Futures Hedging in Electricity Retailing

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Abstract: This paper is concerned with the risk management practices of an electricity retailer motivated by the Dutch electricity market. We examine the effectiveness of the existing base- and peak-load futures contracts as a risk management tool for the electricity retailers. We analytically characterize the retailer's optimal hedging policy as a function of the serial correlation of the prices and the demand profiles of its customers. We find that the retailer typically over-hedges in the futures market, and the over-hedging amount increases when both base- and peak-load contracts are used. Our findings indicate that although the existing contracts in the futures market are quite efficient to replicate the exposure from profiled customers, when industrial consumers and renewable generation are included to the retailer's portfolio, the effectiveness of such contracts decreases substantially. In our motivating example, hedging the risk of the profiled customers with base-load contracts, the firm may reduce the variance of its cash flows by 85.9%. In addition to the base-load contracts, including peak-load contracts into the hedging portfolio of the retailer increases the efficiency of hedging to 89.3%. However, when we consider the aggregate portfolio of the retailer including profiled customers, industrial consumers and renewable contracts, the efficiency of hedging through the existing futures contracts goes down as low as 32.8% during certain periods.

Keywords: Risk Management, Futures Markets, Electricity, Contracting

1. Introduction

Liberalization of the European electricity markets has started in 1990s by unbundling production and transmission services (Hogan, 2002). During the following two decades, reforms quickly transformed the European electricity markets into a competitive business environment. Together with the emergence of competitive markets, electricity retailing gained significant importance and electricity retailers became major players in energy supply chains (Hatami, et al. 2009). Although the operations of electricity retailers received some academic attention (see e.g., Gabriel et al. 2004; and Yusta et al. 2005), according to our best knowledge, the business models of electricity retailers have not yet been fully explored. In this paper, we provide an optimization model for the operations of electricity retailers and develop a framework to manage their price and volume risks using the electricity futures market.

An electricity retailer provides an intermediation service between the users and producers of electricity through the wholesale markets. The retailer exchanges a physical electricity flow for a cash flow in either direction. The direction of the exchange depends on whether the customer is a consumer or producer. Specifically, the customer buys (sells) electricity from (to) the retailer in exchange for a cash flow. Then the retailer procures (sells) this electricity from (to) the wholesale market (the customers), again in exchange for a cash flow (see Figure 1).

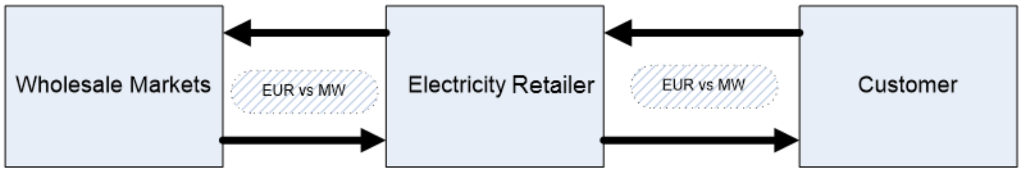


Figure 1: The business model of an electricity retailer

While offering this gateway function the retailer first enters into contracts with its customers for the physical delivery of electricity. Then, the retailer procures/sells electricity in the short-term spot and long-term wholesale markets to fulfill its physical delivery obligations. Retailers serve an important intermediation role as the customers themselves cannot handle this activity in a cost-efficient way due to lack of scale economies, standardized contract sizes, informational problems, etc. More importantly, except for very large clients, customers cannot trade electricity by themselves as they need a physical connection to the grid and a trading permit. In addition, financial and physical contracts are traded with high liquidity only for standard volumes and maturities. For small customers, trading these contracts involve high transaction costs as well as high basis risk. Customers, therefore, prefer to procure/sell electricity through the retailers who may form a portfolio of customers, pool the demand and supply, and efficiently trade in the spot and wholesale markets. This is the central theme of the retailer's business model.

The retailer earns the spread between the cash flows from the customers and the wholesale markets. The margins, however, are quite thin due to the highly competitive nature of the electricity markets. Maintaining this margin against the fluctuating electricity prices is a key business competency of these firms. Retailers typically enter into fixed-price contracts with their customers as the customers demand protection against price risk. Fixed-price contracts involve buying or selling electricity at a fixed price in the future. The uncovered physical positions of the retailer are settled in the day-ahead spot market which is highly volatile. Therefore, the realized margin of the retailer is uncertain and can easily turn negative if the prices move in the wrong direction in the spot market. Fixed price contracts, depending on their volumetric risk, have two common forms: (1) small- and large-use profiled contracts and (2) wind and solar contracts.

Profiled customers mainly consist of small businesses, retail companies and households. The demand of these customers is fairly stable and exhibits daily and weekly cycles. Retailers work closely with these customer groups and form daily and yearly demand profiles as illustrated in Figure 2. For example, in the Netherlands, household consumption peaks around 19h00 and dips during early morning. Also, the consumption is at its minimum level during the summer and peaks in the winter. The retailer faces price risk as well as a relatively small volume risk in these contracts since the consumption profile is also uncertain.

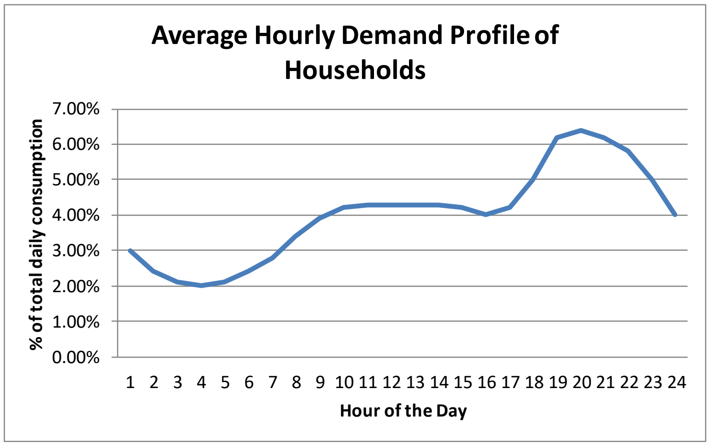


Figure 2: Daily demand profile of household customers

The other major set of assets in the retailer's portfolio are **wind contracts**. These contracts involve buying the entire production volume of the customers at a fixed price over a certain time period. In addition to price risk, these contracts also involve a significant amount of volume risk as wind generation output is highly unpredictable. Given that these contracts are typically signed couple of months in advance of physical delivery, the coefficient of variation of the output in these cases reaches up to 80% in our data set. For all the fixed-price contracts, the relevant futures price in the market is adjusted by a mark-up to obtain the fixed contract sales price.

Overall, the retailer's contract portfolio includes physical short (demand) and long (supply) positions in electricity commodity, which partially offset each other creating a natural hedge. For example, the supply from wind contracts can be used to meet the demand of profiled customers. So, the net exposure of the retailer's portfolio needs to be settled in the spot and wholesale markets.

We assume that the retailers do not speculate in the market via intentionally carrying open positions to make profits from pure financial trading. Instead, as explained above, retailers focus on making money through providing an intermediation service between the customers and the wholesale markets. Eliminating price risk enables them to better perform this service and price their contracts more competitively (i.e., determine their mark-ups.). So, we assume that they are interested in hedging their price risk using the long-term wholesale markets. Ideally, a perfect hedge would precisely offset each short and long positions for each hour of the day through the entire duration of the exposure. A perfect hedge would result in zero risk and a fixed operating margin. However, this is not possible due to two sources of basis risk. First, there exists volumetric risk in the contracts which cannot be replicated in the financial markets. Second, the market only offers base- and peak-load futures contracts which deliver electricity with a standard flat profile. In addition, more complex derivatives such as electricity options are not traded at significant volumes.

As it is very well established in the finance literature, corporate level risk management is motivated by the goal of maximizing firm value. In the presence of capital market frictions, hedging enhances firm value by reducing the firm's exposure to those frictions such as costly financial distress and bankruptcy or by reducing taxes (Smith and Stulz, 1984; Froot et al., 1993; Stulz, 1996; Jin and Jorion, 2000 and Brown and Toft 2002). In the presence of such frictions hedging or risk management is not necessarily a zero-sum game. These frictions result in the breakdown of the Modigliani and Miller (1958) framework and requires the joint optimization of hedging (financial) and operating policies. In this paper, we consider the objective of cash flow variance reduction as a proxy for value-maximization since reducing variance limits the firm's exposure to deadweight costs associated with capital market frictions improving firm value.

Motivated by the operations of a small Dutch electricity retailer, we will consider two types of derivative contracts for hedging. The first contract type is a '**Dutch Power Base Load**' future (DPBL). This future enables the physical delivery of 1 MWh of electricity energy to the Dutch high voltage grid, from 00h00 on the first day of the delivery period until 24h00 on the last day of the delivery period. The second contract is called the '**Dutch Power Peak Load** (8-20) (DPPL8-20), which delivers 1 MWh of electricity energy from 08h00 to 20h00 on all weekdays (including public holidays), during the delivery period. These contracts are traded for different durations and a fixed number of months, quarters and calendars forward. Contracts expire two days before the physical delivery and the delivery is served in the form of a day-ahead spot contract via the clearing house. Both of the contracts are traded in significant liquidity for one year ahead.

As suggested by the motivating retailer's portfolio of physical contracts, we perform our analysis in multiple steps. First, we consider a deterministic demand profile and develop a hedge using the base-load futures only. Next, we include the peak-load contracts to the optimal hedging portfolio of the retailer. In the third step, we extend our analysis with stochastic profiles and explore the optimal hedging portfolio under volumetric risk. Our findings can be summarized as follows:

- We find that the electricity retailer typically over-hedges in the futures market (i.e., the optimal net financial position in the futures market exceeds the expected net physical position in the retail market). The over-hedging amount increases during the more windy periods when there is more wind generation, volumetric risk and increased possibility of price spikes.
- We calculate that hedging the risk of the profiled customers with base-load contracts, the firm may reduce the variance of its cash flows by 85.9%. In addition to the base-load contracts, including peak-load contracts into the hedging portfolio of the retailer increases the efficiency of hedging only to 89.3%.
- When we consider the aggregate portfolio of the retailer including profiled customers, industrial consumers and wind contracts, the efficiency of hedging through the existing futures contracts in

the market substantially decreases. The efficiency goes down as low as 32.8% during certain periods. We also note that peak-load contracts have marginal value for hedging against exposure from profiled customers and renewables.

- Our work also reveals a possible self-hedging behavior embedded in the load profiles of the retailer. As the load level of a particular delivery interval increases, this does not necessarily lead to a higher hedging level in the futures market. A possible negative correlation of that particular interval's load with the rest of the delivery intervals serves as a self-hedging mechanism reducing the firm's reliance on the financial hedge.

The rest of the paper is organized as follows. In Section 2, we provide a review of the related literature. Mathematical model is presented in Section 3 followed by the price and demand estimation models in Section 4. In Section 5, we provide the results of our numerical analysis based on the data from the motivating Dutch electricity retailer. We conclude with a discussion of our results and future research directions in Section 6.

2. Literature Review

Risk management in electricity markets is more complex than in other commodity markets due to the non-storable nature of the electricity power. Uncertain and non-stationary load and generation profiles of market participants further exacerbates the basis risk in the electricity markets complicating risk management activities. Real-time delivery coupled with the lack of real-time metering and the need to constantly balance supply and demand are the most important issues when hedging in electricity markets (Dahlgren, 2003).

Together with the deregulation of electricity markets in 1990's, research on electricity risk management proliferated. One of the earliest works, Bjorgan et al. (1999), examine how a generator can utilize futures markets to minimize the variance of its cash flows from sales. Authors develop an efficient frontier of risk and return and examine the impact of resource constraints on the firm's efficient choices of production and hedging. Follow up research further examines the use and hedging role of electricity futures contracts in different business contexts. Bystrom (2003) explores the role of futures in the Nord Pool market to hedge the short-term positions in the spot market. The author determines the minimum variance hedge ratio and estimates it using different statistical methods that are central for the effectiveness of futures hedging (Zanotti et al. 2010). Tanlapco et al. (2002) find that direct hedging outperforms cross-hedging, even though the relationship between spot and futures prices is weak in electricity markets. Huisman et al. (2009) considers the composition of an efficient portfolio including both peak and off-peak futures contracts.

Another related stream of literature focuses on the efficiency and design of futures contracts for risk management in electricity markets. Collins (2002) argues that the electric power is different from other commodities due to its non-storable nature resulting in a weak relationship between the futures and spot prices during the delivery period. The author suggests index delivery contracts, such that the contract price during the delivery period is indexed to actual daily spot trades at some point in the future. He finds that index delivery contracts may significantly reduce basis risk and improve the efficiency of hedging in electricity markets. Hanly et al. (2018) examine the efficacy of hedging in European electricity futures markets (Nordpool, APXUK, and Phelix) and find that the futures markets provide little risk protection to market participants. Authors' findings agree with Collins (2002) and raise questions about the value and the function of electricity futures as risk management tools in practice. Deng and Oren (2006) also point out the challenges in electricity derivatives about the breadth, liquidity and the use of such contracts as a hedging means.

In this manuscript, we focus on the use of existing liquid base- and peak-load futures contracts in the market by an electricity retailer. Different from the existing literature, an electricity retailer can take both physical long and short positions depending on the nature of its trade in the retail market. In addition, the retailer's physical positions may present significant volumetric risk and non-stationarity over time. We focus on how this non-stationarity and volumetric risk in the physical positions coupled with price risk can be managed in the futures market. We find that the correlation of volumetric risk and price risk over time plays a key role in the efficiency of the futures contract in reducing cash flow variability. Similar to Hanly et al. (2018) and Collins (2002), we also aim to understand the drivers of hedging efficiency in the electricity futures markets. Using real load data from a Dutch electricity retailer, we find that base- and peak-load futures are very effective in hedging the exposures due to profiled customers. However, the hedging efficiency quickly deteriorates when wind contracts are included to the retailer's portfolio.

Similar to our work, Boroumand et al. (2015) also consider the risk management activities of an electricity retailer based on electricity market data from 2001 to 2011 for the French market. They find that optimal hedging portfolio significantly depends on the hours of the day. Unlike our work, authors focus on short-term hedging of spot exposures in the intra-day market. Our focus is on long-term hedging as the retailers usually engage in physical exposures at least couple of months in advance of physical delivery.

It is well known that in the presence of both price and volumetric risk, options can be an effective way of managing risk. Goel and Tanrisever (2011) show that hedging with options can create more value as compared to using only futures in the presence of logistical frictions in commodity markets. The role of options in electricity risk management has also been explored. Gedra (1994) examines the role of put and call options when the market participants have the flexibility to change their generation and consumption levels. Pineda and Conejo (2012, 2013) show that electricity options may provide protection for the generation units against fluctuations in electricity prices and unexpected unit failures. Nikkinen and Rothovius (2019) study the trading dynamics in electricity options in NASDAQ OMX Commodities and observe both hedging and speculative behaviors. In practice, however, low trade volumes and large bid-ask spreads in electricity options limit the use of such derivatives as a hedging tool. Hence, in this manuscript we focus on futures contracts in the market, which are sufficiently liquid for practical purposes. In a recent paper, Roncoroni and Brik (2017) develop a customized contract to hedge financial exposure to both market and idiosyncratic risks. The authors suggest that their custom hedge can be used to benchmark the risk management solutions in practice.

Previous research have also examined the hedging problem for load serving entities (LSEs) which serve to regulated customers with a stochastic load profile. Oum et al. (2006) consider an expected utility maximizing LSE, and derive an optimal hedging portfolio including forward and option contracts. Authors assume that the contracts are priced under the risk-neutral measure while the firm uses the true measure to calculate the expected utility. Oum and Oren (2009) extend Oum et al. (2006) by providing a VaR-constained hedging problem. In the presence of both price and volumetric risk, VaR-constrained optimization problems have also been considered by Woo et al. (2004), Wagner (2001) and Kleindorfer and Li (2005) for hedging the risks of LSEs. More recently, Yang et al. (2015) suggest indirect load control via risk-limiting contracts to manage volumetric risk in consumer profiles and Yu et al. (2018) consider consumer's demand response capabilities. Although our work shares some similarities with risk management in LSEs, risk management for retailers is a broader problem as it considers a wider range of both sales and procurement contracts leading to a richer problem setting.

3. Mathematical Model

During its daily operations the retailer engages in physical procurement and sales of electricity through fixed price contracts which provide a positive expected return. The objective of the firm is to minimize the volatility of cash flows from physical trade using the electricity futures contracts. In Section 3.1, we first consider a retailer portfolio with deterministic profiles, then we gradually add real-life complexities including: stochastic profiles, multiple futures contracts and hedging periods. Before we explore those cases, we start with explaining the general business model of an electricity retailer and some terminology with the market. Below, we list and clearly define some terminology used in our manuscript and also in practice during physical trading of electricity:

Bilateral contract (order):	An agreement between the electricity retailer and the customer for the delivery or acceptance of physical electricity commodity, signed on a particular *agreement date*, for a certain *delivery price* and *delivery period*. We will simply refer to bilateral contracts as a customer order.
Agreement date:	The date that the bilateral contract is signed between the parties.
Delivery period:	In practice, a delivery period is usually a collection of multiple consecutive months, which specifies the time interval during which the electricity will be delivered by the short party.
Delivery (settlement) intervals:	This is the smallest time unit that the spot day-ahead-market clears in practice. This is usually one hour (or less) for most electricity markets. For example, if the delivery period of a contract is June 2022, then it means the delivery period includes 30 x 24 = 720 hourly delivery intervals.
Agreement (contract) price:	A fixed price per MWh that the trade will take place during the delivery period. For all the fixed-price contracts, the relevant futures price in the market is adjusted by a mark-up to obtain the fixed contract sales price.
Demand (supply) profile of a customer order:	Distribution of physical demand (or supply) over the delivery period specified in the bilateral contract. In our case, there are 4 different clusters of customers depending on the nature of their demand profiles: (*i*) profiled customers, (*ii*) wind customers, (*iii*) industrial customers and (*iv*) other producers.

Table 1: Summary of Terminology

We now illustrate the operations of the retailer with an example. Suppose the retailer starts operations at the beginning of January 2022 with an empty physical portfolio. In practice, planning horizon typically spans one to two years into the future. Consider the following 5 orders (contracts) that are received during January 2022 (recall that each order is a bilateral agreement):

- **Order A**: On 2-Jan-2022, the retailer receives a profiled small business contract for delivery of electricity during June 2022.
- **Order B**: On 5-Jan-2022, the retailer receives an order with a specific profile from a wind customer for accepting delivery of electricity during the second quarter of 2022.
- **Order C**: On 7-Jan-2022, a profiled industrial consumer order is received for delivery during March 2022.
- **Order D**: On 20-Jan-2022, an order from a household customer with a specific profile is received for delivery during the third quarter of 2022.
- **Order E**: On 22-Jan-2022, another order is received with a specific profile from a household customer for delivery from February to September 2022.

The timeline for these orders and the planning period are illustrated in Figure 3:

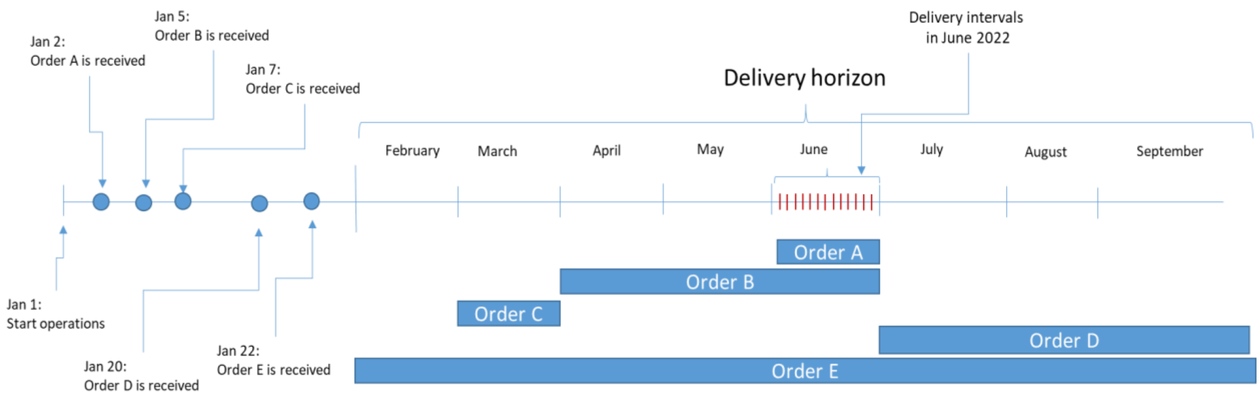


Figure 3: An illustration of the timeline of contract agreement dates and associated delivery periods

In practice, retailers myopically manage the risk created by each contract over time. That is, on January 2nd, the firm takes a counter-position in the futures market to hedge the risk of Order A. On January 5th, the firm observes Order B and then re-optimizes its hedging position given the new net physical position.

Now, we take a closer look at Order A. On 2-Jan-2022, the firm engages in Order A for the delivery of a specific profile during the delivery period June 2022 for a fixed price P. The demand profile specifies the distribution of demand over settlement intervals in June 2022. Contracts are settled on an hourly basis meaning that there are a total of 30 x 24 = 720 settlement intervals for Order A. Demand profiles usually present daily and weekly seasonality. During the day, the demand peaks at around 19h00 and during the weekends demand is usually lower. If there are special days such as holidays during the delivery period, this needs to be considered as well. In Figure 4, we provide the daily profile of Order A for the weekdays. This profile repeats itself during the weekdays and weekends are slightly different. The collection of D_is over all the settlement intervals constitutes the demand profile of the Order A. Note that, depending on the customer class, D_i for a given interval i, may be uncertain.

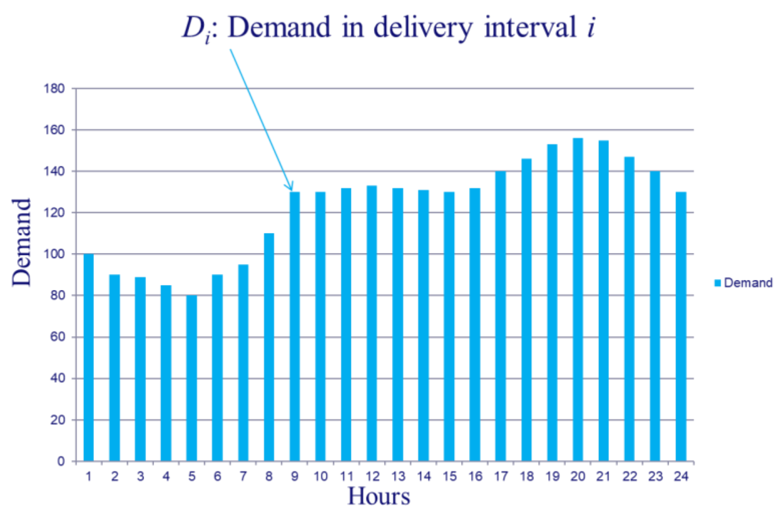


Figure 4: An example daily demand profile for a customer order

On January 2nd, the retailer takes a counter-position in the futures market to offset the risk of the physical exposure. However, for the very reasons discussed in the introduction, the exact demand profile for Order A is not traded in the wholesale markets. Instead, in most markets only base- and peak-load contracts are available, leading to a basis risk in hedging. Suppose the firm buys $Q = 120$ units of base-load futures contract with the same maturity as Order A. Then the remaining exposure of the firm, which will be settled in the spot market during the delivery period, is illustrated in Figure 5 for a particular delivery day in June 2022.

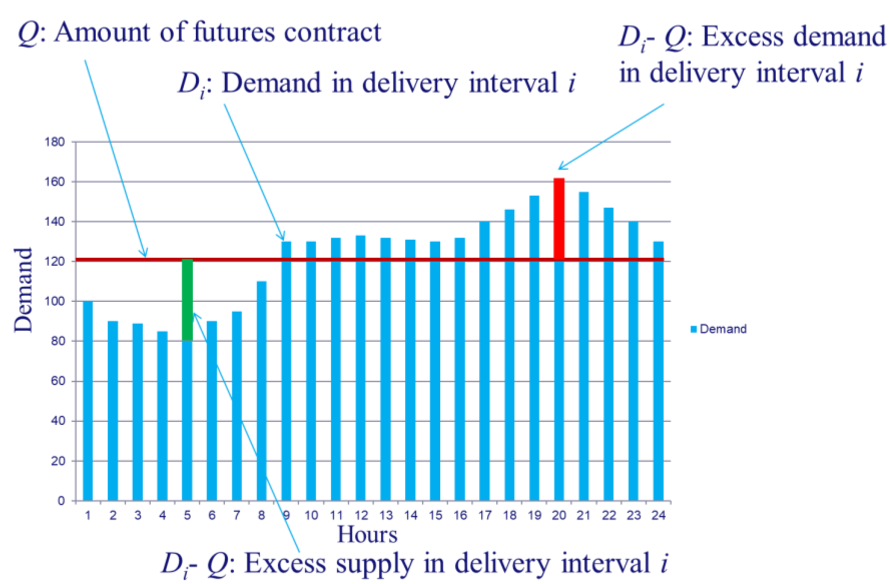


Figure 5: Daily demand profile and the net exposures of the retailer after hedging

Note that the figure shows the demand profile only for one day, the full profile is a collection of similar figures for all the 30 days of June 2022. The blue columns show the volume of demand that is agreed to be met by the retailer, i.e., the physical short positions for each hour of the weekday under consideration in June. The horizontal red line shows the long position taken by the firm to hedge the risk. Q is the volume of the forward hedge. The hedge contract delivers a flat volume over the delivery period, yet the physical exposures fluctuate as dictated by the demand profile. So, for certain delivery intervals, the net exposure of the firm is negative and for some intervals it is positive. These excess and shortage amounts are settled in the day ahead market. The green column on hour 5 shows the excess supply in hour 5 and the red column shows the excess demand in hour 20 which will be settled in the day-ahead market. The price in the day-ahead market is uncertain and is the main driver of the risk of the retailer. By choosing Q the retailer aims to myopically minimize exposure to day-head market risk due to Order A. The problem with peak-load contracts is similar, and the firm is now able to better replicate the physical exposure with two types of contracts. In the next section, we discuss how base- and peak-load contracts are used to hedge against price risk.

3.1 Hedging with Base-load Futures under Deterministic Load Profiles

In this section, we illustrate the optimal hedging position taken by the firm, given a physical exposure of a specific type. The firm may repeat these solutions over time myopically as new orders are observed. We assume that the retailer has a physical exposure at a future *delivery period T*. This may either be a long (agreement to buy electricity at a fixed price) or a short (agreement to deliver electricity at a fixed price) exposure. The delivery period *T* consists of *I* delivery intervals (e.g. hours) where the physical exposure is described by a load profile, D_i, which gives the forecasted demand/load for the delivery interval i =1, …, *I* in period *T*. For instance, the monthly period for June includes *I*=30 $\times$ 24 = 720 hourly delivery intervals. The retailer wants to hedge its net exposure for delivery period *T* with futures contracts. These contracts deliver a constant amount of electricity, at a fixed price $F_{0,T}$, for each delivery interval in period *T*. We assume that the risk-free rate is zero. The timeline of the events and cash flows to the firm are described as follows:

At *t* = 0: The retailer observes its physical exposure for delivery period *T*, i.e., its net load profile D_i, for i =1, …, *I*. In this section, we assume that the load profile is deterministic, and we later extend the analysis to the stochastic load case. The retailer decides the amount of base-load futures, Q, to buy/sell which delivers a constant amount Q during each interval in period *T*.

At the beginning of the delivery period *t* = *T*: The futures contract goes into physical delivery and the firm pays for the futures procurement, i.e., $I \times Q \times F_{0,T}$. In each interval *i*, the retailer receives Q units from the futures contract and the shortage/excess (after meeting the physical demand) is settled in the day-ahead spot market at the prevailing spot price $\tilde{S}_i$, i.e., the firm receives/pays $(Q - D_i)\tilde{S}_i$ in the spot market, for *i*

=1, …, I. Hence, $\tilde{S}_i$ is unknown (random) when the retailer makes hedging decisions at t=0. For ease of exposition, we assume that there is no default risk for the futures contracts, and they are settled at the beginning of the delivery period. This situation is identical to daily mark-to-market when the risk-free rate is zero.

At the end of the delivery period $t = T$: The retailer collects the revenues from the customer, i.e., $\sum_{i=1}^{I} D_i P$.

Then the net cash flows of the retailer for period T is equal to:

$$\tilde{X} = \sum_{i=1}^{I} (D_i P + (Q - D_i)\tilde{S}_i - QF_{0,T}),$$

and the expected value of the total cash flow is:

$$E[\tilde{X}] = \sum_{i=1}^{I} (E[\tilde{S}_i] - F_{0,T})Q + \sum_{i=1}^{I} (P - E[\tilde{S}_i])D_i. \quad (1)$$

The first term in (1) represents the base-load risk premium in the market. The literature has mixed findings about the size and the sign of risk premium in electricity markets. For very short-term contracts Viehmann (2011), and Haugom and Ullrich (2012) show that the base-load risk premiums in EEX and PMJ are not statistically significant. Handika and Trueck (2013) and Dolores and Meneu (2010) provide mixed evidence about the sign and the statistical significance of risk premium in Australian and Spanish electricity markets, respectively. In this paper, we aim to explore the effectiveness of the existing futures contract in managing cash flow volatility under different customer portfolios. While exploring this objective, we also show the effect of hedging on the expected cash flows possibly arising due to non-zero risk premium in the market.

The level of futures hedging affects the volatility of the cash flows, which conditions the firm's exposure to capital market frictions. We do not model those frictions explicitly, however, we aim to minimize the variance of the cash flows so as to reduce the firm's exposure to such frictions and deadweight financial losses. Also note that, in (1) a positive spread $P - E[\tilde{S}_i]$ implies a short exposure (demand) for interval i and a negative spread implies a long exposure (supply). Next, we state the variance of the cash flows:

$$Var(\tilde{X}) = \sum_{i,j}^{I} (Q - D_i)(Q - D_j)Cov(\tilde{S}_i, \tilde{S}_j). \quad (2)$$

The retailer aims to minimize the variance (2) by deciding its hedging position for each order myopically before seeing the future orders with only distributional knowledge about spot prices at the delivery period.

After the spot prices are realized, the retailer buys (sells) the excess demand (energy) in the day-ahead market. Our model is a *stochastic program with simple recourse*, where in the first stage (at the time when the contract is signed) the retailer decides on its hedging position and in the second stage (at the time when the contract goes to delivery), the retailer realizes the randomness in spot prices (and demand if the profile is stochastic) and acts accordingly. Notice that, in our model, the actions of the retailer in the second stage are completely determined by the randomness and there is no optimization at this stage, resulting in a simple recourse model instead of a two-stage stochastic program.[1]

Following theorem describes the variance-minimizing base-load futures position for the retailer. The proofs of all the theorems in this section is postponed to Appendix A for ease of reading.

Theorem 1. *Under a deterministic load profile, the variance-minimizing base-load futures quantity is given by*

$$Q^* = \frac{\sum_{i=1}^{I}\sum_{j=1}^{I} D_i Cov\left(\tilde{S}_i, \tilde{S}_j\right)}{var\left(\sum_{i=1}^{I}\tilde{S}_i\right)}. \tag{3}$$

The variance-minimizing base-load hedging quantity Q^* is equal to a sum of weighted covariance terms with demand loads, divided by the sum of the covariance terms. It is important to note that if the demand profile is constant, i.e., $D_i = c$ for $i = 1, \ldots, I$, then (3) provides a perfect hedge which completely eliminates the price risk. The optimal hedge is determined by the serial correlation of the prices and the spread of the load profile. In particular, an increase in the load of an interval i whose prices are highly correlated with the other intervals has a more pronounced impact on the optimal hedge amount than an increase in the demand in an interval with low price correlation (observe that $\frac{\partial Q^*}{\partial D_i} = \frac{\sum_{j=1}^{I} Cov(\tilde{S}_i, \tilde{S}_j)}{var\left(\sum_{i=1}^{I}\tilde{S}_i\right)}$).

Corollary 1. *Suppose $Q > 0$ and $D_i > 0, \forall i = 1, \ldots, I$. Then,*

(i) The firm's hedging position Q increases with D_i, if $\sum_{j=1}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right) > 0$.

(ii) The firm's hedging position Q decreases with D_i if $\sum_{j=1}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right) < 0$.

Corollary 1 underlines a self-hedging behavior embedded in the load profile of the retailer. As the load level of an interval increases, this does not necessarily lead to a higher hedging level in the futures market. For example, if the total correlation of the price of a delivery interval i, is negative with the rest of the intervals, i.e., $\sum_{j=1}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right) < 0$, then the firm hedges less in the financial market as the load of interval

[1] We note that once a futures contract goes into delivery, trading of this contract ceases in the futures market. So for all practical purposes, it is not possible to update the hedging decision for a particular delivery period during that delivery period. In line with our practical observations, we also assume that demand profiles of the customer orders are independent.

i increases. The negative correlation of interval i with the rest of the intervals serves as a self-hedging mechanism such that the higher load profiles in this period creates a negatively correlated cash flow with the other time intervals reducing the firm's reliance on the financial hedge.

3.2 Hedging with Base-load and Peak-load Futures under Deterministic Demand

In addition to base-load contracts, it is also possible to purchase peak-load contracts that deliver a fixed quantity (Q_P) during peak hours. Peak hours are defined as the hours between 08h00 and 21h00. These contracts only deliver during weekdays. Intuitively, peak-load contracts provide the firm with more flexibility to replicate its demand profile with the futures contracts, and hence improve the hedging effectiveness. Indeed, peak-load is apparent in most demand profiles.

We define set $\boldsymbol{P}$ as the time intervals during which the peak-load contracts deliver electricity. Given the base- and peak-load contract amounts, Q_B and Q_P, the total futures position of the retailer in interval i is defined as:

$$Q_i = Q_B + \alpha_i^P Q_P$$

where $\alpha_i^P = 1$ if $i \in \boldsymbol{P}$ and 0 otherwise. Then, the total cash flows in period T is given by:

$$\tilde{X} = \sum_{i=1}^{I} [Q_B(\tilde{S}_i - F_{0,T}^B) + \alpha_i^P Q_P(\tilde{S}_i - F_{0,T}^P) + D_i(P - \tilde{S}_i)],$$

and the expected cash flow is:

$$E\tilde{X} = \sum_{i=1}^{I} \left[Q_B(E\tilde{S}_i - F_{0,T}^B) + \alpha_i^P Q_P(E\tilde{S}_i - F_{0,T}^P) + D_i(P - E\tilde{S}_i)\right] \tag{4}$$

The first two terms in (4) denote the risk premium in the peak- and base-load markets, respectively, where $F_{0,T}^P$ and $F_{0,T}^B$ are the peak- and base-load futures prices delivering in period T. The last term gives the expected revenue of the firm from the physical delivery of electricity. Similarly, the variance of the cash flows can be written as:

$$Var(\tilde{X}) = \sum_{i,j=1}^{I} c_i c_j Cov(\tilde{S}_i, \tilde{S}_j) \text{ where } c_i = Q_B + \alpha_i^P Q_P - D_i. \tag{5}$$

Now, we are ready to describe the optimal base- and peak-load futures positions, ${Q_B}^*$ and ${Q_P}^*$, that minimizes the variance of the cash flows.

Theorem 2. *Under a deterministic load profile, the variance-minimizing base- and peak-load futures quantities are respectively given by*

$$Q_B{}^* = \frac{\sum_{i,j=1}^{I} D_i Cov(\tilde{S}_i,\tilde{S}_j) \sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\alpha_i^P \alpha_j^P - \sum_{i,j=1}^{I} \alpha_j^P D_i Cov(\tilde{S}_i,\tilde{S}_j) \sum_{i,j=1}^{I} \alpha_i^P Cov(\tilde{S}_i,\tilde{S}_j)}{\sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{S}_j) \sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\alpha_i^P \alpha_j^P - \left(\sum_{i,j=1}^{I} \alpha_i^P Cov(\tilde{S}_i,\tilde{S}_j)\right)^2}, \tag{6a}$$

and

$$Q_P{}^* = \frac{\sum_{i,j=1}^{I} \alpha_i^P D_j Cov(\tilde{S}_i,\tilde{S}_j) \sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{S}_j) - \sum_{i,j=1}^{I} D_i Cov(\tilde{S}_i,\tilde{S}_j) \sum_{i,j=1}^{I} \alpha_i^P Cov(\tilde{S}_i,\tilde{S}_j)}{\sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{S}_j) \sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\alpha_i^P \alpha_j^P - \left(\sum_{i,j=1}^{I} \alpha_i^P Cov(\tilde{S}_i,\tilde{S}_j)\right)^2}. \tag{6b}$$

Analyzing the proof of Theorem 2 (more specifically (8) in the Appendix A), we see that when the peak-load amount is zero, the optimal base-load position reduces to the result in Theorem 1. In a typical demand profile, it is evident that both base- and peak-load quantities should be positive. This implies that using of a peak-load contract leads to a reduction in the optimal base-load amount. This result is formalized in the following corollary.

Corollary 2. *Suppose $D_i > 0, \forall i = 1, \dots, I$. Then, including peak-load contracts to the optimization problem of the retailer leads to a reduction in the optimal base-load levels.*

3.3 Hedging with Base-load Futures under Stochastic Demand

So far, we have assumed that the load profile of the customer order is deterministic. In this section, we analyze the situation when the load profile is stochastic. The load $\widetilde{D}_i$ for each interval is assumed to have a mean $E\widetilde{D}_i$ and a standard deviation σ_{D_i}. In addition, using historical data, we estimate the correlation between the load profile and the prices. For example, we observe that windy periods usually correlate with lower market prices. On the other hand, the consumption of households usually has little correlation with the prices. This is detailed in Section 4 where we explain our price model.

Following the same line of arguments as in Sections 3.1 and 3.2, we obtain the following optimal futures policy for the firm:

Theorem 3. *With stochastic demand and only base-load contracts, the futures procurement quantity Q^* that minimizes the variance of total cash flows is equal to*

$$Q^* = \frac{\sum_{i=1}^{I} \sum_{j=1}^{I} Cov(\tilde{S}_i, \widetilde{D}_j \tilde{S}_j) - P Cov(\tilde{S}_i, \widetilde{D}_j)}{var(\sum_{i=1}^{I} \tilde{S}_i)}.$$

Note that when the load profile is deterministic or uncorrelated with the prices, Theorem 3 reduces to Theorem 1. An interesting observation from Theorem 3 is that when the load is correlated with the prices, then the sales price P affects the optimal hedging level. In particular, the optimal hedging level decreases (increases) with the sales price if the load and prices are positively (negatively) correlated. In addition, we observe that if the load and prices are positively correlated then this may lead to self-hedging, reducing the reliance on the financial hedge. As the correlation between load and prices increases, the firm's optimal futures position may decrease.

3.4 Hedging with Base-load and Peak-load Futures under Stochastic Demand

Finally, in this section we present the most general case in which the retailer hedges a stochastic profile both with base- and peak-load contracts. The optimal policy is described in Theorem 4.

Theorem 4. *Under a stochastic load profile, the variance-minimizing base- and peak-load futures quantities are respectively given by*

$$Q_B{}^* = \frac{\left(\sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{D}_j\tilde{S}_j) - PCov(\tilde{S}_i,\tilde{D}_j\tilde{S}_j)\right)\sum_{i,j=1}^{I}\alpha_i^P\alpha_j^P Cov(\tilde{S}_i,\tilde{S}_j) - \left(\sum_{i,j=1}^{I}\alpha_i^P Cov(\tilde{S}_i,\tilde{D}_j\tilde{S}_j) - P\alpha_i^P Cov(\tilde{S}_i,\tilde{D}_j)\right)\sum_{i,j=1}^{I}\alpha_i^P Cov(\tilde{S}_i,\tilde{S}_j)}{\sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\sum_{i,j=1}^{I}\alpha_i^P\alpha_j^P Cov(\tilde{S}_i,\tilde{S}_j) - \left(\sum_{i,j=1}^{I}\alpha_i^P Cov(\tilde{S}_i,\tilde{S}_j)\right)^2} \text{ and}$$

$$Q_P{}^* = \frac{\left(\sum_{i,j=1}^{I}\alpha_i^P Cov(\tilde{S}_i,\tilde{D}_j\tilde{S}_j) - P\alpha_i^P Cov(\tilde{S}_i,\tilde{D}_j)\right)\sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{S}_j) - \left(\sum_{i,j=1}^{I} Cov(\tilde{S}_i,\tilde{D}_j\tilde{S}_j) - PCov(\tilde{S}_i,\tilde{D}_j\tilde{S}_j)\right)\sum_{i,j=1}^{I}\alpha_i^P Cov(\tilde{S}_i,\tilde{S}_j)}{var\left(\sum_{i=1}^{I}\tilde{S}_i\right)\sum_{i,j=1}^{I}\alpha_i^P\alpha_j^P Cov(\tilde{S}_i,\tilde{S}_j) - \left(\sum_{i,j=1}^{I}\alpha_i^P Cov(\tilde{S}_i,\tilde{S}_j)\right)^2}.$$

Note that a higher sales price, *P*, reduces the risk of a loss and decreases the hedging position. The results also dependent on the type of exposure, which can be positive or negative based on the customer type. We refer to Section 5 for further analyses.

4. Estimation Models

In this section, we describe the stochastic processes used to jointly model the electricity prices and load profile of the retailer. As noted in the literature, electricity prices are characterized by high volatility due to lack of storage and the need to balance supply and demand in real time. Electricity generation and consumption is also subject to transportation constraints and hence, local factors like transmission, plant outages and maintenance costs can influence prices as well. Finally, demand for electricity is highly inelastic, i.e., it is an essential commodity (Geman, 2009).

4.1 Electricity Price Model

The time series of electricity prices present significant differences compared to other financial and demand time series. First of all, the Dutch spot electricity prices are determined daily for the next 24 hours by the power exchange; and hence prices form a multivariate panel time series. In addition, they present seasonality and dependence structure both in the conditional mean and variance (Garcia-Martos et al., 2011).

We denote the spot electricity prices as $S_{i,t}$ where i corresponds to the specific hour of the day $(0 < i \leq 24)$, and t is an index that indicates the day $(t \geq 0)$. Since the prices are published daily, $S_{i,t}$ will be interpreted as 24 different dependent time-series in a panel-data setting. We observe that prices exhibit both cross-sectional and serial correlation, i.e., prices are correlated within the same day i and across different days *t*. We transform the prices into a 24x1 vector-time series $S_t = (S_{1,t} \dots S_{24,t}),\ \ t = 1, \dots, T$ and work with panel data.

We use a three-stage model for our electricity price process. In the first stage, our model captures the seasonality through the estimation of a conditional average. Then, in the second stage, the model aims to capture the cross-sectional structure using a dynamic factor model. Finally, serial correlations are estimated using an ARIMA-based model. This structure is a variation of the model that Garcia-Martos et al. (2011) used to forecast electricity prices and is given as below:

$$S_{i,t} = \mu_{i,t} + \sum_{j=1}^{J} \beta_{i,j} F_{j,t} + \varepsilon_{i,t} \tag{7}$$

Now we provide the details of the three-stage procedure we used to arrive at (7).

<u>Stage 1:</u> The seasonal component, $\mu_{i,t}$, is estimated using multiple regression with multiple dummies as the independent variables. Based on the literature and our analysis of the data the following effects are identified. First, a yearly effect d_{year} is included to capture structural changes that the market has gone through in the past. Next, we introduce the dummy variable d_{month} for the monthly seasonality of the data. Finally, seasonality is captured at the daily level using the dummies d_{day} and $d_{holiday}$. The resulting regression model with the error term $\eta_{i,t}$ are given below:

$$S_{i,t} = \underbrace{\beta_0 + \beta_i + \beta_{year} d_{year,t} + \beta_{month,i} d_{month,t} + \beta_{day,i} d_{day,t} + \beta_{holiday,i} d_{holiday,t}}_{\mu_{i,t}} + \eta_{i,t} \qquad \forall i, t$$

We first simplify this model assuming that certain clusters of hours respond identically to certain effects. In particular, we have observed that the daily effects are identical for Monday through Thursday, and hence we can reduce the number of days to 4. In addition, the yearly effects were only estimated for blocks of hours, because such long-term effects cannot be estimated with hourly indices. The monthly effect was still estimated on an hourly basis, because the impact differs substantially based on the hours. The resulting regression model has an adjusted-R^2 of 42.5%. For all models the residuals, $\eta_{i,t}$, were calculated and tested for normality, independence and homoscedasticity, and as expected, all of these tests showed that there are dependence, non-normality and heteroskedasticity.

Stage 2: In this step, we fit a multivariate dynamic factor model to capture the structure in the correlated residuals, $\eta_{i,t}$, to come up with the second term on the right hands side of Equation (7). This approach, developed by Watson and Engle (1981), is based on state space modeling of multivariate unobserved factors. The factors are characterized using principal components analysis (PCA). From the PCA, we have extracted 4 factors based on their eigenvalues (i.e., $\mathrm{J} = 4$ in equation (7)), which explain approximately 80% of the correlation structure. The first factor, denoted $F_{1,t}$ in (7), corresponds to a general level of volatility that is based on each of the hourly prices. The second factor, $F_{2,t}$, can be interpreted as the general peak/off-peak factor that explains differences between the peak and off-peak hours. The third factor, $F_{3,t}$, accounts for the evening peak that is caused mainly by households. And the fourth factor, $F_{4,t}$, represents the effects due to the transition from peak to off-peak and vice versa. Note that as we are currently analyzing the residuals, $\eta_{i,t}$, these dynamic factors need to be explained in terms of deviations from $\mu_{i,t}$.

After the factors are identified from the residual data of the regression, the unobserved common factors $F_{j,t}$ are assumed to follow a seasonal multiplicative ARIMA model $(p, d, q) \times (P, D, Q)_s$ with GARCH(1,1) disturbances and a seasonality of seven days. The ARIMA accounts for the autocorrelation in the factors with respect to the previous days, the seasonal ARIMA accounts for the autocorrelation with respect to the same day a week before. The GARCH model captures the volatility clustering of the factors. Even though the long-term seasonality has been captured by $\mu_{i,t}$ in the regression, short-term seasonalities also exist. Finally, the random innovations of these factors should be normal and IID. Using the ARIMA model it was possible to remove autocorrelation for a large extent, however not completely. In addition, it was shown that a student-t_3 distribution better fitted the random noise data. Finally, the PCA that was used to estimate the common factors, assures us that the factors themselves are uncorrelated. The seasonal ARIMA specification was evaluated for multiple values; after running multiple models, an AR process proved optimal (for all the factors an $AR(1\ 2\ 3), AR(3)_7$). Finally, the factors were regressed on the individual hourly residuals to find the β −coefficients of equation (7).

Stage 3: The residuals for each hour from the regression of the factors are saved again and the $\varepsilon_{i,t}$ in (7) is modeled with an ARMA(p,q) process. The residual for each hour i is the sum of a moving average of the previous resulting residuals, as well as an autoregressive from the previous impulses which are all i.i.d. normally distributed with mean 0 and variance s. After the model parameters are estimated, we use it as the basis for a Monte Carlo simulation. An important choice here is the specific distribution for the random shocks of the factors and the individual hourly residuals. The standard approach is to assume a mean 0 normal distribution on the residuals. However, this is often not the case for prices in electricity markets (see e.g. Benth et al. 2007, Geman and Roncoroni 2006, and Zhou et al. 2009), since they show a high kurtosis or skewness. A detailed basic analysis of the past hourly prices is presented in Appendix B. In order to address this problem, we fit a number of different distributions, namely student-t distribution and the student t_3-distribution, to describe the data. We also compare the performance of these distributions with respect to the original dataset. In order to choose the best model, the three specifications are used to simulate an identical period as the original data and the results are presented in Appendix C. The model that is best able to capture these effects is the student-t_3 model.

4.2 Demand Model

To generate a demand model, we first take the mean demand to be equal to the best forecast that the retailer can provide for their future demand. This forecast is generated for each separate customer group and varies significantly across the customer groups. In addition, using historical data for 2011, we assign each customer group a coefficient of variation (CV). This CV is used with the forecasted mean demand to calculate the standard deviation, hence the standard deviation can vary hour by hour, but is linearly related to the forecast mean. For profiles, a forecast is based on an hourly quantity which results in different hourly means and standard deviation. On the other hand, wind is forecasted with a constant monthly mean, hence the hourly demand mean and standard deviation are identical for all hours.

In addition, we estimate a correlation coefficient for each customer group with the first pricing factor from the dynamic factor analysis. This correlation coefficient is saved and used to generate the demand scenarios after the price scenarios have been generated. The set of residuals is combined with the new residuals for the demand with the normal distribution and the parameters as shown in the previous paragraph. Using the formula below, the two sets of residuals $(\varepsilon_{1,t}, \varepsilon_{2,t})$, are combined in such a way that they were correlated with ρ.

$$z_{i,t} = \rho\varepsilon_{1,i,t} + \sqrt{(1-\rho^2)}\varepsilon_{2,t}$$

We calculate the new correlated residuals and use them to generate the demand paths. Each hourly demand can be written as the mean expected forecast plus a disturbance $z_{i,t}$ which has an expected value of zero and is correlated with the residual that was used to generate the price path of the first factor. This means that each scenario of prices needs to be combined pair-wise with a scenario of demand. Note that the factor residuals are a daily residual, while the individual demand residuals are hourly.

5. Numerical Results

In this section, we provide a numerical illustration of our hedging results using price and demand data from a Dutch electricity retailer. In Section 5.1, we first analyze how to hedge an exposure from a customer portfolio exclusively comprised of profiled customers, and then in Section 5.2 we extend our numerical analysis to consider for the entire customer portfolio over the full planning horizon of the retailer.

5.1 Hedging a Portfolio of Profiled Customers

A typical electricity retailer carries a significant volume of profiled customers in its physical portfolio. For our case, on 1 August 2011, we downloaded the profiled contracts of the retailer for delivery in October 2011. That is, we assume that the retailer starts operations on 1 August 2011, with a given set of un-hedged profiled customer orders to be delivered in October 2011. The retailer's hourly *mean* profiled demand for October 2011 and the timeline for the hedging decisions are illustrated in Figure 6 and Figure 7, respectively.

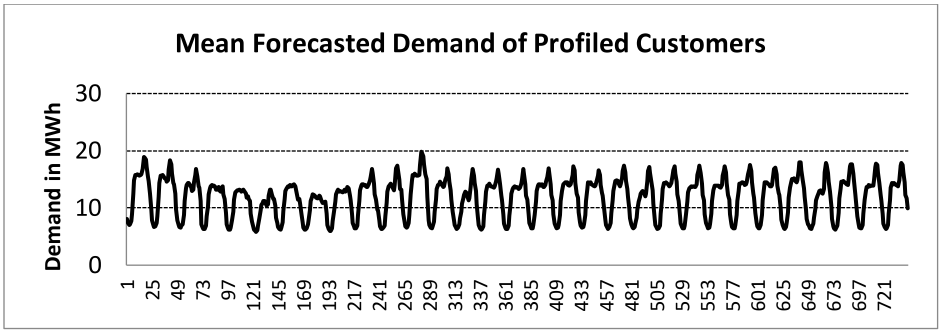


Figure 6: Mean forecasted demand of profiled customers for October 2011

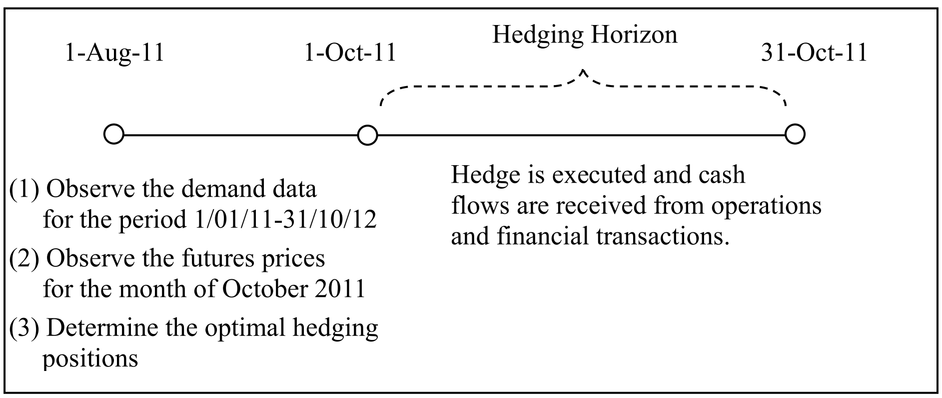


Figure 7: The timeline for the decisions for hedging profiled exposures only

The correlation of the profiled demand with the price is measured to be 10%. On 1 August 2011 the firm observes the futures prices in the market and takes appropriate hedging positions in futures to minimize the variance of its cash flows for the delivery period (October 2011). On 1 August 2011 there were 2 futures contracts relevant for delivery in October 2011 (one base and one peak-load contract). We note that the retailer myopically optimizes the hedging decision given the set of relevant contracts on 1 August 2011. If the firm receives a new order on August 3rd, then it recalculates the next physical exposure and reoptimizes the hedging plan again myopically over time.

Using the derivations in Section 3 and the price model in Section 4, we calculate the optimal futures positions with Monte Carlo simulation. As in Section 3, we discuss our results under 4 scenarios. In scenario 1, we consider the case when the firm hedges with base-load contracts only under deterministic demand.

Then in scenario 2, we consider the same case with both base- and peak-load contracts. Subsequently, in scenarios 3 and 4, we extend our analysis with stochastic demand. Optimal hedging positions for the motivating retailer is summarized in Table 2.

	Scenarios			
	Scenario 1: Base-load with Deterministic Demand	Scenario 2: Base & Peak-load with Deterministic Demand	Scenario 3: Base-load with Stochastic Demand	Scenario 4: Base- & Peak-Load with Stochastic Demand
Base-load (Q_b^*)	12.37	10.27	12.30	10.25
Peak-load (Q_p^*)	-	5.23	-	5.26

Table 2: Variance-minimizing futures positions for profiled customers in October 2011 (MW)

When there is no uncertainty in the demand profile and the firm uses only base-load contracts, the optimal futures position is to long 12.37 MW of October delivery futures contracts. Compared to the 11.9 MW of mean demand in October[2], this implies that the firm is over-hedging relative to the mean level of exposure.[3] This behavior is driven by the nature of the electricity prices. In particular, spikes in electricity prices usually occurs in peak hours when the demand is high, by taking a long position larger than the mean demand, the firm mitigates the effect of these extreme price realizations on the cash flows.

Peak-load contracts provide the firm with more flexibility in matching the demand profile with the futures portfolio of the firm. Accordingly, in the second scenario, the firm shifts a portion of the base-load positions to peak-load contracts. In scenario 2, the total volume of futures position increases to 10.27 + 0.5*5.23 = 12.89MW (peak-load contracts only deliver between 8h00 – 20h00). Since the peak-load contracts better capture the demand profile during the peak hours, they motivate the firm to increase its overall hedging position as compared to hedging with only base-load contracts.

Finally, inclusion of uncertainty in scenarios 3 and 4 has a minor effect since the volatility of profiled demand is only around 10% in practice. Additional volumetric risk makes hedging with futures harder leading to a decrease in the total hedging position.

Table 3 below summarizes the effect of optimal hedging on the cash flows of the firm at the end of the month. When there is no hedging the mean value of the cash flows is € 152,800 with a standard deviation of € 17,549, and a 95% confidence interval is given by [117,721, 187,918]. When the firm optimally hedges with base-load contracts only, then the mean cash flows become € 188,560 while the standard deviation of the cash flows dramatically drops to € 2,481 only. The reduction in the standard deviation is 85.9%, i.e., a

[2] This figure implies an expected energy demand of 11.9*24*31 = 8853.6MWh in October 2011.

[3] Economically, when the firm over-hedges this implies that the firm is more likely to sell excess inventory in the spot market as compared to the under-hedging case. Our interpretation of over- and under-hedging is based on the classical paper of Rolfo (1980) in the finance literature.

big portion of the cash flow variability is eliminated by hedging. It is also important to note that the hedge also changes the mean value of the cash flows due to the risk premium involved in the futures contracts. However, once the cash flows are discounted for the risk-premium this change in mean values is irrelevant in a perfect capital market (Modigliani and Miller, 1958; Stulz, 1996; Titman, 2002). Hence, we focus on the reduction in variance as a measure of the hedging efficiency, since this reduction in variability limits the firm's exposure to capital market frictions and increases firm value.

When the firm uses both base- and peak-load contracts for hedging, then variance can be further reduced to € 1,878. This corresponds to a 89.3% reduction in the standard deviation compared to no hedging. Compared to base-load only case, this is a reduction of 24%.

	Base and Peak hedging	Base-only hedging	No hedging
Mean	€ 182,000	€ 188,560	€ 152,820
St. Dev	€ 1,878	€ 2,481	€ 17,549
Reduction in St. Dev	89.3%	85.9%	
	95% Confidence Interval		
Low	€ 178,242	€ 183,596	€ 117,721
High	€ 185,757	€ 193,523	€ 187,918

Table 3: Summary Statistics of Cash Flows from Profiled Customers

5.2 Hedging the Net portfolio of the retailer

In this section, we consider the entire portfolio of the retailer, which spans from 1 September 2011 to 31 June 2012 as of 1 August 2011. We have excluded the physical contracts beyond 31 June 2012, as these contracts are either not yet fully finalized or there is little volume. Similar to Section 5.1, we downloaded physical exposure data from the retailer's database as of 1 August 2011, for the period from 1 September 2012 to 31 June 2012. Downloaded portfolio includes 4 different types of customers: wind farms, profiled customers, other producers and industrial consumer. The breakdown of these customers in the portfolio is given in Table 4.

Demand	**Correlation with price**	**Coefficient of variation**	**% size in portfolio**
Wind farms	-0.38	96.26%	30.37%
Profiled customers	-0.08	21.01%	15.11%
Other producers	0.13	17.60%	8.18%
Industrial consumers	0.05	17.66%	46.33%

Table 4: Overall physical portfolio composition of the retailer for the period from 1 September 2011 to 31 June 2012 as of 1 August 2011

On 1 August 2011 the firm observes the futures prices in the market and takes appropriate futures positions to myopically minimize the variance of the cash flows (for the current set of orders) over the planning horizon. In practice, the retailer resolves this problem over time as new orders are added to the physical portfolio in a rolling horizon manner. On 1 August 2011, there were 12 relevant futures contracts (6 base- and 6 peak-load contracts) to hedge the portfolio risk. Four of these contracts were traded monthly and two were traded quarterly as presented in Table 5. In addition, Table 5 presents the expected future spot price for the delivery periods of relevant contracts and risk premiums as calculated by our price model. The premium is generally positive for peak load contracts while its sign fluctuates for the base-load contracts. A positive premium implies that hedgers are willing to pay a premium to the speculators for eliminating their risk, and vice versa.

	Base-load Contracts			**Peak-load Contracts**		
Futures Contracts (Delivery Dates)	**Observed Futures Price**	**Expected Future Spot Price**	**Risk Premium**	**Observed Futures Price**	**Expected Future Spot Price**	**Risk Premium**
September 2011	53.03	56.61	-3.58	62.48	80.91	-3.65
October 2011	58.03	61.92	-3.89	73.48	94.18	0.29
November 2011	63.23	56.6	6.63	80.89	101.11	13.81
December 2011	61.21	62.36	-1.15	79.15	95.8	5.32
Quarterly Jan-Mar 2012	63.2	57.84	5.36	79.23	84.95	11.46
Quarterly Apr-June 2012	53.06	53.94	-0.88	63.91	84.93	0.88

Table 5: Price information as of 1 August 2011 (Euro/MWh)

The optimal hedging decision is given by a vector of futures positions for the 6 relevant delivery periods in the future. As in the previous section, we conduct our analysis under four scenarios to explore the effect of uncertainty and the effect of peak- and base-load futures. Optimal solution is tabulated in Table 6.

	Deterministic Portfolio			*Stochastic Portfolio*			*General*
Period	Base-load only	Base-load	Peak-load	Base-load only	Base-load	Peak-load	Mean Net Demand
September	10.8	7.3	5	10.5	8.1	3.3	8.9
October	7.8	4.7	5.5	8.1	4.9	5.7	5.6
November	7.8	4	6.7	8.2	4.6	6.3	5.7
December	7.3	3.8	6.3	7.7	3.8	6.3	5.2
Jan-Mar	6.5	1.7	8.8	7.3	4	5.9	4.9
Apr-Jun	8.7	5.3	6.1	8.9	5.8	5.7	7.25

Table 6: The variance-minimizing base- and peak-load quantities for retailer’s net exposure (MW)

We observe that the difference between the solutions of the stochastic and deterministic cases is more pronounced as compared to the results in Section 5.1. This is because the aggregate portfolio of the firm has more volumetric risk than the profiled customers. For example, the gap between optimal futures positions under the deterministic and stochastic scenarios for the base-load only case goes up to 12.3% for the period Jan-Mar, while it goes down as low as -2.8% for the September period.

Similar to the case with profiled customers only, the firm still over-hedges compared to the mean demand. The relative over-hedging amount ranges between 18.0% (September) to 49.0% (Jan-Mar) under volume uncertainty with both base- and peak-load hedging. Over-hedging amounts are typically higher during the winter period due to the increased likelihood of price spikes and increased volumetric risk due to wind exposure. Hence, the retailer tends to carry a larger net financial position to hedge against those extreme price realizations.

Another interesting observation is the distribution of optimal portfolio between base and peak-load contracts. Unlike in Section 5.1, we observe that the firm may prefer to allocate a significant portion of its hedging portfolio to peak-load contracts. This issue is especially prevalent during the winter load. For the Jan-Mar period with no volume risk, the optimal hedging portfolio includes only 1.7MW of base-load, but 8.8MW of peak-load contracts. Similarly, the same figures for December are 3.8MW and 6.3MW, respectively. During the summer however, a much larger portion of the portfolio is allocated to base-load contracts.

Table 7 summarizes the effect of optimal hedging plan on the cash flows of the firm for the 6 exposure periods between 1 October 2011 and 31 June 2012.

	No Hedge				*Base-load Only*				*Base- and Peak-load*			
Period	**Mean**	**SD**	**[95% C.I.**	**]**	**Mean**	**SD**	**[95% C.I.**	**]**	**Mean**	**SD**	**[95% C.I.**	**]**
September	135730	13200	109330	162130	156920	4192	148536	165304	156260	3967	148326	164194
October	171020	11400	148220	193820	187000	5539	175922	198078	180000	5250	169500	190500
November	228000	9688	208624	247376	205000	4890	195220	214780	194000	4445	185110	202890
December	202000	10100	181800	222200	207000	5701	195598	218402	195000	5316	184368	205632
Jan – Mrt	680000	17700	644600	715400	650000	11900	626200	673800	620000	11346	597308	642692
Apr – Jun	435000	25100	384800	485200	448000	7880	432240	463760	440000	7170	425660	454340

Table 7: Summary statistics of cash flows from the net portfolio

Table 8 shows the percentage reduction in standard deviation of cash flows relative to the no-hedging case.

Period	Base-load Only	Base- & Peak-load
September	68.2%	69.9%
October	51.4%	53.9%
November	49.5%	54.1%
December	43.6%	47.4%
Jan - Mrt	32.8%	35.9%
Apr - Jun	68.6%	71.4%

Table 8: Reduction in standard deviation of cash flows due to hedging

The effectiveness of the hedge significantly depends on the hedging period due to the varying composition of retailer's portfolio over time. During the Jan-Mar period the effectiveness of hedging goes down as low as 35.9% since this period includes a substantial amount of wind contracts. In the Summer and early Fall, as the volume of wind exposures goes down, hedging becomes more effective. Including peak load contracts to the optimal portfolio does not have a large effect on hedging efficiency as in Section 5.1.

6. Conclusions

We examine the risk management practices of an electricity retailer motivated by the Dutch electricity market. An electricity retailer provides a unique intermediation service between the users and producers of electricity through the electricity wholesale markets. While executing their intermediation service in the market, the retailers engage in physical long and short positions, which involve not only price but also demand and generation, i.e., volumetric, risks. These positions depending on the consumption and generation profiles of the customers may also present significant non-stationarity over time. Wholesale markets, on the other hand, due to liquidity issues only offer a number of standardized contracts to the retailers. In this paper, we focus on the effectiveness of the existing base- and peak-load futures contracts in the market as a risk management tool for the electricity retailers. We analytically characterize the retailer's optimal hedging portfolio as a function of the serial correlation of the prices and the demand profiles of its customers. Using electricity prices between 2002 and 2010 and real demand data from a Dutch retailer we construct a GARCH model for electricity prices and test our analytical results.

Our findings indicate that although the existing contracts in the futures market are quite efficient to replicate the exposure from profiled customers, when industrial consumers and renewable generation is included to the exposure of the firm, the effectiveness of such contracts decreases substantially. For our motivating retailer, optimal use of base-load futures contracts can reduce the cash flow variability by 85.9%. Including peak-load contracts into the hedging portfolio further increases the efficiency of hedging to 89.3%. However, when we consider the aggregate net physical exposure of the retailer including profiled customers, industrial consumers and renewable contracts, the efficiency of hedging through the existing futures contracts in the market substantially decreases. The efficiency goes down as low as 32.8% during certain periods when the volume of renewable contracts is large.

Our work also reveals an important over-hedging behavior in the futures market, i.e., the retailer's optimal futures position exceeds the firm's net physical exposures. Including peak-load contracts to the hedging portfolio further increases the over-hedging amount. The over-hedging behavior is particularly driven by the price spikes that typically occur during the winter period in the Netherlands. During the winter periods, high volumetric risk from wind generation coupled with increased probability of price spikes increases the retailer's reliance on the futures market.

We also find a possible self-hedging behavior embedded in the load profiles of the consumers. As the load level of a particular delivery interval increases, this does not necessarily lead to a higher hedging level in the futures market. Basically, a possible negative correlation of that particular interval's exposure with the rest of the delivery intervals serves as a self-hedging mechanism such that the higher load profiles in that period creates a negatively correlated cash flow with the rest of the delivery periods reducing the firm's reliance on the financial hedge.

Overall, we note that both the volumetric risk and the non-stationarity of physical exposures drive the inefficiency of futures hedging in the market. In the future, volumetric risk is expected to increase in the retailers' portfolio as the share of intermittent renewables increases in the market. Designing standard futures and options contracts, which can price in volumetric risk, is essential for the functioning of futures markets as a risk management tool in energy markets (see e.g., Brik and Roncoroni 2016). In our model, we assume that the retailer makes a series of myopic decisions for the future periods as new orders arrive stochastically. That is, whenever a new order is received the firm immediately takes a counter position without any consideration of future arrivals. In principal, better results can be achieved using a dynamic approach by updating our expectation regarding future orders and optimizing the hedge as demand reveals. In principle, developing practical models suitable for real world application will be exceptionally difficult, due to the difficulties in estimating the probability distributions regarding order arrival times, order types and demand profiles. However, developing *stylized* stochastic multi-stage dynamic programming models to capture the dynamic nature of order arrivals and to obtain managerial insights into the nature of hedging in electricity retailing is a promising research direction.

Examining how financial hedging would affect the contracting dynamics between the firms in equilibrium is another interesting research question. As argued by Brown and Toft (2002), by hedging the retailer may reduce its exposure to deadweight cost (in the form of costly external funds, bankruptcy costs etc.) Hence, the retailers which may effectively hedge their risks (both price and volume) may become more competitive in the market being able to accept customer orders with even lower profit margins. In equilibrium, this is likely to result in reduced retail prices possibly making energy cheaper for the final consumer. Another closely related question would be integrating demand pooling (a form of operational hedge) with financial hedging when competing in the commodity market. We believe that this is an interesting an important research question in the context of electricity retailing. Indeed, we observe pooling behavior in the retail electricity market.

7. Appendices

Appendix A– Proofs of Theorems in Section 3

Proof of Theorem 1. Taking the derivative of (2) with respect to Q and setting it equal to 0, we get

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\left(2Q^* - \left(D_i + D_j\right)\right) Cov\left(\tilde{S}_i, \tilde{S}_j\right) = 0.$$

Solving it for Q^*, we obtain (4). Finally, taking the second derivative with respect to Q, we have

$$\frac{\partial^2 Var(\tilde{X})}{\partial Q^2} = 2\sum_{i=1}^{I}\sum_{j=1}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right) = 2Var\left(\sum_{i=1}^{I}\tilde{S}_i\right) \geq 0$$

for all values of Q and hence, Q^* is a global minimizer.

Proof of Theorem 2. Again, taking the partial derivatives of (6) with respect to Q_B and Q_P, we get

$$\frac{\partial Var(\tilde{X})}{\partial Q_B} = \sum_{i=1}^{I}\sum_{j=1}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right)\left(2Q_B + Q_P\left(\alpha_i^P + \alpha_j^P\right) - (D_i + D_j)\right) = 0,$$

$$\frac{\partial Var(\tilde{X})}{\partial Q_P} = \sum_{i=1}^{I}\sum_{j=1}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right)\left(2\alpha_i^P\alpha_j^P Q_P + Q_B\left(\alpha_i^P + \alpha_j^P\right) - (\alpha_j^P D_i + \alpha_i^P D_j)\right) = 0. \tag{8}$$

Solving these equations, we obtain (6). We now need to prove that the solution $({Q_B}^*, {Q_P}^*)$ minimizes the variance by showing the convexity of variance with respect to these variables. We can calculate the Hessian of the variance as

$$\nabla^2 Var(\tilde{X}) = 2\begin{bmatrix} Var\left(\sum_{i=1}^{I}\tilde{S}_i\right) & Cov\left(\sum_{i=1}^{I}\tilde{S}_i, \sum_{i=1}^{I}\tilde{S}_i\,\alpha_i^P\right) \\ Cov\left(\sum_{i=1}^{I}\tilde{S}_i, \sum_{i=1}^{I}\tilde{S}_i\,\alpha_i^P\right) & Var\left(\sum_{i=1}^{I}\tilde{S}_i\,\alpha_i^P\right) \end{bmatrix}.$$

The positive semi-definiteness of the Hessian matrix can then be proven by using the covariance inequality.

Proof of Theorem 3. The proof is similar to the proof of Theorem 1 and we take the first and second derivative of the variance of the cash flow

$$Var(\tilde{X}) = Var\left(\sum_{i=1}^{I}\left(P\widetilde{D}_i + Q\tilde{S}_i - \widetilde{D}_i\tilde{S}_i\right)\right).$$

Setting the first derivative equal to 0, we obtain the optimal Q^* and similar to the proof of Theorem 1, we get the second derivative of the variance to be $2Var\left(\sum_{i=1}^{I}\tilde{S}_i\right)$ which proves the convexity.

Proof of Theorem 4. The proof of this theorem is similar to the proof of Theorem 2, where we take the first and second derivatives of the variance of the cash flow

$$Var(\tilde{X}) = Var\left(\sum_{i=1}^{I}\left(Q_B\tilde{S}_i + \alpha_i^P Q_P\tilde{S}_i + P\widetilde{D}_i - \widetilde{D}_i\tilde{S}_i\right)\right).$$

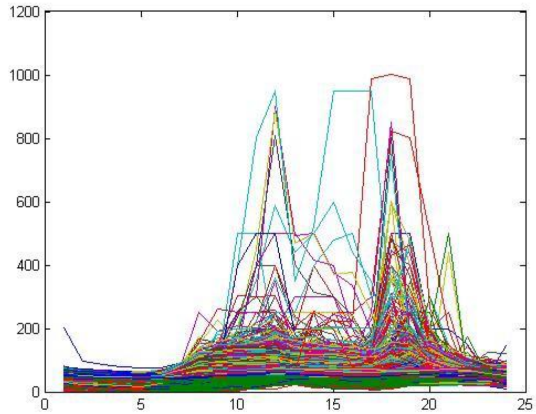
The second derivative of the above variance is the same as in Theorem 2, which proves convexity with respect to Q_B and Q_P.

Appendix B– Basic Distributional Statistics for Hourly Price Data

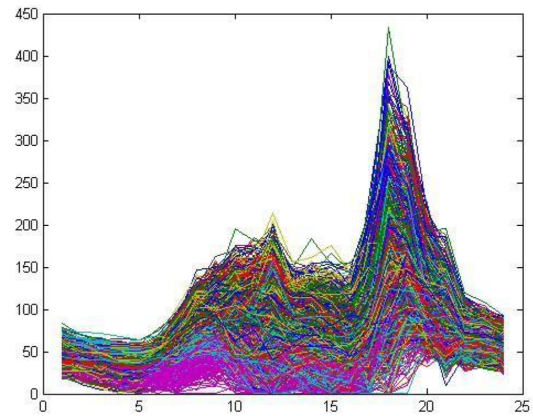
Statistics	*Hours* 1	2	3	4	5	6
Mean	38.52	34.01	30.65	27.20	26.55	30.85
St. Dev	12.59	11.85	11.75	11.65	11.76	12.50
Median	36.83	32.49	29.93	26.80	26.35	30.10
Min	0.01	0.01	0.01	0.01	0.01	0.01
Max	200.97	95.57	85.12	79.93	75.57	75.00
Range	200.96	95.56	85.11	79.92	75.56	74.99
Kurtosis	14.12	3.12	2.94	2.81	2.76	2.86
Skewness	1.29	0.32	0.22	0.22	0.20	0.10
N	2,438	2,438	2,438	2,438	2,438	2,438
Statistics	*Hours* **7**	**8**	**9**	**10**	**11**	**12**
Mean	37.42	50.26	56.65	64.04	68.45	74.30
St. Dev	16.13	24.01	26.12	33.58	40.36	52.45
Median	37.33	48.28	53.00	57.25	60.10	63.06
Min	0.01	0.01	0.01	1.49	7.81	9.31
Max	90.00	250.00	260.20	500.00	800.00	950.00
Range	89.99	249.99	260.19	498.51	792.19	940.69
Kurtosis	3.09	5.86	6.80	26.38	68.77	107.75
Skewness	0.14	0.86	1.25	3.15	5.56	7.85
N	2,438	2,438	2,438	2,438	2,438	2,438
Statistics	*Hours* **13**	**14**	**15**	**16**	**17**	**18**
Mean	65.44	63.68	60.07	55.47	55.09	65.67
St. Dev	32.66	36.02	37.96	34.10	37.53	61.68
Median	59.41	56.39	53.20	50.01	49.61	52.73
Min	21.70	14.01	9.18	8.18	5.98	9.10
Max	499.00	520.01	950.00	950.00	988.00	1000.12
Range	477.30	506.00	940.82	941.82	982.02	991.02
Kurtosis	53.16	52.30	157.54	221.17	294.61	73.77
Skewness	5.02	5.20	8.73	10.23	12.87	7.07
N	2,438	2,438	2,438	2,438	2,438	2,438
Statistics	*Hours* **19**	**20**	**21**	**22**	**23**	**24**
Mean	66.70	61.60	56.83	50.05	48.39	43.11
St. Dev	49.92	30.07	23.67	15.47	13.86	12.85
Median	55.00	54.50	52.37	47.57	46.21	41.00
Min	19.91	18.94	20.00	20.97	19.90	10.00
Max	988.00	500.12	500.00	175.01	125.00	145.00
Range	968.09	481.18	480.00	154.04	105.10	135.00
Kurtosis	84.99	28.32	83.81	6.47	3.90	5.58
Skewness	6.77	3.19	5.44	1.18	0.87	1.03
N	2,438	2,438	2,438	2,438	2,438	2,438

Table 9: The Basic Distributional Characteristics of Hourly Prices

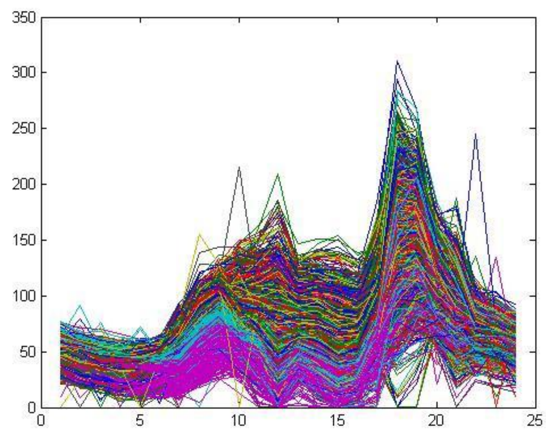
Appendix C– Simulation Results under Various Residual Distributional Assumptions



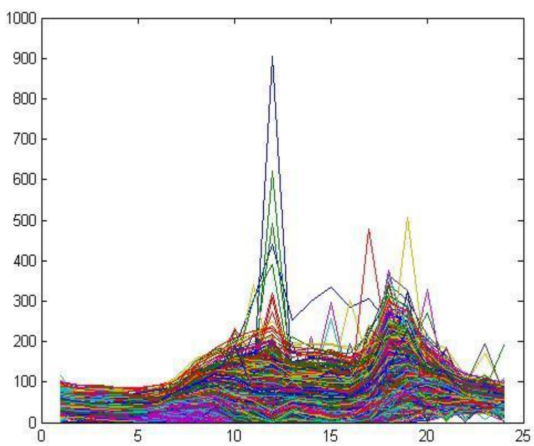
(a) Actual daily price profiles between 2005 and 2011



(b) Simulations under normal residual assumption



(c) Simulations under Student-t residual assumption



(d) Simulations under Student-t_3 residual assumption

Figure 8: The daily price profiles in the period between 2005 and 2011 and the simulation results under various distributional assumptions on residuals of the price model

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