Platonic Solids

Eloise Lardet

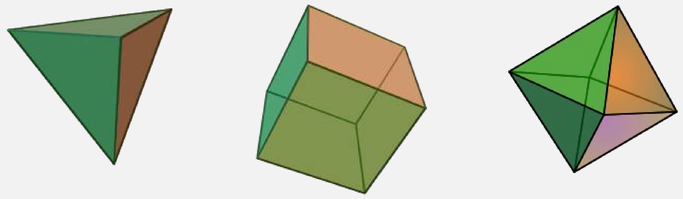
The University of Edinburgh

What are Platonic solids?

You might have heard of “regular polygons”. Examples include the equilateral triangle, square, regular pentagon. They have the same number of edges as vertices, and all angles are the same.
In 2D there are an infinite number of regular polygons.
But what about regular shapes in 3D?
They must satisfy these conditions:

- All the faces must be the same regular polygon
- The same number of faces must meet at each vertex (corner)

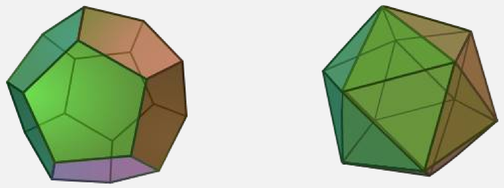
How many polyhedra satisfy these conditions? Only 5!
These are called **Platonic solids**. Let’s meet them…



Tetrahedron
- 4 triangles
- 4 vertices
- 6 edges

Cube
- 6 squares
- 8 vertices
- 12 edges

Octahedron
- 6 squares
- 8 vertices
- 12 edges



Dodecahedron
- 12 pentagons
- 20 vertices
- 30 edges

Icosahedron
- 20 triangles
- 12 vertices
- 30 edges

Figures: Platonic Solids [1]

History

The name “Platonic" comes from the Greek mathematician **Plato**, who lived around 400 BC. He may not have been the first to discover all five solids, but he had a theory about the Universe based on these shapes.

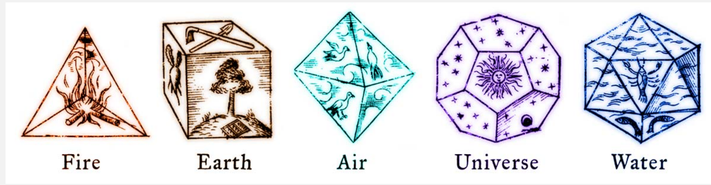


Figure: Plato’s Elements [2]

He believed that the Platonic solids represented the four basic elements (fire, earth, air, and water), with the last one representing the whole Universe.

Euclid was another Greek mathematician who researched Platonic solids. The last book of Euclid’s Elements is all about them. He is thought to be the first person to prove that only five Platonic solids exist.

In the 16th century, **Johannes Kepler** had a new theory about the Universe and the Platonic solids.
He imagined placing a sphere around each of the Platonic solids. The ratios between the sizes of these spheres were remarkably similar to the distances between the orbits round the Sun of the six known planets at the time.

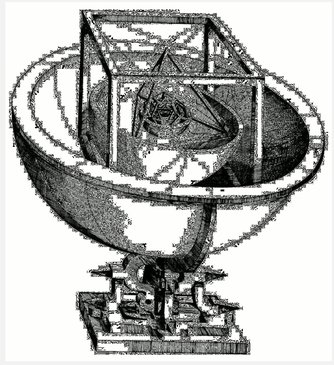


Figure: Kepler’s Mysterium Cosmographicum [3]

He outlined this theory in his book “Mysterium Cosmographicum". This theory was proven to be wrong, but it shows how Platonic solids have had strong influences in science.

Applications

Although they may not be the elements of the Universe, as Plato believed, the Platonic solids do appear all over nature, and have many uses.

- Many **viruses** have the shape of an icosahedron, such as HIV.
- There are tiny single-celled organisms called **Radiolaria** which live in the ocean. All five platonic solids can be seen in Radiolaria skeletons.

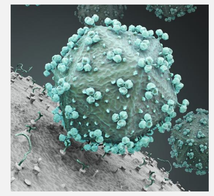


Figure: Icosahedral HIV virus [4]

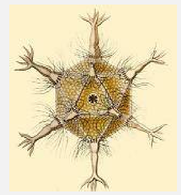


Figure: Icosahedral Radiolarian [5]

- Regular polyhedra are common among **molecules** due to their strong structures and symmetries.
- Cubes, tetrahedra, and octahedra can all be seen in naturally occurring **crystal structures**.

Figure: Fool’s Gold (Pyrite) Cube [6]

Figure: Fluorite Octahedron [7]

- Most **dice** are platonic solids, due to the fairness of each face being rolled.

[1] Platonic Solids: Tetrahedron, Cube, Octahedron, Icosahedron, Dodecahedron. The original uploader was Cyp at English Wikipedia. https://commons.wikimedia.org/wiki/Platonic_solid.

[2] Platonic solids as elements. Originally by Johannes Kepler, Harmonices Mundi, 1619. https://mathigon.org/course/polyhedra/platonic.

[3] Kepler’s Mysterium Cosmographicum. Johannes Kepler, 1597. Mysterium Cosmographicum.

[4] HIV virus attacking cell. 3D render. User martynowi.cz, Shutterstock

[5] Circogonia Icosahedra. Ernst Haeckel, 1904. Kunstformen der Natur. https://upload.wikimedia.org/wikipedia/commons/0/02/Circogoniaicosahedra_ekw.jpg

[6] Isolated Pyrite cubic crystal. User Myriam B, Shutterstock

[7] Piece of blue Fluorite mineral from Seilles, Belgique. User Myriam B, Shutterstock