Building interest rate scenarios

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Risk management

We’re in the business of risk-management

- On behalf of large groups of individuals
- Over very different time horizons
Pension scheme example

Employer → Pension scheme → Scheme members
Pension scheme example

1. How much should be invested so that the probability of running out of cash is low enough?
2. What’s the market-consistent value of the liability?
Economic Scenarios

- Probability of running out of cash:
  - Real world probabilities

- Market consistent value of liability:
  - Risk neutral probabilities
Outline
Outline

- Why use economic scenarios?
- Main challenges
- Empirical facts about interest rates and inflation
- Arbitrage free pricing
- Calibration
Main challenges
Main challenges

- Many choices
- Complexity
  - No analytical formulas
  - Computational resource constraints
    - Size of time-steps
    - Number of scenarios

What are we trying to do?
Many choices

- **Risk factor**
  - Initial Yield Curve
  - Interest Rate
  - Inflation
  - Credit
  - ...

- **Model**
  - Nelson-Siegel
  - Cubic Spline
  - Smith-Wilson
  - ...
  - Deterministic
  - Vasicek
  - G2++
  - ...
  - Jarrow-Yildirim
  - Stoch Volatility
  - ...
  - CIR++
  - JCIR++
  - JLT
  - ...

- **Calibration**
  - Gilts
  - Swaps
  - OIS
  - ...
  - Swaptions
  - Caps/Floors
  - Distib Targets
  - ...
  - YY Inf Swaps
  - LPI Bonds
  - Distib Targets
  - ...
  - Corp Bonds
  - CDS
  - Spreads
  - ...
  - ...

- Dependencies
Size of time steps
Empirical facts about interest rates
A brief history of interest rates
A brief history of interest rates
Gilt spot rates - BoE
Nominal short rates
Real spot rates - BoE
Real short rates
Inflation (RPI)
Inflation (RPI)
Inflation (RPI) by month
Facts about interest rates & inflation

- Interest rates do not vary deterministically
- Nominal rates are (usually) positive, real rates can be negative
- They appear to be observations of a jump-diffusion process
  - Regime shifts
- The short rate is more volatile than the long rate
- Short and long rates exhibit decorrelation
- Seasonality in inflation
General theories of interest rates

- Expectations Theory
- Liquidity Preference Theory
- Market Segmentation Theory
- Arbitrage-Free Pricing Theory

\[ f(0,T) = E[r(T)] + \text{RiskPremium} + \text{ConvexityAdjustment} \]
Forward rates as expectations?
Forward rates as expectations?
Arbitrage free pricing
Arbitrage

- A model that allows for arbitrage leads to combinations of prices that do not make sense
- How do we build an arbitrage free model?
Fundamental Theorem of Asset Pricing

Cash account: \( B(s) = B(t) \exp \left( \int_t^s r(u) du \right) \)

Arbitrage free dynamics

Existence of \( Q (\sim P) \) such that, for \( t \leq s \leq T \),
\[
E_Q \left[ \frac{P(s, T)}{B(s)} \right| \mathcal{F}_t \right] = \frac{P(t, T)}{B(t)}
\]
It follows \((s = T)\) that the price at time \(t\) of a derivative with payoff \(V(T)\) at time \(T\) is

\[
V(t) = E_Q \left[ e^{-\int_t^T r(u)du} V(T) | \mathcal{F}_t \right]
\]

Stochastic discount factor \(D(t, T)\)
A word of caution
Our requirements so far

- Arbitrage-free pricing model
- Realistic probability distributions
- Capable of pricing interest rate and inflation derivatives
Real rates and inflation

Foreign currency analogy

In each economy there are two currencies: the nominal (i.e. domestic) currency and the real (i.e. foreign) currency. The exchange rate between the two currencies is given by the inflation index $I(t)$ (e.g. CPI index): one unit of real currency is equal to $I(t)$ units of nominal currency.

Nominal rates, real rates and inflation are modelled simultaneously
Modelling multiple economies

 GBP
- Nominal: 2 factors
- Real1 (e.g. RPI): 2 factors; Inflation1: 1 factor
- Real2 (e.g. CPI): 2 factors; Inflation2: 1 factor

 USD
- Nominal: 2 factors; Exchange: 1 factor
- Real: 2 factors; Inflation: 1 factor

\[
egin{pmatrix}
  1 & < dz_1, dz_2 > & \ldots & < dz_1, dz_{14} > \\
< dz_1, dz_2 > & 1 & \ldots & < dz_2, dz_{14} > \\
\vdots & \vdots & \ddots & \vdots \\
< dz_1, dz_{14} > & < dz_2, dz_{14} > & \ldots & 1
\end{pmatrix}
\]
Quanto adjustment (Risk-Neutral)

\[
\frac{X(T)}{B(T)} \text{ is a martingale} \iff \frac{dX}{X} = n \, dt + \cdots
\]

\[
B^{\text{real}}(T)I(T)/B(T) \text{ is a martingale} \iff \frac{dI}{I} = (n - r) \, dt + \cdots
\]

\[
X^{\text{real}}(T)I(T)/B(T) \text{ is a martingale} \iff \frac{dX^{\text{real}}}{X^{\text{real}}} = (r - \rho_{XI}\sigma_I\sigma_X) \, dt + \cdots
\]
Model calibration
Calibration

Calibration of a model

Find the best choice of parameters such that the (weighted) squared difference between targets and corresponding model values is as small as possible

$$\Theta^2(\vec{p}) = \sum_{i=1}^{N} w_i \left(V_i^{\text{target}} - V_i^{\text{model}}(\vec{p})\right)^2$$

Example of targets:

- Market prices of financial instruments (e.g. swaptions, inflation swaps, etc.)
- Distributional properties (e.g. short rate dispersion in 3yrs, or long term dispersion of 17yr spot rate)
Calibration methods

Cascade calibration
1. Calibrate Nominal Short Rate Model
2. Calibrate Real Rates & Inflation Model

Full calibration
Calibrate Nominal Short Rate Model and Real Rates & Inflation Model as a single model
Calibration

- It is a non-linear least squares optimisation problem
- Can be solved numerically
  - Levenberg-Marquardt optimiser
Real world modelling

- Each path should look realistic. Paths should be equally likely.
- Ability to reproduce initial yield curve
- Future expectations of key variables (short rates, long rates, instantaneous inflation) should be in line with our views
  - Type of distribution is enforced by the model
- Width of distributions at different maturities should reflect our views
Risk premium and convexity

\[ f(0,T) = E_P[r(T)] + \text{RiskPremium} + \text{ConvexityAdjustment} \]
Specification of target paths

**EXPECTED LONG RATE**

- **F(0,t,t+17)**
- Long term target
- E[R(t,t+17)] (RW)

**Maturity (yr)**

**Level**

**Dispersion**
Realistic distributions

- **Levels**
  - Exponentially weighted average of historical time series

- **Dispersion**
  - Exponentially weighted average of standard deviation of historical time series

Alternative: target quadratic (co)variations
Summary

- We want to price instruments and generate realistic probability distributions
- Modelling involves many choices and assumptions
- Empirical facts
- Foreign currency analogy for real rates and inflation
A little disclaimer

We review the calibration methodology on a regular basis and may modify, adjust or improve the methodology.
Thank you
Any questions?