

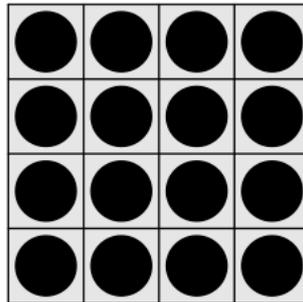
### Episode 3

The main character is Dr Steven O'Hagan, deputy director of UK Mathematics Trust.

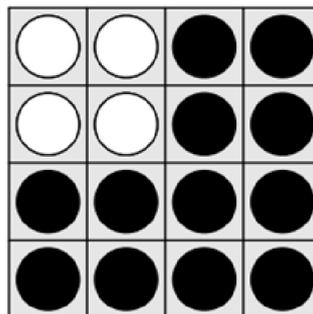
#### The puzzle

##### Question

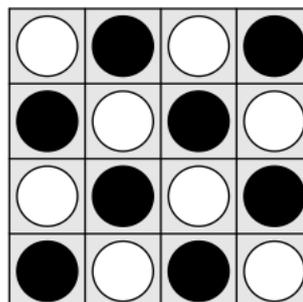
We have sixteen counters which are black on one side and white on the other. They are arranged in a 4-by-4 square. Initially, all the counters are facing black side up (see picture below).



In one move, we must choose a 2-by-2 square within the square and turn all four counters over. For example, we could turn all four counters in this square.



Describe a sequence of moves of minimum length that turns the initial configuration into one where the counters alternate in colour, as shown below.



**How can you be sure this is really the minimal number of moves you need?**



## Aims

- Developing logic and problem solving skills
- Constructing a mathematical argument
- Getting an idea of what mathematical thinking means: it does not consist of repeating tasks, but rather involves studying the underlying structure of patterns, finding an optimal way to reach a solution, constructing a logical argument.

## If your class gives up...

This puzzle is quite challenging, we do not expect that your students will find a solution straight away. If they are giving up, we recommend to give some hints. A sequence of possible hints is mentioned below.

## Prompts and tips (in case you or the students get stuck)

- Start playing a bit with the puzzle to get an idea of what moves help you and what moves do not take you anywhere. Don't worry if you do not get to the solution. We (Francesca and Hannah) also needed a little hint and a few goes.
- Keep in mind Steven's hint: you should not flip the same 2x2 square twice, because this is inefficient.
- Now, try to be more **strategic**. Look at the initial arrangement and the final arrangement. Try to understand whether there are moves that you are **forced** to make. For example, look at the four counters in the corners. You will need to flip two of them.
- Again, compare the arrangement you have now and the final arrangement. Try and understand whether there are other moves that you are **forced** to make.
- After you reach the final arrangement, **try and answer this question**: "How can you be sure that you have really used the minimal number of moves you needed?"

## A question for your students

Ask your students: "Where is the Maths in this puzzle?", and discuss the answers they come up with. Then watch the solution video, where Steven will give his own answer.

## Material

Ideally, you will need sixteen double sided counters. Otherwise, they can draw on paper.