CAS?

Selected techniques

1. Antiderivative
2. Partial fraction decomposition
3. Partial integration
4. Substitution

Antiderivative

Definite Integrals
Solving definite Integrals using Fundamental Theorem of Calculus

$$\int_a^b f(\xi)\,\mathrm{d}x = F(b) - F(a).$$

Three steps:

1. Determine F,
2. Evaluate F at a and b,
3. Evaluate $F(b) - F(a)$.

Implemented "independently", may be copied to other questions.

Antiderivative

Find an antiderivative of f with $f(x) = \frac{1}{x}$ $(x > 0)$

$F(z) =$, $c \in \mathbb{R}$.

Compute the value of the integral $\int_2^3 f(x)\,\mathrm{d}x$ using the antiderivative. Use as constant $c = 0$.

Type in here the value of

$F(3) =$

and

$F(2) =$

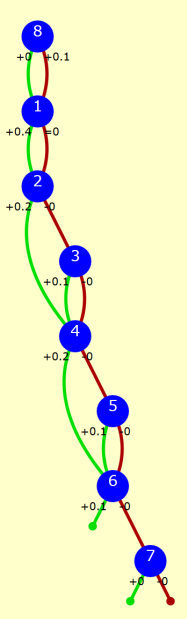
an.

Finally put integral (simplified) value here :

$\int_2^3 f(x)\,\mathrm{d}x =$

Antiderivative

Possible Response Trees



Node 8: Check the integration constant.

Node 1: Check the antiderivative F, if is is not correct use it as $\hat{F}$.

Node 2: $F(b)$ is correct?.

Node 3: $\hat{F}(b)$ is correct?

Node 4: $F(a)$ is correct?

Node 5: $\hat{F}(a)$ is correct?

Node 6: $F(b) - F(a)$ is correct?

Node 7: $F(b) - F(a)$ is numerically correct?

1.1 Partial Fraction Decomposition

Partial Fraction Decomposition

Solve the integral:

$$\int \frac{3 \cdot x + 9}{x^2 + 8 \cdot x + 16}\,\mathrm{d}x =$$

Roots of the denominator polynomial:

Tip: Type `abs(u)` for $|u|$. Use $\{\}$ to specify sets, e.g. $\{-5, 6\}$.

Focus

Dealing binoms. Only

$$(x-\lambda)(x+\lambda) = x^2 - \lambda^2$$

or

$$(x-\lambda)^2 = x^2 - 2\lambda x + \lambda^2$$

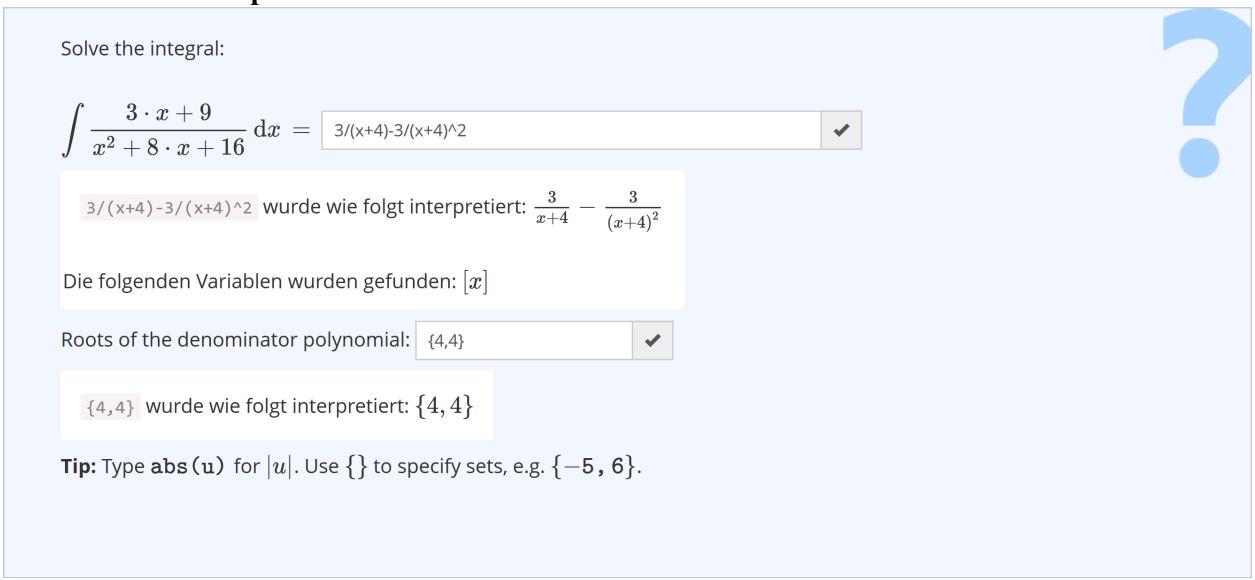
are considered and the detection of these expressions should be trained.

Partial Fraction Decomposition

Implementation / Generation

1. Choose $\lambda_1 \in \{-4, \dots, 4\}$
2. Choose $\alpha \in \{-1, 1\}$ and set $\lambda_2 = \alpha \cdot \lambda_1$, hence two cases can occur
3. Choose $A, B \in \{-3, \dots, 3\}$ and set
$$q := \frac{A}{x-\lambda_1} + \frac{B}{x-\lambda_2} \text{ or } q := \frac{A}{x-\lambda_1} + \frac{B}{(x-\lambda_1)^2}.$$
4. Simplify q and show the integral

Partial Fraction Decomposition



Problem: $\{4, 4\}$ is treated to be algebraic equivalent to $\{4\}$.

Partial Fraction Decomposition

Typical cases

- $\int \frac{x+1}{x^2+4\cdot x+4} dx = \ln(|x+2|) + \frac{1}{x+2} + c$
- $\int -\frac{3\cdot x-11}{x^2-6\cdot x+9} dx = -\frac{2}{x-3} - 3 \cdot \ln(|x-3|) + c$
- $\int -\frac{2\cdot x-12}{x^2-9} dx = \ln(|x-3|) - 3 \cdot \ln(|x+3|) + c$

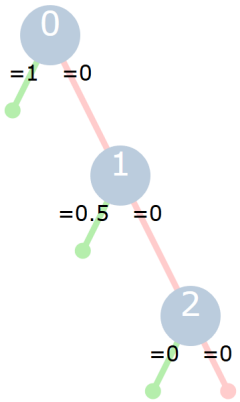
Possible Response Trees

Two trees,

- one to handle the integration,
- one to handle the roots of the denominator.

Partial Fraction Decomposition

Possible Response Tree



Node 0: Check solution ist correct, integration constant specified.

Node 1: Check solution ist correct, integration constant not specified.

Node 2: Check partial fraction decomposition was entered.

1.2 Partial Integration

Partial Integration

Solve the integral:

$\int x^2 \cdot e^x \, \mathrm{d}x \; =$ [ ] $, c \in \mathbb{R}$.

Tip: Type `abs(u)` for $|u|$ and `log(u)` for $ln(u)$.

Focus

Use the antiderivative of elementary functions.

Randomization for f: $\frac{1}{\cos(v)^2}, \frac{1}{\sin(v)^2}, \exp(v), \ln(v), \sin(v), \cos(v)$

Randomization for g: x^n, $n \in \{1, 2\}$.

Partial Integration

Some realizations

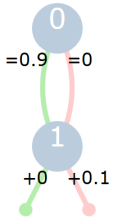
- $\int x^2 \cdot \cos(x) \, \mathrm{d}\, x = \left(x^2 - 2\right) \cdot \sin(x) + 2 \cdot x \cdot \cos(x) + c$
- $\int t^2 \cdot e^t \, \mathrm{d}\, t = \left(t^2 - 2 \cdot t + 2\right) \cdot e^t + c$
- $\int u^2 \cdot \ln(u) \, \mathrm{d}\, u = \frac{u^3 \cdot \ln(u)}{3} - \frac{u^3}{9} + c$

Reference solution

- Not implemented (yet)
- different approaches: decrease the power of the independent variable or avoid $\ln$.

Partial Integration

Implementation

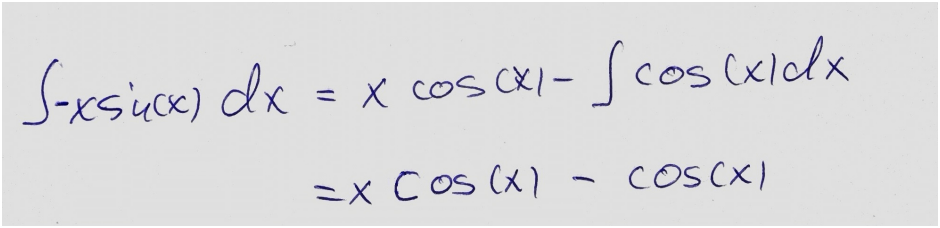


Node 1: Check the result.

Node 2: Check the integration constant.

Improvement

Use the algebraic equivalence input, but how to implemented it (integration as symbol)?



1.3 Substitution

Substitution

Find an antiderivative of $f(u) = -\cos(u) \cdot \sin^3 u + 2 \cdot \cos(u) \cdot \sin(u) + 3 \cdot \cos(u)$

$F(u) =$ [ ] , $c \in \mathbb{R}$.

Focus

- "See" derivatives of elementary functions, since integrand is of the type $(f \circ g) \cdot g'$.
- Randomization for f: $\exp(v), \sin(v), \cos(v)$
- Randomization for g: $a_2x^2 + a_1x + a$, $a_i \in \{-4, \ldots, 4\}$.

Variants

- Only the result.
- Enter the the substitution used, to provide a better feedback.

2 STACK on ILIAS