THE UNIVERSITY of EDINBURGH
School of Mathematics

Have Fun with Maths

This is a workshop about problem solving and strategy games for P4-S3 students. The website contains all the material a teacher would need to run the workshop in their classroom, including explanatory videos, a range of games and puzzles catering for different levels and abilities, a workshop plan, and some tips for teachers/facilitators on how to run the workshop and how to differentiate, and make it suitable for mixed abilities.

Aims of the workshop

The workshop aims at helping children develop problem solving skills, and grasp some basic concepts in Geometry, through playing a fun strategy game. We also aim to show the fun and creative side of Mathematics.

By the end of the workshop, children will see some unexpected applications of Mathematics and find out about Graph Theory, an area of Mathematics studying networks and connections. They will also have a chance to apply their mathematical thinking and problem solving in the context of improving transport networks.

There will be space for creativity, and for children to create their own strategy games and challenge their classmates.

The workshop: an overview

The first part of the workshop will be centred around the Shannon switching game (a strategy game for two players, invented by the American mathematician and electrical engineer Claude Shannon). The seven explanatory videos are meant to guide you through the workshop, explain the rules of the games, and give some hints about strategies. Teachers are very welcome to show them to their class as they see fit. Below is an overview of the video content.

Video 1: Setting the context- Can we do Maths while playing games?

Ideas about Maths are challenged as we reveal that playing games can be part of Mathematics.

Video 2: General rules of the games and examples- game 1- game 2

In this video we explain the rules of the game and we play it on two different diagrams, we prompt students to play, and then we reveal which of the players has a winning strategy.

Rules of the game

Produced by Dr Francesca Iezzi: francesca.iezzi@ed.ac.uk

This is a game for two players. The game is played on a diagram with two special points: *A* and *B.* In the pictures below the special points are denoted by the two houses.

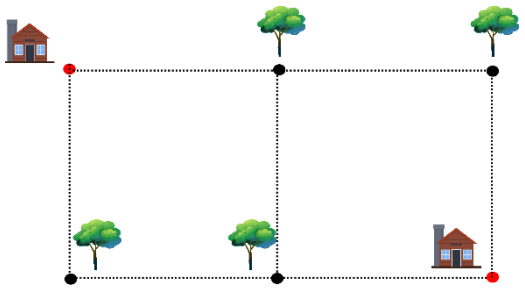
The two players take the roles of *Join* and *Cut*, and alternate moves.

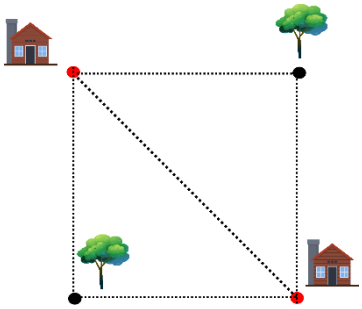
On their turn, *Join* chooses a line and colours it with a marker pen. On their turn, *Cut* chooses a non-coloured line and deletes it by drawing a cross over it. *Join* cannot colour a line which *Cut* has crossed out, likewise *Cut* cannot cross out a line which *Join* has coloured.

Join wins if by the end they manage to create a coloured path between the two houses. Otherwise *Cut* wins.

Note that *Join* does not need to start at one of the houses, and they do not need to colour lines in order. They can choose to colour whatever line they want, as long as it is not crossed out, and to win they need to have a coloured path by the end.

We recommend that you watch the video to see examples of games. The rules become really simple to understand once you have seen people play.





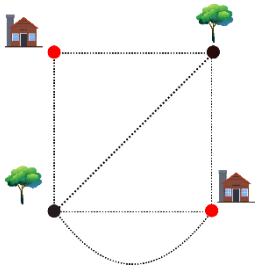
After explaining the rules and showing some examples of children playing, we prompt students to pause the video and play on the first diagram (on the left). We recommend that they switch roles and then play again. After the pause for pupils to play their own games, we reveal which player has a winning strategy. It is *Cut* in the case shown.

Then, we play on the second diagram (on the right) and we reveal which player has a winning strategy (the first player in this case).

The game can be played on all sorts of diagrams. We prompt students to watch the next videos for different diagrams and a greater challenge.

Video 3: game 3

We increase the challenge as we play on a different diagram (below).



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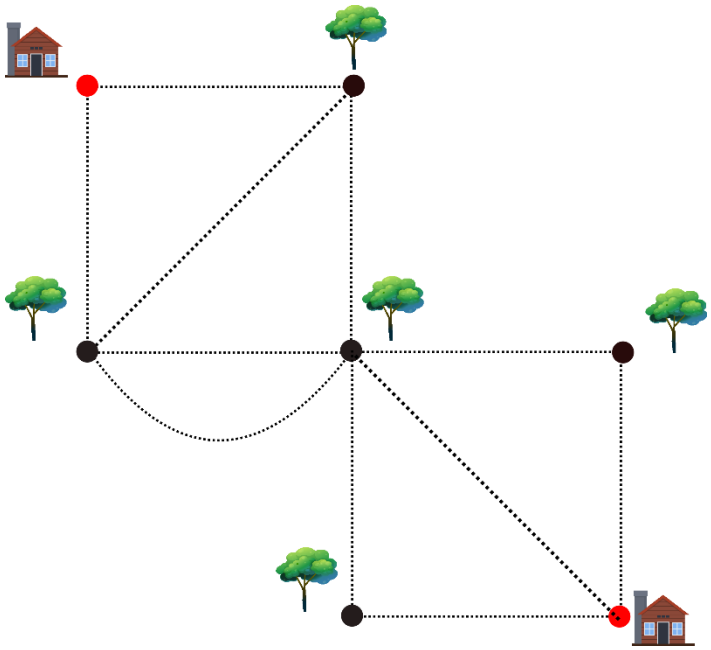
We prompt students to pause the video and play with their classmates for a few minutes. Then we reveal which of the two players has a winning strategy, and we give hints about good and bad moves.

Video 4: game 4

We keep increasing the challenge as we play on the diagram below:

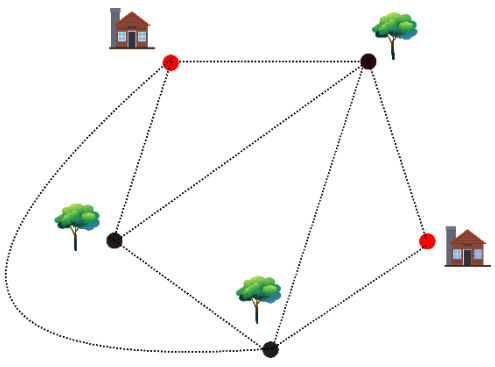


Again, we encourage students to pause the video and play the game with their classmates. After the pause we reveal which player has a winning strategy and why. (It is the player who starts in this case).

We also explain how a mathematician would approach this game, and can ultimately prove, with no doubt, that a certain player has a winning strategy.

Video 5: game 5

We now play on the diagram below.



Again, we encourage students to pause the video and play the game with their classmates. After the pause we reveal which player has a winning strategy (*Join* in this case) and we give examples of good and bad moves.

Then we prompt students and teachers to look at the website for many other diagrams, play the game on those diagrams, and try to understand which player has a strategy.

Video 6: Where is the Maths?

Here we explain why Maths is at the core of these games. The game has been invented by the Mathematician and electrical engineer Claude Shannon, and has been studied by mathematicians at University. Geometry lies at the core of the game, as strategies depend entirely on the shape of the diagrams.

We also prompt students to be creative, draw their own diagrams, challenge their classmates, and try to understand which player has a winning strategy. They can email us their diagrams. We will use the most interesting for future challenges.

Video 7: What's the Point?

Graph Theory in our everyday life

In this video we relate the games we just played to everyday life.

At the very core of the game is the branch of Mathematics called graph theory. In advanced Mathematics, a *graph* is a collection of points and lines. The diagrams where we played the game are examples of *graphs*, but we encounter other examples of graphs every day: tube maps or train route maps, for example (here the points represent the stations and the lines represent connections).

We will give examples of very simple mathematical questions in the context of planning a journey, and we prompt students and teachers to look at our website and try the problems and exercises about transport maps and networks.

A workshop plan

Below we advise on how we would run the workshop. Teachers are obviously welcome to follow our advice as much or as little as they want, and to use the resources as they see fit, based on the background of their students.

We would advise you to watch video 1 to set the scene, or else ask students what they associate Maths with and whether they have ever thought of playing a game as doing maths.

Then watch video 1-5 in order. For each of the videos 2-5, pause to let students play for as long as necessary, let students discuss possible strategies, then watch the second part where we give hints about strategies. You may want to let students play again for a minute, so that they can assimilate our tips if necessary.

Teachers may also decide to explain the rules and strategies themselves, instead of showing the videos if they so wish.

After watching video 5, you can let children play on other diagrams (there will be many on our website), as you see fit. For each of the diagrams they play with, let them discuss strategies.

Then, ask students whether they had fun, whether they think they did Maths and why/why not. Watch video 6 to see how maths underpins the games. Let students draw their own diagrams and investigate strategies.

If you, or any of the students, are interested, the website contains a document which explains how we can detect whether *Join* has a winning strategy.

Then watch video 7 to show some practical applications and let children have a go at problems related to transportation networks (we give some on the website).

The Material you will need

Nothing more than printouts, pencils and erasers. All the games and puzzles will be uploaded on the website for you to print out. Students can just play on the printouts, using pencils and erasers.

If you want to minimise paper use, you may consider laminating the handouts, and using dry whiteboard pens and erasers.

How to differentiate and cope with a range of abilities

The resources lend themselves quite well to differentiation. In the videos we start with two simple games, to make it easier for younger children or less confident? students.

If you are teaching an older or high achieving class, you may wish to go very fast on the first two games.

There will be further resources on the website, (a range of diagrams and a variety of exercises on transport networks), catering for different stages and abilities. The difficulty level is indicated by the number of stars at the top, where (*) means simpler, and (***) means more difficult.

The most skilled or keen students will enjoy drawing their own diagrams and thinking of strategies.

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Students who do not like competition can play in pairs, and work together towards finding the winning strategy, rather than competing to win.

Ultimately, most of the students will be able to play the game, explore strategies and engage in creative problem solving.

Just if you are interested- further reading

If you are interested to know more, and ready to read a paper on advanced Mathematics, a description of the game and how to find out which player has a strategy can be found here.

http://www.cs.cmu.edu/afs/cs/academic/class/15859-f01/www/notes/shannon.pdf

But be warned, this paper is not easy to read at all!

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