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School of Mathematics

Putting educational research into practice in a fully online course

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Plan

- About the course
- Overall design
- Task design
- Evaluation

Fundamentals of Algebra and Calculus

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Year 1 Curriculum

Semester 1	Semester 2
Introduction to Linear Algebra *N*=609	Calculus and its Applications *N*=599
Fundamentals of Algebra and Calculus *N*=113	Proofs and Problem Solving *N*=332
Option	Option

FAC: a new course

- Level 7, 20-credit, semester 1
- 113 students from Schools of Maths, Informatics, Economics, Physics and Astronomy, and others
- Introduction to Linear Algebra a corequisite

FAC: a new course

- Aimed at incoming students with lower entry qualifications

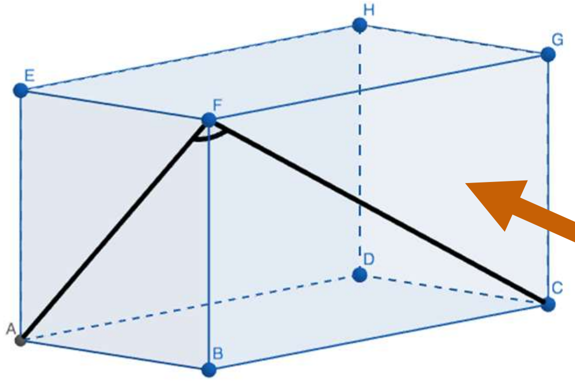
Qualifications (approx)	FAC Advice
AH Maths < B, or A-level Further Maths < B	FAC strongly recommended
AH Maths B, or A-level (Further) Maths A	FAC permitted
AH Maths A, or A-level Maths A*	FAC inappropriate

Topics in the course

Algebra
Vectors
Polynomials and rational functions
Functions and graphs
Complex numbers
Sequences and series

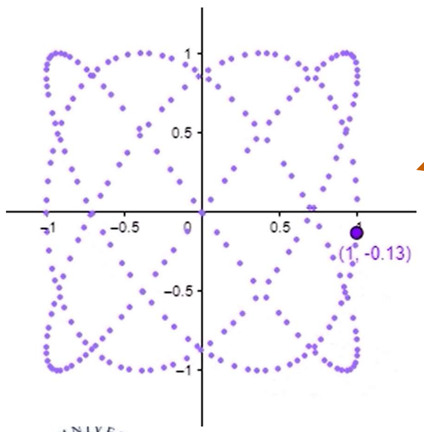
Calculus
Principles and techniques of differentiation
Further techniques and applications of differentiation
Principles of integration
Methods of integration
Applications of integration

A flavour of the topics…

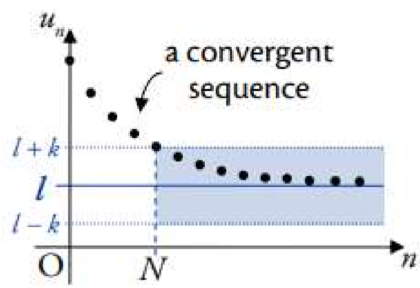


Algebra
Vectors
Polynomials and rational functions
Functions and graphs
Complex numbers
Sequences and series

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$



$$\sqrt{-1} = ?$$



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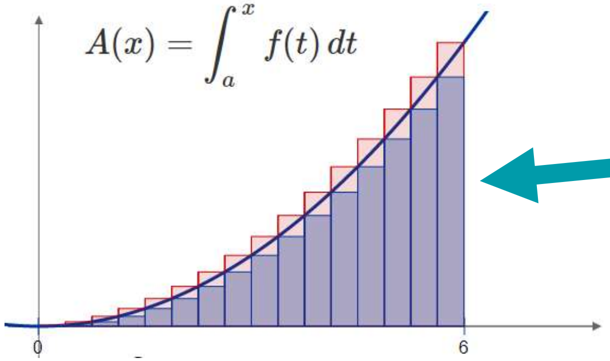
A flavour of the topics…

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Product rule, quotient rule, chain rule



Calculus
Principles and techniques of differentiation
Further techniques and applications of differentiation
Principles of integration
Methods of integration
Applications of integration

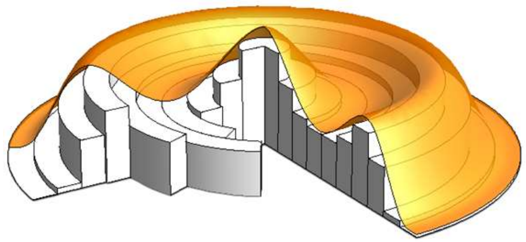
Inverse, implicit and parametric functions

Curve sketching, optimisation

Substitution

Integration by parts

Rational functions



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(Almost) entirely online

Introductory lecture

MathsBase

Autonomous Learning Groups

Mostly: STACK

A typical question

STACK offers

- randomized questions
- input validation
- robust grading

Fully factorise the polynomial $p(x) = 3\,x^4 + 16\,x^3 + 3\,x^2 - 46\,x + 24$, given that $x = -3$ is a root.

$p(x) =$ (x^2+2*x-3)*(x+4)*(3

Your last answer was interpreted as follows:

$$\left(x^2+2\,x-3\right)\,\left(x+4\right)\,\left(3\,x-2\right)$$

The variables found in your answer were: $[x]$

Check

Your answer is partially correct.
Your answer is not factored. You could still do some more work on the term $x^2 + 2\,x - 3$.
The factor $3\,x - 2$ is correct.

The factor $x + 4$ is correct.

Marks for this submission: 0.50/1.00.

A typical week

◄ Week 3: Polynomials and rational functions | Week 5: Functions ►

Week 4: Principles of integration

Your progress

- Getting started
- 1. The area under a curve
- 2. Antiderivatives
- 3. Evaluating definite integrals
- 4. Finding areas
- 2. Antiderivatives
- 3. Evaluating definite integrals
- 4. Finding areas
- Week 4 Practice Quiz
- Week 4 Final Test

Restricted Not available unless: You achieve a required score in **Week 4 Practice Quiz**

Overall course design

Distributed practice	“Textbook in the quiz”	Specifications grading / Mastery	Timing of feedback

Distributed practice

- Consistent finding in cognitive science

  Spaced practice is better than massed practice

- So, we have broken up some large topics and spaced them out through the semester

PSYCHOLOGICAL SCIENCE

Research Article

Spacing Effects in Learning

A Temporal Ridgeline of Optimal Retention

Nicholas J. Cepeda,[1,2] Edward Vul,[2,3] Doug Rohrer,[4] John T. Wixted,[2] and Harold Pashler[2]

[1]York University; [2]University of California, San Diego; [3]Massachusetts Institute of Technology; and [4]University of South Florida

ABSTRACT—*To achieve enduring retention, people must usually study information on multiple occasions. How does the timing of study events affect retention? Prior research has examined this issue only in a spotty fashion, usually with very short time intervals. In a study aimed at characterizing spacing effects over significant durations, more than 1,350 individuals were taught a set of facts and—after a gap of up to 3.5 months—given a review. A final test was administered at a further delay of up to 1 year. At any given test delay, an increase in the interstudy gap at first increased, and then gradually reduced, final test performance. The optimal gap increased as test delay increased. However, when measured as a proportion of test delay, the optimal gap declined from about 20 to 40% of a 1-week test delay to about 5 to 10% of a 1-year test delay. The interaction of gap and test delay implies that many educational practices are highly inefficient.*

As time progresses, people lose their ability to recall past experiences. The amount of information lost per unit of time gradually shrinks, producing the well-known increasingly gradual forgetting curve. Far less is known about the course of forgetting after a person has experienced multiple exposures to the same piece of information. Multiple exposures are obviously very common, and are probably essential for most long-term instruction. Thus, an understanding of how the gap between two exposures affects subsequent forgetting is fundamentally important if one wishes to temporally structure learning events in a

Effects of the gap between exposures on later memory are usually termed *distributed-practice* or *spacing* effects, and there is a large literature on such effects going back to the 19th century (for reviews, see Cepeda, Pashler, Vul, Wixted, & Rohrer, 2006; Crowder, 1976; Dempster, 1988). A spacing experiment should involve multiple periods of study devoted to the same material, separated by some variable time gap, with a final memory test administered after an additional retention interval (RI) measured from the second exposure (see Fig. 1). Many spacing studies have shown that no gap results in worse final test performance than does a brief gap. Several studies involving modest time intervals ranging from minutes to days have found that memory at the final test is best for intermediate gap durations (e.g., Balota, Duchek, & Paullin, 1989; Glenberg, 1976; Glenberg & Lehmann, 1980; Young, 1966; see Cepeda et al., 2006, for a meta-analysis focused on this point).

Given the enormous size of the literature on spacing effects, readers may wonder why there would be a need for further and more systematic exploration. Indeed, the literature is large: A recent review of distributed-practice studies involving verbal recall (Cepeda et al., 2006) examined more than 400 reports. However, only about a dozen of these looked at RIs as long as 1 day, with just a handful examining RIs longer than 1 week. Although psychologists have decried the lack of practical application of the spacing effect (Dempster, 1988; Rohrer & Taylor, 2006), the fault appears to lie at least partly in the research literature itself: On the basis of short-term studies, one cannot answer with confidence even basic questions about the timing of learning. For example, how much time between study sessions is

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Weekly schedule

1	Vectors
2	Principles and techniques of differentiation
3	Polynomials and rational functions
4	Principles of integration
5	Functions and graphs
6	Further techniques and applications of differentiation
7	Complex numbers
8	Methods of integration
9	Sequences and series
10	Applications of integration

“Textbook in the quiz”

- Testing can enhance learning

  the testing effect

- Where to place questions?
  - Interspersed in the text?
  - At the end of the chapter?

  →Either, or both!

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Journal of Applied Research in Memory and Cognition

journal homepage: www.elsevier.com/locate/jarmac

ELSEVIER

The Effect of Question Placement on Learning from Textbook Chapters

CrossMark

Oyku Uner*, Henry L. Roediger III

Washington University in St. Louis, United States

Retrieval practice enhances learning of short passages, but its effectiveness for authentic educational materials such as textbook chapters is not well established. In the current experiment, students studied a 40-page textbook chapter on biology. Retrieval practice with correct-answer feedback was manipulated within subjects: some questions appeared only after a chapter section, others only after the whole chapter, and yet others at both times. Two groups served as controls: the reread group read the feedback presented in the retrieval practice condition, and the other group simply read the chapter once. Students took a final test two days later. Practicing retrieval resulted in greater recall relative to the two control groups. On the final test, the two single testing conditions produced comparable benefits, but testing twice produced the greatest benefit. Retrieval practice is effective in learning from authentic text material and placement of the initial test does not matter.

General Audience Summary

In educational settings, testing is typically used to assess knowledge of students; however, research has shown that testing can be a powerful tool to enhance learning. This outcome is referred to as the retrieval practice effect, or the testing effect. Most laboratory studies examining this effect use simple materials, but it is not clear whether testing can be an effective study strategy when students read entire textbook chapters, which is the task faced by many students in introductory courses. Because a textbook chapter is lengthy and complex, a critical issue is where to place practice tests: after each section, after the whole chapter, or both? In the current study, we asked students to study a biology textbook chapter and we tested them two days later with short-answer questions from the chapter. One group of students read the chapter once, another group read the chapter and then reread critical information from the chapter, and a final group read the chapter and answered practice questions on it. The questions could occur after each section, after the entire chapter, or both. We found that answering questions once while reading the chapter increased recall two days later relative to the two control groups. Where the questions were placed did not matter on the final test; however, answering questions twice increased recall more than answering questions once. When studying from textbook chapters, students can use self-testing to improve their grades. Whether they test themselves during reading of the chapter or after reading the chapter does not matter, so long as feedback is provided. To receive the greatest benefit, students should test themselves more than once.

Keywords: Retrieval practice, Testing effect, Learning from text, Question placement

Fundamentals of Algebra and Calculus (2018-2019)[SV1-SEM1]

Dashboard ▸ My courses ▸ www.learn.ed__66007_1 ▸ Week 4: Principles of integration ▸ 2. Antiderivatives

INDEFINITE INTEGRALS

i 1 2 3

STANDARD ANTIDERIVATIVES

i i 4 5 6 7 8

INTEGRATION BY INSPECTION

i 9 10 11 12

MIXED PRACTICE

13 14 15 16 17

Information

Flag question

Antiderivatives and indefinite integrals

Remember that any function we can differentiate tells us about a corresponding antiderivative.

For example, $\frac{d}{dx}(x^2) = 2x$ so we know x^2 is an antiderivative of $2x$.

However, notice that we also have $\frac{d}{dx}(x^2+1) = 2x$. So x^2+1 is also an antiderivative of $2x$.

In fact, if $F(x)$ is an antiderivative of $f(x)$ then so is $F(x)+C$ where C is any constant. We can see this because differentiating both $F(x)$ and $F(x)+C$ gives $f(x)$.

We saw in the last section that antiderivatives are related to definite integrals:

Evaluation Theorem

If $f(x) = G'(x)$ (i.e. G is an antiderivative of f) then

$$\int_a^b f(x)\,dx = G(b) - G(a).$$

Because of this connection, we also talk about indefinite integrals:

The **indefinite integral** $\int f(x)\,dx = F(x) + C$ means $F'(x) = f(x)$.

It represents the most general antiderivative of f, so must always include an arbitrary constant (usually $+C$).

Note that:

- we say that $f(x)$ is the **integrand.**
- The dx is very important because it indicates the variable we are integrating with respect to.
- the function F is often just referred to as the **integral** of f.

The notation is very similar for definite and indefinite integrals -- the only difference is whether we attach limits to the integral sign. However, notice that the result of an indefinite integral is a *function*, whereas the definite integral gives a *number*.

Example

Returning to the example above, we can write the indefinite integral

$$\int 2x\,dx = x^2 + C$$

to represent the fact that x^2+C is the most general antiderivative of $2x$.

Question **1**

Tries remaining: 1

Marked out of 1.00

Flag question

Which of the following are antiderivatives of x^6?

- (a) $\frac{x^7}{7} + 6$
- (b) $\frac{x^7}{7} + 1$
- (c) $\frac{x^7}{7}$
- (d) $\frac{x^6}{7}$
- (e) $6x^5 + x$
- (f) x^7

Check

Question **2**

Tries remaining: 1

Marked out of 1.00

$\int x^2\,dx =$

Check

Simple questions to check understanding

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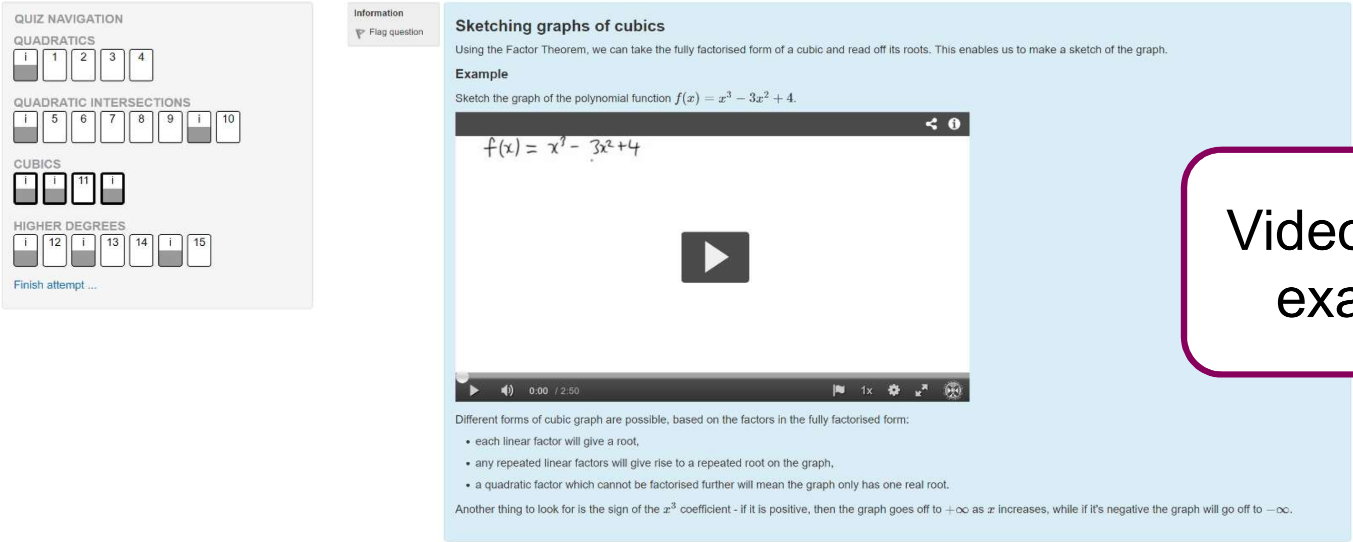
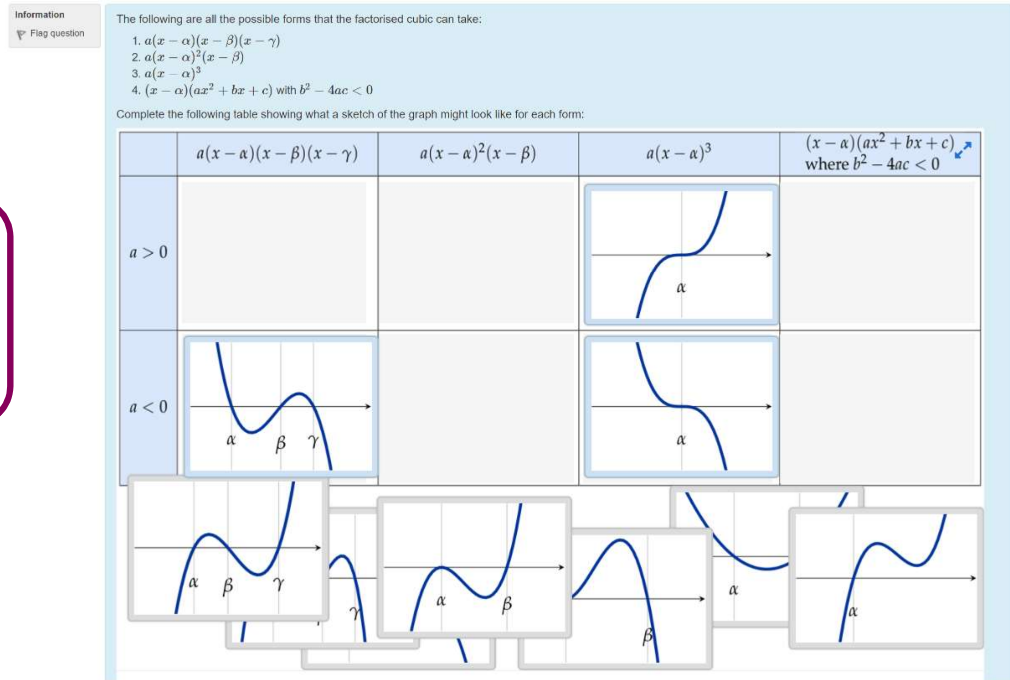


Video worked examples



Matching/sorting activities

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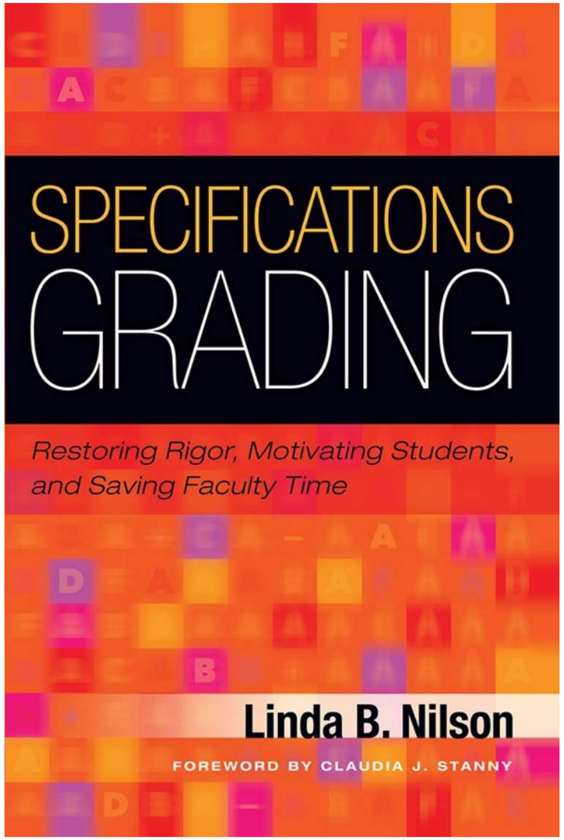
Specifications grading

- Individual assessments are graded pass/fail
- Some amount of resubmission is allowed
- Letter grades are based on performance across multiple assessments



Specifications grading

Practice Quiz

Unlimited attempts

Full feedback

80% score required to unlock Final Test

Final Test

Single attempt

80% required to pass

Final Test (Resit)

One more attempt

80% required to pass

Each week gets either a

Mastery (80%+)

or

Distinction (95%+)

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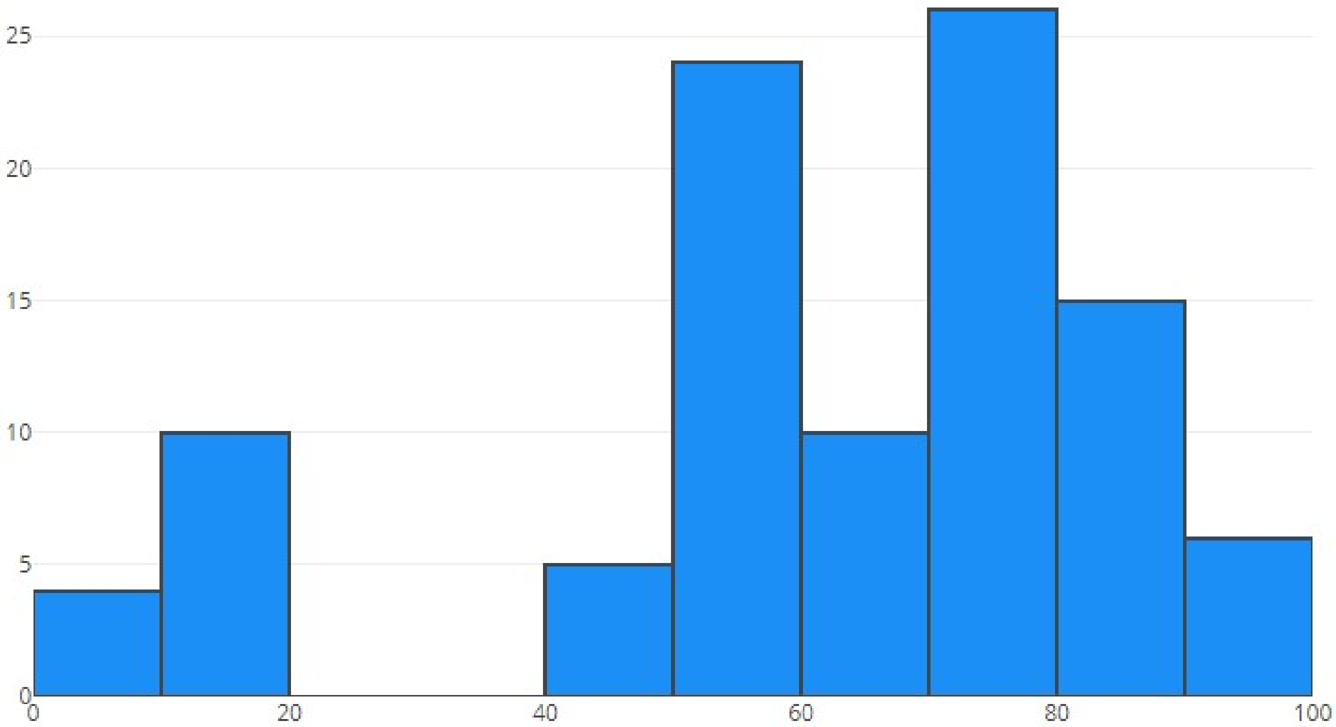
What grade will you get?

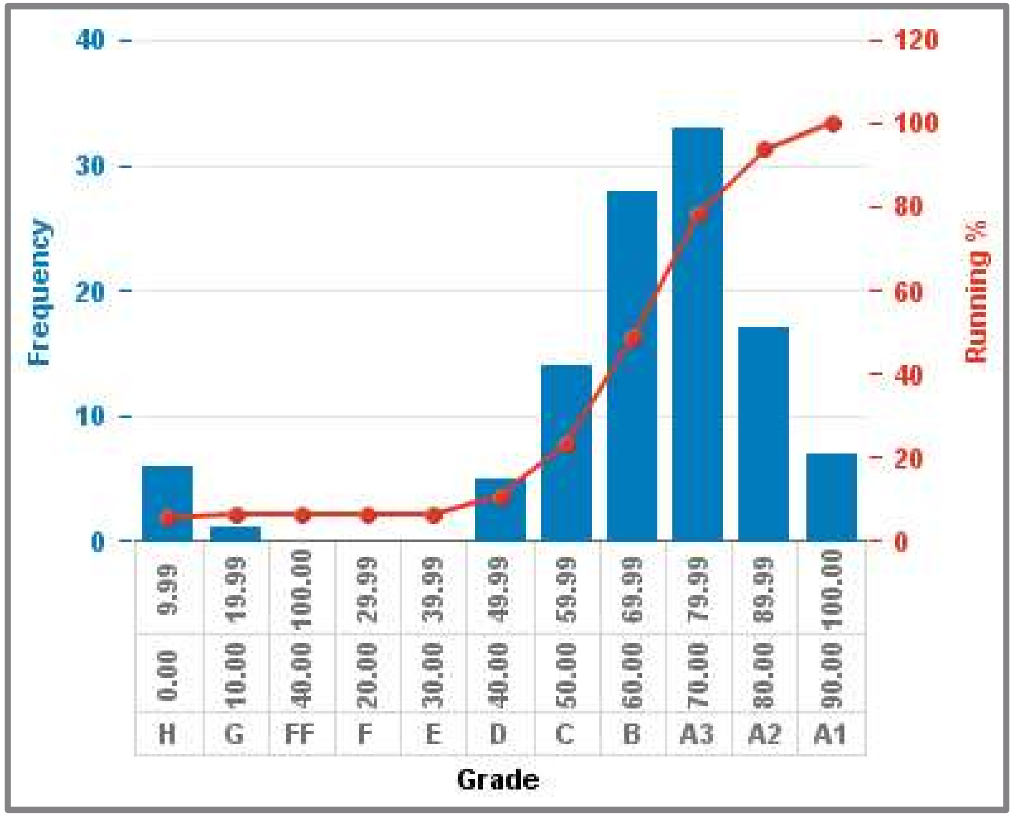
Number of units Mastered (80%+)	Number of Distinctions (95%+)	Percentage awarded for the Unit Score	Equivalent Grade
Less than 7	-	0	F
7	-	45%	D
8	2 or 3	55%	C
9	4 or 5	65%	B
10	6 or 7	75%	A1
10	8 or 9	85%	A2
10	10	100%	A3

Possible grade distribution

(based on extrapolating from results in weeks 1-5)



Actual results...



(N=113)

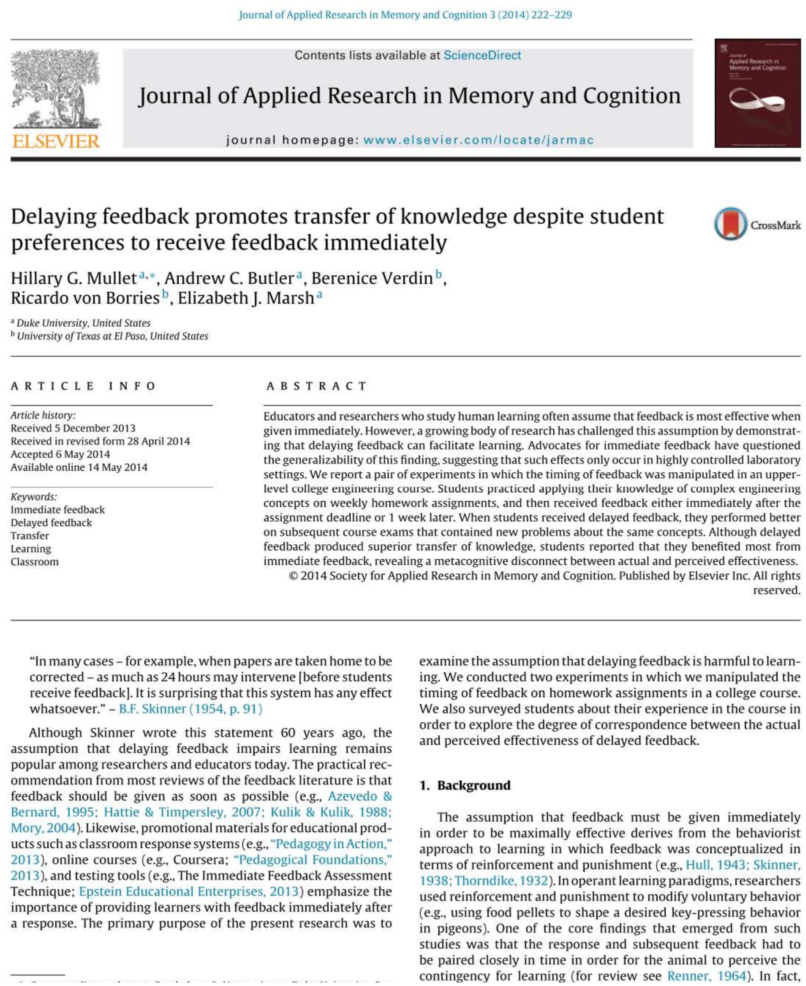
- Mean: 67
- Median: 70
- Pass rate: 94%

Timing of feedback

- Is it best to give feedback as soon as possible?

  "delaying feedback can facilitate learning"

- We delay the feedback on the Final Test until after the deadline.



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ELSEVIER

Delaying feedback promotes transfer of knowledge despite student preferences to receive feedback immediately

CrossMark

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Delayed feedback
Transfer
Learning
Classroom

ABSTRACT

Educators and researchers who study human learning often assume that feedback is most effective when given immediately. However, a growing body of research has challenged this assumption by demonstrating that delaying feedback can facilitate learning. Advocates for immediate feedback have questioned the generalizability of this finding, suggesting that such effects only occur in highly controlled laboratory settings. We report a pair of experiments in which the timing of feedback was manipulated in an upper-level college engineering course. Students practiced applying their knowledge of complex engineering concepts on weekly homework assignments, and then received feedback either immediately after the assignment deadline or 1 week later. When students received delayed feedback, they performed better on subsequent course exams that contained new problems about the same concepts. Although delayed feedback produced superior transfer of knowledge, students reported that they benefited most from immediate feedback, revealing a metacognitive disconnect between actual and perceived effectiveness.

"In many cases – for example, when papers are taken home to be corrected – as much as 24 hours may intervene [before students receive feedback]. It is surprising that this system has any effect whatsoever." – B.F. Skinner (1954, p. 91)

Although Skinner wrote this statement 60 years ago, the assumption that delaying feedback impairs learning remains popular among researchers and educators today. The practical recommendation from most reviews of the feedback literature is that feedback should be given as soon as possible (e.g., Azevedo & Bernard, 1995; Hattie & Timpersley, 2007; Kulik & Kulik, 1988; Mory, 2004). Likewise, promotional materials for educational products such as classroom response systems (e.g., "Pedagogy in Action," 2013), online courses (e.g., Coursera; "Pedagogical Foundations," 2013), and testing tools (e.g., The Immediate Feedback Assessment Technique; Epstein Educational Enterprises, 2013) emphasize the importance of providing learners with feedback immediately after a response. The primary purpose of the present research was to examine the assumption that delaying feedback is harmful to learning. We conducted two experiments in which we manipulated the timing of feedback on homework assignments in a college course. We also surveyed students about their experience in the course in order to explore the degree of correspondence between the actual and perceived effectiveness of delayed feedback.

1. Background

The assumption that feedback must be given immediately in order to be maximally effective derives from the behaviorist approach to learning in which feedback was conceptualized in terms of reinforcement and punishment (e.g., Hull, 1943; Skinner, 1938; Thorndike, 1932). In operant learning paradigms, researchers used reinforcement and punishment to modify voluntary behavior (e.g., using food pellets to shape a desired key-pressing behavior in pigeons). One of the core findings that emerged from such studies was that the response and subsequent feedback had to be paired closely in time in order for the animal to perceive the contingency for learning (for review see Renner, 1964). In fact,

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Task design

Faded worked examples	"Give an example"	Retrieval practice

Faded worked examples

Worked Example

↓

Problem Solving

Worked Example

↓

Worked Example with last step as a task

↓

Worked example with more steps to complete

↓

Problem solving

Faded worked examples

- Using a faded sequence of worked examples

  “The fading procedure fosters learning”

- The course uses this structure for many key procedures…

The Journal of Experimental Education, 2002, *70*(4), 293–315

From Example Study to Problem Solving: Smooth Transitions Help Learning

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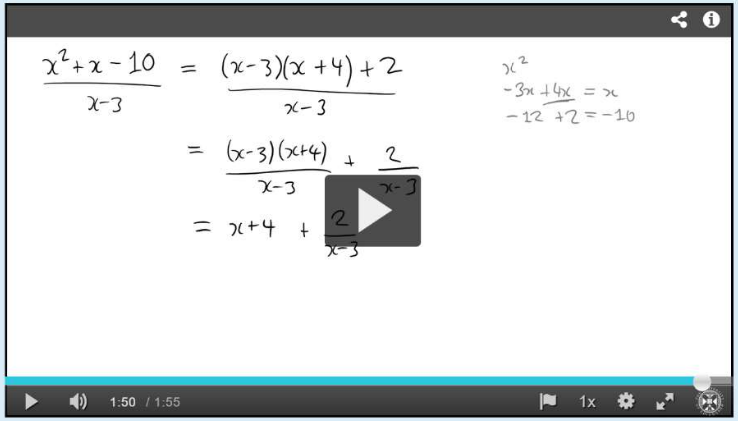
ABSTRACT. Research has shown that it is effective to combine example study and problem solving in the initial acquisition of cognitive skills. Present methods for combining these learning modes are static, however, and do not support a transition from example study in early stages of skill acquisition to later problem solving. Against this background, the authors proposed a successive integration of problem-solving elements into example study until the learners solved problems on their own (i.e., complete example → increasingly more incomplete examples → problem to-be-solved). The authors tested the effectiveness of such a fading procedure against the traditional method of using example–problem pairs. In a field experiment and in 2 more controlled laboratory experiments, the authors found that (a) the fading procedure fosters learning, at least when near transfer performance is considered; (b) the number of problem-solving errors during learning plays a role in mediating this effect; and (c) it is more favorable to fade out worked-out solution steps in a backward manner (omitting the last solution steps first) as compared with a forward manner (omitting the first solution steps first).

Key words: fading, learning, problem solving, transfer, worked-out examples

WORKED-OUT EXAMPLES consist of a problem formulation, solution steps, and the final solution itself. Researchers have shown that learning from such examples is of major importance for the initial acquisition of cognitive skills in well-structured domains such as mathematics, physics, and programming (for an

Example

Divide $x^2 + x - 10$ by $x - 3$.



Now apply this procedure to the following examples:

Question 2

Tries remaining: 1

Marked out of 1.00

Flag question

$$\frac{2x^2+7x}{x+3} = \frac{(x+3)\,(2x+1)-3}{(x+3)}$$

$=$??? + ?/(x+?)

Check

Question 3

Tries remaining: 1

Marked out of 1.00

Flag question

$$\frac{3x^2+10x-4}{x+4} = \frac{(x+4)(\quad)+\quad}{(x+4)}$$

$=$??? + ?/(x-?)

Check

Question 4

Tries remaining: 1

Marked out of 1.00

Flag question

$$\frac{x^2-4}{x-1} =$$??? + ?/(x-?)

Check

Worked Example

↓

Worked Example with last step as a task

↓

Worked example with more steps to complete

↓

Problem solving

Worked Example

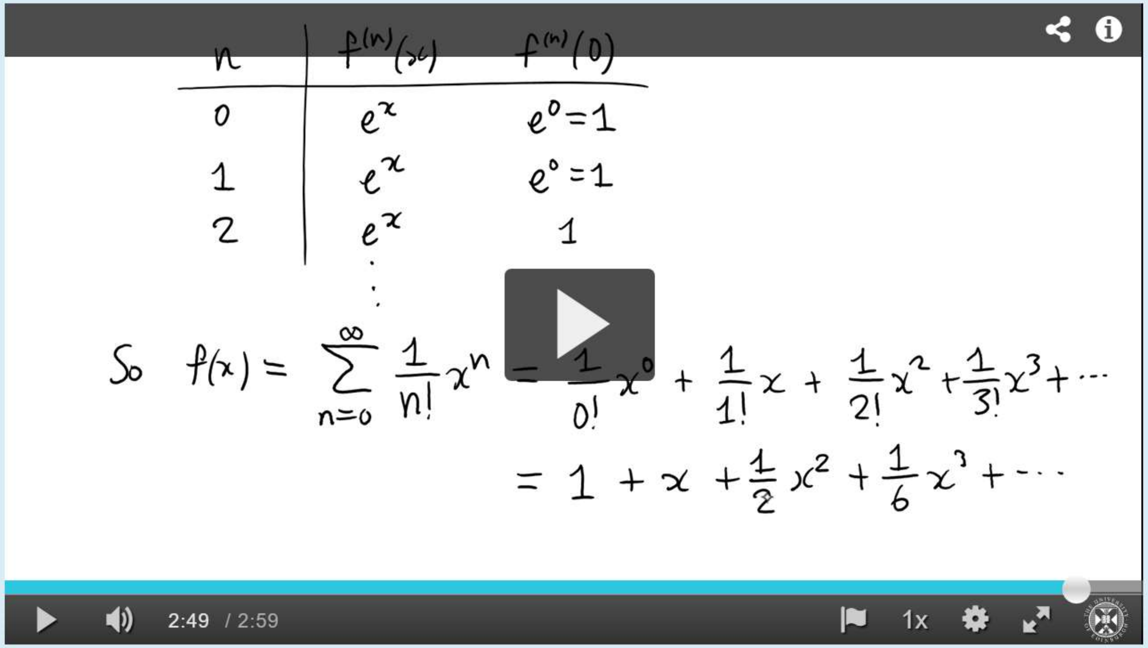
Maclaurin series

The **Maclaurin series** of f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \cdots$$

Example

Find the Maclaurin series for $f(x) = e^x$.



Worked Example

Worked Example with last step as a task

Find the Maclaurin series of $f(x) = \sin(x)$.

We compute the derivatives and evaluate them at $x = 0$:

$$\begin{aligned} f(x) &= \sin(x) & f(0) &= 0 \\ f'(x) &= \cos(x) & f'(0) &= 1 \\ f''(x) &= -\sin(x) & f''(0) &= 0 \\ f^{(3)}(x) &= -\cos(x) & f^{(3)}(0) &= -1 \\ f^{(4)}(x) &= \sin(x) & f^{(4)}(0) &= 0 \end{aligned}$$

and from here we see that the cycle of values $0, 1, 0, -1$ will repeat.

So the Maclaurin series begins:

[ ] (enter the first four nonzero terms)

The general term is

- (a) $\dfrac{(-1)^{2n+1} x^{2n+1}}{(2n+1)!}$
- (b) $\dfrac{(-1)^{n} x^{n}}{n!}$
- (c) $\dfrac{(-1)^{n} x^{2n+1}}{(2n+1)!}$
- (d) $\dfrac{(-1)^{n} x^{2n}}{(2n)!}$

Check

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Worked Example

Worked Example with last step as a task

Worked example with more steps to complete

Find the Maclaurin series of $f(x) = \cos(x)$.

We compute the derivatives and evaluate them at $x = 0$:

$$\begin{aligned} f(x) &= \cos(x) & f(0) &= 1 \\ f'(x) &= -\sin(x) & f'(0) &= 0 \\ f''(x) &= -\cos(x) & f''(0) &= -1 \\ f^{(3)}(x) &= \sin(x) & f^{(3)}(0) &= 0 \\ f^{(4)}(x) &= \cos(x) & f^{(4)}(0) &= 1 \end{aligned}$$

and from here we see that the cycle of values $1, 0, -1, 0$ will repeat.

So the Maclaurin series begins:

[ ] (enter the first four nonzero terms)

The general term is [ ]

Note: for the last question, the general term would be typed as `(-1)^n*x^(2*n+1)/(2*n+1)!`

Check

Worked Example

Worked Example with last step as a task

Worked example with more steps to complete

Worked example with more steps to complete

Find the Maclaurin series of $f(x) = \ln(x+1)$.

We compute the derivatives and evaluate them at $x = 0$:

$f(x) = \ln(x+1)$ $\quad$ $f(0) = 0$

$f'(x) =$ ______ $\quad$ $f'(0) =$ ______

$f''(x) =$ ______ $\quad$ $f''(0) =$ ______

$f'''(x) =$ ______ $\quad$ $f'''(0) =$ ______

So the Maclaurin series begins:

______ (enter the first four nonzero terms)

The general term is ______

Check

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Worked Example

Worked Example with last step as a task

Worked example with more steps to complete

Worked example with more steps to complete

Problem solving

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Find the Maclaurin series of $f(x) = e^{-5\,x}$.

(a) The first five nonzero terms are:

(b) The general term is:

Check

“Give an example”

- Inviting students to construct examples

  “Simply ‘giving’ examples and construction techniques is rarely sufficient for most learners. Most learners need to (re)construct examples in order to populate their example space”

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Educ Stud Math (2008) 69:183–194
DOI 10.1007/s10649-008-9143-3

Shedding light on and with example spaces

Paul Goldenberg · John Mason

Published online: 25 July 2008

Abstract Building on the papers in this special issue as well as on our own experience and research, we try to shed light on the construct of *example spaces* and on how it can inform research and practice in the teaching and learning of mathematical concepts. Consistent with our way of working, we delay definition until after appropriate reader experience has been brought to the surface and several ‘examples’ have been discussed. Of special interest is the notion of *accessibility* of examples: an individual’s access to example spaces depends on conditions and is a valuable window on a deep, personal, situated structure. Through the notions of *dimensions of possible variation* and *range of permissible change*, we consider ways in which examples exemplify and how attention needs to be directed so as to emphasise examplehood (generality) rather than particularity of mathematical objects. The paper ends with some remarks about example spaces in mathematics education itself.

Keywords Examples · Exemplification · Example spaces

1 Examples

The widespread use of ‘examples’ in mathematics textbooks from the earliest recorded time is a manifestation of the common insight that it is through the appreciation of familiar examples that abstractions become reified (Sfard 1994). Just as with natural language, meaning arises mainly from encountering instances in use, while definitions provide a reference against which to test those uses. Put another way, definitions function as generalisations or abstractions whose meaning emerges through experience of particular

“Give an example”

For each case below, type in a quadratic, e.g. `2x^2+3x+1`, whose graph has exactly the given number of intersections with $y = x^2$. If it is not possible, then enter `none`.

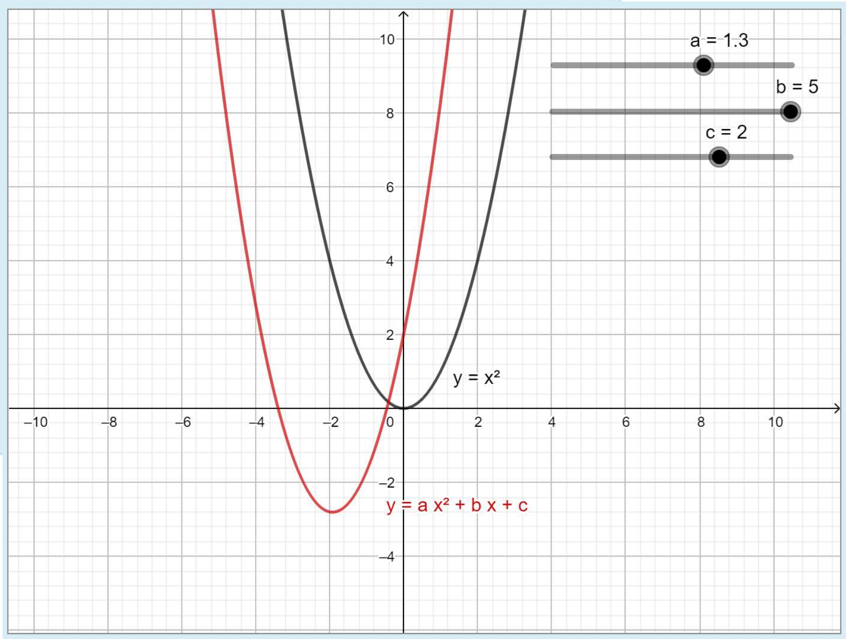
No intersection: $y =$

1 intersection: $y =$

2 intersections: $y =$

3 intersections: $y =$

Check



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"Give an example"

In each case below, give an example of an arithmetic sequence with the stated property, by entering an expression for the general term. If it is not possible, enter `none`.

(a) Increasing

$u_n =$ [ ]

(b) Decreasing

$u_n =$ [ ]

(c) Bounded above

$u_n =$ [ ]

(d) Decreasing and bounded below

$u_n =$ [ ]

Check

Retrieval practice

“retrieval of information from memory *produces better retention than restudying …*

the testing effect”

Review

Cell PRESS

The critical role of retrieval practice in long-term retention

Henry L. Roediger III[1] and Andrew C. Butler[2]

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Learning is usually thought to occur during episodes of studying, whereas retrieval of information on testing simply serves to assess what was learned. We review research that contradicts this traditional view by demonstrating that retrieval practice is actually a powerful mnemonic enhancer, often producing large gains in long-term retention relative to repeated studying. Retrieval practice is often effective even without feedback (i.e. giving the correct answer), but feedback enhances the benefits of testing. In addition, retrieval practice promotes the acquisition of knowledge that can be flexibly retrieved and transferred to different contexts. The power of retrieval practice in consolidating memories has important implications for both the study of memory and its application to educational practice.

Introduction

> A curious peculiarity of our memory is that things are impressed better by active than by passive repetition. I mean that in learning (by heart, for example), when we almost know the piece, it pays better to wait and recollect by an effort within, than to look at the book again. If we recover the words the former way, we shall probably know them the next time; if in the latter way, we shall likely need the book once more.
>
> William James [1]

Psychologists have often studied learning by alternating series of study and test trials. In other words, material is presented for study (S) and a test (T) is subsequently given to determine what was learned. After this procedure is repeated over numerous ST trials, performance (e.g. the number of items recalled) is plotted against trials to depict the rate of learning; the outcome is referred to as a learning curve and it is negatively accelerated and is fit by a power function. Thus, most learning occurs on early ST trials, and the amount of learning decreases with additional trials. The critical assumption is that learning occurs during the study phases of the ST ST ST… sequence, and the test phase is simply there to measure what has been learned during previous occasions of study. The test is usually considered a neutral event. For example, researchers in the 1960 s debated whether learning ignored the possibility that learning occurred during the retrieval tests [2–5]. Exactly the same assumption is built into our educational systems. Students are thought to learn via lectures, reading, highlighting, study groups, and so on; tests are given in the classroom to measure what has been learned from studying. Again, tests are considered assessments, gauging the knowledge that has been acquired without affecting it in any way.

In this article, we review evidence that turns this conventional wisdom on its head: retrieval practice (as occurs during testing) often produces greater learning and long-term retention than studying. We discuss research that elucidates the conditions under which retrieval practice is most effective, as well as evidence demonstrating that the mnemonic benefits of retrieval practice are transferrable to different contexts. We also describe current theories on the mechanisms underlying the beneficial effects of testing. Finally, we discuss educational implications of this research, arguing that more frequent retrieval practice in the classroom would increase long-term retention and transfer.

The testing effect and repeated retrieval

The finding that retrieval of information from memory produces better retention than restudying the same information for an equivalent amount of time has been termed the testing effect [6]. Although the phenomenon was first reported over 100 years ago [7], research on the testing effect has been sporadic at best until recently (but see Box 1 for some classic studies). In the last 10 years, much research has shown powerful mnemonic benefits of retrieval practice [8–10] . The data in Figure 1 come from a study in which two groups of students retrieved information several times

Glossary

Expanding retrieval schedule: testing of retention shortly after learning to make sure encoding is accurate, then waiting longer to retrieve again, then waiting still longer for a third retrieval and so on.
Feedback: providing information after a question. General (right or wrong) feedback is not very helpful if the correct answer is not provided. Correct answer feedback usually produces robust gains on a final criterion measure.
Negative suggestion effect: taking a test that provides subtly wrong answers (e.g. true or false, multiple choice) can lead students to select a wrong answer, believe it is right, and thus learn an error from taking the test.
Retrieval practice: act of calling information to mind rather than rereading it or hearing it. The idea is to produce ‘an effort from within’ to induce better retention.

Retrieval practice

- This can be combined with the idea of distributed practice

  "spaced retrieval practice can have a meaningful, long-lasting impact on educational outcomes"

Educ Psychol Rev (2016) 28:853–873
DOI 10.1007/s10648-015-9349-8

CrossMark

INTERVENTION STUDY

Spaced Retrieval Practice Increases College Students' Short- and Long-Term Retention of Mathematics Knowledge

Robin F. Hopkins[1] · Keith B. Lyle[1] · Jeff L. Hieb[2] · Patricia A. S. Ralston[2]

Published online: 12 December 2015

Abstract A major challenge college students face is retaining the knowledge they acquire in their classes, especially in cumulative disciplines such as engineering, where ultimate success depends on long-term retention of foundational content. Cognitive psychologists have recently recommended various techniques educators might use to increase retention. One technique (*spaced retrieval practice*) involves extending opportunities to retrieve course content beyond a customarily short temporal window following initial learning. Confirming the technique's utility requires demonstrating that it increases retention in real classroom settings, with commonly encountered educational content, and that gains endure into subsequent semesters. We manipulated spaced versus massed retrieval practice in a precalculus course for engineering students and followed a subset of students who proceeded into a calculus class the following semester. Spacing versus massing was manipulated within- and between-subjects. Within-subjects, students retained spaced content better than massed content in the precalculus course. Between-subjects, students for whom some retrieval practice was spaced, compared to those for whom all practice was massed, performed better on the final exam in the precalculus class and on exams in the calculus class. These findings suggest that spaced retrieval practice can have a meaningful, long-lasting impact on educational outcomes.

Keywords Memory · Spacing · Retrieval practice · Mathematics · Engineering

Complete the following table of standard derivatives. Try to do this without checking your notes -- which of the most important standard derivatives can you remember?

Function	Derivative
x^n	

Check

At the start of integration (Week 4) – recall practice of differentiation (Week 2)

Summary

Distributed practice

"Textbook in the quiz"

Specifications grading / Mastery

Timing of feedback

Faded worked examples

"Give an example"

Retrieval practice

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Kinnear, G. (2019)
Delivering an online course using STACK
http://doi.org/10.5281/zenodo.2565969

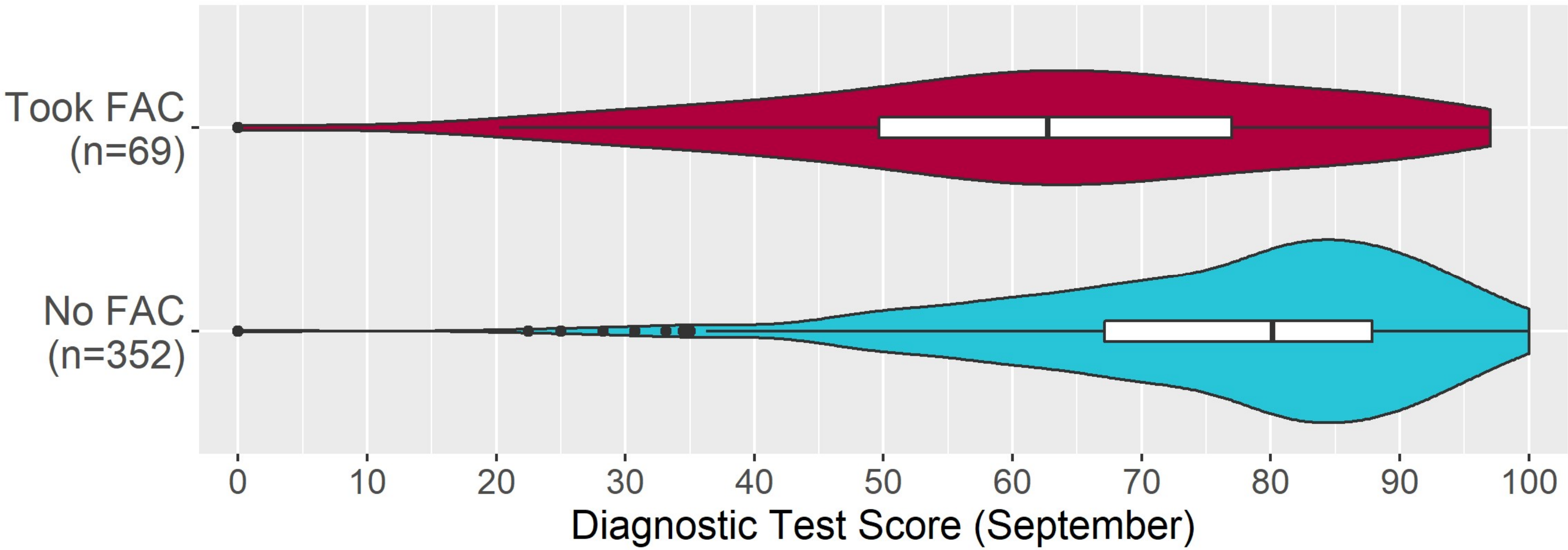
Evaluation

Year 1 Curriculum

DT Semester 1	DT Semester 2
Introduction to Linear Algebra	Calculus and its Applications
Fundamentals of Algebra and Calculus	Proofs and Problem Solving
Option	Option

1 2

Diagnostic Test - Before



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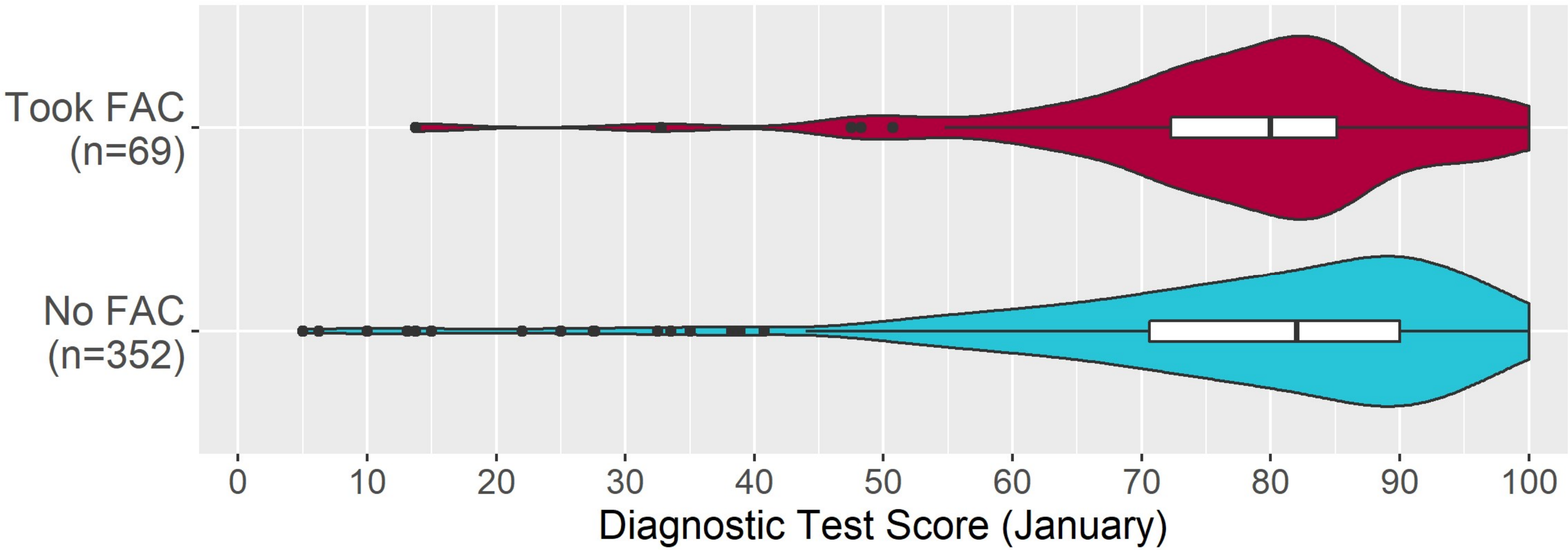
Diagnostic Test - After



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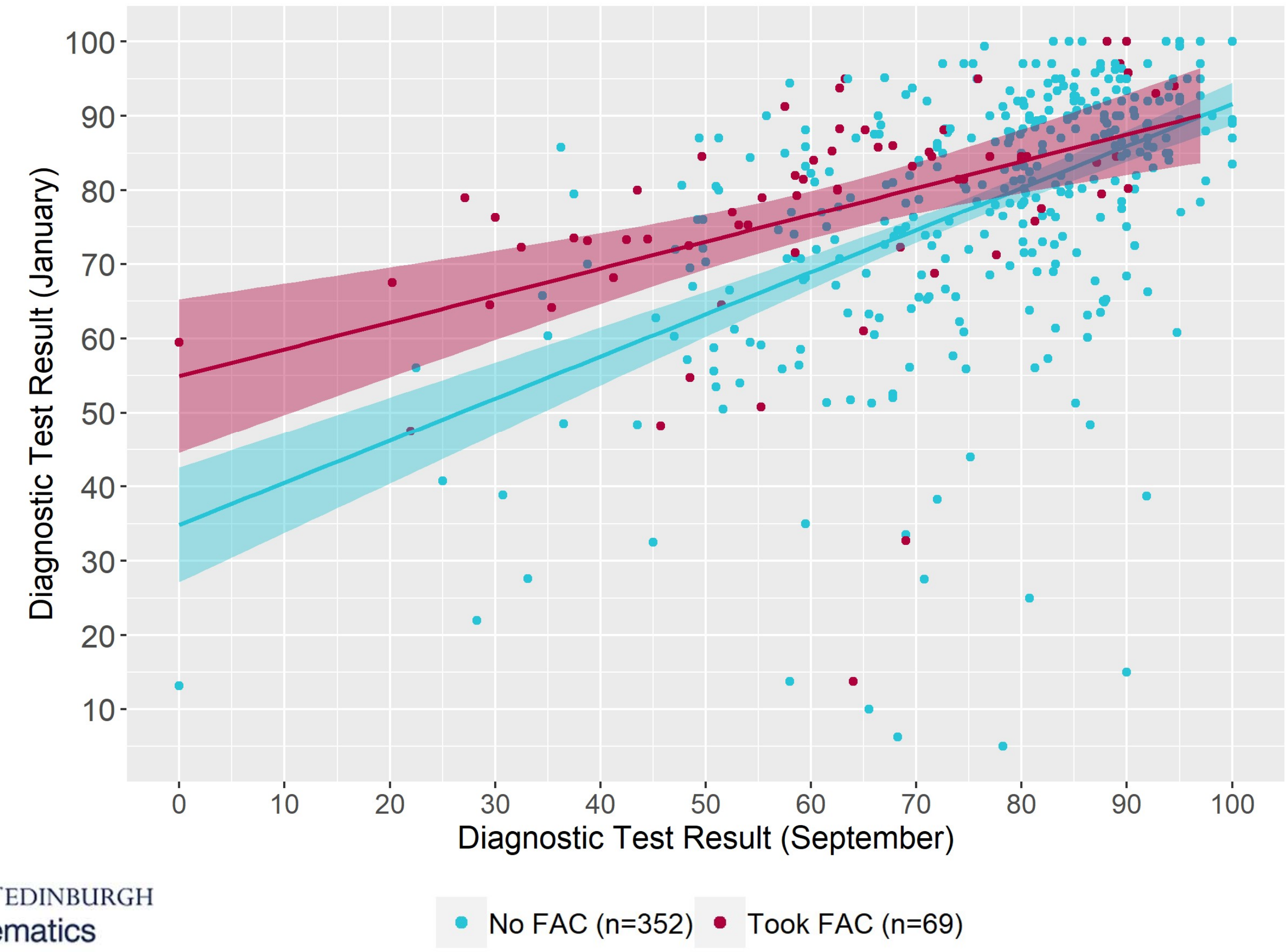
Diagnostic Test Gains

	Pre-test	Post-test	Gain
FAC	62.1	77.4	15.3
Non-FAC	76.1	78.1	2.0

Diagnostic Test Gains



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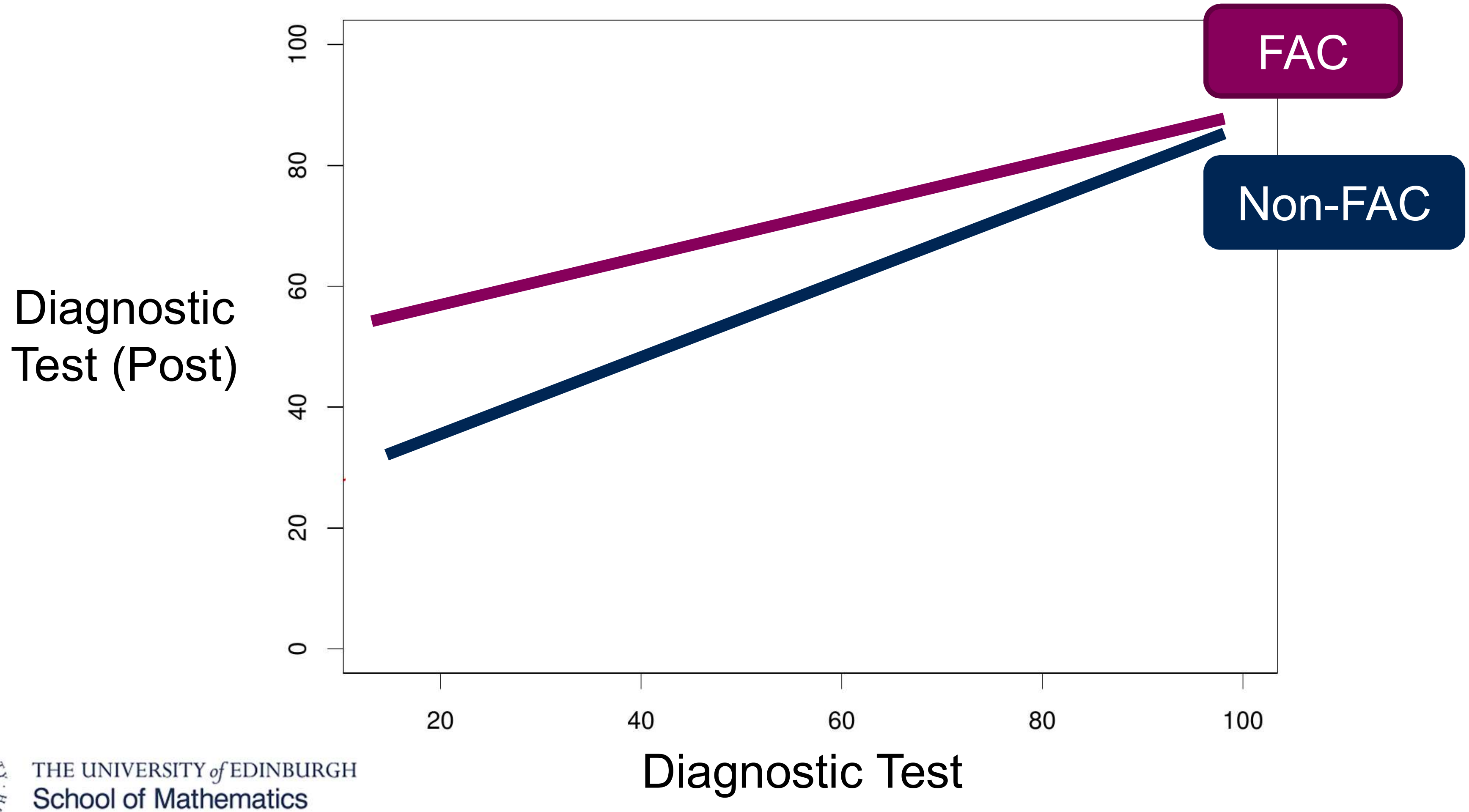
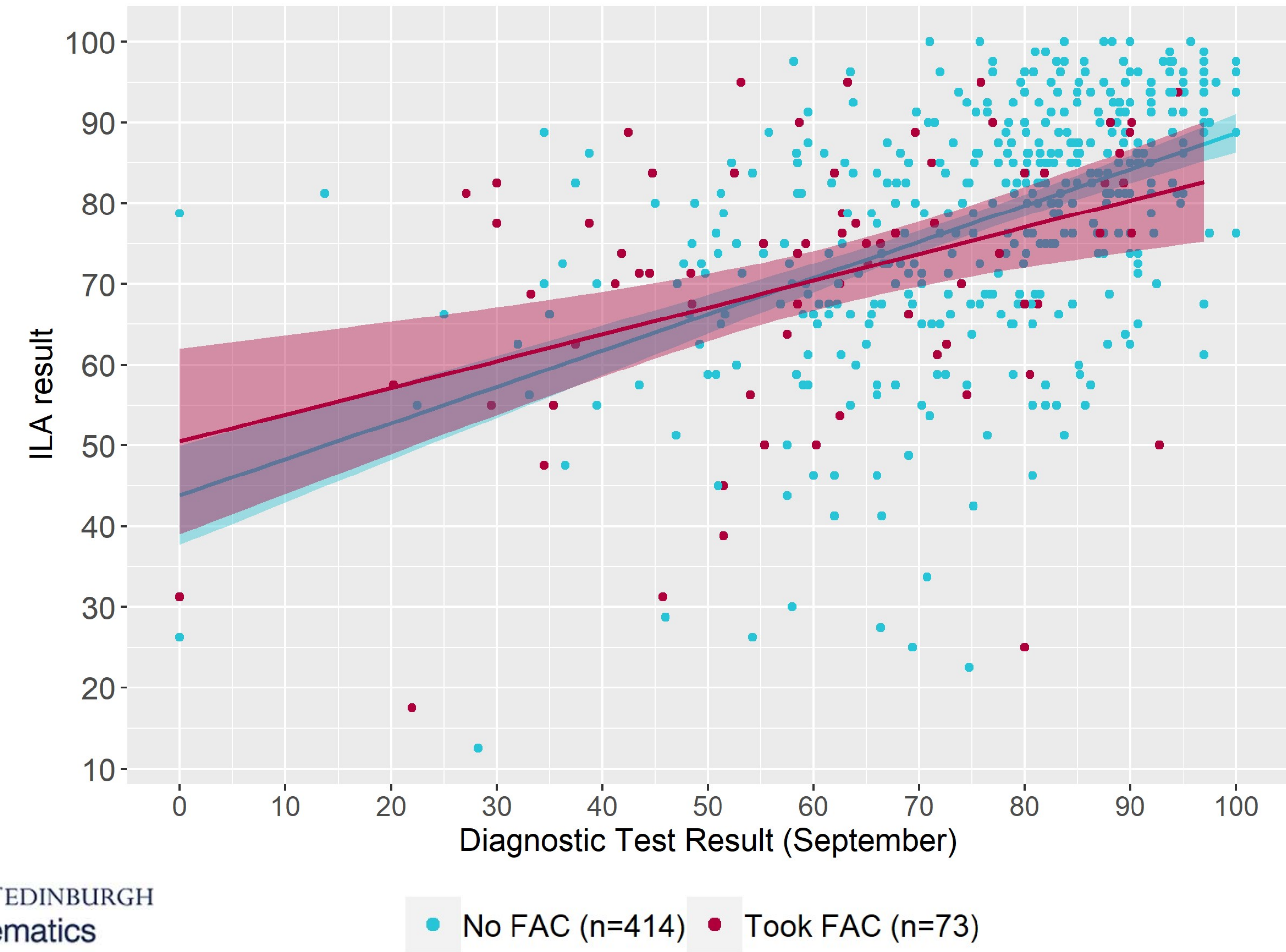
Effect of FAC



ILA Results



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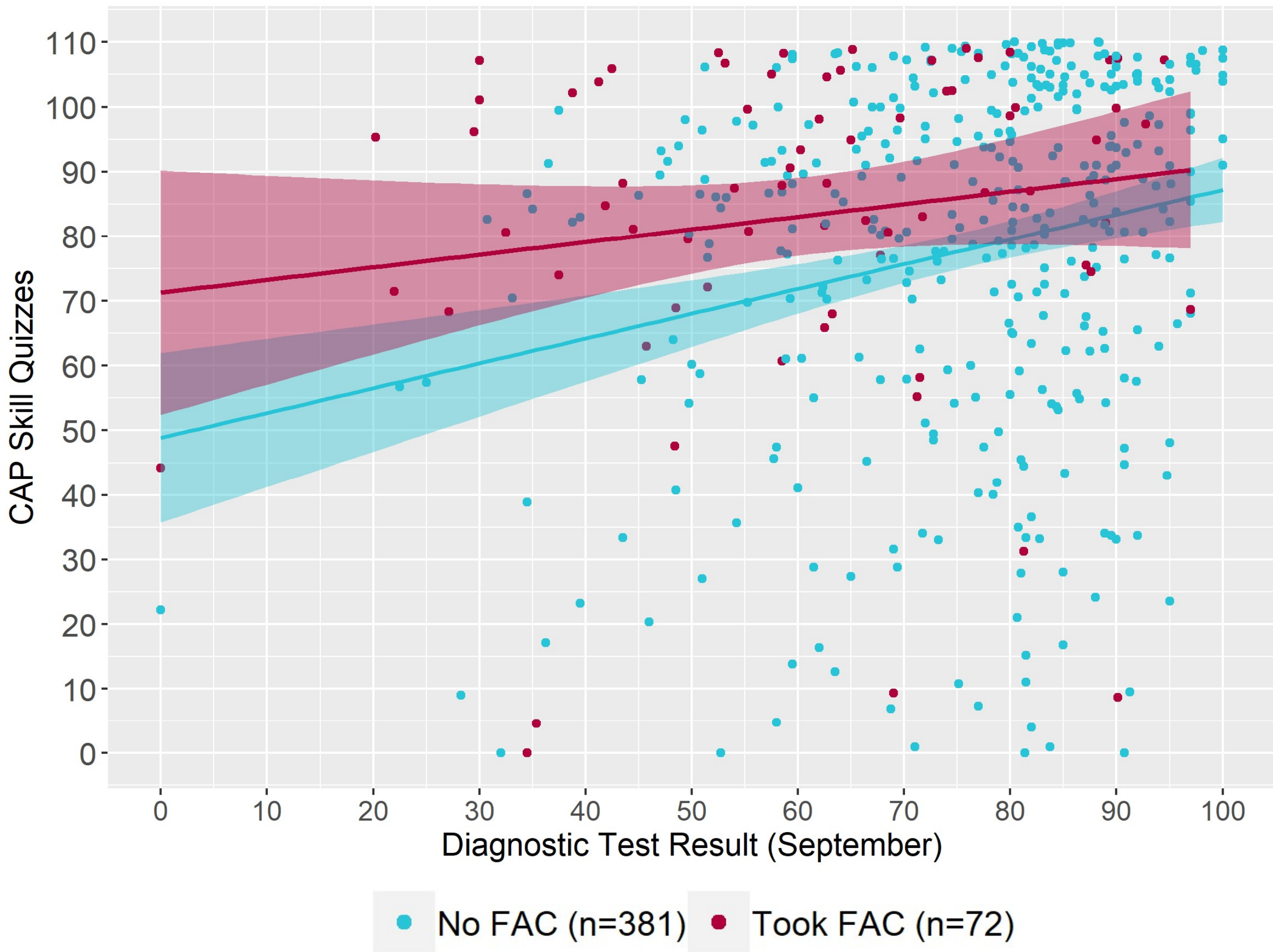
CAP Coursework (Online)



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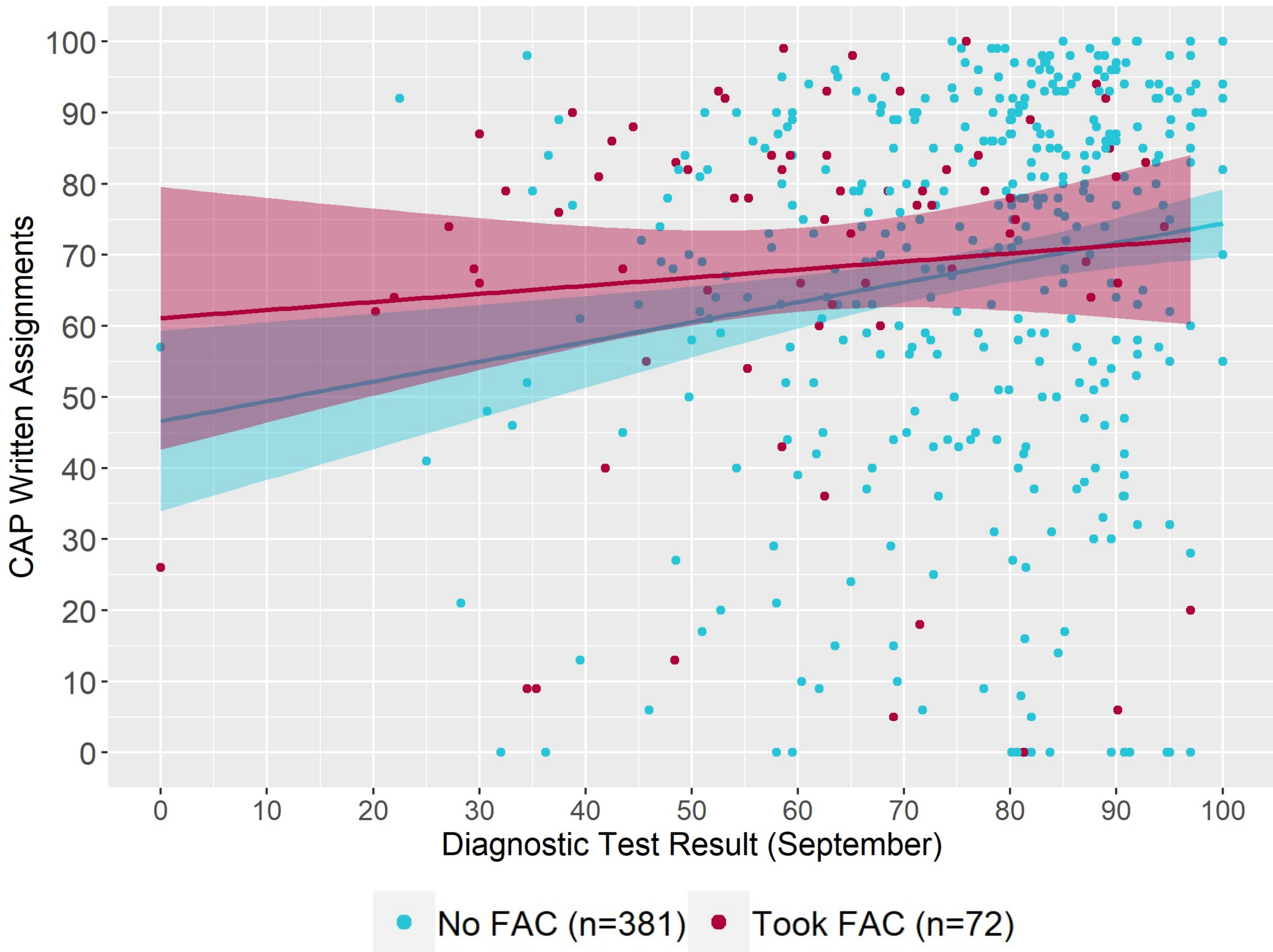
CAP Coursework (Written)



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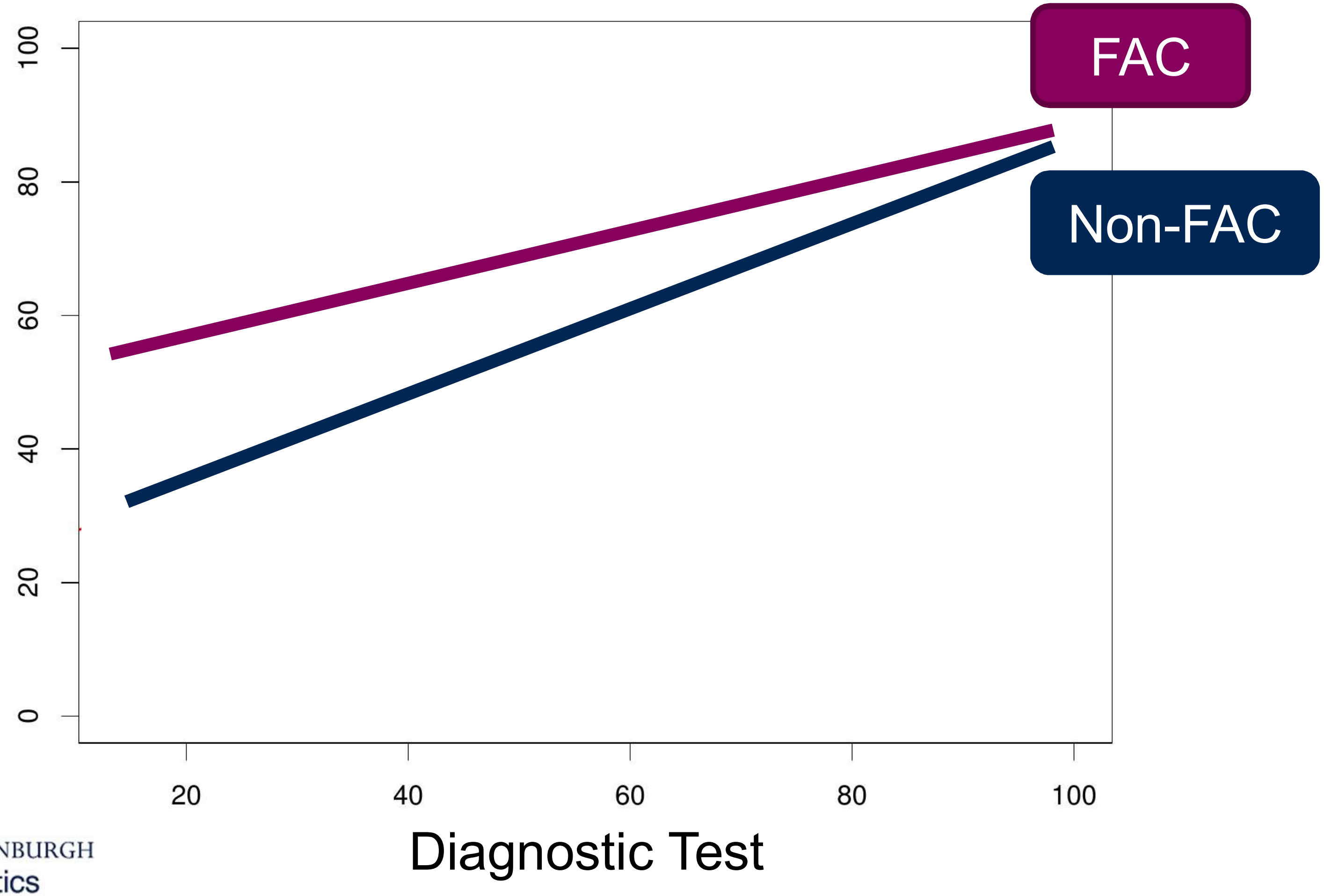
CAP Exam

Calculus and its Applications

?



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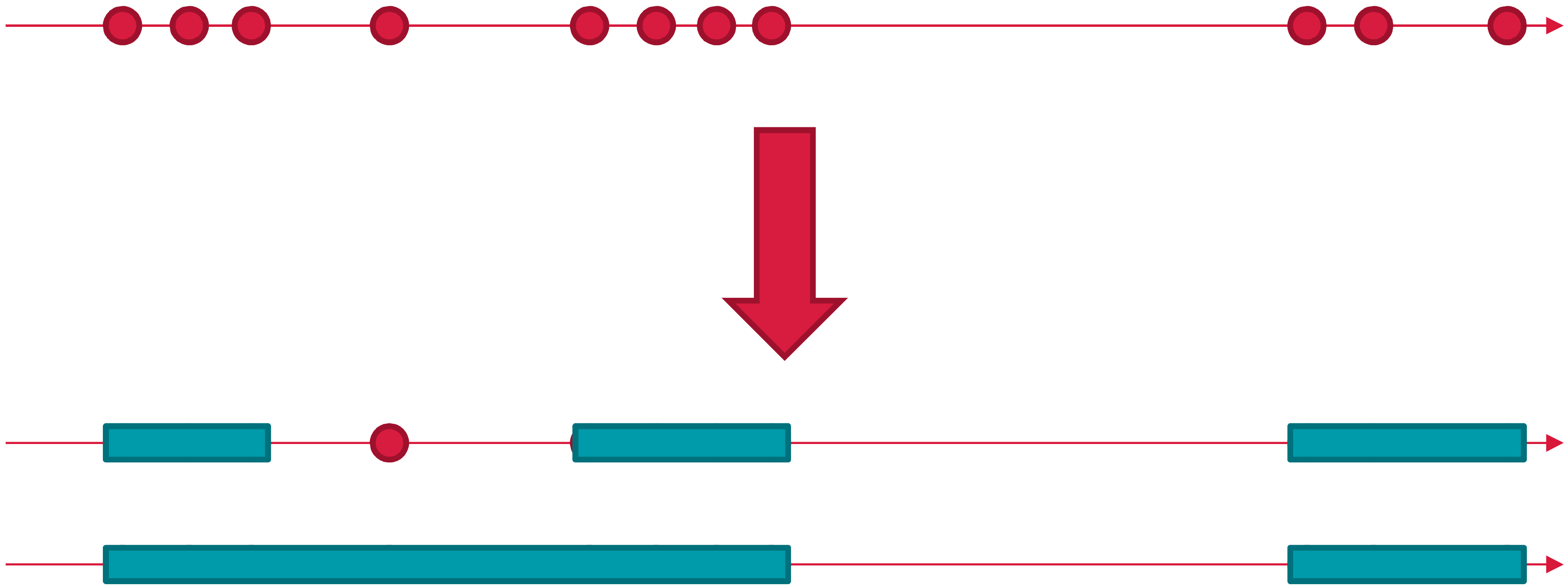
Time on task?



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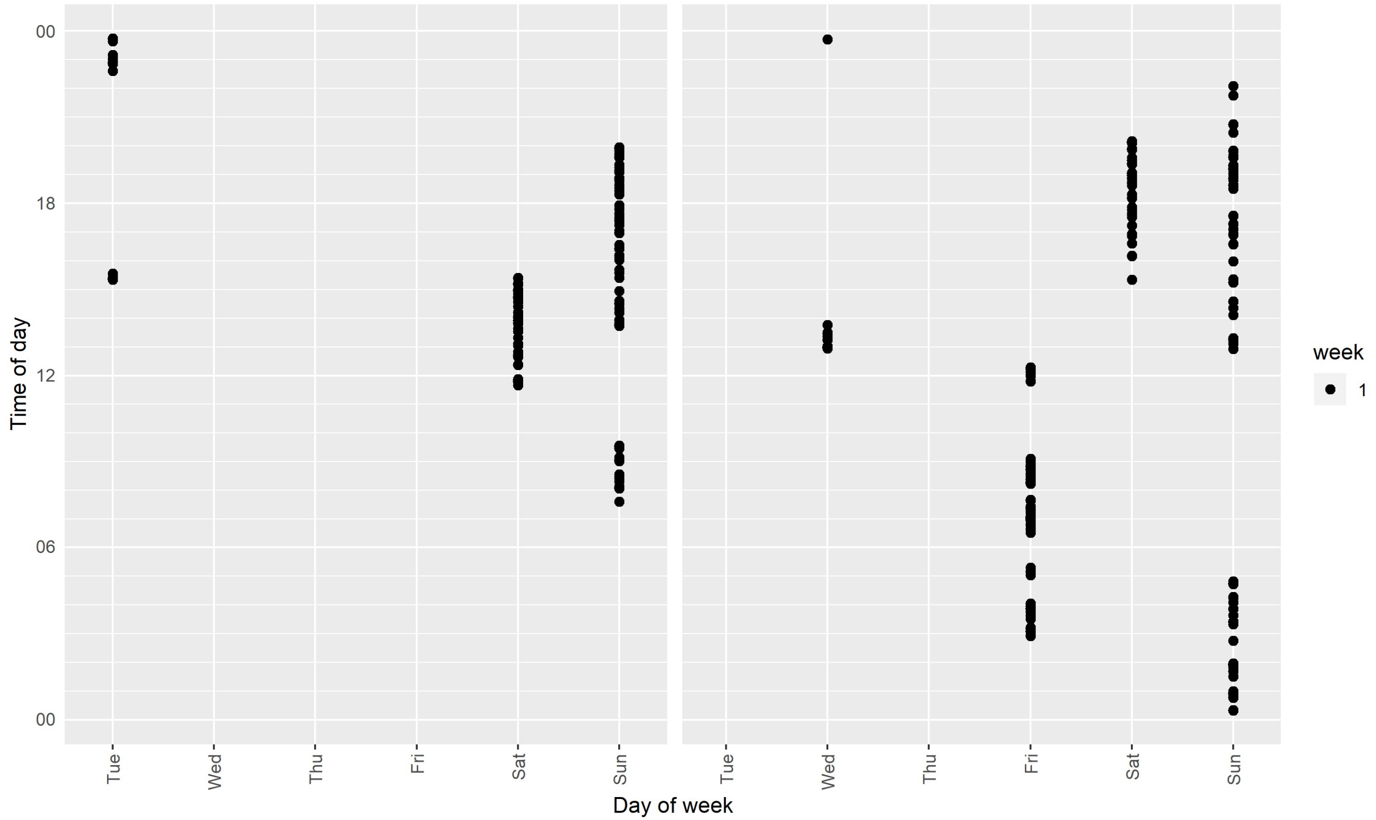
Example students



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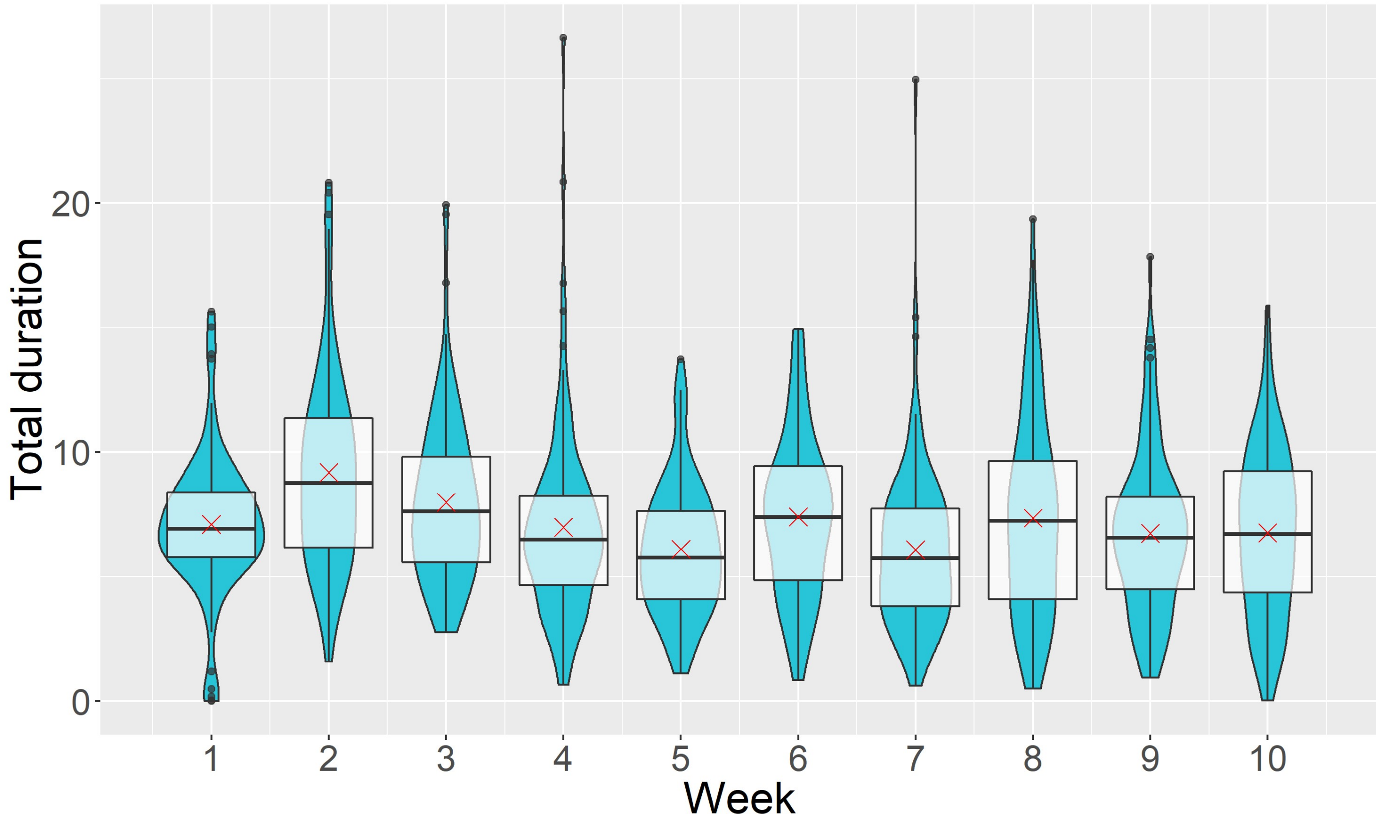
Hours spent each week



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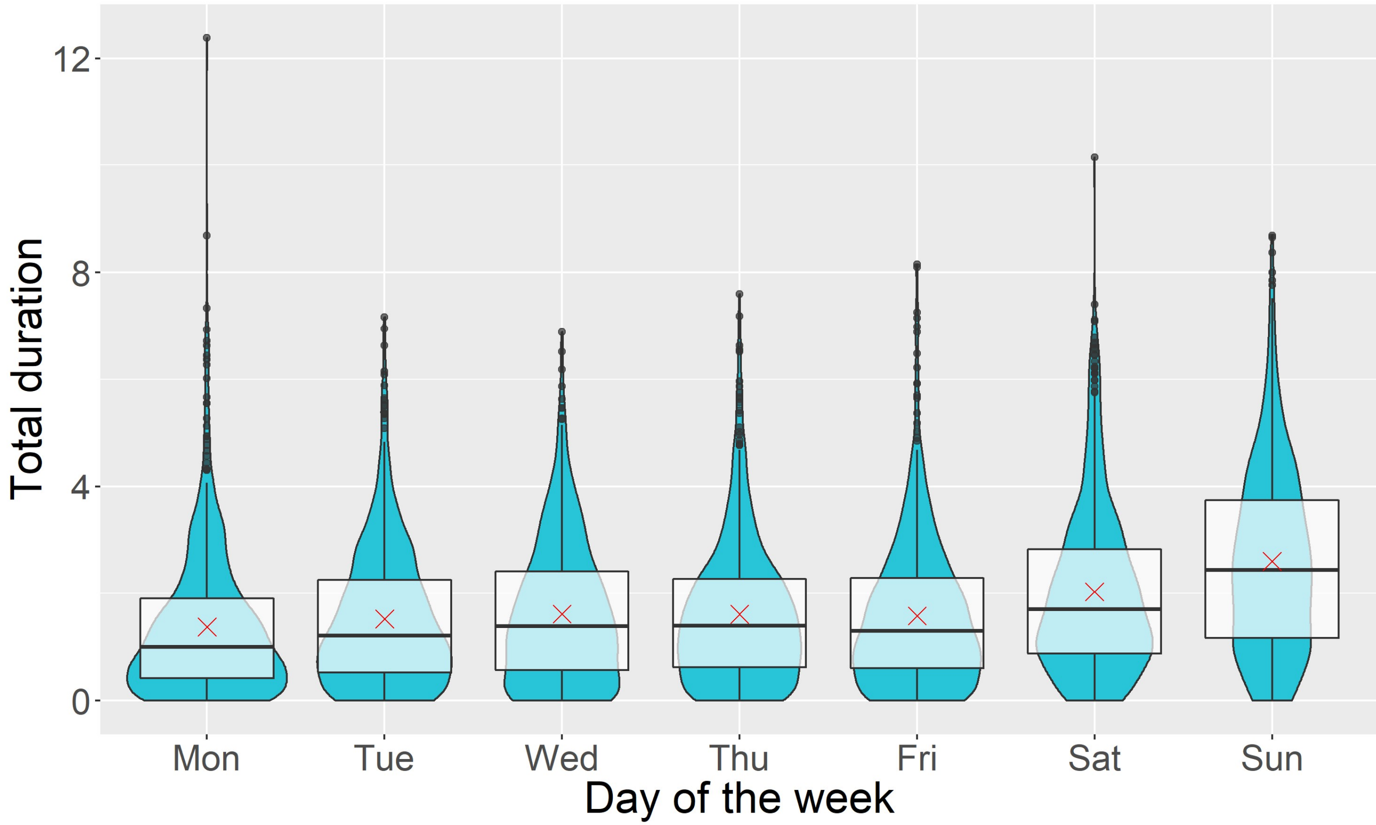
Activity by day of week



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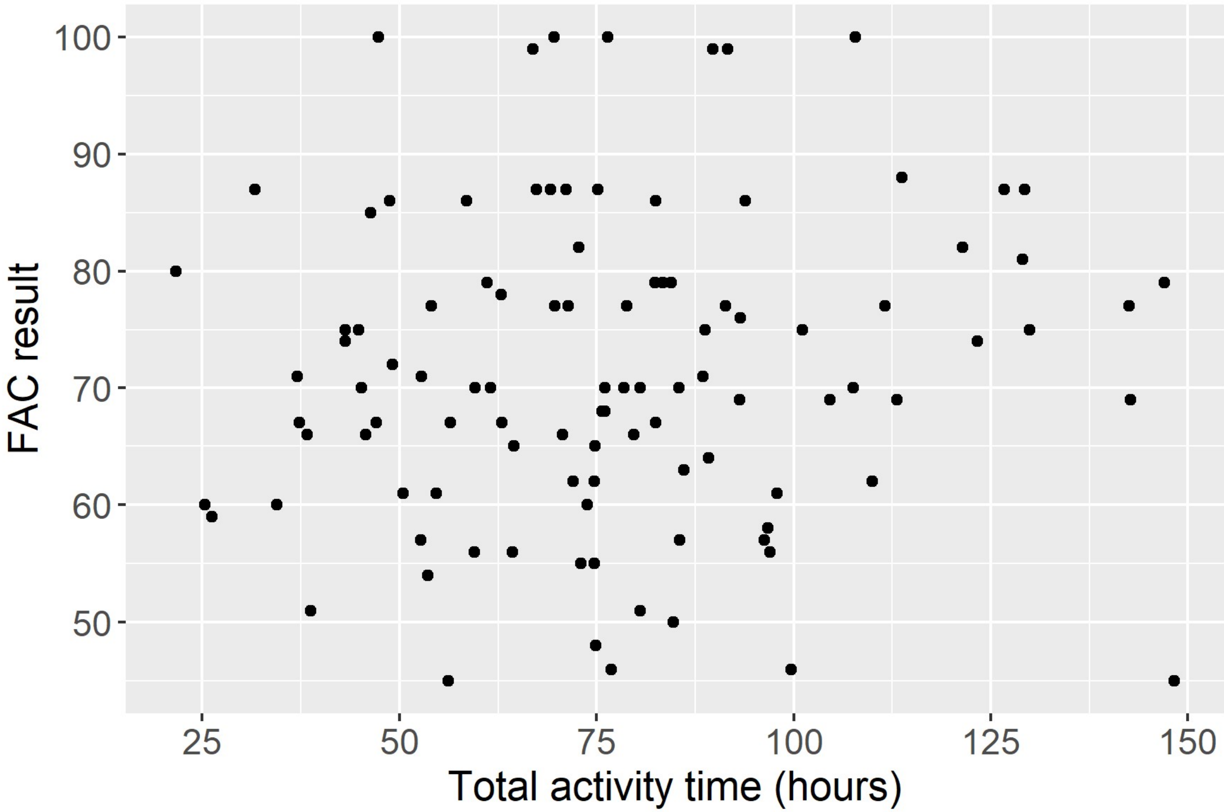
Activity versus results



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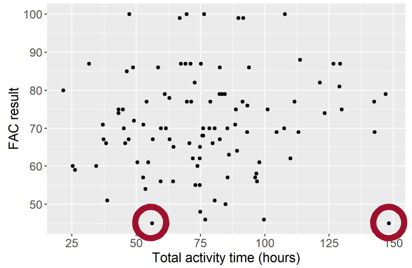
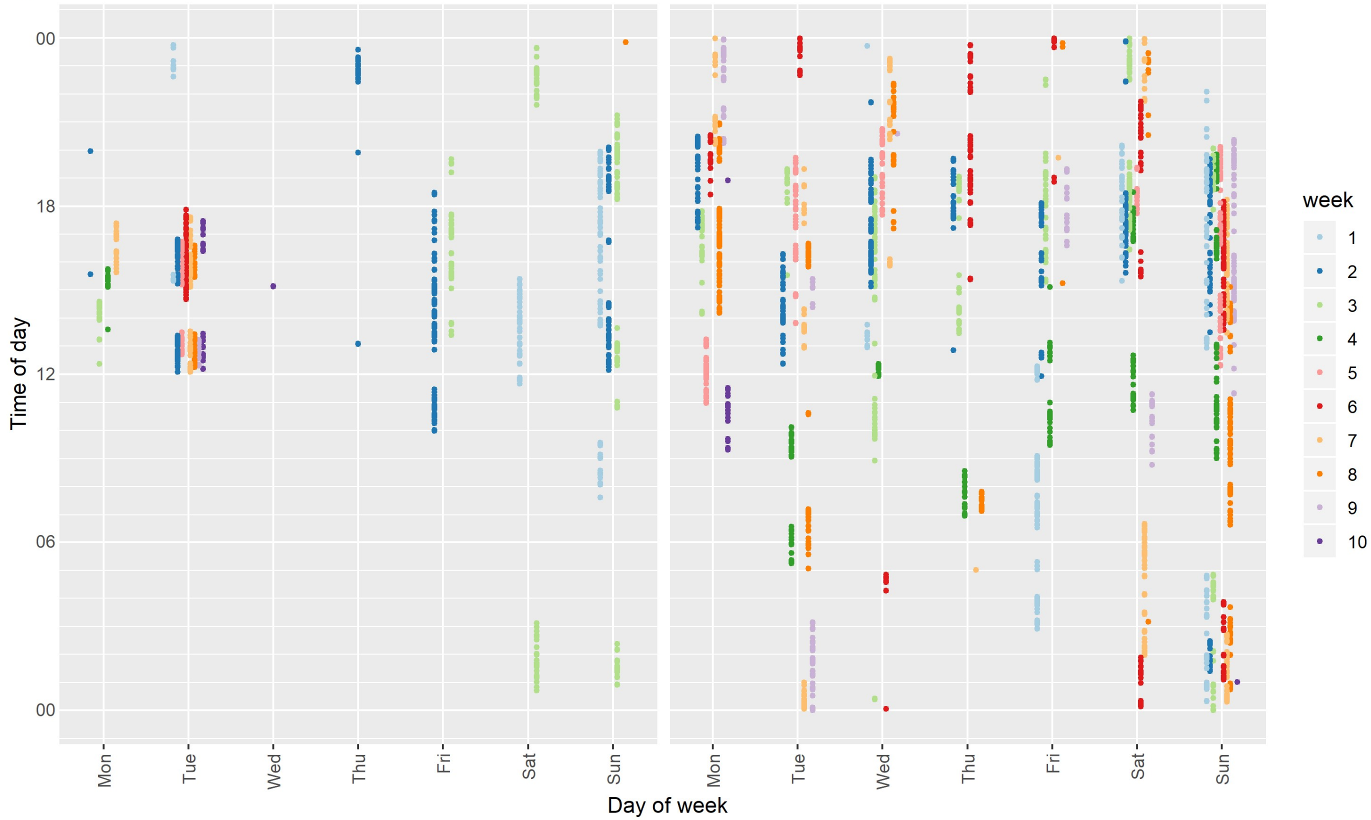


Example students



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Thoughts for next year

- Tweak the weekly test regime
- Make use of new STACK features
  - Interactive diagrams
  - Line-by-line reasoning
- Do more with autonomous learning groups

Conclusions

- FAC has been successful in boosting students' skills
- The design worked well, underpinned by STACK
- Lots of scope for further analysis…

Thank you!

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