Contours

Stories from the School of Mathematics 2014–15







Interviews with:
Sir Michael Atiyah
Gonçalo dos Reis
Anna Lisa Varri
Michael Wemyss
Burak Büke
Tibor Antal
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THE UNIVERSITY OF EDINBURGH

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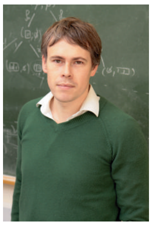
Contents

This is the second edition of Contours magazine, written by students in the School of Mathematics. Yes, by mathematics students. Throw out those hoary prejudices. Mathematicians write wonderfully! We communicate! This whole issue reflects that. Communication between mathematics and other disciplines; between students; between lecturers and students; between professional mathematicians and the public.

In this issue you'll see some of the incredible spectrum of the application of mathematics that is happening right here, right now. From the mysteries of the universe, through problems in efficient management and effective markets, across to fundamental research at the boundary between medicine and mathematics. And you'll get advice and insight from one of the greatest mathematical minds of the last century and some of the hip young gun slingers of the next!

Mathematics is being created all around in the city. By our students in Innovative Learning Week. By the postgraduates throughout the School. And at one of the world's great Mathematics Conference Centres, right at our doorstep. Swathes of the world is becoming more and more quantitative, and wherever that is happening, you will find the Mathematical Sciences.

So, find out what we are all talking about! Be part of the mathematical movement! Read on!



Professor Iain Gordon
Head of School

Innovative Learning Week

Classes were suspended for a week in February to make way for over 300 special events across the University – here are the highlights from mathematics

Matt Parker in 4D

Julia Collins

One of the most anticipated events during ILW starred award-winning comedian and mathematician Matt Parker and his lecture entitled *Things to See and Hear in the Fourth Dimension*, based on his new book.

Cubes featured heavily in the talk, with Matt first impressing us by cube-rooting large numbers, and next solving a Rubik's cube in just over a minute while telling the audience fun facts about the famous toy. For example, that there are world records for solving it blindfolded, underwater and with one's feet! He showed us how to think about a 4D cube (aka tesseract) and how to picture one by looking at a 3D projection of it, just as we are used to looking at 2D drawings of 3D cubes. It was mesmerising to watch Matt's videos of a 3D net being folded up into a 4D cube, or a 4D cube rotating in space.

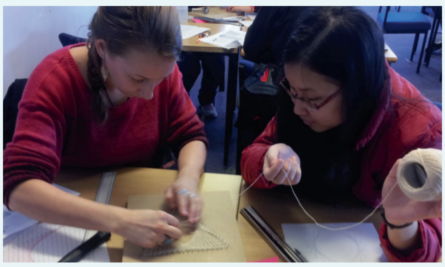
Matt also showed us strange shapes that can be made from joining together the edges of a square. For example, joining opposite sides to make a cylinder and then turning one end of the cylinder inside out before joining it to the other end results in a crazy 4D object called a Klein bottle, which has no inside or outside and makes for a wonderfully geeky hat. My favourite shape was probably the Möbius strip, which is made by joining the ends of a strip of paper with a twist, resulting in an object that only has one side and one edge. I recommend experimenting with the strip, cutting it down the middle and seeing if you can predict what will happen!

Of course it was the cube which formed the grand finale of the talk, with Matt blowing our minds by describing how it was possible to create a 4 dimensional Rubik's cube. And if that wasn't enough, somebody on the internet has now created a 5D Rubik's cube for us to get our heads around. ■

Maker Faire

Julia Collins

Fourth year students Tessa Hayman, Martina Matrtajová and Rae Wilson put together a 'mini Maker Faire' for the School of Mathematics, encouraging people to learn about mathematical concepts through making beautiful things. On the Wednesday of ILW they led a workshop teaching people to make mathematical origami, using modular pieces to build Platonic solids (regular 3D shapes) and then using giant pieces of paper with circular folds to create hyperbolic planes. On the Thursday was the Mathematical Jewellery workshop, where people used beads to create dodecahedra and octahedra, and considering different colouring patterns that were possible on such shapes. Finally, the Friday was all about string art, and seeing how we could create shapes like circles and parabolas by arranging string in straight lines across a board.



It was wonderful to see people take these basic ideas and stretch their imagination in different directions, and most of all to see people from many different Schools and backgrounds engaging with mathematics in a new way. ■

The LEGO calculator challenge

Julia Collins

Is it possible to build a working mechanical calculator with only LEGO? This was the challenge we set students from across the University, with a little help from LEGO expert Alex Allmont. At the beginning of the day Alex showed us some prototype calculators he had made, exhibiting techniques from binary adders using switches and falling balls, to complicated gear systems capable of adding in base 10. We watched an amazing video from the 1940s showing the intricate and clever things that can be done just using gears and ratchets.

After a tasty lunch of pizza, people were set free to play with Mindstorms™ kits to design their own calculators. It was great to see people get creative with different sorts of adding machines, looking at using logic gates, or adding in different bases, while other people decided to look at other mathematical ideas such as integration. One team couldn't stop themselves from getting distracted and building a car, but even this turned out to be a gateway to a machine that could measure distances.

The day was a great example of an innovative activity that could only be done because of ILW funding, where students from across disciplines worked together to figure out the mathematics, engineering and physics that go into machines that we so easily take for granted. And we now have lots of LEGO kit sitting in the School of Maths for anyone who wants to build on these ideas! ■

Alumni reception

Imogen Morris

Former students return to the School every year to talk about their experiences of maths after university. Their careers have been surprising, interesting and never quite turned out how they expected. But most of all, each of them have managed to end up doing something they love.

Robin Carmichael (1977) avoided all the applied maths and computing courses at university, but ended up working for IBM, where his reasoning and problem solving skills were highly valued. Roger Thomas (1969) also got a job with IBM, but later moved to Scottish Telecom and realised that his skills lay in managing people. He has now set up his own company coaching others to cope with positions of responsibility. Patricia Hiddleston (1956) was the only woman in her honours class at university. She went on to win the Napier Medal in her final year, then moved to Zambia where she became a teacher before doing her PhD by distance learning. Today she travels the world as an independent education consultant, writing curriculums and reforming exam systems, not showing any signs of slowing down despite being in her eighties!

Michael Douglas (1989) combines mathematics with policing, calculating the speeds and angles involved in road collisions. His mathematics degree was what enabled him to have the career he wanted – when he left school he couldn't apply for the police force due to the height restrictions. Nora Mogey (1983) advised the audience to take risks in life and not miss out on opportunities, while Hamish Leiper (2009), who works for Standard Life, told us not to measure our progress against others, but to be true to ourselves. Last but not least, Cecilia Macintyre (1980) got the job she wanted at the Medical Statistics Unit by simply writing to ask them rather than waiting to see a job advert. ■

Find out more about the alumni and their careers in the extended article online:

www.maths.ed.ac.uk/contours

Sir Michael Atiyah
Mathematical Physics

Rising to the challenge

With a career in mathematics spanning over 60 years, including a Fields Medal in 1966, Sir Michael Atiyah sat down with us to share some advice

Lara Anisman
Yeow Liiyung

The bold, welcoming smile that greeted us as we were invited in to Sir Michael Atiyah's office in JCMB had us feeling immediately awestruck; we were in the presence of one of the world's leading mathematicians of the past 60 years.

This was a man who, at the age of 85, was retired professionally yet still insistent on throwing himself into the deep end of mathematical research – currently working on 'fine-tuning some of the early Maxwell equations' with some younger researchers here at Edinburgh, alongside his recent foray into the world of neuroscience with Semir Zeki, among others, writing a paper on the perception of beauty in mathematics. Sir Michael certainly won't be letting age slow him down.

There is an air to Michael that is unmistakable, one of self-assuredness and wisdom about the world of Mathematics. We ask him about the inspiration behind his first mathematical paper, a short note on twisted cubics which his relatively inexperienced, undergraduate curiosity compelled him to write whilst studying at Cambridge. "In my first year at Cambridge I came top of the class, so I thought it was likely I was going to have a mathematical career. In 2nd year I began to attend more advanced lectures and then I started to ask questions; tried to solve them. Through some of these courses I had learned about 3-dimensional geometry and lines and found them very nice! I wrote a short little paper about it, just two sides on a piece of paper. It gave a boost to my morale. It was very encouraging as a student to find you could contribute. It was my paper. It's my favourite area of mathematics – every time I go back to it, I find it has elementary links to other things."

> "Keep struggling! Without the struggle, there is no progress. If everything were easy, we would not learn anything new"

Surely, however, even for someone of Michael's skill, there had to be some area of mathematics that had him stuck every once in a while? There was indeed a point in his career when he began to consider other areas of interest. "That was during my post-grad, a period where you were mostly isolated in research and when you faced problems, you were stuck. I attended lectures in archaeology and architecture at that time and thought they were both quite fun. But then again, there were no assignments or exams for me, I just went in to sit at the lectures – there were lots of pictures! It all came to a stop though, when I won my first medal." This was the Smith's prize in 1954 for a sheaf-theoretic approach to ruled surfaces.

Michael does agree that he finds mathematical research challenging, but that in itself is part and parcel of the work. For some, the most difficult part of research is choosing what questions to answer. "Normally your supervisor will suggest a few topics for you, but then you have to read up on a lot of things and learn things you previously did not know before. So it's best to go about choosing your own topic, so that you at least have a little background information on it already. Having said that, choose something you really like because you'll be working on it for a long time!"

He feels that choosing geometry as his home base for research turned out very well for him. It set the stage for his works in K theory, Index theory and

Gauge theory with the allowance to wiggle around other areas of interest. New theories such as superspace, supergravity and string theory were some of the areas of theoretical physics which developed using the ideas he introduced. When asked if he would have gone into a different area if the clocks were rerun, he responded "I'm a geometer first and foremost, and certainly have no regrets about the way things have turned out. However, who's to say where I'd be in a different time frame? Choices like this depend on what the cutting edge topic is at the moment. For instance, if I were to start all over again now, I might go into neuroscience – that fascinates me now."

And indeed, neuroscience is exactly what he's working on now, having recently published a paper on the correlation of beauty in mathematics and activity in the emotional part of the brain. "You see, I already have a PhD and a career, unlike most postgrads and job-finders who need to contribute more quickly and directly to science to secure a job. I feel like I can turn to look at more interesting, risky questions now."

Before we closed the interview, we asked for Atiyah's advice to struggling young mathematicians. "Keep struggling! Without the struggle, there is no progress. If everything were easy, we would not learn anything new. Don't be afraid of making mistakes, because you will learn from them. For example, if everything went smoothly and you've proved what you wanted to prove, what do you learn at the end? Nothing new. But if there was a mistake somewhere, it would challenge our understanding and allow us to perceive the topic in a different way. Now, I'm not saying go out there and deliberately make mistakes! Always be careful in your work! You could think of finding the proof as ascending the peak of a mountain. You see the peak of the mountain from far off, and try to map out a way to get there. But of course initially you must be able to make your way through the jungle of knowledge and theorems, ensuring you have the right tools for the job. In the end, you may not even make it to the peak you've set yourself to climb. You may end up lost in the jungle, or even find yourself at the base of a different peak altogether. Who knows, you may want to start climbing the new peak and end up on top of that!" ■

The Michael and Lily Atiyah Portrait Gallery on level 3 of JCMB features photos of 70 mathematicians through the ages who have influenced the Atiyahs. It includes an interactive display with commentaries written by Sir Michael.

You can also see these online: http://edin.ac/1GsYrg5

Gonçalo dos Reis
Probability & Stochastic Analysis

Complex games

Research in financial mathematics often involves dealing with randomness. This is where stochastic analysis comes in, and things get complex...

Anna Greer
Abby Li

Dr. Gonçalo dos Reis is from a small village in Portugal. When he was young, his mother wanted him to become a doctor, so he decided to get a PhD in Mathematics. It was not what his mother expected but he did become a doctor. She does not find this funny though.

At university, Gonçalo studied Mathematics. Though initially disliking statistics and probability at the beginning – something he attributes to a lecturer he disliked – he gradually gained an interest in stochastic processes and eventually pursued a PhD in stochastic analysis.

Gonçalo's current research is still based on probability, finance and stochastic analysis. At the moment, he is planning to collaborate with fellow Edinburgh mathematician Lukasz Szpruch on a project about a topic called interacting particle methods.

Imagine several people acting in a market, each independently trying to maximize their wealth according to their own criteria. This basic problem is well understood, but let's add a twist: now the agents interact with each other, using each other as reference points for their own investment problems. In this new framework, each person is considered as a particle interacting with the others.

Take the example of pension funds: a fund manager watches the performance of other hedge and pension funds via a performance market index. Managers are paid bonuses if they manage to reach a minimal level of performance according to the index, which is built with everyone's performance in mind.

"My particle system is not just me going around and if I hit you I hit you," says Gonçalo. "I'm also paying attention to more or less everyone going around in the system. So this means that the particles are interacting much more. In terms of optimal investment and strategies, these kind of games can be transformed into something very complex."

Gonçalo's current area of interest is so-called mean field games. We have a fixed number of agents and a game for which we cannot solve the optimization problem. Then we rewrite the problem and start looking at the average of what all the agents are doing; the trick is by looking at the average we can create an artificial game where the number of agents can be increased to infinity. At this point we can consider the agents' actions under a probability distribution. We do not know the position of the particle, only the probability of that particle being in a certain division of space. Once there are an infinite number of agents, the optimal investment problem changes completely and is often easier to solve. What is making this complicated is how the agents interact with each other. "This is one of the sexiest topics of the last ten years," he says. ■

Gonçalo helps to run the several finance MSc programmes at the School of Mathematics. He says it is very important to go beyond what the students simply "need to know". Students on the MSc programmes are encouraged to develop the skills to write and articulate ideas effectively, he says. This way, when students later attend interviews for future positions, they will be able to speak confidently and clearly.

Anna Lisa Varri
Applied and Computational Mathematics

Stars in motion

Understanding the movement of stars within a dense cluster is a hard problem – but it can give a window into the history of the universe

Amelia Abeyawardene
Veronika Chronholm

We met up with Anna Lisa Varri for a chat about her research in theoretical astrophysics. Born and raised in Milan, Italy, she came to Edinburgh in 2012 to take up a position as a Postdoctoral Fellow.

Could you tell us something about your research?

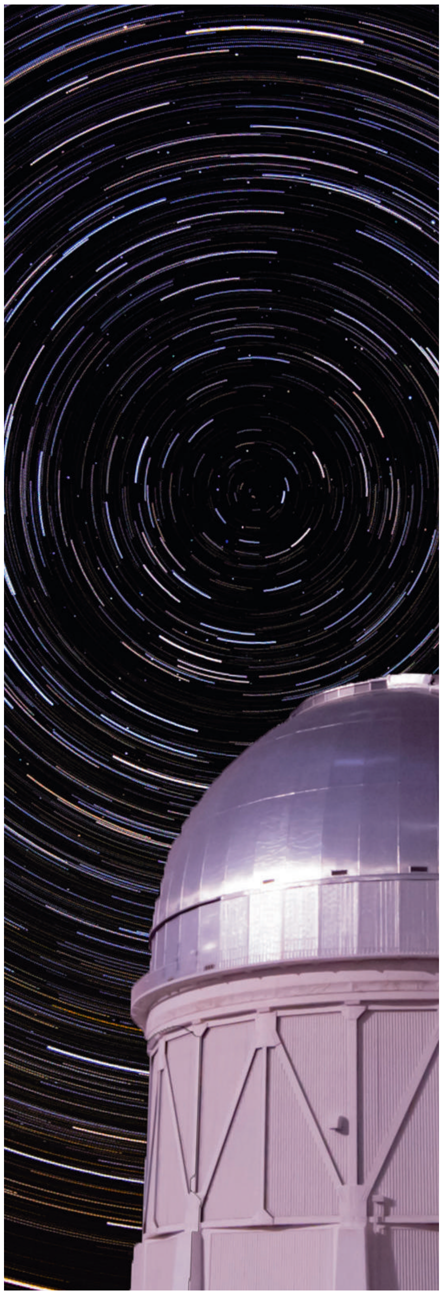
My field is stellar dynamics, a branch of theoretical astrophysics. I'm interested in studying the motion of stars with respect to the effect of their own gravity; in particular, star clusters, which are collections of 10^5 to 10^6 stars. These have a concentrated region, and then they are progressively less dense towards the outskirts. They are bound together by their own gravity, that is, they move in a gravitational field that is generated by the stars themselves. From a mathematical point of view, this is a highly nonlinear problem. You have to evaluate the equation of motion if you want a model that gives the motion of the stars at any given time. This is doable, but it is computationally expensive.

It's interesting to explore the full extension of the lives of star clusters. These are ancient objects, typically around 12 billion years old. We believe that they are almost as old as the universe itself. People used them initially as a lower limit for the age of the universe. Now we have cosmology and a number of more sophisticated tools to explore the very early stages of the universe's formation, but globular star clusters still do a good job.

Can studying star clusters tell us what the universe looked like when it was very young?

Yes and no. Not at the cosmological level because they can't give us insight into inflation or anything associated with the instants after the big bang. But there are a number of models from the galaxy point of view that use them. These describe how the stars were formed throughout the duration of the universe. Some models build up progressively from small to large until you have a full-grown galaxy. Others work the other way around and produce a full-scale galaxy, which then has a discretized resolution.

We also use globular clusters to ▸



REIDAR HAHN/FERMILAB

study stellar evolution. If you want to focus on an individual star within the star cluster, you can explore its life with respect to how much mass it had at the beginning and how it evolved.

The challenging part is to connect the effect of the gravity of the cluster itself with the effect of the gravity of the external galaxy. What's interesting is that you can use pen and paper to get an approximate description.

How much do you have to simplify things so they can be done with pencil and paper?

A lot, but it's still a very useful guide. If you run a simulation with very advanced theories of physics thrown in, you might miss the key principle of the problem. In the star cluster community we refer to this kind of simulation as a 'kitchen sink model'. Each time you find something surprising, you have to take a step back and attempt it with pen and paper, which allows you to reanalyze things with a clear mind. Then you can rerun the simulation and compare the results to your approximate, simplified methods.

How do you test your models against real star clusters, which are very far away?

First you talk with observational astronomers, who typically have the required data. You receive information about the brightness (or luminosity) in the given area of a star cluster. Basically, you draw a circle that encloses a given fraction of the stars in the cluster and measure the luminosity of that. As you increase the size slightly and repeat the process, you eventually get a radial profile of luminosity per unit of the surface. This is a fundamental quantity that is normally measured from

> "Each time you find something surprising, you have to take a step back and attempt it with pen and paper"

the earth with telescopes. We can then compare this with the predictions from our theoretical models.

I'm not normally involved in observational astronomy, but I once was for a week. My role was just to keep the 'real' astronomer awake during the night, because I had no clue about the actual work that you have to do in an observatory!

Are there any observatories in Edinburgh?

Up on Blackford Hill, next to King's Buildings, there is a historical site which is still used for teaching observational astronomy. The oldest observatory in Edinburgh is the one on Calton Hill, but I don't think it's used for teaching any more. These sort of historical sites in the urban area are no longer used for research, because of the light pollution and their very small telescopes. I think one of the misconceptions about being an astronomer or astrophysicist is that we still look through the telescope in an old-fashioned way; however, everything is done through the computer at this stage. I'm often asked about constellations if I'm out with friends. I'm always a disappointment, because I have no idea apart from being able to point out a few major constellations that you probably know yourselves!

So at the research front it's not really about what you can see with the naked eye in the night sky?

Not at all, you really need to get to a level of precision which is not at all achievable with a small telescope that you can have in your backyard, unfortunately. But in some areas we can still take advantage of the data collected by amateur astronomers, and there are projects like Galaxy Zoo online which provide people

with images to interpret. So professional astronomy isn't completely separated from normal people.

Are you ever struck with amazement at the size and age of the universe?

Yes, it's mesmerizing! Sometimes you get used to the daily routine of having your specific goals, but occasionally, you realize that you're actually spending your days thinking about the universe and it's a luxury. You can be overwhelmed by thinking of the profundity and depth of the astronomical object you're studying and that is really… something.

What progress do you hope will be made in astrophysics, within your lifetime?

I am very curious about the existence of the intermediate black hole, and this is something that I believe we could get a final word on in a reasonable timescale by combining information from the Hubble telescope and the Gaia probe.

Who are your biggest inspirations in your field?

I would say my mentor here in Edinburgh: Professor Douglas Heggie. He's a retired professor of mathematical astronomy and he devoted his scientific career to star cluster dynamics. He's one of the few researchers in the community who values analytical tools and doesn't just rely on simulations.

My peers have also given me support and motivation along the way. I'm lucky because the star cluster community is really friendly. There is a tradition of open source code. You are encouraged to use tools from other people and improve them if you can.

Can you tell us about a memorable moment in your research?

During my masters I faced an issue with perturbation problems related to star cluster dynamics. I remember that one night I came across a way to use a different technique to solve a problem, which was formally and mathematically equivalent, even though the physics was different. At this point I was stressed and tired, but I ended up working for another couple of hours until I saw that everything was in place.

Do you think there is a way that astrophysics could be made more accessible to women?

There is an imbalanced distribution of genders at any stage of the career and the higher you get, the worse it is. You start out with almost 50:50 at the undergrad level for physics and maths and then progressively lose women along the way. At the level of faculty and full professors it's ridiculous: 1% to 3% women in the majority of research institutions in the world.

One thing that would help is female role models, to show women that being a researcher is actually achievable. From a practical perspective, you need action that would make the life of a female scientist easier. For example, fellowships or grants that would allow you to return after a break in career, such as a maternity break. These sorts of implementations are taking place and I'm optimistic about that for the future of the present undergraduate generation in particular, so be reassured. ■

- Help astronomers to classify galaxies on the **Galaxy Zoo** website: www.galaxyzoo.org
- Find out about opportunities to visit and tour the **Royal Observatory** at: www.roe.ac.uk/vc

Burak Büke
Operational Research and Optimization

Call centre mathematics

"Operational research is for people who like theory, but who do not want to lose sense of real-world problems," says lecturer Burak Büke

David Selby

Burak has a background in industrial engineering, having graduated from Boğaziçi University, Istanbul, before pursuing a PhD in operational research at the University of Texas. Burak's research interests are tied to real-world applications. "I develop technical tools to assist with decision-making problems in real life; to design and operate systems efficiently."

Operational research is a very dynamic field, he explains. "It has changed a lot. What is popular today may not be popular three years from now."

In the 1970s, the major area of study was scheduling problems — assigning jobs to times — yet by the 1980s most problems had effectively been solved and were no longer studied. As another example Burak cites his PhD topic, semiconductor manufacturing, interest in which had already waned slightly when he completed his thesis.

And these days?

"It might be surprising, but in the area of applied probability nowadays, call centre research is a very big area of research," says Burak. "Call centres have a big impact in everyday life; one in every 30 people is working in a call centre. And call centre mathematics is very difficult mathematics indeed."

Consider a computer company, whose customer service agents are dedicated to software problems, dedicated to hardware problems or cross-trained to handle both. On calling the support helpline, customers press [1] to join the software queue or press [2] to join the hardware queue. As agents become available, customers are assigned to them in an attempt to resolve their problems. This is known as an *M-system*.

The question that arises for the call centre companies is: if a customer joins the software queue, do we assign them to a dedicated agent or to a cross-trained agent? How many dedicated customer support staff should we have on each desk? How much should we invest in cross-training agents to deal with both kinds of problem?

Sometimes a software customer discusses an issue with a software agent and it turns out not to be a software problem after all,

Burak teaches courses in the School's suite of **Operational Research MSc** programmes, including one on Operational Research in the Airline Industry. This course looks at everything from how airlines should set prices and decide their schedules, through to finding the quickest way to have passengers board a plane.

The Edinburgh OR MSc is an intensive one year programme. After working on the core skills of Operational Research in Semester 1, students can tailor their degree by selecting from a broad range of optional courses in Semester 2 — there are optional specializations in Computational Optimization, Finance and Risk. The courses are run by lecturers from the School of Mathematics, with selected courses delivered by industry professionals.

Beginning in June, the MSc students work on a three month project forming the basis of their dissertation. Many projects take the form of a consultancy exercise for a sponsoring organisation, which may be an industrial or commercial company, a government body or research lab.

More details can be found at:

http://msc.maths.ed.ac.uk/or

but actually something to do with the computer's hard disk. So they are rerouted to the back of the hardware queue. This phenomenon was tackled in a paper published by Burak and one of his postgraduate students last year.

"In the literature, it is generally best to route a software customer to software-only agents if possible," explains Burak. This leaves more cross-trained agents free to deal with a surge from either queue. "But we figured out that if you have this problem of misclassification, the usual answers in the literature do not work." In fact, it might be better to send customers to cross-trained agents, who know how to deal with issues, even misclassified ones, straight away without wasting anybody's time.

Who should study operational research? "Not everyone. Many people want to think abstractly. Abstract reasoning is needed in OR but not as much as in other areas such as algebra or topology. I would recommend OR to people who like doing technical research: proving theorems, writing codes and so on."

"Call centres have a big impact in everyday life; one in every 30 people is working in a call centre"

"Operational research is mostly interested in economic problems – meaning business-related or decision making. But my wife is a chemist working in theoretical chemistry, and most of the algorithms she works with – applying and designing new molecules – are developed by OR. So it is not just industry and economics where OR is used." ■

Michael Wemyss
Algebra and Number Theory / Geometry and Topology

Justifying pure mathematics

For a pure mathematician, interactions with other researchers – and with students – are vitally important

Carl Pierer
Anja Stein

Any 2nd Year student on a Mathematics degree will be familiar with the inspiring force of Michael Wemyss' enthusiasm. It seems that such an infectious way of teaching, since it is so rarely found, is hard to achieve. But Michael denies this. Quite the contrary, he says, it is very easy for him to motivate people and it is something he's been doing every day for the past decade.

The perspective Michael has on lectures is quite different from the student's. During the interview it becomes clear that for Michael, communication flows in both directions during a lecture. It is not just the lecturer imparting knowledge, but also the students responding to the way this is done. Students might not be aware of this, but the lecturer pays attention to the students' reactions. Michael points out that during the lecture students are focused on what is going on rather than on what facial expression they put on. When students smile and nod, this is very exciting for the lecturer as it means that they enjoy and understand what they are being taught. Conversely, there is one thing that terrifies Michael when he gives a lecture: "When there is utter silence you know that something is going totally wrong." He explains that silence is eerie because it usually means that the audience doesn't have a clue what is going on. If there is some whispering, people are talking to each other in the hope that their neighbour understands the material. But when this interaction ceases, then the situation becomes very uncomfortable for the lecturer.

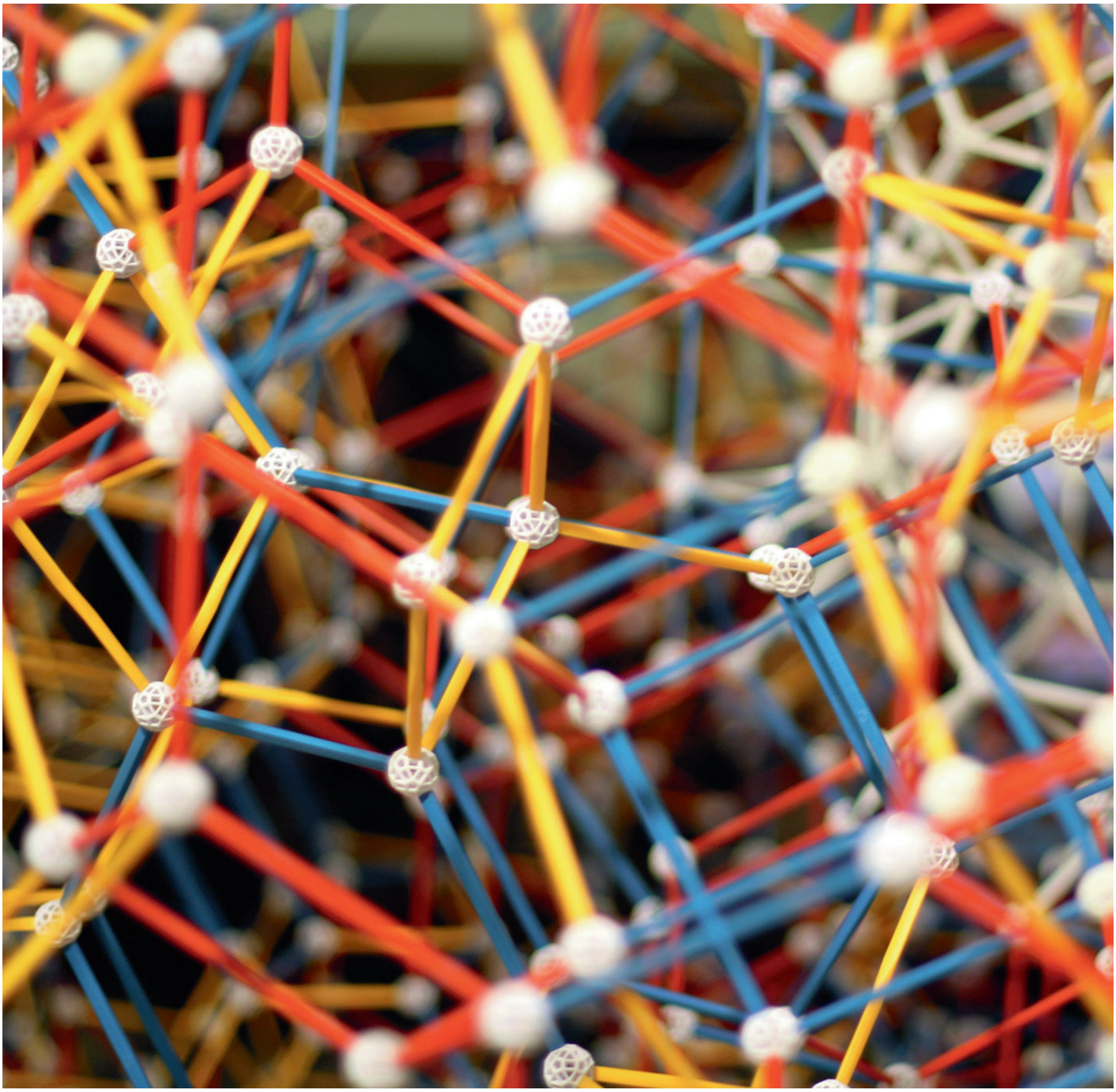
> "Doing research can be a lonely experience and it is good to know that people are in a similar situation as you"

Michael is convinced that this component of human interaction during lectures is fundamental to understanding mathematics. Having a human being to talk to about mathematics, even on a high research level, can give you a lot more than studying research papers. He says: "Some things are just difficult to convey formally. Someone needs to talk you through the methods, the contexts and the notations: mathematics is written very densely."

Michael's lectures in the Fundamentals of Pure Mathematics course started off with the question of why pure mathematics should be studied at all. Often, he tries to relate the results of group theory back to this question. The answer that comes up time and again is that pure mathematics understands the structural properties of a problem, thereby potentially solving a number of problems with the same structure. These skills of abstraction, logical reasoning as well as the analytical skills to process important information are learned through the study of pure mathematics. So while the actual results might not be immediately relevant in the real world, the skills involved are of great importance.

Michael's research is in algebraic geometry and he is active in the GLEN (Glasgow-Liverpool-Edinburgh-Newcastle) seminar for algebraic geometry. The seminar is meant as forum for researchers in the same field to communicate about their work, because "doing research can be a lonely experience and it is good to know that people are in a similar situation as you" but also because of the additional element human interaction adds to the understanding of mathematics. When talking about research seminars, Michael points out that the difference between undergraduate and graduate level mathematics is that in the former you go into lectures expecting to understand most of it if not everything. For the latter, you have to revise your expectations drastically. It's already pretty good, he says, if you can understand the first 10 minutes of a one-hour talk. Indeed, when you first go to graduate seminars, you will hardly understand anything. But, he continues, this shouldn't put undergraduates off from attending the seminars. Rather, if this experience is made now, you will have more time to familiarise yourself with what is happening and you won't experience the shock upon transition from undergraduate to graduate studies. While those seminars are highly challenging, even for researchers in the field, Michael would very much welcome undergraduates to attend them.

Towards the end of the interview, in response to the question of what he would recommend to students interested in pursuing research, Michael states that "You have to enjoy doing what you are doing and immerse yourself in it". It seems, however, that this is hardly possible if you face the prospect of not understanding most of what is going on at a graduate level. So Michael concludes: "If you want make the transition, acknowledge that it will be difficult and it will take a long time to comprehend things being said in seminars. I guess my most important advice is to be brave." ■



GRAEME TAYLOR

Tibor Antal
Probability and Stochastic Analysis

The beauty in mathematical modelling

How can mathematical research contribute to the fight against cancer?

Anna Greer
Abby Li

Dr. Tibor Antal is one of a small group of researchers in the School working on Mathematical Biology. Many of his current projects relate to cancer modelling, in particular to metastasis formation. He describes metastasis as a dangerous process, in which cancer cells in the human body travel to distant organs and then new cancer cells are created there. Tibor is working on some mathematical models to predict whether the patient has metastasis by investigating the probability of the patient having the disease, the number and the sizes of the new metastatic groups of cells.

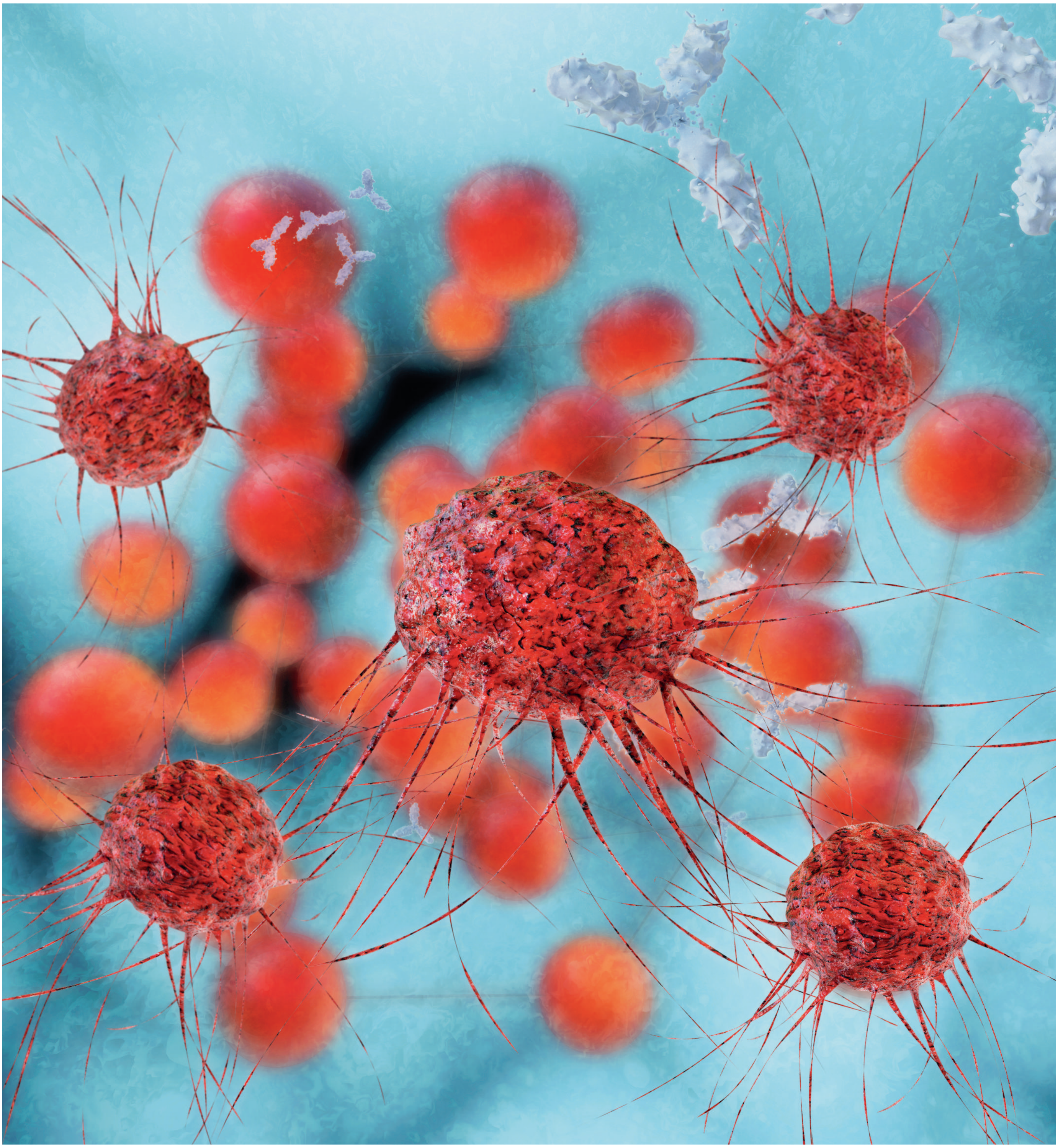
An interest in mathematical modelling and the ambition to help cure cancer are his major motivations behind this choice of research. He finds it amazing that his research findings might one day contribute to medical progress and benefit people's health, and he will definitely spare no pains for this.

He often collaborates with cancer researchers who are biologists, oncologists, and pathologists. It is an advantageous cooperation because he can draw on knowledge from biology to develop mathematical models and the others can learn about the predictions of these models. As he explains how a model works, the collaborators can have a deeper understanding about the model which was already in their heads.

Many properties of the cancer cell can be effectively illustrated by relatively simple models. Even so, it turns out even tackling the simplest model is very complicated. Tibor is simulating the process of tumor metastasis by stochastic modelling, which introduces some elements of randomness into the model. Furthermore, he would like to dig into concrete matters so as to solve the models more precisely and analytically within some certain conditions. Many rules can be specified to check whether everything fits into the model and to study the behaviour of the cell from different aspects. Although not many people are working on this topic and the progress is not very fast, he is working hard step by step to make a difference in this field.

However, mathematics is not Tibor's only interest. In fact, he studied and researched physics until he came to Edinburgh in

During a Probability lecture, Tibor told us a story about **Buffon's Needle**. A soldier in the army had to stand for days at one place, so to pass the time the only thing he could do was to drop a needle on the floor, and count the number of times that the needle crossed the lines on the tiles and the times it did not. Just from the numbers, and a bit of mathematics, he could estimate π. This was magic to Tibor! How could this estimate π which you knew only as the ratio between the perimeter and the diameter of a circle? How could you estimate π from this randomness? He was entirely fascinated by it and he has idealized mathematicians since hearing the story in high school.



2010. He decided to devote himself to research in mathematics because loves to use mathematics to model nature and the universe. He is fascinated by the beauty of mathematics, especially mathematical models, and he thus would like to adopt it as a tool to study other issues. When he arrived in Edinburgh, he realized he has to teach mathematics differently to physics. While Tibor relies more on intuition to teach physics, in mathematics every step also requires rigorous justification.

Besides teaching and research activities, he also has duties in administration. The roles are distributed in the department so the staff are working together to make sure the department functions. As it is not a physical job, he places importance on physical activities as well — he does swimming and cycling in his spare time — which helps him to deal with pressure at work.

For him, doing mathematics allows him to get a glimpse of the internal beauty of something magical. As for us, the idea that his research can help cure cancer is magical indeed. ■

Karen Ogilvie
Applied and Computational Mathematics

We are all inverted pendulums

In the same way that you have to constantly keep moving your hand to balance a metre stick upright, you have to make adjustments in how you move while running or walking. So how can we mathematically model an inverted pendulum?

Veronika Chronholm
Richard Schmoetten

While it is relatively easy to model a normal pendulum, the inverted variety requires mathematics on a different level: the field of asymptotics. This is used to describe equations and functions around singular, unstable points — and it is the field of expertise of PhD student Karen Ogilvie.

Karen studied in Edinburgh as an undergraduate, and was so inspired by her personal tutor that she stayed for a doctorate. She admitted that, since starting postgraduate research, her perception of mathematics has changed quite radically. While the typical undergraduate course contains carefully chosen bits of beautiful mathematics, the liberty of reading about any topic relating to your research comes at a cost. Solving particular problems that have never been answered before involves many more dry technicalities than the proofs commonly presented in lecture theatres. "Problems are so specific now, you have to go through a lot of not-very-pretty analysis to get to the result you want," she laments.

The funding for Karen's PhD is provided by the American National Institute of Standards and Technology. Her work is meant to fill in gaps in the "Digital Library of Mathematical Functions" (DLMF), an online resource used by engineers and researchers across the globe. The inverted pendulum, for example, is approximated by a differential equation called the Mathieu Equation. Karen studies the limiting behaviour of such equations to provide a numerical model for behaviour under extreme conditions in physical applications.

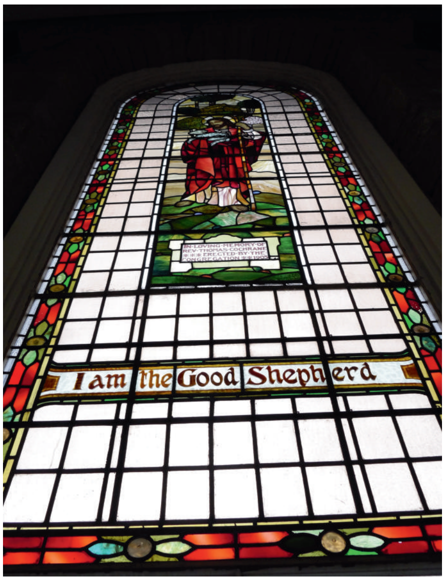
The DLMF exemplifies, in Karen's view, an interesting relationship between applied mathematics and physics: "The way it works in my head is that physics conforms to mathematics, but then again the questions in mathematics crop up because physicists are interested in certain problems! For me the reason to study mathematics is the application to real world problems."

Nonetheless, research in mathematics is prone to doubt: "If even brilliant academics publish incorrect proofs,

how can you ever be sure that your answer is right? It's a scary thought! There are no answers to check your work against." But the greatest danger to an aspiring academic is the feeling that your very own work is irrelevant. Here Karen mentions again the importance of a good relationship to your supervisor, both for intellectual guidance and the occasional pep talk. "Part of the problem is that it's hard to see the impact of your work on the world; you can only hope that it goes out there and eventually leads to something." ■

The church of mathematics

A former church with a prominent stained glass window, ICMS is now home to a different kind of religion: mathematics



Lisa Marquand

The International Centre for Mathematical Sciences, or ICMS for short, sits proudly on South College Street. Its impressive exterior is the perfect host to the equally impressive mathematical activities within.

I was met by Jane Walker, the Centre Manager, and was led about the facilities, pleasantly surprised by the background hum of mathematicians deep in discussion; research in action. We went from seminar room to impressive lecture theatre, walking past mathematicians scribbling away at blackboards, immersed in a week-long program on stochastic systems simulation and control, one of the many workshops that ICMS holds.

It became evident that the centre caters for a wide array of mathematical topics, with workshops ranging from mathematics for health and disease, to game theory, to harmonic analysis, to the mathematics of weather forecasting. Jane explained that ICMS hosts mathematicians from around the globe, providing an opportunity for collaboration and innovation. They see many mathematicians coming through their doors, with a range of 30 to 90 people attending each workshop, and have accomplished names lecturing to the public, such as the Fields Medalist Martin Hairer or mathemagician Colm Mulcahy.

Professor David Abrahams, the Scientific Director, was keen to get across the main aims of the centre: to foster mathematical activity through both workshops and public lectures. Mathematics is, as he puts it, a 'living breathing subject', and it is important to discover the links between various disciplines. Rather inspiringly, David explains how his aim is to have ICMS seen as Northern Britain's centre for mathematics not only for postgraduate and higher, but for undergraduate students and school level too.

Professor David Abrahams, ICMS Scientific Director

On one side, ICMS exists to nurture research potential. As well as organizing workshops, they run initiatives like Research in Groups where 4 to 5 mathematicians can join together and work for a month on a project, enabling intense research without the distraction of teaching and administration. Another example is the Indo-UK Initiative that ICMS are facilitating, where workshops would be held in both the UK and India, helping to improve connections throughout the mathematical community. The centre also shares the building with the MIGSAA CDT programme (see pages 20-21), where PhD students can study, work and are taught through video conferences.

On the other side, ICMS is a gateway to mathematics for the wider community. They collaborate with various companies in the UK to aid the transfer of mathematical knowledge into the business world, working with sectors such as defence, financial services, and pharmaceuticals. And they try to create a wider appreciation for the beauty of mathematics, running public engagement initiatives such as a Soap Bubble Geometry workshop for primary school children.

With David Abrahams keen to influence the younger generation of mathematics, I am sure that the ICMS will be holding some very interesting public lectures and other activities in the months to come, and personally look forward to being inspired further by the many speakers, and the International Centre of Mathematical Sciences itself. ■

The launch of MIGSAA

Imogen Morris
David Selby

In February MIGSAA held its official launch event, bringing together mathematicians from the University of Edinburgh and Heriot-Watt, as well as visitors from across the UK.

Tony Carbery, the Director of MIGSAA, opened the event with a detailed description of MIGSAA's purpose, and the way it works. It is one of a few centres which have been funded to address certain priority areas where more research needs to be done, or in which more people need to be trained. In mathematics some of the priority areas are weather, mathematical biology and, of course, analysis. MIGSAA combines research into many of these practical applications with simultaneous research into pure analysis.

The highlight of the afternoon, for the mathematicians at least, was the two lectures on mathematics at the end of the event. Keith Ball from the University of Warwick gave a lecture on convolution to illustrate one of the surprising places where analysis appears. "Analysis is not just a brand of maths. It is a set of tools and ideas that are used all over mathematics." he says. He starts by listing some of the places analysis ends up. Probability and PDEs are not so surprising, but analysis also turns up in geometry (for example, in the proof of the Poincare conjecture), in number theory (which has an analytic branch involving the Riemann zeta function), and in combinatorics, where it is crucial for additive analysis. "All of analysis appears everywhere, not just parts of it."

Convolution is a particularly nice example. Convolution takes an average of a function from different places. It appears in the solution of the heat equation from physics, and in mathematical entropy.

David Abrahams from the University of Manchester and ICMS gave us an overview of some of the topics in analysis he is interested in. He is an applied mathematician, broadly interested in mathematical aspects of wave phenomena. "I have to be constantly feeding myself real life problems", he says. Most of his lecture was on the method of matched asymptotic expansions and was quite difficult to follow. He did, however, mention something that I assume mathematics students, like myself, will find rather surprising: in asymptotic analysis there are a few useful truths that have never been proved, and maybe never will be, but that always work. ■

A student's perspective

Imogen Morris

Luke Dyer is a PhD student at MIGSAA in his second semester, with supervisor Martin Dindos. He is the student representative at MIGSAA, and enthusiastic about analysis. He is easy to talk to, and our interview turns into a stimulating conversation where questions occur naturally.

Luke is currently working on solving PDEs (partial differential equations). I ask him how this works and he explains that the domain across which the PDE is defined is important to the solution, because there are some domains where the PDE cannot be solved, such as a 'cusp'. He describes a cusp as an area between two curves that seemingly approach on opposite sides of an asymptote, but that actually meet at a point. It reminds me a little of a picture of the onion domes in Moscow. Luke says that there just isn't enough space in the part squeezed between the two lines.

Luke Dyer,
MIGSAA Student

One of the reasons that Luke chose to apply to MIGSAA was because it gave him a chance to learn about topics around his potential research field before deciding on a PhD project. Analysis is a huge topic and one of the most applicable areas of mathematics: it has applications in physical contexts, and also to other areas of mathematics, such as probability, PDEs, combinatorics and more unexpectedly, geometry and number theory. This is one of the reasons it is such a fascinating subject to study. Luke mentions to me that he often finds himself exploring areas that aren't so relevant, or going off on a tangent, because there are so many interesting paths to explore in analysis.

Another reason why Luke chose MIGSAA was because he liked the idea of working with other students,

The Maxwell Institute Graduate School in Analysis and its Applications (MIGSAA) is a new centre for postgraduate research, based in Edinburgh.

- It is a Centre for Doctoral Training (CDT) and the result of a £4.5M investment by the Engineering and Physical Sciences Research Council (EPSRC) and the Scottish Funding Council.
- MIGSAA is the only mathematics CDT in Scotland and one of only eight in the UK.
- It unites mathematicians at the University of Edinburgh and Heriot-Watt University (together forming the Maxwell Institute) in the training of research postgraduates, and will welcome upwards of 60 PhD students over 5 yearly intakes.
- In a traditional PhD, supervisors and topics are chosen at the very beginning, while in a CDT the first year features a broad range of courses so that students can get to know different topics and staff before choosing a research project.
- Training is in a cohort of about 12 students each year so that students can support each other in their learning.
- To apply for the CDT students need a strong first class degree in mathematics (or similar subject). A Master's degree is not necessarily a requirement. The key qualification is to love mathematics!
- Females are particularly encouraged to apply, as gender ratios at postgraduate level are currently at only 20:80 (F:M).

For more information, visit: www.maxwell.ac.uk/migsaa

rather than in isolation. In their first semester at MIGSAA, students are sorted into groups of three, and work on a group project together as well as taking lectures and workshops together. This makes it easy for students to get to know one another, discuss what they are working on and support each other in learning difficult new concepts.

According to Luke, analysis is no longer intuitive at the level he is working at. But it still has a unique character. Luke describes how analysis has particular 'tricks' involved, that only seem to work in analysis. For example, sometimes it isn't straightforward to prove something like '$f(x) = 0$' directly. However, we can prove that '$|f(x)| < \varepsilon$ for all $\varepsilon > 0$' and this effectively proves the same thing.

Luke wasn't always certain he wanted to do a PhD. He also considered jobs in computer programming. However, he realized that it was maths he loved, and the best way to continue that passion was to go into research. Many of us aren't sure that we are intelligent enough to do a PhD, but Luke reassuringly says that he wasn't either. "It's best to let other people decide" he says. He recommends chatting to your Personal Tutor if you are worried. He also tells me that it is normal to be stuck on a problem, or not understand a definition immediately. "That's how you learn" he says. You can't be learning if you understand everything without trying.

He also points out a favourite quote from Michael Atiyah's 'Advice to a Young Mathematician', the article he was inspired by as an undergraduate: "Only the mediocre are supremely confident in themselves". In Luke's opinion perseverance and hard work are often underestimated. Unless you are willing to put in the effort, no amount of intelligence will do you any good. "It's passion and dedication that make you want to come in in the morning and learn something new" he says. ■

Backbone and support

A look inside the Mathematics Teaching Organisation, and the role of Student Learning Advisor

Imogen Morris
Carl Pierer

The efficiency and swiftness with which the School of Mathematics deals with administrative tasks is much appreciated by the student body. Most of the time, the School is among the first across the university to return exam results. Responsible for this is the Mathematics Teaching Organisation (MTO), but their responsibilities include more than the administration of exams.

Fiona Curle, who manages the MTO, explains that it deals with all teaching-related administration, which includes both undergraduate and postgraduate taught courses. This ranges from dealing with workshop allocations and attendance recording to exam timetables. It becomes clear during the interview, however, that the exam period is the most stressful and challenging time for the seven people working in the MTO. Fiona explains how an exam is processed on the side of the MTO. Staff ensure that papers are received from lecturers and sufficient copies are submitted to central administration before the examination date. When the exam is finished, the scripts are returned to the MTO for distribution to the markers, then the marks are entered into the assessment software for the examination boards to ratify. Fiona has introduced an efficient procedure for this. First, a member of MTO staff will download the marks spreadsheet from the examination software, which will then be checked by a second person, another person validates the marks as they go into the University's EUCLID database, and a fourth person uploads them. Furthermore, Fiona mentions that "the same person won't do two [consecutive] steps". Only at the end of this process are the marks available to the students.

There are two things Fiona would like students to know. First, the course secretary is there to help and students shouldn't hesitate to contact them. Second, she feels that students could use Path more; the website is designed for students to help them figure out which courses they want to take.

While the MTO deals with all teaching related administration and provides support for students experiencing difficulties with formal issues, Dr Pamela Docherty is there to help students with structural issues of studying Mathematics.

Pamela Docherty, Student Learning Advisor

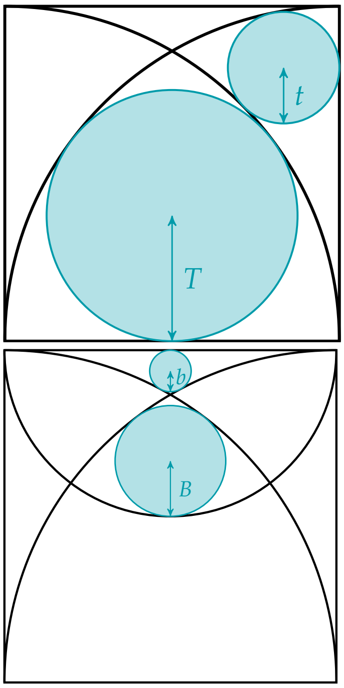
Once a mathematics student at Edinburgh herself, Dr Pamela Docherty is now helping current students with their studies. Since she is the first ever 'Student Learning Advisor' at the School of Mathematics, she was given the freedom to shape her job. Pamela explains that three of the things she tries to accomplish in her job are helping students; improving the student experience, and encouraging students to learn in a better way.

Pamela says she tries to give evidence-based advice informed by reputable studies. In a recent mid-semester review meeting with a group of students she mentioned a Scientific American article that reported reading a passage more than twice doesn't help you remember it better. Other advice she has learned from reading studies is that it's better to study several different subjects in a day rather than focusing on a single one, as it reinforces your memory. It's also important to keep work balanced through the semester. No one can catch up on all the work they've missed at last minute, before the exam, so catching up as soon as we fall behind is the only way to stay on top of it all. Another tip Pamela mentions is to avoid rewriting notes, or highlighting. Only methods that involve testing ourselves will help retain memory, or deepen understanding. Even trying to write out a definition before looking it up in the textbook can help.

Most importantly, Pamela recommends that we look ahead. Many students only worry about the next assignment, she says. But activities that can motivate us and reinforce our love of maths are just as important, even if they are less urgent. We should take the effort of going to inspiring lectures and extracurricular events that in the future will help us decide on our path in life. ■

Puzzles

Stuck? Solutions are on the website:
www.maths.ed.ac.uk/contours

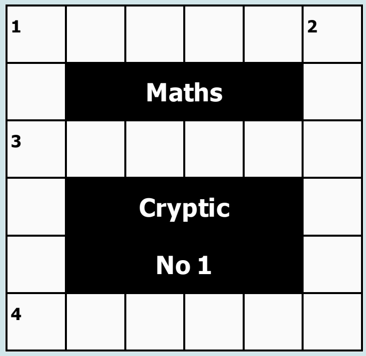


Sangaku problems

Show that $4T=9t$ and $3B=8b$.

These are examples of problems which were hung on tablets in shrines and temples during the *sakoku* period (1639-1853) in Japan. Since the country was secluded in those years, Japanese mathematicians used a distinct kind of mathematics known as *wasan* to solve the problems.

In the School, Professor José Figueroa-O'Farrill offers a 4th year project which involves studying some sangaku, and their connections with Western approaches to similar problems.



Across

1. Rings from void one (6)
3. Round decrepit relic found at the end of a field (6)
4. Maclaurin has such views about the Royal Institution (6)

Down

1. Function for confused car companies (6)
2. Principles: they're not necessarily right! (6)

Easy Klein Bottle Knitting/Crochet Pattern

Imogen Morris

The materials specified make a miniature Klein bottle suitable for hanging from a bag or keychain. My Klein bottle took about an hour to make, but that will vary from person to person.

Abbreviations
ch = chain
skp = skip next stitch
dc = US double crochet (UK treble crochet)
sl st = slip stitch

Materials

- Four double pointed 2.5 mm needles
- One 2.5 mm crochet hook
- Jamieson's Shetland Spindrift (or any 4ply yarn)

As the size of the Klein bottle is not too important, you can use any weight of wool with the appropriate needles and hook.

Knit the 'handle'
Cast on 9 stitches. Join in the round, being careful not to twist the stitches. Knit garter stitch (plain knit) until about 12 cm long. (It doesn't matter if you knit a bit longer: a Klein bottle is a topological shape after all! But make sure it is at least 12cm.)

Work increases
Divide the stitches equally onto 3 needles (You may want a stitch marker to mark the start of each round. A safety pin is useful for this.).
Rows 1-2: Knit front and back loop of the first stitch of each needle, and knit all other stitches. You should end up with 15 stitches.
Rows 3-6: Knit front and back loop of the first and the last stitch of each needle, and knit all other stitches. You should end up with 33 stitches.

Crochet the 'outer surface'
Cast off all stitches except the last. Switch to the crochet hook.
Rows 1-2: ch4 skp1 (*dc ch1 skp1*) Repeat from * to * until end of round, join onto 3rd chain of 1st ch4 with sl st.
Row 3: ch4 skp1, dc ch1 skp1, dc ch4. Now put the end of the knitted 'handle' under the chain 4, and skp3 dc so that it is now sticking out through the newly made gap. (*dc ch1 skp1*) Repeat from * to * until end of round, join onto 3rd chain of 1st ch4 with sl st.
Rows 4 onwards: * (dc ch1 skp1) twice then (dc ch1 skp3) * Repeat from * to * until end of round, join onto 3rd chain of 1st ch4 with sl st. This will decrease stitches. Keep going until you have an opening around the same size as the opening of the end of your knitted 'handle' (which should be sticking out of the side of your Klein bottle). Then bind off, and sew the two openings together, so that you have a seamless, complete Klein bottle.

Tips
I used a 'long tail' cast on, but the method is not important for this pattern.
Your crochet stitches will probably be wider than your knit stitches (at least mine were) so you might find you are joining, say, 7 stitches to 9, at the end.
Most importantly, Klein bottles are quite forgiving of mistakes, so don't worry too much about it being perfect, and enjoy yourself!

THE UNIVERSITY of EDINBURGH
School of Mathematics