European Electricity Day-Ahead Markets: A Review of Models and Solution Methods

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Abstract

Recently, designing and clearing day-ahead electricity markets have received significant attention from both academia and practice alike. Together with the increasing penetration of intermittent renewables to the electricity supply mix, efficiently designing and clearing day-ahead electricity markets have become a central concern for policy makers in Europe. Although the literature is expanding rapidly, our review indicates that the research in the area is relatively fragmented. In view of this, we provide a structured review of the day-ahead markets in Europe by unifying three building blocks: current market models, the optimization techniques used to clear those markets and the directions of future research in market design and market clearing approaches. To the best of the authors' knowledge, there is no article that reviews and compares order types, objective functions, and solution methodologies in European day-ahead markets. Hence, this paper is the first exhaustive review of exchange-type day-ahead electricity markets in Europe to take both the theoretical and technical aspects of the problem into account from both operational and strategic perspectives. We also highlight the key differences, strengths, and deficiencies in developed mathematical models in the literature to provide a holistic picture that can be used by the academic community to choose the appropriate model based on the problem characteristic and data structure.

Keywords: OR in energy, Electricity Markets, Optimization, Market Organization

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1. Introduction

During the last ten years, the size and the complexity of the European day-ahead markets (DAMs) have grown substantially. In 2019, spot trade in European Power Exchange reached a record volume of 577 TWh [1]. These ever-increasing trade volumes as well as the introduction of complex bidding structures have significantly increased the need for sophisticated optimization techniques to clear the day-ahead markets. For example, the current Turkish DAM clearing problem includes more than 15000 orders out of which nearly 800 are block orders [2]. The European DAM coupling problem is much bigger and involves more than 110000 orders with 53 separate bidding regions of six power exchanges, including NordPool, APX, Belpex, EPEXSPOT, OMIE and GME [3]. In most cases, these auctions need to be cleared in less than one hour every day [2, 4-6]. Although an exact solution is desirable, finding a good feasible solution within the time limits of the auction is essential for the market to function well. Increasing the efficiency of the DAM auctions, even by a fraction of a percent, would have enormous welfare implications in Europe. In this paper, we aim to review the literature on DAM design and solution methodologies, while pointing to future research directions in this promising research area.

In Europe, spot electricity trading is organized in three closely linked markets: the *day-ahead market* (DAM), the *intra-day market* (IDM) and the *imbalance market* (IM). The DAM is the main spot market and is used to determine electricity prices for the next day. DAM serves as a very short term (i.e., one-day-ahead) forward market where the prices are determined through a double-sided blind combinatorial auction. IDM operates in tandem with DAM, and market participants use it to manage the changes in their operating and consumption plans after the DAM is closed but before the physical delivery of electricity takes place. It is particularly desirable to have well-functioning day-ahead and intra-day markets so that intermittent renewables can be successfully integrated to the grid [7]. Finally, after trading both at DAM and IDM, if there are still some supply and demand mismatches these imbalances are cleared in the IM. Overall, the successful operation and integration of these three spot markets constitute the backbone of the liberalized electricity markets as well as the grid security in Europe. Figure 1 presents the timeline of trading in these spot markets. The day-ahead market closes 24 hours prior to physical delivery. In the next *n* hours, the intraday market starts to operate. Finally, after the closure of the intraday market, the imbalance market starts to operate in a real time fashion.

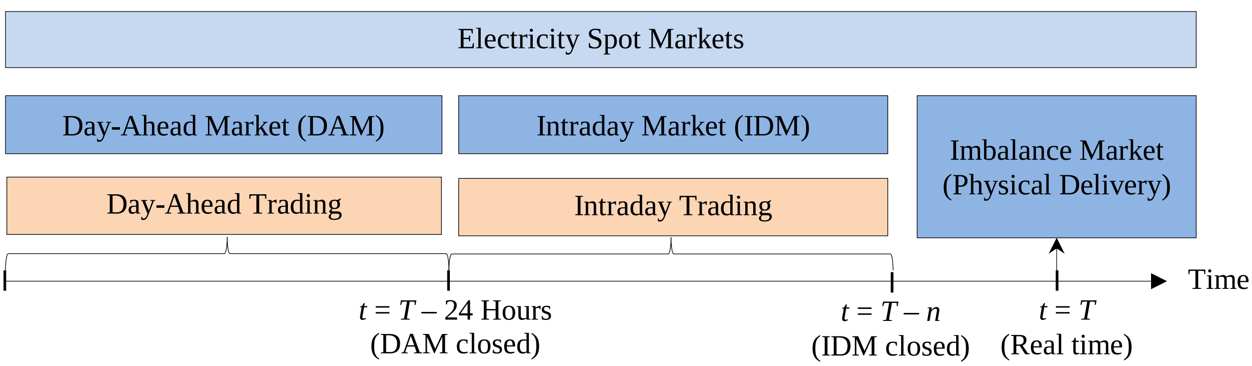

Figure 1. Electricity spot markets

DAMs are crucially important to spot electricity markets for a number of reasons. DAM clearing prices provide a reference for the derivative markets through which more than 80% of the electricity trade takes place [8]. It is essential to provide a reliable reference for those markets in order for the physical electricity market to operate smoothly. Regulatory authorities also index and revise electricity tariffs/subsidies based on DAM prices. A reliable DAM is essential for the development and execution of governmental subsidy policies. Lastly, day-ahead markets reduce the need for trading in IDM and IM markets, which are costly markets for trading energy and used as a last resort.

There are two types of DAMS, namely pool- and exchange-type DAMs, which are used in liberalized electricity markets. Pool-type models are often used in US markets whereas exchange-type models are common in Europe. See [12] for a detailed discussion of the differences between these two models. In this paper, we focus on exchange-type DAMs and the corresponding clearing methodologies which have been developed to determine the market prices in Europe. European DAMs were established and have evolved independently in different European countries based on the expectations and needs of market participants over time. Historically, DAMs developed through the introduction of increasingly complex bidding structures [3, 5, 6, 9, 10, 11] and industry constraints [3, 4, 6, 10, 12, 13] to better enable market participants to reflect their consumption and generation dynamics through their orders in the market. The introduction of such complex bidding structures and constraints has led to complicated combinatorial auctions and has created the need for more advanced optimization techniques to clear those markets [10]. During the last decade, through the price coupling of regions (PCR) initiative, most of the local DAMs (NoordPool, CWE, etc) have gradually connected to creating a single DAM covering most of the European countries [14]. These countries are shaded in Figure 2.

Figure 2. Map of PCR countries in 2019 (The Europe map is taken from Freepik.com)

European DAMs are organized as a special form of combinatorial auctions. We provide an example of the daily operations of the Turkish DAM in Figure 3 to present the steps involved in the market. As shown in Figure 3, the procedure is a double-sided blind combinatorial auction. Its primary steps include bid submission, bid validation, market clearing, objection evaluation and the determination of electricity prices for each period of the next day.

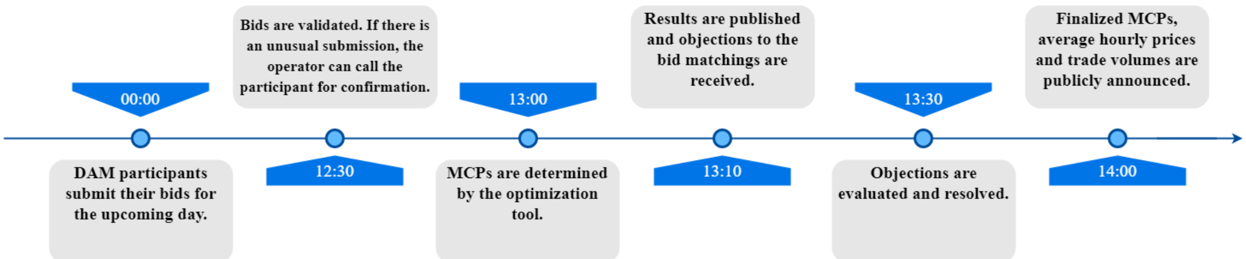

Figure 3. Daily Operations in Turkish DAM [2]

In commodity markets, it is common for the traders' valuation of a bundle of items to exceed the sum of the individual valuation of those items. This is known as the *complementarity effect* in economics. In electricity markets, the setup costs of ramping up or down thermal power plants or the consumption economics of large-scale electricity customers create a similar complementarity effect, across multiple consecutive trading hours, which motivates bidders to use complementary bidding [15-17]. DAMs allow for complementary bidding through block orders, linked orders and many other forms of complex bidding structures [1, 2, 4]. However, the combinatorial auctions in electricity markets differ from the ones studied in economics [18, 19] and supply chain management [20-22] literature due to a number of key features, including (*i*) the presence of complex and paradoxical orders and (ii) the participation of multiple buyers and sellers in the auction. As a result, special solution methods ranging from genetic algorithms [2] to exact decomposition methods [4, 23-26] and iterative heuristics [4] are developed to tackle the combinatorial auctions in electricity markets.

The literature captures the combinatorial nature of the problem through variants of the Mixed Integer Programming (MIP) models, such as Mixed Integer Linear Programs (MILP), Mixed Integer Quadratic Programs (MIQP) or more general Mixed Integer Non-Linear Programs (MINLP). Most solution methods for the DAM clearing problem (exact and heuristic approaches) aim to exploit the special problem structure. For instance, one would formulate a Mixed Complementary Problem to manage a wide range of complex order types [5]. As another example, one would exploit the block order structures by developing a genetic algorithm where the blocks correspond to genes [2]. Exploring the specific structure of the problem may result in discovering hidden characteristics like convexity and total unimodularity (among others), thus leading to more efficient solution methods.

In this paper, our aim is to provide a review of the latest DAM designs used in Europe as well as the academic literature developed to model and clear those DAMs. For a recent and more general review of the operational and policy subjects in the electric power industry see [27]. The focus of this review article is solely on European DAMs. Our contributions to the literature can be summarized as follows:

- We provide a systematic categorization of the recent DAM optimization models based on the order types, constraints and alternative objective functions. We highlight the drivers of the problem complexities including paradox orders, integer variables and non-convexities in the objective function.
- We categorize and discuss the existing exact and heuristic solution methods developed for clearing DAMs. We emphasize the need for heuristic approaches given the practical time limits to clear the markets and we suggest ways to explore the problem structure to develop new heuristic methods.
- We identify and discuss important open research questions related to both designing and clearing of European DAMs. We believe that our systematic articulation of the literature will provide both the academicians and practitioners with valuable insights and guidelines on how to design and clear DAMs as the electricity markets keep evolving to address the ever-changing needs of the market participants.

The rest of the paper is organized as follows: Section 2 provides the details of DAM formulations including the order types, objective functions and handing paradox orders in different markets. Section 3 is dedicated to the review of the solution methods used in the literature, and in Section 4 we discuss future research opportunities.

2. Formulation: Orders, Objectives and Handling Paradox Orders

In this section, we explain the order types, handling paradox orders, and objective functions that the DAMs use. We first define the concept of different order types and discuss how each order varies in terms of complexity. Next, we elaborate on paradox orders and demonstrate the basic challenges concerning such orders by providing a detailed illustrative example. Finally, we present various objective functions, the comparison between each formulation, and the advantages of each approach.

2.1 Order Types

Participants of the DAM auctions place orders to buy or sell a specific amount of energy during a particular hour or (hours) of the next day. The most common approach to trade electricity in DAMs is the *hours orders* (see e.g., [4, 25, 28, 29]). Hourly orders are represented by multiple quantity-price pairs, which are basically piece-wise (or step-wise) linear functions which determine the level of quantity that the bidder is willing to trade at a given price. With these orders, the bidder aims to trade electricity for a single specific hour of the next day; the order may be partially accepted if its price condition is satisfied. Hourly orders are the simplest form of orders, among others, and a DAM auction solely consisting of these orders is linear in decision variables, thus it can be trivially solved using a greedy approach. Single orders make up around 75-80% of the total volume of a typical DAM. Figure 4 presents an example of an hourly order with piece-wise linear interpolation. If the MCP is 310 TL, the accepted volume of the hourly order is 180MW in Figure 4.

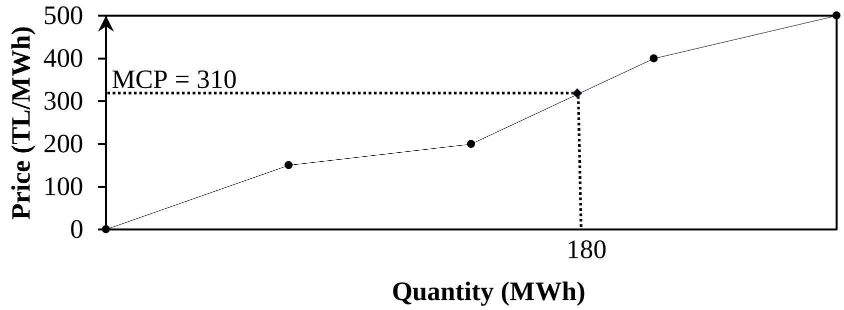

Figure 4. An example price-quantity function for hourly orders

The second most common orders are *Block orders,* which are the main complicating factor when clearing DAM auctions [30]. The clearing rules for such orders introduce an important dimension of complexity to the DAM clearing problem (see e.g., [10, 29, 31, 32]). In practice, complementarity effects across multiple consecutive operating hours require that some orders are placed as blocks covering multiple hours. Most coal-powered and natural-gas-powered plants need to make expensive setups to begin and cease operations. Hence, it is desirable for those plants to minimize the number of startups via block orders during their operations. Naturally, these orders have a *fill-or-kill property*; that is, either the total order amount is accepted or the order is not accepted at all. This operating behavior is captured through block orders in the DAM auctions. Acceptance of block orders may also be conditional on the acceptance of other blocks (i.e., linked blocks) which further complicates the combinatorial auction. For instance, in Figure 5, none of child blocks A.1, A.2 and A.3 would be accepted unless the parent block A is accepted. Linking block orders helps generators reflect their thermal generation capabilities when bidding in the market.

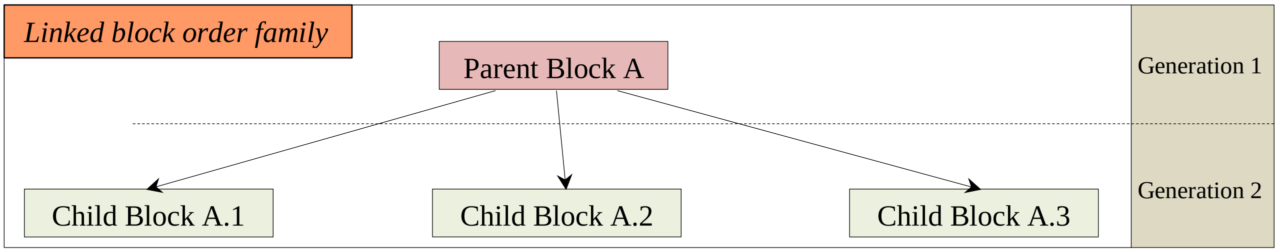

Figure 5. An example of a parent block linked with three child blocks

Some European exchanges allow for orders called *Exclusive Blocks* [3], *Block with minimal acceptance ratio (MAR)* [10] and *Convertible Block* [33]. Exclusive blocks consist of a set of block orders among which only one block may be executed. The execution of a block order in an exclusive group depends on whether it is in the money and its contribution to the objective function. These types of exclusive blocks are frequently used in thermal power plants and combined heat and power installations because their flexibility is an imperative factor in considering price fluctuation outside of peak periods [34]. *Block with MAR* orders specify a minimal quantity through MAR such that the quantity accepted for this order should be greater than equal to this minimal quantity. In general, MAR corresponds to the minimum stable output levels of thermal power plants. In the special case of blocks with a MAR of one, it simply indicates a regular fill-or-kill block order [14]. Lastly, convertible blocks are basically block offers with eligibility for conversion to hourly offers if they are not cleared as a block at the problem solution and if the required clearing price is achieved in at least one hour in the specified block period [35].

Another order type is *Flexible orders*, which are similar to hourly orders except that they do not specify a trading hour (so that they can be considered for any hour of the day) [4]. On the other hand, they also have a fill-or-kill property possibly leading to paradoxical situations as in the case of block orders. Supply flexible orders are accepted if the highest MCP of the day is larger than the order price [2].

Some DAMs in Europe have specific order types which reflect the market dynamics of that country, such as the *Complex Orders* which the French market uses [9] and the *Prezzo Unico Nazionale (PUN) Orders* of the Italian market [5]. A complex order consists of simple step-wise hourly orders with a Minimum Income Condition (MIC) or a Load Gradient Condition (LGC), with or without a scheduled stop condition [13]. The MIC simply refers to the coverage of the costs by the amount of money collected in all periods. If the condition is not satisfied, the entire MIC order set is rejected. The literature argues that the presence of orders with MIC necessitates the use of iterative methods when clearing the market [10]. LGCs are complex orders with their set of hourly sub-orders along with a load gradient constraint. On the other hand, PUN orders are the peculiar demand merit orders that mainly the Italian market uses. These orders are cleared at the PUN price where all customers pay the PUN, calculated as the demand-weighted average of zonal prices [5]. Figure 6 illustrates a simplified overview of various order types in the DAM.

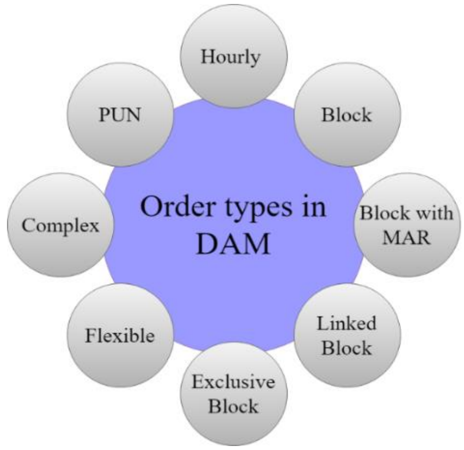

Figure 6. Order Types in DAM.

Table 1 summarizes the order types that can be found in European DAMs and how they affect the modeling of the DAM clearing problem. As indicated in Table 1, while hourly bids are the simplest order type that can be modeled linearly, block, flexible, and linked orders can be modeled using integer programming only. Block with MAR, exclusive, and complex orders, on the other hand, can be modeled using mixed-integer linear programming. Finally, the most complicated order type is PUN order where mixed complementarity problems should be used.

Table 1. Summary of Order Types

Order Type	Modeling	Acceptance Rule	Frequently Using Participants	Covering Hours
Hourly	Linear	Partial	Traders, Hydro, Renewable	Single
Block	Integer	Fill-or-Kill	Traders, Coal, Natural gas, Hydro	Multiple
Flexible	Integer	Fill-or-Kill	Traders, Hydro	Single
Block with MAR	MILP	Partial and Fill-or-Kill	Natural gas	Multiple
Linked	Integer	Fill-or-Kill	Traders, Natural gas	Multiple
Exclusive	MILP	Partial and Fill-or-Kill	Traders, Natural gas	Multiple
Complex	MILP	Partial and Fill-or-Kill	Renewable	Multiple
PUN	MCP	Partial	Natural gas	Multiple

The problem of determining the DAM clearing price would be simple if all orders were placed as hourly orders. However, the presence of various orders (block, flexible, linked, exclusive, etc.) introduces an extra complication to the problem where each order brings its own brand of complexity. For instance, block orders introduce integrality requirements and non-convexity to the problem, making it very difficult to solve. Moreover, complex orders require separate ex-post tests and iterative methods to clear them that further complicate the problem. From an algorithmic perspective, in the presence of exclusive blocks, strong duality fails and it would be impossible to obtain a market equilibrium with linear prices [10]. In addition, the clearing conditions of MIC and PUN orders require an iterative procedure [5, 13]; this also complicates the market clearing problem accordingly. Finally, the standard clearing rules for block and flexible orders result in paradoxical situations; efficiently resolving these paradoxes lead to multi-criteria problems [9].

Table 2 summarizes the DAM literature based on the order types considered in the literature. We would like to note that early papers in this field mostly considered basic order types such as hourly, block and flexible. However, as markets develop and as market participants demand more flexibility to reflect their consumption and generation constraints, more complicated orders have been introduced to the DAM auctions. For example, exclusive blocks are modeled by [13] and linked blocks are considered by [4]. So far, only one study in the literature considers blocks with MAR [10]. In addition, studies exploring the Italian market consider PUN orders [5] and multiple recent studies consider complex orders [11].

Table 2. Summary of Literature Based on Order Types

Article	Stepwise hourly	Interpolated hourly	Block	Block with MAR	Linked blocks	Exclusive blocks	Flexible	Complex	PUN
[31] O'Neill et al. (2005)	✓		✓						
[36] O'Neill et al. (2007)	✓		✓						
[23] Basagoiti et al. (2008)	✓		✓					✓	
[37] Meeus et al. (2009)	✓								
[29] Meeus et al. (2009)	✓		✓						
[12] Van Vyve (2011)	✓		✓						
[33] Biskas et al. (2014a)	✓		✓		✓		✓	✓	
[28] Madani and Van Vyve (2014a)	✓		✓						
[38] Tjeransen and Rudlang (2014)	✓		✓						
[9] Madani and Van Vyve (2014b)	✓		✓					✓	
[4] Martin et al. (2014)	✓	✓	✓		✓		✓		
[25] Madani and Van Vyve (2015)	✓	✓	✓						
[32] Derinkuyu (2015)		✓	✓		✓		✓		
[13] Dourbois and Biskas (2015)	✓		✓		✓	✓	✓	✓	
[10] Chatzigiannis et al. (2016)	✓		✓	✓	✓	✓	✓	✓	
[5] Dourbois and Biskas (2017)	✓		✓				✓	✓	✓
[11] Madani and Van Vyve (2017)	✓		✓					✓	
[39] Madani and Van Vyve (2018)	✓		✓					✓	
[6] Dourbois et al. (2018)	✓		✓				✓	✓	✓
[3] Lam et al. (2018)	✓	✓	✓		✓	✓	✓	✓	✓
[2] Derinkuyu et al. (2020)	✓	✓	✓		✓		✓		
[44] Zak et al. (2012)	✓		✓						

2.2 Handling Paradox Orders

The handling of paradoxical situations is one of the main design challenges of DAMs. Paradoxical situations may arise when executing intuitive economic rules for clearing block and flexible orders. For example, an auction rule such as fully accepting a block order that is in-the-money and fully rejecting a block order that is out-of-the-money may result in paradoxical situations [2, 11, 40, 41]. In Table 3, we provide an illustrative example with four hourly orders and one block order valid for hours 7 and 8.

Table 3. Illustration of a Paradoxical Situation

Hourly Orders

Trade Position	Trading Hours	Price (TL/MWh)	0	100	150	170	180	190	200
Buy	Hour 7	Quantity (MWh)	100	90	80	70	60	50	0
Sell	Hour 7	Quantity (MWh)	0	30	60	70	100	150	250
Buy	Hour 8	Quantity (MWh)	100	90	80	70	60	50	0
Sell	Hour 8	Quantity (MWh)	0	30	60	70	100	150	250

Block Bid

Trade Position	Trading Hours	Price (TL/MWh)	Quantity
Sell	Hour 7 and 8	150	60

When clearing this auction in Table 3, suppose the block order is rejected. Then, the equilibrium price and quantity for hour 7 are illustrated in Figure 7 (the situation is identical for hour 8 and is omitted for brevity).

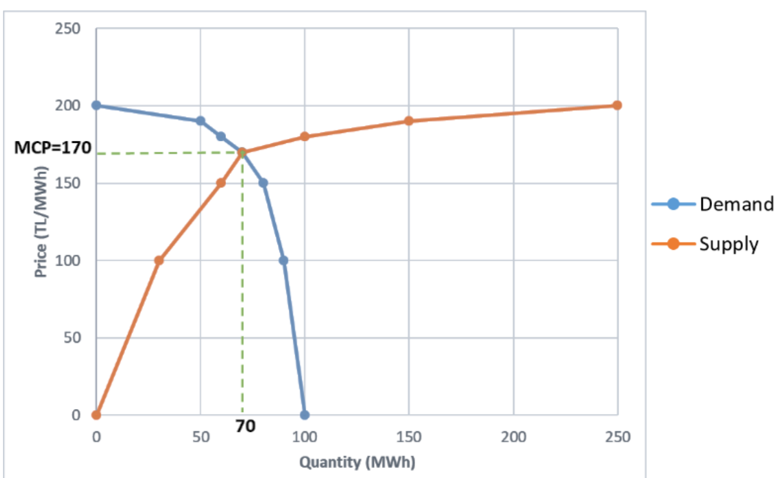

Figure 7. Clearing of the auction in Table 3 when the block order is rejected

The market-clearing price in this case is 170 TL and the clearing quantity is 70 MWh for both hours. However, recall that the price of the block order is 150 TL to sell electricity during hours 7 and 8.

This implies that the block should be accepted because it is in-the-money (i.e., its price is below the average clearing prices of hours 7 and 8). Now, suppose the block order is accepted. In this case, the block order shifts the supply curve and we reach a new equilibrium as depicted in Figure 8.

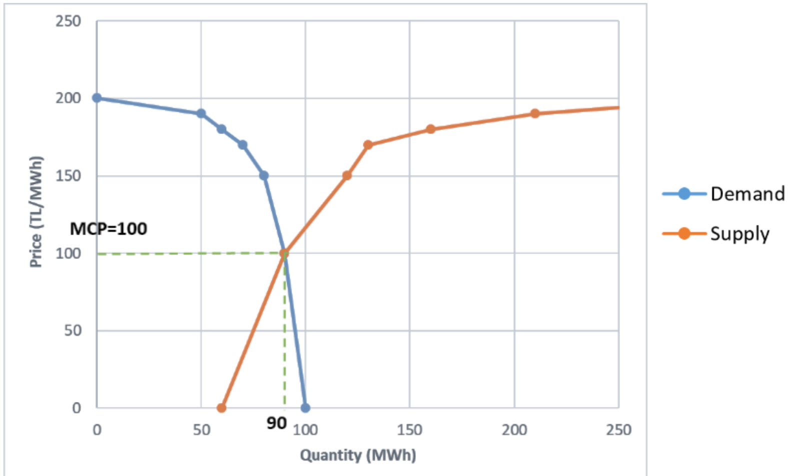

Figure 8. Clearing of the auction in Table 3 when the block order is accepted

The MCP is 100 TL in this new equilibrium and the clearing quantity is 90MWh. The new equilibrium, however, implies that the block bidder incurs a loss as its price is now above the MCP and the order has to be rejected. This leads to a paradox as neither accepting nor rejecting the block is feasible. To resolve this paradox and obtain a feasible solution, DAMs can relax the block acceptance rule in two possible ways. The Turkish DAM allows for paradoxically accepted blocks (PABs) (i.e., as in the latter case above, a block may be accepted although it is not in the money). In this case, the loss incurred by the block bidder is compensated through side payments collected from market participants.

The other alternative to resolve paradoxes is to allow paradoxically rejected orders (PRBs) (i.e., rejecting a block order which is in-the-money). Allowing for PRBs is adopted by most of the European exchanges including NoordPool and CWE (but with the exception of Energy Exchange Istanbul). Unlike the PAB rule, the PRB rule does not result in side payments as the bidder of the rejected order does not produce or consume energy. However, the PRB rule results in lost revenue as the bidder of the rejected order loses the opportunity to make money [9]. Overall, neither PABs nor PRBs are desirable situations for the exchanges, and minimizing the volume of such orders appears as an implicit side objective in many European markets.

The exploration of the impact of paradox order rules on the operations of DAMs is an important research gap in the literature. Except [2, 12, 28], the literature on DAM auctions is mostly mute on the impact of paradox order rules on the total welfare, problem complexity, solution time and bidder behavior.

In a market with a significant volume of PABs or PRBs, the market-clearing price would not be meaningful. Hence, it is necessary to understand the impact of such paradox rules and to minimize the volume of such orders. In Section 4, we provide a detailed discussion of this matter.

2.3 Objective Functions

Maximizing market surplus is the most widely accepted objective function in DAM auctions; it is the sum of all producer and consumer surpluses [4, 36, 39]. Here, the *producer surplus* refers to the cumulative difference between the total sales revenue and the order price of the producer. Similarly, *consumer surplus* is defined as the cumulative difference between the consumer's total willingness to pay and the cost of purchasing energy [2]. As illustrated in Figure 9, the total surplus corresponds to the region between the demand and supply curves to the left of the market equilibrium point.

The objective of maximizing the total market surplus is non-linear in decision variables [25]. Besides the total surplus, the literature also considers other objective functions such as price minimization [32]. Under the price minimization objective, the resulting problem is linear in the decision variables and,

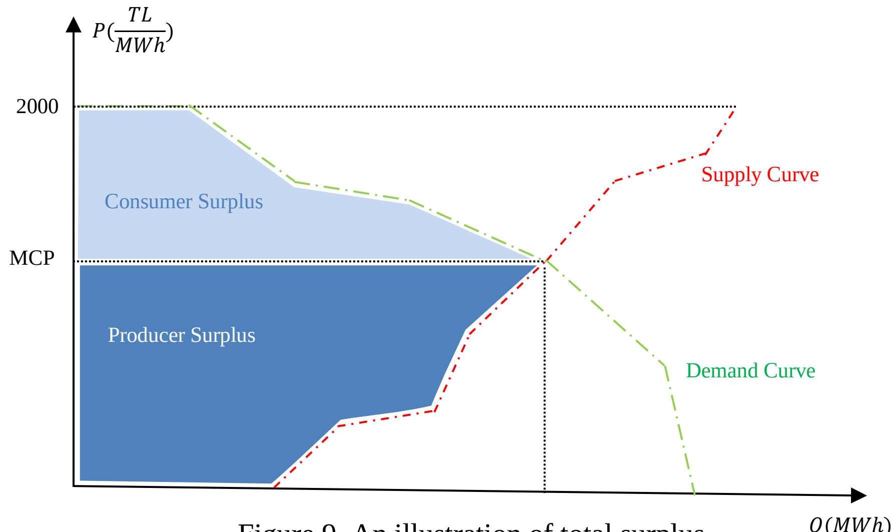

Figure 9. An illustration of total surplus

hence, easier to solve. This objective may be desirable by policy makers who are primarily interested in minimizing market prices. However, minimizing market prices may be detrimental to the overall economy as it unfairly favors the demand side, and hurts the supply side, of the market. The Turkish DAM operator had been using this objective function prior to 2017.

Some research in the literature considers multi-objective approaches for modeling and clearing DAMs. While maximizing the total surplus, DAMs also aim to minimize the impact of paradox orders on the market due to PAB or PRB rules. For example, [12] proposes a new market model that maximizes welfare and minimizes uplift roughly. Here, the term *uplift* refers to losses and foregone profit opportunities due to PRBs. Similarly, [11] explores the objectives of maximizing the traded volume and minimizing the opportunity costs of PRBs. They find that these objectives may result in significantly different equilibriums as compared to maximizing market surplus in a non-convex context. Their formulation relies on an exact linearization of MICs, which makes it possible to examine the tradeoff between various objective functions. Dourbois et al. [6] extend the objective of market surplus by considering the welfare loss due to congestion rents. Finally, Madani and Van Vyve [9] compare and contrast the following three objective functions for DAMs: Surplus maximization with complex MIC orders, the traded volume maximization and opportunity costs minimization. Their numerical experiments indicate significant improvements on the upper bound on the maximum traded volume when the welfare maximization and traded volume maximization problems are solved consequently. Table 4 summarizes the literature based on the used objective functions. While the majority of articles incorporate surplus maximization into objective functions, a few papers employ other approaches. For instance, opportunity costs arising due to PRBs and trade volume have also become important side objectives in DAM auctions and they are considered by a number of recent articles. The idea of price minimization, however, is an outdated idea which lacks a clear economic justification, and hence this objective is abandoned by both the academics and practice alike.

Table 4. Summary of Objective Functions Used in DAMs

Article	**Surplus Maximization**	**Price Minimization**	**Opportunity cost minimization**	**Congestion rent minimization**	**Maximum uplift minimization**	**Traded volume maximization**	**Total offer cost minimization**
[31] O'Neill et al. (2005)	✓						
[36] O'Neill et al. (2007)	✓						
[23] Basagoiti et al. (2008)	✓						
[37] Meeus et al. (2009)	✓			✓			
[29] Meeus et al. (2009)	✓						
[12] Van Vyve (2011)	✓				✓		
[33] Biskas et al. (2014a)							✓
[28] Madani and Van Vyve (2014a)			✓				
[38] Tjeransen and Rudlang (2014)	✓						
[9] Madani and Van Vyve (2014b)	✓		✓			✓	
[4] Martin et al. (2014)	✓						
[25] Madani and Van Vyve (2015)	✓						
[32] Derinkuyu (2015)		✓					
[13] Dourbois and Biskas (2015)							✓
[10] Chatzigiannis et al. (2016)	✓						
[5] Dourbois and Biskas (2017)	✓						
[11] Madani and Van Vyve (2017)	✓		✓			✓	
[39] Madani and Van Vyve (2018)	✓						
[6] Dourbois et al. (2018)	✓			✓			
[3] Lam et al. (2018)	✓						
[2] Derinkuyu et al. (2020)	✓						
[44] Zak et al. (2012)	✓						

3. Solution Methods

In this section, we review the main solution methodologies that are used to clear DAM auctions. The solution to the DAM clearing problem gives the set of accepted order volumes and the market price through which the accepted orders are cleared. Hence, the problem can be seen as choosing the best market-clearing prices along with a method to address the paradoxical orders. For hourly orders, the accepted quantity can be linearly interpolated between the two relevant break points of the order. However, for block orders the quantity offered is fixed and one needs to make a fill-or-kill decision, which in general is modeled by a binary decision variable taking the value of 1 or 0 for filling and killing, respectively. The existence of these binary decisions results in a non-convex model and hence,

i. One cannot resort to the readily available efficient methods for convex and, more specifically linear, programming. The problem has a combinatorial nature and needs to be addressed using the tools of integer programming, which are in general less efficient than its continuous counterparts.

ii. In the absence of binary variables, one would be able to use the strong duality of convex programming to calculate the market-clearing price at each hour as the dual variable (Lagrange multiplier) associated with the flow balance constraints. However, the integer variables introduce a duality gap and render this direct approach inapplicable.

For DAMs, it is crucial that a good solution, preferably the best solution, is computed within a short time (generally measured with minutes). There has been a considerable effort to devise fast methods to solve the DAM clearing problems. The literature on solution methods can initially be classified into two as (*i*) exact methods, which aim to find the optimal solution and (*ii*) heuristic methods, which aim to find an acceptable solution in a reasonable time. A sketch of our classification of the existing methods along with some notable papers is given in Figure 10. In the next two sections, we discuss the exact and heuristic solution methods used in the literature, respectively.

3.1 Exact Solution Methods

The majority of the research in the literature has used exact methods to clear DAM auctions. We further classify the exact methods into five main types: (*i*) Direct methods, (*ii*) Iterative approaches, (*iii*) Primal-Dual methods, (*iv*) Decomposition methods and (*v*) other methods that do not fit in the first four types.

(i) Direct Approaches

Early work on the European DAM clearing problem relies on commercial solvers such as *IBM CPLEX* to solve the MIP formulation under consideration. This approach is not efficient enough to deal with the size of the problems which arise in real life; nevertheless, there are valuable insights that one can obtain from this early work. For example, O'Neill et al. [31] explore the primal and dual versions of the DAM clearing problem, focusing on the duality gap in the presence of block orders. They show that once the integer programming model is solved (fixing the integer variables to their optimal values), one can get a linear program whose dual variables provide the market-clearing prices.

Meeus et al. [29] provide an MILP formulation for European DAM auctions and analyze how restrictions on paradoxically rejected block orders affect the solution time. They perform a detailed numerical analysis based on simulation in order to calculate the proportion of paradoxically rejected orders. They find that there is no significant correlation between block size and paradoxically rejection probability unless all block sizes are close to the maximum allowed size.

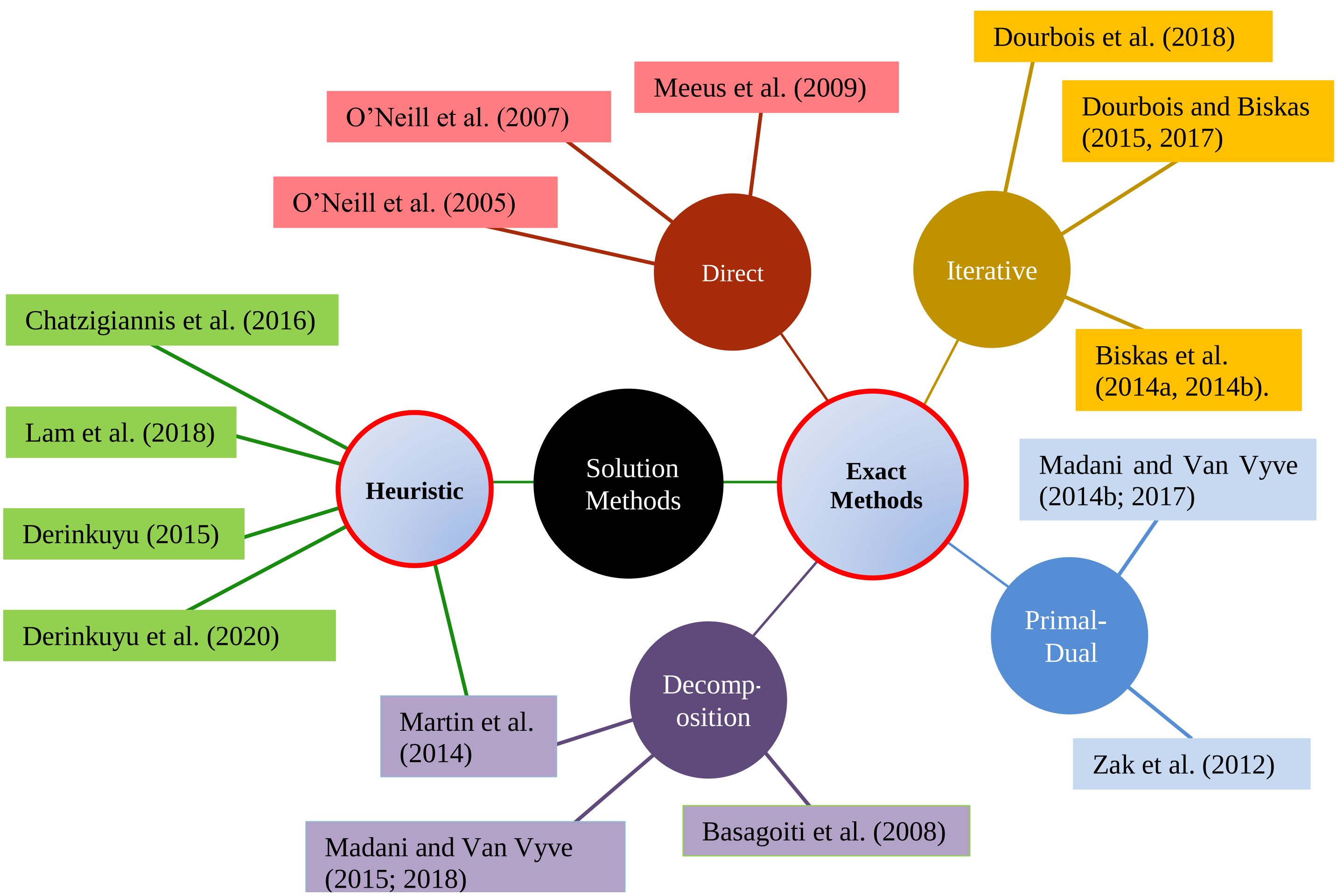

Figure 10. Graphical representation of various methods used in DAM auctions

(*ii*) *Iterative Approaches*

This class of methods addresses the DAM clearing problem by first relaxing the complicating aspects of the problem (such as fill-or-kill conditions, flexible orders, etc.) and solves a simpler problem. After one obtains an initial approximate solution then the complicating constraints are imposed on the problem in an iterative manner to achieve feasibility (e.g., to handle paradox orders as well as PUN and MIC orders). An earlier example of such an iterative approach is the *COSMOS* market coupling algorithm used by the Central Western Europe (CWE) project (see e.g., [42]) prior to switching to *EUPHEMIA* algorithm. The underlying idea in *COSMOS* was to solve a quadratic problem by first completely disregarding the fill-or-kill structure of the block orders. Then, the solution at hand is modified to guarantee that the block orders are handled properly. *EUPHEMIA*, the algorithm currently in use in European exchanges, also has iterative features even though it can be better classified as a decomposition algorithm (as discussed below).

One of the first iterative methods in the literature is proposed by Biskas et al. [33, 43], where a new formulation is introduced in the former work and a detailed solution methodology is discussed in the latter. The iterative approach in [43] first solves an MILP to attain MCPs for each hour of the trading period, and then it iteratively identifies and removes paradoxical orders. The authors also present a comprehensive numerical study which tests the practical applicability of their approach. Extending [43], Dourbois et al. [13] model all the relevant order types and constraints present in the current European DAMs and provide an iterative algorithm to clear the market. Their algorithm solves an MILP and iteratively eliminates the possible paradoxically accepted blocks while also checking the clearing conditions of the MIC orders (PRBs are allowed in the model). One of the main contributions of this paper is to incorporate the full functionality of *EUPHEMIA* (except for modeling of PUN orders) to the model. Authors test their model using similar-sized auctions to European DAM coupling.

Dourbois and Biskas [5] develop another iterative algorithm for clearing DAM auctions with multiple bidding zones. Their method consists of two main steps. In the first step, the MCPs are calculated for every bidding zone and each trading period, where the clearing conditions of the block, PUN and MIC orders are considered. The non-intuitive bilateral exchanges[1] are also incorporated into the market clearing process. Next, similar to [43], the existence of non-intuitive bilateral exchanges and paradoxically accepted orders are checked sequentially.

Another paper which combines iterative approaches with decomposition methods is Dourbois et al. [6]. This work presents three novel methods for clearing the European DAM coupling. The first iterative

[1] A non-intuitive bilateral exchange occurs if a local bidding zone with a higher price transports energy to a local exchange with a lower price. [6]

method incorporates solving a master problem and a sub-problem iteratively. The solution of the master problem returns the clearing status of block and flexible orders. The values of these decision variables are passed to the sub-problem, providing upper bounds for the corresponding continuous decision variables. Next, the algorithm solves the sub-problem to compute the cleared quantities of all orders for every bidding zone and period. Similar to previous iterative methods, the algorithm performs a couple of sequential checks to determine the existence of non-intuitive bilateral exchanges, PABs, MIC and PUN orders. The other two methods discussed in [6] employ a single-stage optimization model, where a Mixed Complementarity Problem is solved with and without considering the non-intuitive exchanges.

(*ii*) *Primal-Dual Modeling Approaches*

The DAM clearing problem requires that one determines the accepted quantity for each order (the set of paradoxically accepted or rejected orders). All these quantities appear as the ‘primal variables’ of the MIP formulations in the literature. On the other hand, another important quantity is the market clearing price for each hour, which can be obtained as the dual variables. This interaction between primal and dual formulation of the problem is used to come up with models that can be efficiently solved with commercial software.

One of the earliest models to make use of the primal-dual nature of the problem is Zak et al. [44], which propose a bi-level formulation for DAMs where the primal and dual variables are incorporated into the model to consider the primal-dual market rules. The upper-level and lower-level problems contain an MIP and a parametric dual linear problem, respectively. The latter one corresponds to the upper level problem where all binary variables are fixed. This bi-level formulation is transformed later into a single level MIP because of the duality relationship between upper- and lower-level formulations. Commercial MIP solvers are able to solve this problem to optimality.

Madani and van Vyve [9] consider the European DAM coupling problem with complex and block orders. They propose a primal-dual approach, which takes full advantage of the parallel computing features of commercial solvers. Such a multi-computing feature enables them to investigate the trade-off between various objectives such as total surplus and trade volume maximization. The authors find that the solution for the surplus maximization problem does not necessarily maximize the total traded volume. However, it provides a good heuristic. They also provide an exact linear formulation of non-convex MIC orders without using auxiliary variables. As one of their key contributions, the authors show the possibility of effectively using parallel computing routines of state-of-the-art solvers for DAM clearing problems.

In a more recent work, Madani and van Vyve [11] extend [9] and provide MILP formulations of DAM clearing problems with the objectives of (*i*) maximizing trade volume and (*ii*) minimizing the opportunity costs of PRBs. The authors clearly illustrate that these two objectives result in different solutions as compared to maximizing total surplus. They also provide extensive numerical analysis illustrating the efficiency of their proposed solution approaches.

(*iii*) *Decomposition methods*

In another line, models have been proposed to decompose the clearing problem into a master problem and smaller sub-problems. The problem is solved iteratively by combining the results of these smaller sub-problems using techniques such as cutting planes. One of the first decomposition methods proposed in the literature is by Basagoiti et al. [23]. They suggest a method relying on Lagrangian Relaxation to decompose the problem into smaller problems. Their model assumes the existence of block orders, but the other complicated order types are not considered. Martin et al. [4] are one of the first to propose a decomposition type approach for the market-coupling problem between European day-ahead electricity exchanges. They introduce two types of sub-problems: (*i*) the FixFlow problem, which is a quadratic programming problem to take care of the flow of electricity and (*ii*) the QPPrice problem, where a clearing price for each hour is determined again using a quadratic model. They then use cutting planes to solve the master problem.

Meeus et al. [37] suggest an MILP formulation to model stepwise order curves, and a mixed-integer quadratically constrained program (MIQCP) with non-linear convex quadratic constraints to model piecewise-linear order curves. For the MILP formulation, they take full advantage of the power of a commercial solver (*CPLEX* 12.5) to solve large-scale problem instances to optimality. However, in the presence of piecewise linear order curves, *CPLEX* fails to solve the resulting MIQCP. Concerning this non-linear formulation, the authors devise a Benders-like decomposition procedure and strengthen the classical Benders infeasibility cuts. Madani and van Vyve [25] provide a reformulation of the European market as an MILP for the determination of the DAM prices in the presence of stepwise order curves. Their new formulation relies on the strong duality theory rather than using complementarity constraints and auxiliary variables in [37].

In a more recent work, Madani and van Vyve [39] examine another tractable formulation for the DAM clearing problem to handle the situations where every bidder is allowed to state a minimum profit condition. The presence of both block orders with a minimum acceptance ratio and minimum profit requirements adds significant complexity to the problem. Their proposed method is similar to the one used in [25]. It is based on a Benders-like decomposition algorithm where cuts are generated within the branch and bound tree. They have successfully used Benders decomposition with strengthened cuts as described in [25] to handle newly introduced minimum profit orders on large problem instances. The main feature of this method is that it avoids generating cuts after each iteration, because solving the augmented master problem to optimality is computationally expensive.

As mentioned above, the algorithm used by most European exchanges, *EUPHEMIA*, also employs a decomposition-based approach to incorporate a variety of problem features. *EUPHEMIA* considers a master problem that aims to maximize the social welfare along with three types of sub-problems, price determination, PUN Search and Volume Indeterminacy sub-problems. The price determination sub-problem deals with setting up the market-clearing price while considering the paradoxically accepted and rejected orders. The PUN Search sub-problem sets traded PUN volumes and prices. The final Volume Indeterminacy sub-problem determines the actual traded volume which is compatible with the results of previous sub-problems. The decomposition is combined with the integer nature of the problem using a branch-and-cut approach.

3.2 Heuristic Solution Methods

Clearing DAM auctions is a difficult problem for which commercial solvers are not guaranteed to find an optimal solution or even a feasible solution within the time limits of the auction [2]. Most of the market operators need to find a high-quality solution in a short time, usually less than one hour. This practical situation makes using heuristics a necessity for market operators. The heuristic methods developed in the literature usually spin around either iterative or neighborhood search approaches. Iterative methods are particularly desirable to clear paradoxical blocks, PUN orders and orders with MIC.

By exploiting their decomposed model structure, Martin et al. [4] suggest an Iterative Bid Cut heuristic to clear the market coupling problem in Europe. The authors first relax all the pricing constraints to obtain a relaxed mixed integer quadratic program (MIQP). The solution to this problem may violate the PRB rule; to restore feasibility, the authors iteratively solve a pricing problem that includes additional constraints (bid cut) that cuts off the infeasible bid selection. The iteration terminates when a feasible selection is found. On average, the heuristic finds a high-quality solution in less than five seconds. Empirical tests suggest that the solutions determined by the heuristic are optimal in many cases

Similar to [4], Chatziginannis et al. [10] also consider the market coupling problem in Europe. They extend [4] by including nearly all market details (including profile block orders and exclusive blocks). They also consider the line set power flow as well as ramping constraints. The authors provide an iterative heuristic similar to [36], which eliminates PABs and checks the clearing conditions of the orders with MIC. The authors argue that the presence of orders with MIC necessitates the use of iterative methods when clearing the market.

Derinkuyu [32] models the Turkish DAM auction with the objective of minimizing price, which is linear in decision variables. He provides a multi-step heuristic to find a good solution. First, a constructive heuristic relaxes the complicating block and flexible order clearing rules and solves a linear program. The algorithm then sequentially adds the block and flexible orders based on the current MCP and order prices. The clearing conditions of the linked orders are checked until a feasible solution is obtained. Next, the feasible solution obtained by the constructive heuristic is improved by an IP-based neighborhood search. The final feasible solution is fed to a commercial solver to obtain further improvements. The paper shows that the final feasible solution fed to the commercial solver is optimal for 45 out of 56 problem instances. Using this initial solution, the commercial solver proves optimality for the 53 out of 56 problem instances, and for the remaining three instances the worst optimality gap is only 2.7%. Overall, the multi-step heuristic proves to be effective in providing good feasible solutions.

Lam et al. [3] present an iterative heuristic to handle the nonconvexity due to the fill-or-kill condition of block, complex and PUN orders. The authors formulate the problem as an MIQCP and accommodate for the piecewise-linear orders. They use an iterative approach to reduce the number of paradoxically rejected orders. They verify the robustness of their model through two case studies.

More recently, Derinkuyu et al. [2] model the most recent auction rules for the Turkish DAM and develop heuristics based on the tabu search and genetic algorithms to maximize the total surplus for the market. These algorithms are currently used by EXIST to clear the Turkish market. The paper also provides performance benchmarks based on real market data. They show that the Turkish market operator generates significantly more total surplus using their heuristics. Although the commercial solver fails to generate even a feasible solution for 4.10% of all test instances, the authors' heuristics are guaranteed to generate a feasible solution. The authors further test the robustness of their results under the PRB and PAB rules as well as under very large problem instances (as compared to the current market size). They find that the effectiveness of heuristics relative to the commercial solver substantially increases with problem size. In addition, they observe that under the PRB rule the heuristics perform better. The authors also note that the tabu search usually performs better than genetic algorithms when it is difficult to find an initial feasible solution.

Table 5. Summary of solution methodologies presented in each article

	Solution Methodology	
Article	**Exact**	**Heuristic**
[36] O'neill et al. (2007)	✓	
[23] Basagoiti et al. (2008)	✓	
[43] Biskas et al. (2014b)	✓	
[9] Madani and Van Vyve (2014b)	✓	

[4] Martin et al. (2014)	✓	✓
[28] Madani and Van Vyve (2014a)	✓	
[25] Madani and Van Vyve (2015)	✓	
[32] Derinkuyu (2015)		✓
[13] Dourbois and Biskas (2015)	✓	
[10] Chatzigiannis et al. (2016)		✓
[11] Madani and Van Vyve (2017)	✓	
[5] Dourbois and Biskas (2017)	✓	
[39] Madani and Van Vyve (2018)	✓	
[6] Dourbois et al. (2018)	✓	
[3] Lam et al. (2018)		✓
[2] Derinkuyu et al. (2020)		✓

4. Discussion and Directions for Future Research

The goal of this paper is to review the recent literature on the DAM clearing problem in the European electricity markets with a comparative view in mind to suggest promising future research directions. In this section, we point out gaps in the literature where we think there is a great potential to contribute to the literature and practice. Based on our discussion above, we present these ideas in four parts. The first part discusses the ideas to improve the problem formulation and suggests some potential ideas for designing efficient solution methods. In the second part, we focus on handling paradox orders and their impact on market clearing as well as the bidding behavior of market participants. Then, we discuss some empirical research questions exploring the impact of market design and bidder behavior. Finally, in part four, we suggest new ideas for more efficient DAM design to better reflect the needs of market participants.

A) Formulation and Solution Methods

As presented in Section 2, the DAM clearing problem presents many challenging features for modeling and optimization. First, the problem needs to handle hourly orders covering a 24-hour time period, and these periods are connected by the existence of complex orders such as block, linked and flexible orders. The fundamental fill-or-kill property of the block orders imposes a non-convexity expressed by binary decision variables in the models. Even though there is a significant amount of research in decomposing the problem and handling the nonconvex structure, we believe that there is still significant room for improvement to run the market more efficiently. First, we believe that decomposition methods which solve each hour as a sub-problem and combine the results of these sub-problems in a master problem is one of the potential ways to reduce the solution times. The key point here is to design decomposition strategies that end in efficiently solvable, small sub-problems.

To handle the nonconvexity due to binary decisions, many heuristic methods in addition to exact methods are proposed in the literature. With the exception of [2], the heuristics proposed are solution strategies designed specifically to exploit the structure of the DAM problem (i.e., they rely on 'human-inferred properties' of the problem under consideration). Generic heuristics, such as Variable Neighborhood Search, Adaptive Large Neighborhood Search and SimHeuristic, aim to solve a problem while automatically learning the structure of the problem through iterations and, to the best of our knowledge, have not been used to address the DAM clearing problem. We believe that these *metaheuristics* can yield fruitful results in addressing the binary structure of the DAM clearing problem. For instance, it may be convenient to model the sequence of block order decisions as chromosomes in Genetic Algorithm rather than modeling it in a trajectory-based heuristic. Using granular neighborhood search methods may also enhance the performance of such generic methods when clearing the DAM auctions.

The current methods for solving the DAM clearing problem rely on the integer programming formulation, which uses the quantities accepted for each order as the decision variables. This method has the advantage of dealing with balance constraints explicitly. A more drastically different approach can be to consider the dual of this problem as the main model, and think of clearing prices for each hour as the decision variables. The most challenging aspect of this approach would be to determine paradoxically rejected and accepted block orders given a clearing price, as it is not trivial to determine what happens with each block order. If an efficient mechanism is available to determine the best strategy to accept and reject the block orders given the clearing price, then the 'master' problem can be stated as a nonconvex model with only 24 decision variables. Methods of global optimization will be available to address the problem.

Machine Learning (ML) may also be used for generating initial feasible solutions. Apparently, the structure of the optimal clearing prices is closely related to certain features of the market orders for a given day. For instance, accepting block orders whose bid prices are close to the market clearing price obtained by matching single orders is a clear pattern in the optimal solution. In addition, block orders with high volume and competitive prices are the most likely candidates for PABs or PRBs. There are other hidden patterns of the optimal clearing strategies, which can be identified by learning through the previous orders and the clearing prices.

B) Handling Paradox Orders

Even though many models exist that capture the rules employed for handling paradox orders, the impact of paradox order rules on the operations of DAMs has not been fully analyzed. With the exception of [2, 9, 12], the literature on DAM auctions is mostly mute on the impact of paradox order handling rules (PAB and PRB rules) on the total welfare, problem complexity, solution time and bidder behavior. An examination of these issues may significantly contribute to the DAM literature. For instance, [2] models the Turkish DAM under PRB and PAB rules to see how the paradox order rules affect the solution complexity. They find that it is faster to clear the market under the PRB rule. [9] examines the tradeoff between the total surplus and opportunity costs of PRBs. However, more research is needed to understand the impact of such rules on the objective function (total surplus) and problem complexity.

In a market with a significant volume of PABs or PRBs, the market-clearing price would not be meaningful. Hence, it is necessary to understand the impact of such paradox order rules and to minimize the volume of such orders in equilibrium. Multi-criteria optimization techniques may be an effective way to handle paradox orders while maximizing total surplus. One may explicitly consider minimizing the volume of PABs or PRBs as a secondary objective, or construct an efficient frontier as a function of PAB/PRB trading volumes. In this way, a decision maker can choose the best combination of social welfare and PAB/PRB volume. However, we expect that constructing such an efficient frontier would be computationally intensive.

C) Empirical Research Directions

The impact of auction rules on the bidding behavior of market participants also deserves academic attention. Based on our interviews with the Turkish DAM participants, we found strong anecdotal evidence that, under the PAB rule, bidders are motivated to place block orders that can potentially generate paradoxes. Because a supply PAB is paid based on the order price that is above the MCP, market participants consider the PAB rule and associated side payments as an opportunity to trade electricity above the market price. A similar motivation also applies to the demand side of the market. On the other hand, the PRB rule may motivate bidders to lower their order prices (on the supply side), resulting in lower equilibrium prices. Overall, the equilibrium behavior of bidders under PAB and PRB rules can be explored analytically and empirically, generating valuable insights to the managers of power plants and policy makers.

The introduction of new bidding structures may also affect the bidding behavior of market participants. For example, allowing for linked blocks while better reflecting the generation dynamics of thermal power plants may also motivate arbitrage trading in the market. It would be a promising research direction to explore the impact of such new bidding structures on market participants as well as on the efficiency of the auction.

D) Market Design

DAMs have evolved through the continuous introduction of more complex order types in order to address the needs of market participants. Today there is still a need for new bidding structures to allow market participants to reflect their generation and consumption dynamics and flexibility. With the introduction of large-scale storage systems, batteries become active trades in DAMs [45]. These battery operators try to engage in two-legged trades in the DAM to buy (and store) and then sell electricity at different hours of the same day. While they are executing these two-legged trades, they face the risk of one leg of the trade being accepted while the other is rejected at the DAM auction. In such a case, their trade becomes uncovered and they risk loss because the open leg needs to be covered in the intra-day or in the imbalance market. Hence, traders find a special type of bidding structure desirable (for which we suggest the name *dual blocks),* in which the legs of the trade are either both accepted, or both rejected. Dual blocks would ensure that storage operators securely arbitrage between peak and off-peak hours, possibly multiple times during a day. We expect that such a bidding structure would significantly increase the utilization and economic value of battery operators.

Block orders with price flexibility, which we name *Block with PFlex*, may also be a relevant bidding structure in the market. In this case, the bidders place a block order with a range of price; if the market clearing price falls into this range then the order is accepted. These orders may reduce the number of PRBs and PABs in the market. From the policy maker's perspective, minimizing such orders are important for meaningful price formation in the market.

Another innovative DAM design would consider a clearing mechanism with two separate prices for buying and selling electricity. In the absence of block orders, these two prices are equal to each other. However, in the presence of block orders, these prices may not converge to each other. Through this new definition of market clearing prices, the market can avoid the notion of paradoxical orders and the acceptance/rejection decisions can be determined based on the price corresponding to the order type (buy or sell). This new design may provide more total surplus in the market, and we believe the design of such markets is a promising research direction.

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