Finding the Potential Response Trees in the Potential Response Forest

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Feedback literature

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Feedback Both Helps and Hinders Learning: The Causal Role of Prior Knowledge

Emily R. Fyfe and Bethany Rittle-Johnson
Vanderbilt University

Feedback can be a powerful learning tool, but its effects vary widely. Research has suggested that learners' prior knowledge may moderate the effects of feedback; however, no causal link has been established. In Experiment 1, we randomly assigned elementary schoolchildren (N = 108) to a condition based on a crossing of 2 factors: induced strategy knowledge (yes vs. no) and immediate, verification feedback (present vs. absent). Feedback had positive effects for children who were not taught a correct strategy, but negative effects for children with induced knowledge of a correct strategy. In Experiment 2, we induced strategy knowledge in all children (N = 101) and randomly assigned them to 1 of 3 conditions: no feedback, immediate correct-answer feedback, or summative correct-answer feedback. Again, feedback had negative effects relative to no feedback. Results provide evidence for a causal role of prior knowledge and indicate that minimal feedback can both help and hinder learning.

Keywords: feedback, problem solving, prior knowledge, mathematics learning

Feedback literature

- Hinders or helps?
- High vs low achievers
- Immediate vs delayed
- Correct vs worked solution vs none
- Increased over time

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- Misconceptions literature not incorporated

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$$\frac{1}{10} + \frac{2}{3} = \frac{3}{13}$$

(van Dooren et al., 2015)

$$(x+y)^2 = x^2 + y^2$$

(Kirshner & Awtry, 2004)

STACK feedback

- Potential Response Trees
  - Send student answers to CAS
  - Identify patterns of common errors
  - Provide personalised feedback

STACK feedback

Tidy question | Question tests & deployed versions

Enter your answers as fractions in lowest terms, or as integers.

1. $\frac{1}{3} + \frac{1}{6} =$ 2/9

Your last answer was interpreted as follows: $\frac{2}{9}$

Incorrect answer.
It looks like you simply added the numerators and the denominators. To add fractions you need to find a common denominator and then add the numerators.

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Potential Response Tree output

Potential Response Forest

- Sources of common student errors
  - Expert experience
  - Research literature
  - … and responses to STACK questions?

Pilot study

- Online STACK test with randomisation
- Foundation module ($N = 93$)
- Simple differentiation questions ($N_Q = 30$)

Differentiation Rule Tested	Number of Questions	Mean Score %	SD %
Single Function	8	87.23	11.55
Sum Rule	3	91.40	10.23
Second Derivative	4	67.20	6.56
Product Rule	5	64.94	9.20
Quotient Rule	5	46.02	13.82
Chain Rule	5	67.96	11.21

Example question

Find the following derivative:

$$\frac{d}{dz}[\cos(z) \cdot \cos(2 \cdot z)].$$

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$$\frac{d}{dz}(\cos(z) \cdot \cos(6 \cdot z)) = -6 \cdot \cos(z) \cdot \sin(6 \cdot z) - \sin(z) \cdot \cos(6 \cdot z)$$

$$\frac{d}{dz}(\cos(z) \cdot \cos(4 \cdot z)) = -4 \cdot \cos(z) \cdot \sin(4 \cdot z) - \sin(z) \cdot \cos(4 \cdot z)$$

$$\frac{d}{dz}(\cos(z) \cdot \cos(2 \cdot z)) = -2 \cdot \cos(z) \cdot \sin(2 \cdot z) - \sin(z) \cdot \cos(2 \cdot z)$$

$$\frac{d}{dz}(\cos(z) \cdot \cos(3 \cdot z)) = -3 \cdot \cos(z) \cdot \sin(3 \cdot z) - \sin(z) \cdot \cos(3 \cdot z)$$

Example question

Find the following derivative:

(z) · cos(2 · z)].

W
Response 20
ans1: -2*sin(2*z)*cos(z)-sin(z)*cos(2*z) [score]
ans1: -6*sin(6*z)*cos(z)-sin(z)*cos(6*z) [score]
ans1: (-sin(z))*(cos(3*z)+3*cos(z)) [score]
ans1: -sin(z)*cos(4*z)-4*cos(z)*sin(4*z) [score]
ans1: -6*cos(z)*sin(6*z)-sin(z)*cos(6*z) [score]
ans1: -2*cos*z*sin*2*z-(cos*2*z*sin*z) [score]
ans1: -6*cos(z)*sin(6*z)-cos(6*z)*sin(z) [score]
ans1: (-4*cos(z)*sin(4*z))-(sin(z)*cos(4*z)) [score]
ans1: (-4*cos(z)*sin(4*z))-(sin(z)*cos(4*z)) [score]
ans1: cos(z)*-6*sin(6*z)-sin(z)*cos(6*z) [score]
ans1: -z*sin*(z)*-4*sin(4*z) [score]
ans1: -6*sin(6*x)*cos(x) - sin(x)*cos(6*x) [score]
ans1: -(sin(z)*cos(6*z)+6*cos(z)*sin(6*z)) [score]

$) \cdot \sin(6 \cdot z) - \sin(z) \cdot \cos(6 \cdot z)$

$) \cdot \sin(4 \cdot z) - \sin(z) \cdot \cos(4 \cdot z)$

$) \cdot \sin(2 \cdot z) - \sin(z) \cdot \cos(2 \cdot z)$

$) \cdot \sin(3 \cdot z) - \sin(z) \cdot \cos(3 \cdot z)$

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14	$\frac{d^2(a-nx)}{dx^2}$	$\frac{d^2f}{dx^2} = \frac{df}{dx}$	90.9
20	$\frac{d}{dx}(\cos(n)\cos(nz))$	$\frac{d}{dx}f(x)g(x) = f'(x)g'(x)$	71.4
13	$\frac{d^2(e^{nx}-e^{-nx})}{dz^2}$	$\frac{d^2f}{dx^2} = \frac{df}{dx}$	65.7
12	$\frac{d^2(\cos(nx))}{dx^2}$	$\frac{d^2f}{dx^2} = \frac{df}{dx}$	63.3
15	$\frac{d^2(-nx+\frac{a}{x}+\frac{a}{x^3})}{dx^2}$	$\frac{d^2f}{dx^2} = \frac{df}{dx}$	62.9
8	Differentiate $\ln(nx)$	$\frac{d}{dx}\ln(nx) = \frac{1}{nx}$	44.4
18	Differentiate $x^{\frac{a}{b}}e^{-nx}$	$\frac{d}{dx}f(x)g(x) = f'(x)g'(x)$	42.9
26	Differentiate $(nx+a)^2$	$(x+y)^2 = x^2+y^2$	41.2
17	Differentiate $x^a\cos(nx)$	$\frac{d}{dx}f(x)g(x) = f'(x)g'(x)$	39.4
30	Differentiate $\sin^n(x)$	$\frac{d}{dx}\sin^n(x) = n\sin^n(x)$ and $\frac{d}{dx}\sin^n(x) = \cos^n(x)$	37.8
28	Differentiate $\frac{a}{(nx+b)^c}$	$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$	29
29	Differentiate $\sqrt{nx+a}$	$\sqrt{ab} = \sqrt{a}+\sqrt{b}$ and $(x+y)^a = x^a+y^a$	28.1
21	Differentiate $\frac{x}{a-x^2}$	$\frac{a}{b+c} = \frac{a}{b}+\frac{a}{c}$ and $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$	27
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25	$\frac{d}{dx}[\frac{\ln(x)}{x^{\frac{a}{b}}}]$	$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$	22
23	Differentiate $\frac{x^{\frac{a}{b}}+c}{x^{\frac{a}{b}}-x}$	$\frac{a}{b+c} = \frac{a}{b}+\frac{a}{c}$ and $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$	20.6
27	Differentiate $(a-x^b)^2$	$(x+y)^2 = x^2+y^2$	20
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Reproduced published findings

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Kirshner & Awtry (2004)

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Luneta & Makonye (2010)

We can decide what PRTs to programme…

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And which not to bother with…

We can theorise errors to decide how to feedback.

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Implications

- We can analyse catalogues of STACK responses to identify common errors and their prevalence.
- We can theorise common errors (slips, rule ignorance, overgeneralisation, visual salience, natural number bias, and so on).
- We can write more and better PRTs.
- We can contribute to the literature on misconceptions and the literature on feedback.

Thank you!

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Thank you to Michael Bennett.
This talk is based on his third year mathematics project.