The EDF Day Ahead Unit Commitment Approach

Scheduling à la Française – It's all French to me!

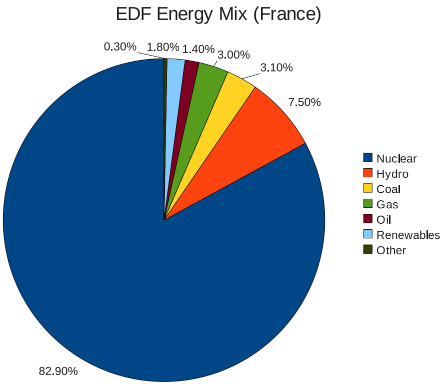
THE UNIVERSITY of EDINBURGH

School of Mathematics

Agenda

EDF System Overview

- Largest electric utility company in France



EDF System Overview

- 58 nuclear units and 47 other thermal units (coal, gas, oil)
- 50 hydro valleys: interconnected reservoirs (150) and hydro plants (448)
- Wind, solar and biomass $< 2\%$ but significantly growing

The day-ahead model has 48h in half-hour periods (96 periods)

EDF System Overview

1. Data collection finished, optimization timeframe of 15 min
2. Human postprocessing of schedules
3. Grid operator and engineers on site prepare for operation
4. Schedule is active, hourly intra-day reoptimization with at most 30 changes in total, 3h ahead

Model for Thermal Units (incl. Nuclear)

1. Output bounds, min up/down times
2. Discrete output levels, ramp rates
3. Min const. time after increase and variation prohibition after decrease
4. Startup and shutdown curves
5. Daily limits on variation, startup, shutdown
6. IP instead of MIP, so as to be solved by DP

Model for Hydro Valleys

1. Turbines discharge water from upstream reservoirs, some can pump
2. The plants' turbines are modeled individually, with fixed output (approx.)
3. Discrete output levels for plants, turbines ranked according to power rates (sequence constraint)
4. A plant can either produce or pump, with 30min pause in between
5. A turbine has to be switched (on/off) for at least 1h
6. There are plant-wide ramp rates
7. Preservation constraints balance the water flow, objective is the value of water

Abstract Model of the Whole System

$$\begin{aligned} \min_{(p_i,r_i)\in\mathcal{P}_i} \quad & \sum_{i\in I} c_i(p_i) \\ \text{s.t.} \quad & \sum_{i\in I} p_{it} = D_t, \ \forall t \in T \\ & \sum_{i\in I} r_{it} \geq R_t, \ \forall t \in T \end{aligned}$$

Notation

I – units (gens,valleys)
T – planning horizon
$\mathcal{P}_i$ – feasible schedules
R_t – required reserve
p_i – output schedule (all $t \in T$)
r_i – reserve (all $t \in T$)
D_t – demand

The Solution Method

- APOGEE solver, proprietary software developed 1993 under A. Renaud, later modified by C. Lemaréchal
- Based on Lagrangian decomposition of the system into
  1. Individual thermal units
  2. Individual hydro valleys (multiple plants, turbines, reservoirs)
- Solve the Lagrangian by Uzawa type subgradient methods (in new version: bundle methods)

The Primal

$$\min_{(p_i, r_i)\in\mathcal{P}_i} \quad \sum_{i\in I} c_i(p_i)$$
$$\text{s.t.} \quad \sum_{i\in I} p_{it} = D_t, \ \forall t \in T$$
$$\sum_{i\in I} r_{it} \geq R_t, \ \forall t \in T$$

A Lagrangian Dual

$$\max_{\lambda \text{ free}, \ \mu\geq 0} \ \min_{(p_i, r_i)\in\mathcal{P}_i} \sum_{i\in I} c_i(p_i) + \sum_{t\in T} \lambda_t \left(D_t - \sum_{i\in I} p_{it} \right) + \sum_{t\in T} \mu_t \left(R_t - \sum_{i\in I} r_{it} \right)$$

- Inner problem separable by generation units / valleys
- Thermal unit subproblems solved by DP, hydro valley solutions approximated by LP (simplex method)
- Outer problem can be solved by subgradient method

Uzawa's Subgradient Method

Initialize $k = 1$, $\lambda = \mu = 0$ and a tolerance $\delta > 0$. Choose sequences $\left(\epsilon_p^k\right)_{k \in \mathbb{N}}$ and $\left(\epsilon_r^k\right)_{k \in \mathbb{N}}$ with $\sum_{k=0}^{\infty} \epsilon^k = \infty$ and $\sum_{k=0}^{\infty} \left(\epsilon^k\right)^2 < \infty$.

1. For all $i \in I$ solve $\min_{(p_i, r_i) \in \mathcal{P}_i} \left[c_i(p_i) - \sum_{t \in T} \lambda_t^k p_{it} - \sum_{t \in T} \mu_t^k r_{it}\right]$ to find p_i^{k+1} and r_i^{k+1}.
2. Update $\lambda_t^{k+1} = \lambda_t^k + \epsilon_p^k \left(D_t - \sum_{i \in I} p_{it}^{k+1}\right)$.
3. Update $\mu_t^{k+1} = \max\left\{\mu_t^k + \epsilon_r^k \left(R_t - \sum_{i \in I} r_{it}^{k+1}\right), 0\right\}$.
4. If $\left\|\lambda_t^{k+1} - \lambda_t^k\right\|_2 < \delta$ and $\left\|\mu_t^{k+1} - \mu_t^k\right\|_2 < \delta$ terminate. Otherwise set $k = k + 1$ and go to 1.

A Variable Metric Bundle Method

Initialize $k = 1$ and $\lambda^1 = \mu^1 = \hat{\lambda} = \hat{\mu} = 0$ and p^1, r^1. Choose a penalty $s > 0$ and tolerance $\delta > 0$.

1. For all $i \in I$ solve $\min_{(p_i, r_i) \in \mathcal{P}_i} \left[c_i(p_i) - \sum_{t \in T} \lambda_t^k p_{it} - \sum_{t \in T} \mu_t^k r_{it} \right]$ to find p_i^{k+1} and r_i^{k+1}.
2. If $L(p^{k+1}, r^{k+1}, \lambda^k, \mu^k) > L(p^k, r^k, \lambda^k, \mu^k)$ set $\hat{\lambda} = \lambda^k$,$\hat{\mu} = \mu^k$ and increase s. Else decrease s.
3. To find λ_t^{k+1} and μ_t^{k+1}, solve

$$
\begin{aligned}
\max \quad & r - \frac{1}{2s} \|\lambda - \hat{\lambda}\|_2^2 - \frac{1}{2s} \|\mu - \hat{\mu}\|_2^2 \\
\text{s.t.} \quad & r \leq L(p^l, r^l, \lambda^l, \mu^l) + \lambda^T (D_t - \sum_{i \in I} p_{it}^l) + \mu^T (R_t - \sum_{i \in I} r_{it}^l), \ \forall l = 1, \ldots, k \\
& \mu \geq 0
\end{aligned}
$$

4. If $\left\| r^{opt} - L(p^{k+1}, r^{k+1}, \lambda^{k+1}, \mu^{k+1}) \right\|_2 < \delta$ terminate. Otherwise set $k = k + 1$ and go to 1.

Why it fails.

- Primal problem is nonconvex $\rightarrow$ duality gap is nonzero
- Subgradient method: solving the dual yields no primal point
- Bundle method: After convergence of the dual, the primal point is noninteger
- This convex combination of schedules (think CG in the primal!) is called pseudo schedule
- For hydro valleys only the LP relaxation is solved

How to proceed.

Use the dual solution as lower bound and perform a heuristic search for a primal solution.

APOGEE:

1. Solve the dual to obtain a pseudo schedule and a lower bound. Formerly by subgradients, more recently by a bundle method. Solve subproblems by DP (thermal) and LP (hydro).
2. Primal recovery: perform an augmented Lagrangian based heuristic search for primal feasible solutions. Solve subproblems by DP (thermal) and IP heuristics (hydro).

Phase 2: Augmented Lagrangians

Consider the convex split variable problem (J_1, J_2 differentiable, $\mathcal{U} \cap \mathcal{V} \neq \emptyset$)

$$\min_{u \in \mathcal{U},\ v \in \mathcal{V}} J_1(u) + J_2(v) \ \text{ s.t. } \ u - v = 0.$$

Its augmented Lagrangian

$$L(u, v, \lambda) = J_1(u) + J_2(v) + \lambda^T (u - v) + \tfrac{c}{2}(u - v)^T (u - v)$$

is not separable due to the quadratic term. However, with a maximizing price λ the primal minimization yields a feasible solution.
Idea: make quadratic term separable by linearizing it.

An Auxiliary Problem

Consider the convex problem (with l_1 differentiable, l_2 l.s.c.)

$$\min_{x \in \mathcal{X}} l_1(x) + l_2(x).$$

The auxiliary function at a given point $\bar{x} \in \mathcal{X}$ with some strongly convex, differentiable K [e.g. $K(x) = \frac{1}{2}x^T x$] and $\epsilon > 0$ is

$$G^{\bar{x}}(x) := \frac{1}{\epsilon}K(x) - \frac{1}{\epsilon}x^T K'(\bar{x}) + x^T l_1'(\bar{x}) + l_2(x).$$

Lemma (Auxiliary Problem Principle), Proof in [1]

Assume that $\bar{x}$ minimizes $G^{\bar{x}}$:

$$G^{\bar{x}}(\bar{x}) = \min_{x \in \mathcal{X}} G^{\bar{x}}(x),$$

then

$$l_1(\bar{x}) + l_2(\bar{x}) = \min_{x \in \mathcal{X}} l_1(x) + l_2(x).$$

Auxiliary Problem for the Augmented Lagrangian

We can apply this to the Lagrangian

$$L(u,v,\lambda) = \underbrace{J_1(u) + J_2(v) + \lambda^T(u-v)}_{l_2(u,v)} + \underbrace{\tfrac{c}{2}(u-v)^T(u-v)}_{l_1(u,v)}$$

to get a separable auxiliary function

$$\begin{aligned} G^{(u^k,v^k)}(u,v) \quad &:= \quad \tfrac{1}{\epsilon}K_1(u) - \tfrac{1}{\epsilon}u^T K_1'(u^k) + [\lambda + c(u^k - v^k)]^T u + J_1(u) \\ &+ \quad \tfrac{1}{\epsilon}K_2(v) - \tfrac{1}{\epsilon}v^T K_2'(v^k) - [\lambda + c(u^k - v^k)]^T v + J_2(v) \end{aligned}$$

The Algorithm

Initialize $k = 1$ and λ^k, u^k, v^k, a tolerance $\delta > 0$ and a steplength $0 < \rho < 2c$.

1. $u^{k+1} = \text{argmin}_{u \in \mathcal{U}} \left[\frac{1}{\epsilon} K_1(u) - \frac{1}{\epsilon} u^T K_1'(u^k) + [\lambda^k + c(u^k - v^k)]^T u + J_1(u) \right]$
2. $v^{k+1} = \text{argmin}_{v \in \mathcal{V}} \left[\frac{1}{\epsilon} K_2(v) - \frac{1}{\epsilon} v^T K_2'(v^k) - [\lambda^k + c(u^k - v^k)]^T v + J_2(v) \right]$
3. $\lambda^{k+1} = \lambda^k + \rho(u^{k+1} - v^{k+1})$
4. If $\left\| u^{k+1} - v^{k+1} \right\|_2 < \delta$ stop. Otherwise set $k = k + 1$ and go to 1.

The choice for $K_{1,2}(\cdot)$ is $K(x) := \epsilon K^c \|x\|^2$.

The Split Variable Unit Commitment Problem

$$
\begin{aligned}
\min \quad & \sum_{i \in I} c_i(p_i) \\
\text{s.t.} \quad & \sum_{i \in I} q_{it} = D_t, \ \forall t \in T \\
& \sum_{i \in I} s_{it} \geq R_t, \ \forall t \in T \\
& (p_i, r_i) \in \mathcal{P}_i \\
& p_{it} = q_{it}, \ \forall t \in T, \ i \in I \\
& r_{it} = s_{it}, \ \forall t \in T, \ i \in I
\end{aligned}
$$

- q_i and s_i satisfy the static constraints (demand, reserve)
- p_i and r_i satisfy the dynamic constraints (technical, temporal)
- the linking constraints need to be relaxed to achieve separability by units
- apply the augmented Lagrangian based decomposition heuristic to solve this problem

The Phase 2 Algorithm

Initialize $k = 1$ and λ^k, μ^k from Phase 1 and choose a tolerance $\delta > 0$

1. For all $i \in I$ find p_i^{k+1}, r_i^{k+1} by solving

$$\min_{(p_i, r_i) \in \mathcal{P}_i} \left[c_i(p_i) - \sum_{t \in T} \left(\bar{\lambda}_{it}^k p_{it} + \bar{\mu}_{it}^k r_{it} \right) + K^c \sum_{t \in T} \left((p_{it} - p_{it}^k)^2 + (r_{it} - r_{it}^k)^2 \right) \right]$$

2. For all $t \in T$ find q_t^{k+1}, s_t^{k+1} by solving

$$\begin{aligned} \min_{q_{it}, s_{it}} \quad & \left[\sum_{i \in I} \left(\bar{\lambda}_{it}^k q_{it} + \bar{\mu}_{it}^k s_{it} \right) + K^c \sum_{i \in I} \left((q_{it} - q_{it}^k)^2 + (s_{it} - s_{it}^k)^2 \right) \right] \\ \text{s.t.} \quad & \sum_{i \in I} q_{it} = D_t, \ \sum_{i \in I} s_{it} = R_t. \end{aligned}$$

3. Set $\lambda_{it}^{k+1} = \lambda_{it}^k + c(q_{it}^{k+1} - p_{it}^{k+1})$ and $\mu_{it}^{k+1} = \mu_{it}^k + c(s_{it}^{k+1} - r_{it}^{k+1})$.
4. If $\left\| q_{it}^{k+1} - p_{it}^{k+1} \right\|_2 < \delta$ and $\left\| r_{it}^{k+1} - s_{it}^{k+1} \right\|_2 < \delta$ stop. Otherwise set $k = k + 1$ and go to 1.

Here the shifted duals are $\bar{\lambda}_{it}^k = \lambda_{it}^k + c(q_{it}^k - p_{it}^k)$ and $\bar{\mu}_{it}^k = \mu_{it}^k + c(s_{it}^k - r_{it}^k)$.

Performance

- In operational practice phases 1 and 2 require $\approx$10 min
- Due to nonconvexity the final schedule satisfies load balance only approximately
- The final gap is $\approx$3%
- Marginal costs are taken from the phase 1 solution

EDF in the Future

- MIP for hydro subproblems in phase 1 (currently LP in phase 1 and heur. in phase 2)
- Get meaningful marginal costs from phase 2
- Improve phase 2: better heuristics (gen. alg.) to reduce the gap

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