WILLIAM GORDON BROWN.

Biographical Note by Dr C. G. Knott, F.R.S., General Secretary.

William Gordon Brown, the author of the accompanying paper, was born at Edinburgh on August 12, 1895. He was educated at George Watson's College (1901–1914), where he distinguished himself as a pupil of outstanding ability in all lines of study, and exceptionally brilliant in mathematics. In his last year at school he gained the Glasse Bursary and also one of the College bursaries, and he was evidently marked out for a University career. Unfortunately, the outbreak of the war cruelly interrupted the natural development of an intellectual life of the highest order. Along with a group of his schoolmates, William Brown at once enlisted in the 4th Royal Scots. Later, as a private in the Royal Naval Division, he served in Gallipoli, whence he was invalided to Alexandria in 1915, and finally home in 1916. In August of that year he crossed to France, and met his death on November 13, 1916, in the attack on Beaumont-Hamel. He was a general favourite among his comrades, among whom he was recognised as a personality of rare distinction both in character and in intellectual power.

Dr Pinkerton, formerly mathematical master in Watson's College, now Rector of the Glasgow High School, describes William Brown in these words: "Brown was a most infectious pupil. I always called him my Newton. He was far ahead of his fellows at school, and I taught him, in addition to class-work, to explore any region of mathematics or physics that had even the remotest connection with what turned up in my work. He responded to a marvellous extent; and the width of his knowledge as a schoolboy and the methods by which he had arrived at it were probably unique. He had an intellect of the first order; more important still, he was a boy of a simple sincerity of character, of a modest and charming manner, and with an influence on other boys that sprang from a deep respect for self-forgetfulness, courage, directness, and patriotism in a school sense—later all these made him a willing servant of his country, and made him give up everything to be true to himself."

While still at school, William Brown, following a hint from Dr Pinkerton, read with critical appreciation Norman Campbell's book on *Electricity* in the People's Library Series. This led to study of more advanced books, and all through his war experiences he not only kept up an interest in electro-magnetic theory and relativity, but became himself an investigator of important problems. Inspired by J. J. Thomson's *Recent*

Advances in Electricity and Magnetism, he tackled in his own way the many difficult questions suggested by that treatise, and applied his mind to the mastery of vector algebras as an aid in electro-magnetic investigation. From the notes he sent home at intervals it was abundantly evident that, with scanty help from the ordinary sources, he was himself constructing his own vector methods, largely retreading paths which, unknown to him, had been trodden by his predecessors, Hamilton, Tait, Heaviside, M'Aulay, Gibbs, and others. So impressed were his scientific friends with the brilliancy shown by Gordon Brown that an effort was made to get him appointed to a less dangerous post; but he himself would not listen to the proposal.

His correspondence is particularly interesting as throwing light on the way his mind was working. For example, in a letter written to his mother from Haslar R.N. Hospital of date December 5, 1915, we find the following:—

"Up to the present I have written a first scroll of (1) the first part of a paper on 'The Faraday-tube Theory of Electro-magnetism,' of which you will find an elementary discussion in Thomson's *Electricity and Matter* I have in the house as a prize. I think this first part, which deals with the theory in general, and shows that on simple assumptions it leads in general (as it ought) to what are called the Five Equations of Electro-magnetism, of which the first four are the epitome of Maxwell's theory, and the Fifth Equation is due to Lorentz or possibly (in justice) to Heaviside—I think this part of the paper may be of interest in itself, and is almost certain to be correct and original. The second part I am only just commencing. . . . Its purpose would be to apply the Faraday-tube theory to special problems, such as the law of gravitation, the structure of the atom, and the density of energy in radiation in temperature equilibrium with matter. . . . Besides working the first part of the paper, I have written a rough note on the 'Energetics of the Electron,' which is fairly short, and more on the lines of the work I had published in August." (See *Phil. Mag.*, August 1915, "Note on Reflections from a Moving Mirror.")

The paper referred to above is the one now being published; and a more particular account of its genesis was given to Dr Pinkerton, in a letter written about the middle of January 1916. This letter brings out the writer's keen critical faculty, and his determination not to accept anything which was not fully understood by him:—

"DEAR DR PINKERTON,—I am ashamed to trouble you again about my scribbling, but as I am now having typed a paper which you yourself—

indirectly—suggested, I should like to ask your advice about it. You may remember recommending to your classes in Watson's some years ago Dr Norman Campbell's little book on *Electricity* in the People's Library Series. . . . I worked more or less on the lines of the bibliography in Dr Campbell's book, at least at first, but I was always curious to learn more about the theory of Faraday-tubes which he sketches. I eventually read the original investigations in Sir J. J. Thomson's *Recent Advances*, but could not understand his proofs in the light of Dr Campbell's ideas (and also those of Heaviside and Maxwell). When I referred at last to Dr Campbell's *Modern Theory of Electricity* I was disappointed to find him refer readers for proofs to J. J. Thomson's work. The more I began to understand the ideas of the *Recent Advances* proofs, the less inclined was I to admit their justification for Dr Campbell's purposes. I came to the conclusion that the Faraday-tube theory, which perhaps Thomson regarded more as an aid to thought than as a serious physical hypothesis, is too good for such a subordinate place as a mere easy guide to Maxwell; but it had not yet been sufficiently developed in a consistent manner so as to take rank as a physical theory.

"The paper I have written, after struggling with these difficulties for about two years, proposes to supply this development. It is written mostly in Heaviside's vector notation. . . . New proofs directly from dynamical hypotheses are given of three of the five laws of electro-magnetism. There is also a good deal of other matter in the paper . . . for instance, a modified theory of stresses, and an extension of Campbell's discussion of propagation so as to include an explanation of aberration, etc. It was writing this section which suggested my note in the *Phil. Mag.* of last August. . . .

"If you and Professor Gray mean to communicate my note on 'Mass as a Linear and Vector Operator' to some society, I could send you a slightly fuller typewritten copy; but I am afraid it is not up to R.S.E. standard at any rate.

"I am now on convalescent leave after having dysentery at Gallipoli, so have a good deal of time on my hands.

"Hoping you are keeping well during the winter,—Your affectionate pupil,
W. G. Brown."

The note here referred to on mass as a linear vector operator will be discussed more fully below.

Meanwhile W. G. Brown had been grappling with the Theory of Relativity, as is shown both in his letters to his parents and in the following letter of date July 8, 1916, written to Professor Whittaker from Blandford Camp:—

"DEAR SIR,—In reading rather hurriedly E. Cunningham's *The Principle of Relativity*, I was much struck with the similarity between the relativity view of the electro-magnetic field and that of the Faraday-tube theory. I found it, in fact, very easy to write down expressions for the electro-magnetic field in terms of a system of lines of force in a four-dimensional space of the relativity type. As I have not noticed any reference to this matter anywhere (for though the method I have used is very much borrowed from Bateman and Cunningham, I think they seem to regard lines or tubes of force from a different point of view), I have taken the liberty of sending you my results. You will understand that I am not in a position to consult books just now, and so have no idea what has been done in this line previously.

"I supposed a system of moving lines of force to be determined by the equations

$$\left.\begin{aligned}\alpha &= m\\ \beta &= n\end{aligned}\right\}, \qquad (1)$$

where α, β are real functions of the coordinates $x, y, z, u\ (=ict)$; and m, n are parameters, to every pair of integral values of which corresponds a moving line of force.

"Now, on the theory of lines of force, the component of electric displacement in any direction is the number of tubes which pass through unit area normal to that direction; and the component of magnetic force is the number of tubes per unit time which cut unit length in that direction ($\mathbf{H} = V\mathbf{q}\mathbf{D}$, where $\mathbf{q}$ is the velocity). From these I find by taking the density of tubes in the elements

$$dydz,\ dzdx,\ dxdy,\ dxdu,\ dydu,\ dzdu,$$

$$\left.\begin{aligned}E_x &= \frac{\partial(\alpha, \beta)}{\partial(y, z)} \text{ etc., etc.}\\ H_x &= \frac{\partial(\alpha, \beta)}{\partial(x, u)} \text{ etc., etc.}\end{aligned}\right\} \qquad (2)$$

"Or, the electro-magnetic six-vector is the product of the two four-vectors

$$\left(\frac{\partial\alpha}{\partial x}, \frac{\partial\alpha}{\partial y}, \frac{\partial\alpha}{\partial z}, \frac{\partial\alpha}{\partial u}\right)\left(\frac{\partial\beta}{\partial x}, \frac{\partial\beta}{\partial y}, \frac{\partial\beta}{\partial z}, \frac{\partial\beta}{\partial u}\right).$$

"From these follow at once

$$\left.\begin{aligned}&\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \frac{1}{c}\frac{\partial E_x}{\partial t}, \text{ etc.}\\ &\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0\end{aligned}\right\} \qquad (3)$$

"By choosing for α and β an appropriate form and by arbitrarily defining the positive direction of the tubes as that of the expressions (2), we could represent moving point charges, and from a discussion of their distribution in the four-dimensional space obtain the modifications in the kinematic scheme (3) due to the presence of electrification.

"The remaining (dynamical) scheme of four equations is of course deducible from the volume density $\frac{1}{2}(H^2-E^2)$ or kinetic potential.

"I noticed particularly that the velocity of the tubes was only defined (by the equations (1)) for directions normal to that of the tubes themselves, that is, only its component in the direction of the momentum is determinate, as in Cunningham's theory of the stresses; but, while on the simple æther theory only one velocity is considered at any point, when we are dealing with lines or tubes of force each separate set has its peculiar velocity, though the momentum corresponding to a tube is always perpendicular to it.

"With regard to electro-magnetic inertia, I have calculated the mutual electro-magnetic momentum of a pair of point charges moving obliquely to the line joining them, and find it not parallel to the velocity—as indeed is implied in Heaviside's paper for low velocities (*Phil. Mag.*, 1889). I do not know, however, if this point is of interest. Hoping you will pardon my troubling you so much,—I am, yours truly, W. G. BROWN."

On August 1, 1916, Professor Whittaker, writing from Edinburgh, replied as follows:—

"DEAR MR BROWN,—Thank you for sending me an account of your results regarding lines of force.

"Probably you have noticed that your two functions α and β are connected with the old vector-potential and scalar-potential of the electro-magnetic field; thus, if a_x, a_y, a_z are the three components of the vector-potential, and ϕ is the scalar-potential, so that the components of electric force are

$$d_x = -\frac{1}{c}\frac{\partial a_x}{\partial t} - \frac{\partial \phi}{\partial x}, \text{ etc.,}$$

and the components of the magnetic force are

$$h_x = \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}, \text{ etc.,}$$

let us write down the 'differential form' or 'Pfaff's expression

$$a_x dx + a_y dy + a_z dz - c\phi dt.$$

Then if this differential form is reduced to the form $\alpha d\beta$ (there is a big theory dealing with the reduction of Pfaff's expressions) we have

$$a_x = \alpha \frac{\partial \beta}{\partial x}, \quad a_y = \alpha \frac{\partial \beta}{\partial y}, \quad a_z = \alpha \frac{\partial \beta}{\partial z}, \quad \phi = -\frac{\alpha}{c}\frac{\partial \beta}{\partial t},$$

and therefore

$$d_x = \frac{1}{c}\frac{\partial(\alpha, \beta)}{\partial(x, t)}, \quad h_x = \frac{\partial(\alpha, \beta)}{\partial(y, z)},$$

which are the same as your expressions, except that the electric and magnetic rôles are interchanged.

"I have been so exclusively occupied with pure mathematics for the last four years that I can't say whether any of your work has been anticipated or not.—Yours very truly, E. T. Whittaker."

Before this reply from Professor Whittaker reached him, William Brown was with the Expeditionary Force in France. In later letters to his parents he discussed the significance of the Relativity Theory, and on September 16 asked them to send him, if possible, a copy of Minkowski's *Raum und Zeit.* In acknowledging books received on October 15, he asked for news of the *Relativitäts Prinzip.* Before the lapse of another month this young life had passed from among us, and the world was the poorer by the loss of an intellect brilliant in mathematical power and promise.

W. G. Brown's notes on the investigation described above have not been recovered, but there is no doubt that his mind was taking a firm grip of the mathematical methods associated with the modern theory of Relativity.

To show the extent to which he had mastered the quaternion method, I conclude with giving a brief account of his note on Mass as a Linear Vector Operator. It is an interesting and novel generalisation of Tait's early work on the Rotation of a Rigid Solid (see Tait's *Quaternions*, §§ 406 *et seq.*):—

Let σ be the velocity of the body (more strictly of a definite point in the body), and let the operator μ take the place of the mass factor in ordinary dynamics, being a self-conjugate linear vector function whose axes are fixed in the body. Then the momentum is

$$\eta = \mu\sigma.$$

The force β is assumed to be given by the Newtonian law

$$\text{force} = \frac{d}{dt}(\text{momentum}),$$

or

$$\beta = \dot{\eta} = \frac{d}{dt}(\mu\sigma).$$

Since the quantities β and μ are referred to axes fixed in space, it is necessary to consider the variation $\dot{\mu}$.

Translational motion of the body will not affect the space relations of μ, but rotation will. Any vector a in the body will in virtue of rotation ω change at the rate

$$\dot{a} = \mathrm{V}\omega a\,,$$

and, as is well known in quaternions, the rate of change of $\mu\sigma$ will be

$$\frac{d}{dt}(\mu\sigma) = \dot{\mu}\sigma + \mu\dot{\sigma}$$
$$= \mathrm{V}\omega\mu\sigma - \mu\mathrm{V}\omega\sigma + \mu\dot{\sigma}\,.$$

Hence the equation of linear motion is

$$\beta = \mathrm{V}\omega\mu\sigma - \mu\mathrm{V}\omega\sigma + \mu\dot{\sigma} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The moment of momentum of the body with regard to a fixed origin is of the form

$$\gamma = \mathrm{V}\rho\mu\sigma + \phi\omega\,,$$

where ρ is the vector position of a given point of the body and ϕ is the linear vector function which, operating on the angular velocity ω, gives the angular momentum about the extremity of ρ. (See Tait's *Quaternions*, p. 323.)

If ψ is the torque acting on the body, the equation of rotational motion is

$$\psi + \mathrm{V}\rho\beta = \dot{\gamma}$$
$$= \mathrm{V}\dot{\rho}\mu\sigma + \mathrm{V}\rho\frac{d}{dt}(\mu\sigma) + \dot{\phi}\omega + \phi\dot{\omega}$$
$$= \mathrm{V}\sigma\mu\sigma + \mathrm{V}\rho\beta + \mathrm{V}\omega\phi\omega + \phi\dot{\omega}$$

or

$$\psi = \mathrm{V}\sigma\mu\sigma + \phi\dot{\omega} + \mathrm{V}\omega\phi\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The total activity equation is obtained from (1) and (2) in the form

$$\mathrm{S}\sigma\beta + \mathrm{S}\omega\psi = \frac{d}{dt}\{\tfrac{1}{2}\mathrm{S}\sigma\mu\sigma + \tfrac{1}{2}\mathrm{S}\omega\phi\omega\},$$

the integral of which may be represented in the form

$$\mathrm{W} = \mathrm{T} - \mathrm{T}_0\,,$$

where W is the work done by the forces β and ψ, and $\mathrm{T}(=\mathrm{T}_1+\mathrm{T}_2)$ is the kinetic energy. It will be seen that T is a quadratic function of the components of the velocity and of the angular velocity, the two parts T_1 and T_2 being quite separate, and no products of the components of different type existing. This feature involves or is involved in the self-conjugateness of the operators.

If σ is parallel to one of the axes of μ, so also is $\mu\sigma$, and $\mathrm{V}\sigma\mu\sigma$ vanishes. Equation (2) then reduces to

$$\psi = \phi\dot{\omega} + \mathrm{V}\omega\phi\omega\,,$$

the well-known equivalent of Euler's three equations. Stability or instability will be determined by the usual considerations.

Again, if the momentum $\eta(=\mu\sigma)$ remains constant in amount but changes direction with uniform angular speed, we may write

$$\beta = \dot{\eta} = V\omega\eta = V\omega\mu\sigma\,.$$

But

$$\beta = V\omega\mu\sigma - \mu V\omega\sigma + \mu\dot{\sigma} \text{ by (1)}.$$

Hence

$$\dot{\sigma} = V\omega\sigma\,,$$

and the body will describe a circular path with uniform speed.

The radial component of force is

$$\begin{aligned} -SU\rho\beta &= -SU\rho\omega\mu\sigma \\ &= +S\mu\sigma V\omega U\rho \\ &= \frac{S\sigma\mu\sigma}{T\rho} = -\frac{2T_1}{r}\,, \end{aligned}$$

where r is the tensor of ρ, and T_1 the kinetic energy of translation, the same relation as in ordinary dynamics.

The tangential component of the force is

$$-SU\sigma\beta = -SU\sigma V\omega\mu\sigma = \frac{S\omega\sigma\mu\sigma}{T\sigma}\,,$$

vanishing when σ is parallel to a principal axis of μ.

If vibrations are set up about a position of stable equilibrium, any damping action will bring the body to move so that the axis of greatest mass will lie along the tangent to the path, and the motion will be sustained as in ordinary dynamics.

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