Chapter 7
A Decision Support Model for Weekly Operation of Hydrothermal Systems by Stochastic Nonlinear Optimization

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Abstract This chapter formulates and solves an optimal resource allocation problem of thermal and hydropower plants with multiple basins and multiple connected reservoirs. The stochastic factors of the problem are here represented by natural hydro inflows. A multivariate scenario tree is in this case obtained taking into account the stochastic inputs and their spatial and temporal dependencies. The hydropower plant efficiency depends on its water head and the reservoir volume depends nonlinearly on the headwater elevation, leading to a large-scale stochastic nonlinear optimization problem, whose formulation and solution are detailed in the case study. An analysis of exhaustive alternatives of computer implementation is also discussed.

Keywords Medium-term hydrothermal scheduling · Nonlinear optimization · Water head · Stochastic optimization · Scenario tree · Cost minimization · Hydro reservoirs

Notation

For clarity purposes parameters are represented in uppercase letters and variables are represented by lowercase letters.

Indices

p	time period
p'	time subperiod
t	thermal unit
h	storage hydro or pumped storage hydro plant

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r	hydro reservoir
$rr(r)$	reservoir upstream of reservoir r
$rh(r)$	reservoir r upstream of storage hydro plant
$hr(r)$	storage hydro plant upstream of reservoir r
ω	inflow scenario
$a(\omega)$	ancestor scenario of scenario ω in previous period

Parameters

$D_{pp'}$	demand of subperiod p' in period p (MW)
$\mathrm{DU}_{pp'}$	duration of subperiod p' in period p (h)
P_p^{ω}	probability of scenario ω in period p (p.u.)
$\underline{TP}_{pt}, \overline{TP}_{pt}$	minimum and maximum output of thermal unit t in period p
VC_t	Variable cost of thermal unit t (€/MWh)
FOR_t	Forced outage rate of thermal unit t (p.u.)
$\underline{H}_t, \overline{H}_t$	Minimum and maximum yearly operation hours of thermal unit t (h)
$\mathrm{PDH}_t, \mathrm{PEH}_t$	Penalty by deficit or surplus of yearly operation hours of thermal unit t (€/kh)
$\underline{\mathrm{HP}}_{ph}, \overline{\mathrm{HP}}_{ph}$	Minimum and maximum output of storage hydro plant h (MW)
$\underline{\mathrm{PP}}_{ph}, \overline{\mathrm{PP}}_{ph}$	Minimum and maximum consumption of pumped storage hydro plant h (MW)
η_h	Efficiency of pumped storage hydro plant h (p.u)
$\underline{R}_{pr}, \overline{R}_{pr}$	Minimum and maximum operational reserve volume of reservoir r in period p (hm^3) (GWh)
$\underline{C}_{pr}, \overline{C}_{pr}$	Minimum and maximum capacity of reservoir r in period p (hm^3) (GWh)
$\mathrm{IR}_r, \mathrm{FR}_r$	Initial and final reserve volume of reservoir r (hm^3) (GWh)
$\mathrm{PDFR}_r, \mathrm{PEFR}_r$	Penalty by deficit in final reserve (and in minimum and artificial reserve) and surplus in final reserve (and in maximum reserve) of reservoir r (k€/hm^3) (€/MWh)
$\underline{G}_{pr}$	Minimum release of reservoir r in period p (hm^3) (GWh). This accounts for other uses of water for water supply, environmental and ecological concerns like fish and wildlife maintenance and recreational activities
I_{pr}^{ω}	Unregulated inflow of reservoir r in period p of scenario ω (m^3/s) (GWh)
TH_h	Tailrace elevation of hydro plant h(m)
RH_r	Reference elevation of reservoir r(m)
A_h, A'_h	Fixed and linear term of production function of plant (hWh/m^3) (hWh/m^4)
B_r, B'_r, B''_r	Fixed, linear, and quadratic terms of reserve volume of reservoir r (hm^3) (hm^3/m) ($\mathrm{hm}^3/\mathrm{m}^2$)

Variables

$tp^{\omega}_{pp't}$	Output of thermal unit t in subperiod p' of period p of scenario ω (MW)
$hp^{\omega}_{pp'h}$	Output of storage hydro plant h in subperiod p' of period p of scenario ω (MW)
$pp^{\omega}_{pp'h}$	Consumption of pumped storage hydro plant h in subperiod p' of period p of scenario ω (MW)
s^{ω}_{pr}	Spillage of reservoir r in period p of scenario ω (hm^3) (GWh)
$\mathrm{ar}^{\omega}_{pr}$	Artificial reserve of reservoir r in period p of scenario ω (hm^3) (GWh)
r^{ω}_{pr}	Reserve volume of reservoir r at the end of period p of scenario ω (hm^3) (GWh)
dfr^{ω}_{r}, efr^{ω}_{r}	Deficit and surplus of final reserve of reservoir r in scenario ω (hm^3) (GWh)
dmr^{ω}_{pr}, emr^{ω}_{pr}	Deficit of minimum reserve and surplus of maximum reserve of reservoir r in period p of scenario ω (hm^3)
doh^{ω}_{t}, eoh^{ω}_{t}	Deficit of minimum yearly operation hours and surplus of maximum yearly operation hours of thermal unit t in scenario ω (h)
g^{ω}_{pr}	Release of reservoir r in period p of scenario ω (hm^3)
g^{ω}_{ph}	Release of storage hydro plant h in period p of scenario ω (hm^3)
$\mathrm{pf}^{\omega}_{ph}$	Production function of hydro plant h in period p of scenario ω (hWh/m^3)
$\mathrm{tv}^{\omega}_{ph}$	Tailrace volume of hydro plant h in period p of scenario ω (m)
rh^{ω}_{ph}	Reservoir elevation of reservoir r in period p of scenario ω (m)
wh^{ω}_{pr}	Headwater elevation of reservoir r in period p of scenario ω (m)

7.1 Introduction

Nowadays, under a deregulated framework in many countries electric companies manage their own generation resources and need detailed operation planning tools. In the next future, high penetration of renewable intermittent generation is going to change the electric system operation. Pumped storage hydro and storage hydro plants will play a much more important role due to their flexibility and complementary use with intermittent generation.

Operation planning models considering multiple interconnected cascaded hydroplants belonging to multiple basins can be classified into

- hydroelectric models that deal exclusively with hydropower plants and
- hydrothermal coordination models (HTCM) that manage the integrated operation planning of both hydropower and thermal power plants.

By nature, the later models are high-dimensional, dynamic, nonlinear, stochastic, and multiobjective. Solving these models is still a challenging task for large-scale systems. One key question for them is to obtain a feasible operation for each hydro plant, which is very difficult because the models require a huge amount of data, by the complexity of hydro subsystems and by the need to evaluate multiple hydrological scenarios. For these models no aggregation or disaggregation process for hydropower input and output is established. Besides, thermal power units are considered individually. Thus, rich marginal cost information is used for deciding hydro scheduling.

An HTCM determines the optimal yearly operation of all the thermal and hydropower plants taking into account multiple cascaded reservoirs in multiple basins. The objective function is based on cost minimization because the main goal is the medium-term hydro operation. However, the objective function can be easily modified to consider profit maximization if marginal prices are known (Stein-Erik and Trine Krogh 2008), which is a common assumption for price-taker companies.

This model is connected with other models within a hierarchical structure. At an upper level, a stochastic market equilibrium model (Cabero et al. 2005) with monthly periods is run to determine the hydro basin production. At a lower level, a stochastic simulation model (Latorre et al. 2007a) with daily periods details hydro plant power output. This later model analyzes for several scenarios the optimal operational policies proposed by the HTCM. In Fig. 7.1 it is represented the hierarchy of these three models. Adjustment feedbacks are allowed to assure the coherence among the output results.

The model presented in this chapter has two main uses. On one hand, Fig. 7.2 represents the typical horizon for yearly operation planning. It is a 2-year long scope beginning in October and ending in September, which corresponds to 2 consecutive hydrological years needed by the existence of multiannual reservoirs. This timeframe is used to avoid initial and terminal effects on the planning horizon because the natural planning period of interest is defined from January to December. On the other hand, Fig. 7.3 represents the second possible use of the model for obtaining optimal and "feasible" decisions under uncertainty in hydro inflows for the immediate future (for example, next 2 weeks). The operational decisions span for 2 years but only the first 2 weeks are actually implemented. Future decisions beyond these 2 weeks are not known with certainty. Once these 2-week decisions have been implemented, the model is reformulated with a new 2-year rolling horizon and solved again.

A recent review of the state of the art of hydro scheduling models is done in (Labadie 2004). According to stochasticity treatment models are classified into deterministic and stochastic ones.

Deterministic approaches are based on network flows, linear programming (LP), nonlinear programming (NLP) (Dembo et al. 1990), or mixed integer linear programming (MILP), where binary variables come from commitment decisions of thermal units or hydro plants or from piecewise linear approximation of nonlinear and nonconvex water head effects. For taking into account these nonlinear effects a successive LP solves are typically used. This process does not necessarily converge

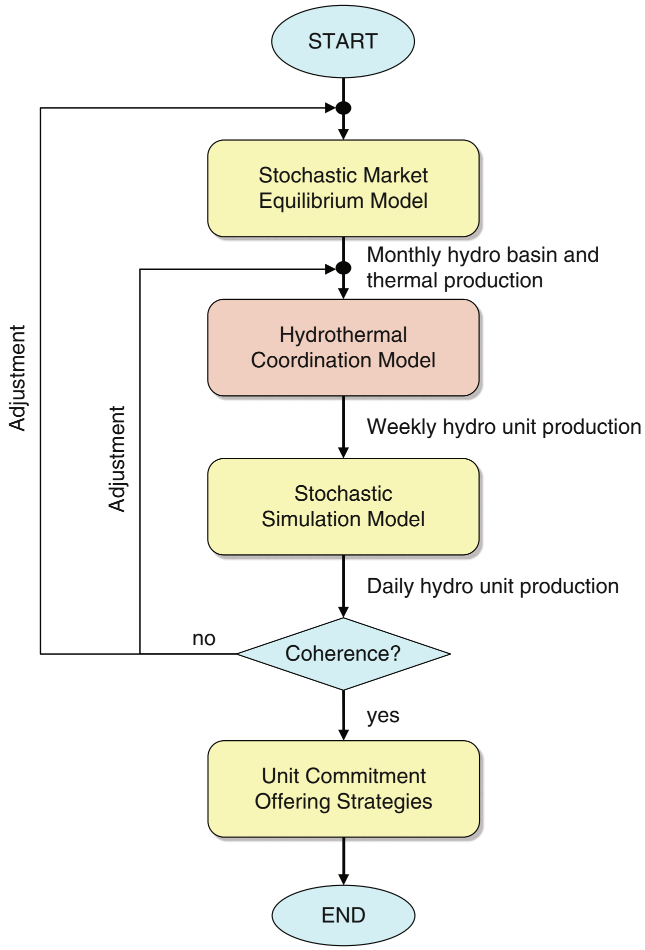


Fig. 7.1 Hierarchy of operation planning models

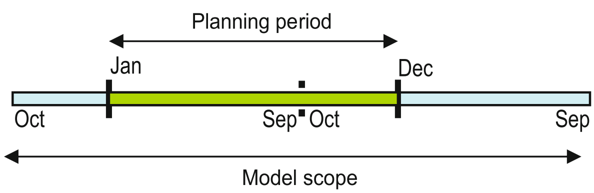


Fig. 7.2 Model scope for yearly operation planning

to the optimal solution; see (Bazaraa et al. 1993). This non-convergent behavior will also be tested with our model.

Stochastic approaches are represented by stochastic dynamic programming (SDP), stochastic linear programming (SLP) (Seifi and Hipel 2001), and stochastic nonlinear programming (SNLP). For SLP problems decomposition techniques like

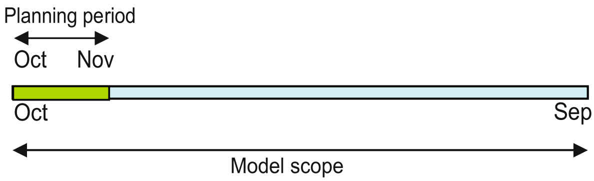


Fig. 7.3 Model scope for next future decisions under uncertain inflows

Benders (Jacobs et al. 1995), Lagrangian relaxation, or stochastic dual dynamic programming (SDDP) (Pereira and Pinto 1991) can be used.

This model has the following main characteristics:

- Specially suited for large-scale hydroelectric systems
- Deals with multireservoir, multiple cascaded hydro plants
- Consider nonlinear water head effects
- Takes into account stochastic hydro inflows
- Uses a robust solution method

The chapter is organized as follows. In Section 7.3 it is described the modeling of the system including all the equations of the mathematical optimization problem. The model implementation is introduced in Section 7.4. Then, the results for a real case study are presented and, finally, some conclusions are extracted.

7.2 System Modeling

The electric demand is modeled in a weekly basis with two constant load levels (peak and off-peak hours). Thermal units are treated individually. Commitment decisions of these units are considered as continuous variables given that the model is used for medium-term analysis. For hydro reservoirs a different modeling approach is followed depending on the following:

- Relevance of the reservoir
  Important large reservoirs are modeled in water units [volume in hm^3 and inflow in m^3/s]. They are modeled with nonlinear water head effects. On the contrary, smaller reservoirs are represented with a linear dependency; therefore, the model do not become unnecessarily complex.
- Owner company
  Hydropower plants belonging to other companies or state-operated reservoirs or the own small reservoirs are aggregated and modeled with one equivalent and independent reservoir each one, given that the reservoir and plant characteristics of some of them are generally ignored. They use energy units [volume and inflow in GWh].

Unregulated hydro inflows are assumed to be the dominant source of uncertainty in current Spanish electric system. In this system, stochasticity in hydro inflows have produced a hydroenergy availability ranging from 33.2 TWh in 2003 to 12.9 TWh in 2005 and hydroenergy generated has accounted for 20% in 2003 to only 9% in 2005 of the total energy demand, see (REE, http://www.ree.es).

Temporal changes in reservoir reserves are significant because of

- stochasticity in hydro inflows,
- highly seasonal pattern of inflows, and
- capacity of each reservoir with respect to its own inflow.

Stochasticity in hydro inflows is represented for the optimization problem by means of a multivariate scenario tree. This tree is generated by a neural gas clustering technique (Latorre et al. 2007b) that simultaneously takes into account the main stochastic inflow series and their spatial and temporal dependencies. The algorithm can take historical or synthetic series of hydro inflows as input data. Very extreme scenarios can be artificially introduced with a very low probability. The number of scenarios generated is enough for medium-term hydrothermal operation planning.

In Fig. 7.4 it is represented a scenario tree with eight scenarios. They correspond to the knee point of the quantization error function that measures the distance between the inflow series and the scenario tree versus the number of scenarios.

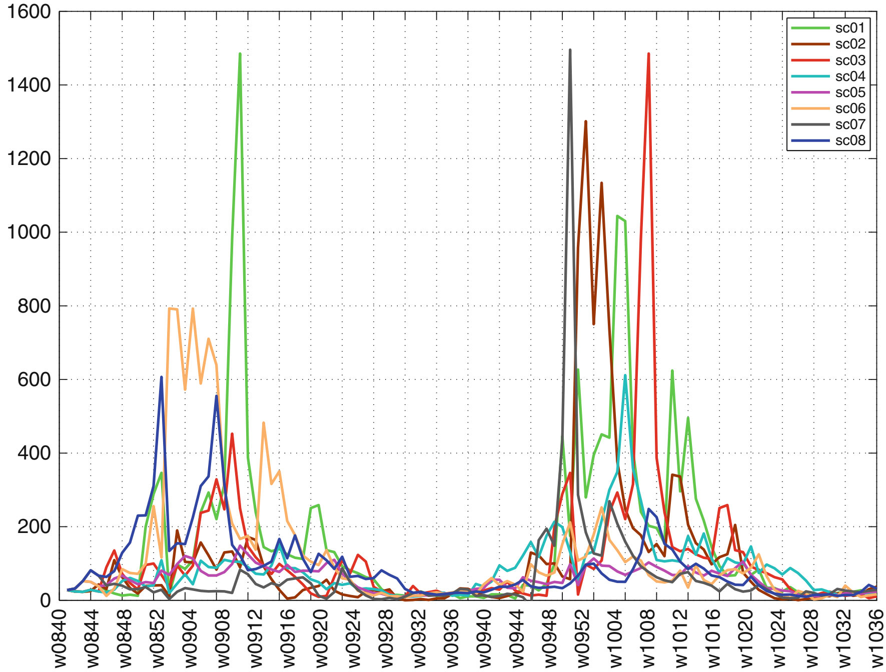


Fig. 7.4 Scenario tree with eight hydro inflows' scenarios from week 40 of year 2008 to week 39 of year 2010 expressed in (m^3/s)

7.3 Model Formulation

This HTCM is formulated as a stochastic nonlinear optimization problem as described in the following sections. The corresponding notation can be found at the beginning of the chapter. For clarity, uppercase letters are used for parameters and lower-case letters for variables.

The main results for each subperiod or load level (e.g., peak and off-peak) of each period (e.g., week) and scenario are storage hydro and pumped storage hydro plants operation and thermal unit operation, reservoir management, basin production, and marginal costs. As a byproduct the optimal water release tables for different stochastic inflows and reservoir volumes are obtained. They are computed by stochastic nested Benders' decomposition technique (Birge and Louveaux 1997) of a linear approximation of the stochastic nonlinear optimization problem. These release tables are used by the lower level daily stochastic simulation model, as seen in Fig. 7.1.

7.3.1 Constraints

The constraints introduced into the model are the following:

- Balance between generation and demand including pumping (MW)
  Generation of thermal units and storage hydro plants, $tp^{\omega}_{pp't}$ and $hp^{\omega}_{pp'h}$, respectively, minus consumption of pumped storage hydro plants, $pp^{\omega}_{pp'h}$, is equal to the demand $D_{pp'}$ for each scenario ω, period (week) p, and subperiod (load level) p':

$$\sum_{t} tp^{\omega}_{pp't} + \sum_{h} hp^{\omega}_{pp'h} - \sum_{h} \left(pp^{\omega}_{pp'h}/\eta_h \right) = D_{pp'} \qquad \forall p, p', \omega, \tag{7.1}$$

  where η_h is the efficiency of pumped storage hydro plant h.
- Minimum and maximum yearly operation hours for each thermal unit in each scenario (h)
  These constraints are relaxed by introducing deficit and surplus variables, doh^{ω}_{t} and eoh^{ω}_{t}, respectively, that are penalized in the objective function; see Section 7.3.2. Those slack variables can be strictly necessary in the case of many scenarios of stochasticity where the larger the variability of hydro inflows the larger the change in a subset of thermal units.
  This type of constraints are introduced to account for some aspects that are not explicitly modeled into this model like unavailability of thermal units, domestic coal subsidies, CO_2 emission allowances, long-term capacity payments, etc.

$$\underline{H}_t - doh^{\omega}_{t} \leq \frac{\sum_{pp'} \mathrm{DU}_{pp'} tp^{\omega}_{pp't}}{\overline{TP}_t} \leq \overline{H}_t + eoh^{\omega}_{t} \qquad \forall t, \omega \tag{7.2}$$

  being $\mathrm{DU}_{pp'}$ the duration of subperiod p' of period p.

- Minimum and maximum yearly average operation hours for each thermal unit (h)
  Observe that this constraint does not have deficit and surplus variables because it corresponds to average generating hours:

$$\underline{H}_t \le \frac{\sum_{pp'\omega} P_p^\omega \, \mathrm{DU}_{pp'} tp_{pp't}^\omega}{\overline{TP}_t} \le \overline{H}_t \qquad \forall t, \tag{7.3}$$

  where P_p^ω is the probability of scenario ω in period p.
- Water inventory balance for large reservoirs modeled in water units (hm^3)
  Reservoir volume at the beginning of the period $r_{p-1,r}^{a(\omega)}$ plus unregulated inflows I_{pr}^ω plus spills from upstream reservoirs $\sum_{r'\in rr(r)} s_{pr'}^\omega$ minus spills from this reservoir s_{pr}^ω plus turbined water from upstream storage hydro plants $\sum_{r'\in rr(r)} g_{pr'}^\omega$ plus pumped water from downstream pumped storage hydro plants $\sum_{r'\in rr(r)} p_{pr'}^\omega$ minus turbined g_{pr}^ω and pumped water from this reservoir p_{pr}^ω is equal to reservoir volume at the end of the period r_{pr}^ω.
  An artificial inflow ar_{pr}^ω is allowed and penalized in the objective function; see Section 7.3.2. Hydro plant h that takes water from reservoir r is $rh(r)$ or releases it to reservoir r, $hr(r)$. The initial value of reservoir volume is assumed known. No lags are considered in water releases because 1 week is the time period unit:

$$r_{p-1,r}^{a(\omega)} + \mathrm{ar}_{pr}^\omega + I_{pr}^\omega + \sum_{r'\in rr(r)} \left(s_{pr'}^\omega + g_{pr'}^\omega + p_{pr'}^\omega \right) - s_{pr}^\omega - g_{pr}^\omega - p_{pr}^\omega = r_{pr}^\omega \quad \forall p, r, \omega, \tag{7.4}$$

  where $a(\omega)$ is the ancestor scenario of scenario ω in previous period.
- Energy inventory balance for reservoirs modeled in energy (GWh)
  Reservoir volume at the beginning of the period $r_{p-1,r}^{a(\omega)}$ plus unregulated inflows I_{pr}^ω minus spills from this reservoir s_{pr}^ω minus turbined water from this reservoir g_{pr}^ω is equal to reservoir volume at the end of the period r_{pr}^ω. An artificial inflow ar_{pr}^ω is allowed and penalized in the objective function. The initial value of reservoir volume is assumed known:

$$r_{p-1,r}^{a(\omega)} + ar_{pr}^\omega + I_{pr}^\omega - s_{pr}^\omega - g_{pr}^\omega = r_{pr}^\omega \quad \forall p, r, \omega. \tag{7.5}$$

- Hydro plant generation (GWh) as a function of the water release
  The hydro output can be expressed as the product of the water release g_{ph}^ω, the head of the plant ph_{ph}^ω, the gravity acceleration g, the efficiency of the turbine η, and of the generator η' and the water density ρ:

$$\sum_{p'} \mathrm{DU}_{pp'} hp_{pp'h}^\omega = g_{ph}^\omega \cdot ph_{ph}^\omega \cdot g \cdot \eta \cdot \eta' \cdot \rho \quad \forall p, h, \omega. \tag{7.6}$$

The last four terms can be approximated by the production function variable $\mathrm{pf}^{\omega}_{ph}$ (also called efficiency)

$$\mathrm{pf}^{\omega}_{ph} = ph^{\omega}_{ph} \cdot g \cdot \eta \cdot \eta' \cdot \rho \tag{7.7}$$

and, therefore, the energy produced by the plant is the product of the water release and the production function.

The production function is usually given by level curves that relate the power output of a plant with the net head for an amount of water released through the turbine. Figure 7.5 shows a typical hill diagram, similar to another found in (Diniz et al. 2007). It may be observed that given a net head (vertical dashed line) for the reservoir, there is an optimum water outflow (thick line).

Equation (7.6) is a nonlinear nonconvex constraint that considers the long-term effects of reservoir management and can be rewritten as

$$\sum_{p'} \mathrm{DU}_{pp'} hp^{\omega}_{pp'h} = g^{\omega}_{ph} pf^{\omega}_{ph} \qquad \forall p, h, \omega. \tag{7.8}$$

- Total reservoir release g^{ω}_{pr} is equal to the sum of reservoir releases from all the downstream hydro plants (hm^3)

$$g^{\omega}_{pr} = \sum_{h \in hr(r)} g^{\omega}_{ph} \qquad \forall p, r, \omega. \tag{7.9}$$

- Pumped water p^{ω}_{pr} in (hm^3) is equal to the pumped storage hydro plant consumption $pp^{\omega}_{pp'h}$ in (GWh) divided by the production function PF_h:

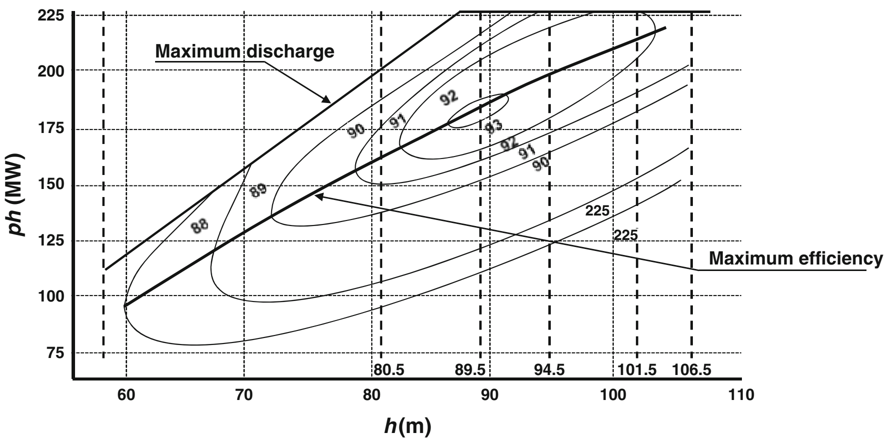


Fig. 7.5 Hill diagram of a hydro plant

$$p_{pr}^{\omega} = \sum_{p'} \sum_{h \in hr(r)} \mathrm{DU}_{pp'} \frac{pp_{pp'h}^{\omega}}{PF_h} \qquad \forall p, r, \omega. \tag{7.10}$$

- Achievement of a preestablished final reservoir volume FR_r with deficit dfr_r^{ω} and surplus variables efr_r^{ω} (hm^3)(GWh)
  This final reserve is determined by running a medium-term market equilibrium model, as seen in Fig. 7.1:

$$r_{pr}^{\omega} + dfr_r^{\omega} - efr_r^{\omega} = \mathrm{FR}_r \qquad \forall r, \omega. \tag{7.11}$$

- Minimum and maximum reservoir volume per period with deficit dmr_{pr}^{ω} and surplus variables emr_{pr}^{ω} (hm^3) (GWh)
  Those bounds are included to consider flood control curve, dead storage, and other plant operation concerns. The deficit variables will be strictly necessary in the case of many scenarios where inflow variety is higher:

$$\underline{R}_{pr} - dmr_{pr}^{\omega} \le r_{pr}^{\omega} \le \overline{R}_{pr} + emr_{pr}^{\omega} \qquad \forall p, r, \omega. \tag{7.12}$$

- Computation of the plant water head (m) and the production function variable (hWh/m^3) as a linear function of it.
  Production function variable pf_{ph}^{ω} is a linear function of the water head of the plant that is determined as the forebay elevation of the reservoir rh_{pr}^{ω} minus the tailrace elevation of the plant th_{ph}^{ω}. Tailrace elevation of the plant is the maximum of the forebay elevation of downstream reservoir rh_{pr}^{ω} and the tailrace elevation of the plant TH_h. This value depends on the outflow through the power plant. However, in this medium-term model it has been assumed as constant:

$$\begin{aligned} pf_{ph}^{\omega} &= A_h + A'_h \left(rh_{pr}^{\omega} - th_{ph}^{\omega} \right) && \forall p, h, \omega, \\ tv_{ph}^{\omega} &\ge \max \left(rh_{pr}^{\omega}, TH_h \right) && \forall p, h, \omega. \end{aligned} \tag{7.13}$$

- Computation of the reservoir headwater elevation (m) and the reservoir volume (hm^3) as a nonlinear function of it.
  Reservoir headwater elevation wh_{pr}^{ω} is determined as the forebay elevation rh_{pr}^{ω} minus the reference elevation. Reserve volume r_{pr}^{ω} is a quadratic function of the reservoir headwater elevation wh_{pr}^{ω}:

$$\begin{aligned} wh_{pr}^{\omega} &= rh_{pr}^{\omega} - \mathrm{RH}_r && \forall p, r, \omega, \\ r_{pr}^{\omega} &= B_r + B'_r \left(wh_{pr}^{\omega} \right) + B''_r \left(wh_{pr}^{\omega} \right)^2 && \forall p, r, \omega. \end{aligned} \tag{7.14}$$

- Variable bounds, i.e., reservoir volumes between limits for each hydro reservoir and power operation between limits for each unit

$$
\begin{array}{ll}
0 \leq \underline{TP}_{pt} \leq tp^{\omega}_{tp't} \leq \overline{TP}_{pt} & \forall p, p', t, \omega, \\
0 \leq \mathrm{HP}_{ph} \leq hp^{\omega}_{hp'h} \leq \overline{\mathrm{HP}}_{ph} & \forall p, p', h, \omega, \\
0 \leq \underline{PP}_{ph} \leq pp^{\omega}_{pp'h} \leq \overline{PP}_{ph} & \forall p, p', h, \omega, \\
0 \leq s^{\omega}_{pr} & \forall p, r, \omega, \\
0 \leq ar^{\omega}_{pr} & \forall p, r, \omega, \\
0 \leq pf^{\omega}_{ph} & \forall p, h, \omega, \\
0 \leq g^{\omega}_{prh} & \forall p, r, h, \omega, \\
\underline{G}_{pr} \leq g^{\omega}_{pr} & \forall p, r, \omega, \\
0 \leq pp^{\omega}_{pr} & \forall p, r, \omega, \\
0 \leq dfr^{\omega}_{r} \leq \mathrm{FR}_r & \forall r, \omega, \\
0 \leq efr^{\omega}_{r} & \forall r, \omega, \\
0 \leq dmr^{\omega}_{pr}, emr^{\omega}_{pr} & \forall p, r, \omega, \\
0 \leq doh^{\omega}_{t}, eoh^{\omega}_{t} \leq 8760 & \forall t, \omega. \\
r^{\omega}_{0r} = \mathrm{IR}_r &
\end{array}
\tag{7.15}
$$

7.3.2 Objective Function

The multiobjective function in [€] minimizes

- thermal variable costs plus,
- some penalty terms for deviations from ideal reservoir levels, i.e., deficit or surplus of final reservoir volumes, exceeding minimum and maximum operational rule curves, artificial inflows, and
- penalty terms for relaxing constraints like minimum and maximum yearly operation hours of thermal units.

It is important to notice the difficulties of finding a feasible solution for all the scenarios, so the penalties introduced into the objective function just accommodate these deviations in the best possible way. Different solutions and trade-offs can be obtained by changing these penalties and analyzing the stochastic optimization problem in a multicriteria decision-making framework:

$$
\begin{aligned}
\min \sum_{pp't\omega} \; & P^{\omega}_{p} \mathrm{DU}_{pp'} \mathrm{VC}_t tp^{\omega}_{pp't} \\
+ \sum_{r\omega} \; & P^{\omega}_{p} \left(\mathrm{PDFR}_r dfr^{\omega}_{r} + \mathrm{PEFR}_r efr^{\omega}_{r} \right) \\
+ \sum_{pr\omega} \; & P^{\omega}_{p} \left(\mathrm{PDFR}_r dmr^{\omega}_{pr} + \mathrm{PEFR}_r emr^{\omega}_{pr} \right) \\
+ \sum_{pr\omega} \; & P^{\omega}_{p} \left(\mathrm{PDFR}_r ar^{\omega}_{pr} \right) \\
+ \sum_{pt\omega} \; & P^{\omega}_{p} \left(\mathrm{PDH}_t doh^{\omega}_{t} + \mathrm{PEH}_t eoh^{\omega}_{t} \right).
\end{aligned}
\tag{7.16}
$$

7.4 Model Implementation

According to (Labadie 2004) "the keys to success in implementation of reservoir system optimization models are (1) improving the levels of trust by more interactive of decision makers in system development; (2) better 'packaging' of these systems; and (3) improved linkage with simulation models which operators more readily accept." Following guideline (2) this model has been implemented with a spreadsheet-based graphical user interface that improves easiness and usability. It is able to represent any general reservoir system topology, given that it is not customized. The optimization problem is written in GAMS 23.3, see (Brooke et al. 2008), and automatically executed from the interface. The scenario tree generator is also embedded into the hydrothermal coordination model.

As guideline (3) suggests the optimal decisions obtained with this model are passed to another stochastic simulation model (Latorre et al. 2007a) to evaluate decisions at a daily level; see Fig. 7.1.

7.5 Case Study

The case study represents the Spanish electric system with 118 thermal units, 5 main basins with 49 hydro reservoirs, 56 hydro plants, and 2 pumped storage hydro plants. The hydro subsystem is very diverse. Hydro reservoir volumes range from 0.15 to 2433 hm^3 and hydro plant capacities go from 1.5 to 934 MW. We consider different number of scenarios of unregulated hydro inflows.

In the following sections we have done different runs to analyze the electric system and some modeling issues.

7.5.1 Computational Results

In this section we show the use of different nonlinear solvers for different case studies. For avoiding numerical problems a careful natural scaling of variables around 1 has been done and simple expressions are used in the nonlinear constraints, which are very efficiently managed by nonlinear solvers. The nonlinear problem is solved providing initial values and bounds for all the variables from the solution given by the linear solver CPLEX 12.1 (ILOG-CPLEX, http://www.ilog.com/products/cplex/) by an interior point method. Several nonlinear solvers have been tested with different cases to check their robustness and solution time. The tested solvers have been CONOPT3 3.14 based on a generalized reduced gradient method (Drud 1994), IPOPT 3.7 based on a primal–dual interior point filter line search algorithm (Wchter and Biegler 2006), KNITRO 5.1.2 using an interior point (Byrd et al. 2006), MINOS 5.51 based on a project Lagrangian algorithm (Murtagh and Saunders 1987). The default options and algorithms have been used for all the solvers.

The model has been run in a PC with a processor running at 1.83 GHz and with 1 GB of RAM memory. The problem with eight scenarios has been the biggest one

Table 7.1 Size of linear and nonlinear problems

	R	V	E	R	V	E	NE
1 scen	30,952	57,883	157,705	36,984	55,283	157,705	1248
4 scen	120,269	224,925	612,883	143,701	214,825	612,883	4848
8 scen	234,641	438,837	1,195,803	280,345	419,137	1,195,803	9456

Table 7.2 Solutions provided by linear and nonlinear solvers

	CPLEX		CONOPT		IPOPT		KNITRO		MINOS	
	O.F. (M€)	time (s)	O.F. (M€)	time (s)	O.F. (M€)	time (s)	O.F. (M€)	time (s)	O.F. (M€)	time (s)
1 scen	15,717.452	6	15,689.029	275	15,689.086	600[3]	15,689.103	601	15,689.875	78
4 scen	15,750.979	43	15,730.440	3202	15,730.502	4200	15,728.997	2309	15,730.796	2557
8 scen	15,764.817	132	15,750.062	6513	15,746.309	9010	15,754.388	5600	15,747.004	7628

that has fitted into the PC memory. Table 7.1 summarizes the sizes of the problems where R is the number of constraints, V number of variables, E nonzero elements, and NE nonlinear nonzero elements, and Table 7.2 the objective functions in (M€) and the solution times in seconds.

The solution of the nonlinear nonconvex problem by a linear approximation has been tested. The linearization is made by fixing the value of the production function obtained in previous iteration. The results are shown in Fig. 7.6. It can be concluded that the linear iterations do not converge necessarily to the NLP solution.

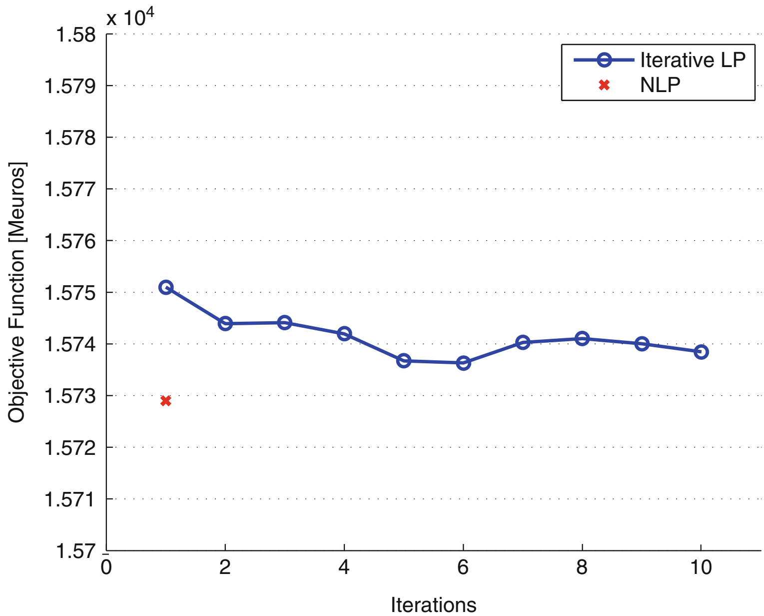


Fig. 7.6 Iterations of LP problem and the NLP solution

The relevant conclusion to extract from this analysis is that large-scale stochastic hydrothermal coordination problems can nowadays be solved by several general-purpose NLP solvers.

7.5.2 Hydro Reservoir Operation Planning

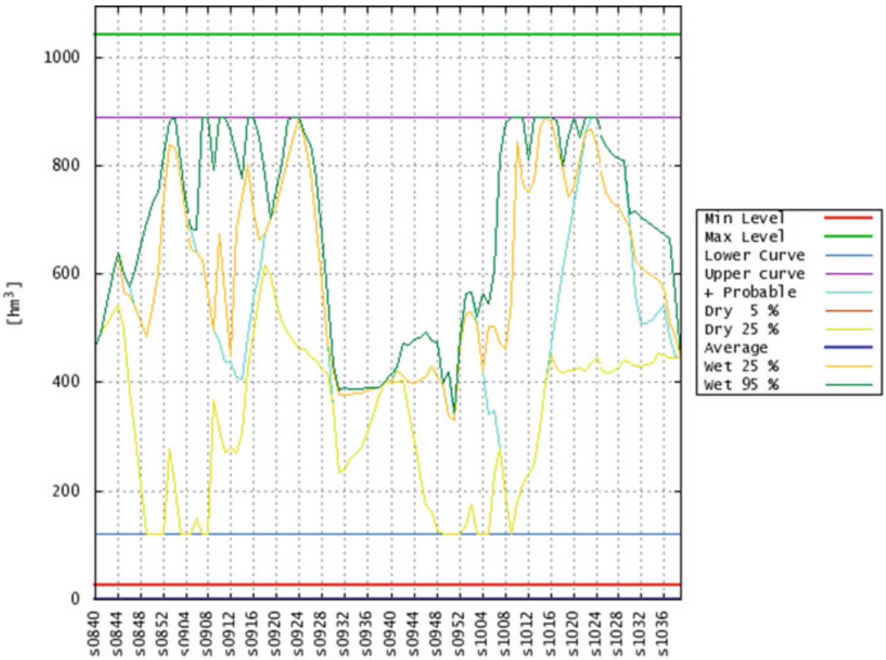
As it has been seen from the previous tables the impact of the nonlinear approximation is not very important regarding the objective function (less than a 0.2%). However, the operation of the hydro plants makes a crucial difference between them. The following figures show the different operation of a large hydro reservoir (with a maximum volume of approximately 900 hm^3) due to use of the linear or nonlinear modeling of water head effects in the four-scenario case. The curves represented in each graph correspond to minimum and maximum volume level, lower and upper operating rule curves, mode (most probable scenario), and five quantiles.

Although in a stochastic framework the only relevant decisions are those corresponding to the first stage, given that these here and now decisions will be implemented, it can be observed that the operation of the reservoir is smoother in the nonlinear approach than in the linear one, see, for example, curves in the weeks from s0852 to s0926 of Fig. 7.7. Besides, there is a strong difference in the optimal volume of the reservoir in the weeks from s0932 up to s1004. In the linear case, reservoir volume is remarkably lower because the production function does not depend on the water head and therefore the linear model is indifferent to this volume. This rational and realistic operation of the reservoir in the nonlinear model fully justifies the importance of using the nonlinear approximation and shows that feasibility is as important as optimality in stochastic hydrothermal planning models.

7.5.3 Scenario Analysis and Stochastic Measures

Figure 7.8 represents on the left y-axis the value of the objective function for the different scenarios with anticipative decisions and on the right y-axis the relative natural inflows (value 1 corresponds to the mean value). Changes in the objective function are around 10% among scenarios. Figure 7.9 plots the quadratic regression function of both variables, determining the impact of hydro inflows in the objective function.

Additionally, we have conducted a scenario analysis and determined some stochastic measures; see (Birge and Louveaux 1997) for their definition, whose values appear in Table 7.3. In this case it can be seen that the expected value of perfect information (EVPI) that measures the impact of the non-anticipative decisions and the value of the stochastic solution (VSS) are very small. The reason is that the branching process of the tree is done in early stages (at the end of the first month) and the scenarios are almost independent among them.



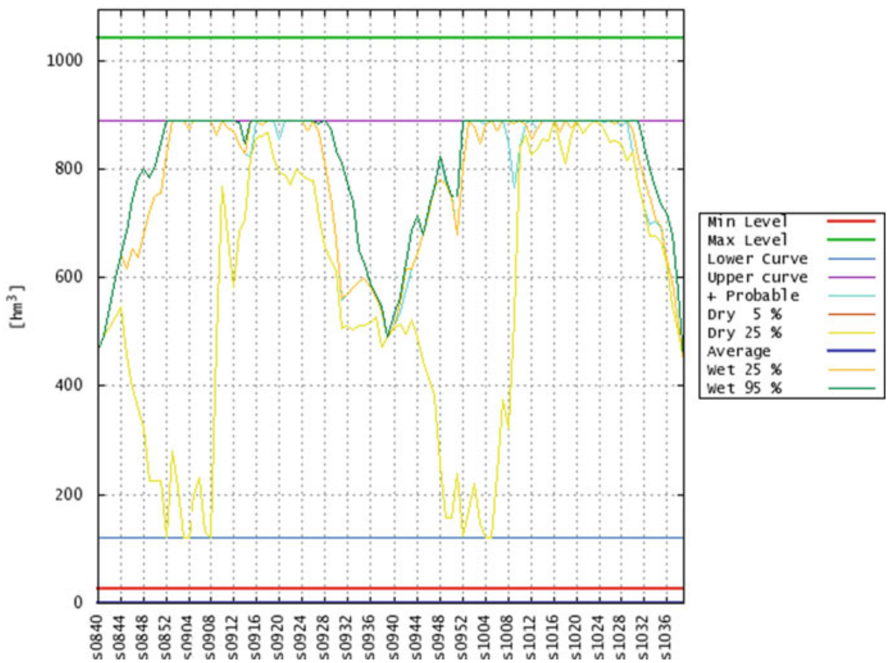


Fig. 7.7 Reserve volume for the planning horizon under stochastic hydro inflows with the linear (*above*) and nonlinear (*below*) approximation

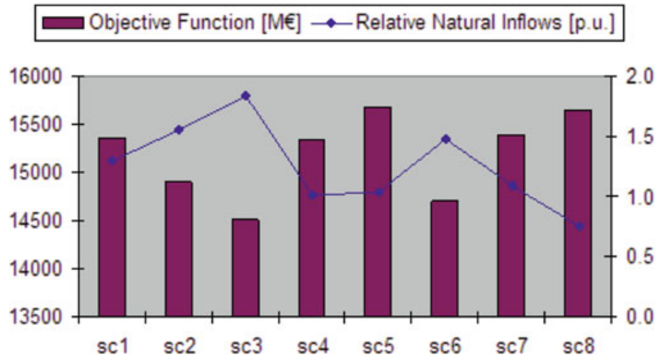


Fig. 7.8 Scenario analysis

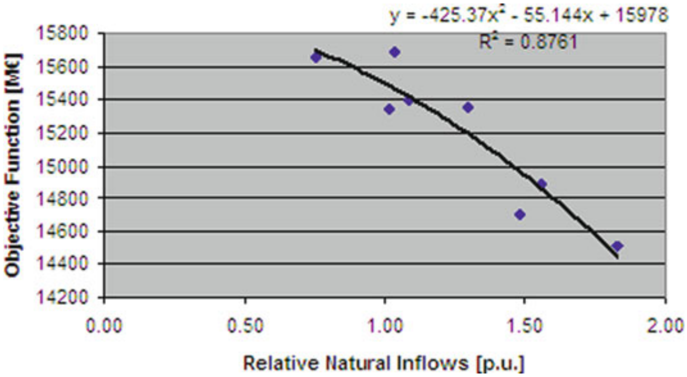


Fig. 7.9 Relation between natural inflows and total variable costs

Table 7.3 Stochastic measures

Expected value with perfect information	(EVWPI)	15,764.709
Expected value solution	(EV)	15,716.937
Stochastic solution	(SS)	15,764.817
Expected result of the expected value solution	(EEV)	15,764.755
Expected result of the stochastic solution	(ESS)	15,764.848
Value of the stochastic solution	(VSS)	0.062
Expected value of perfect information	(EVPI)	0.108

Another reason for that comes from the electric system. Since the last 5 years the Spanish electric system has had a strong investment in CCGTs. As a result nowadays there are only three thermal technologies: nuclear, coal, and natural gas, being natural gas the less competitive from a variable cost point of view. Stochasticity in hydro inflows may represent a variation of approximately 20 TWh of energy from a dry to a wet year and it is fully replaced by electricity produced by CCGTs. Those units are relatively new and therefore have similar heat rates. As the total variable cost of the system behaves linearly with respect to the stochasticity in hydro inflows, see Fig. 7.9, the stochastic measures, that account for changes in the objective function with respect to the mean value or with respect to the anticipative solution, are negligible.

This tendency can be observed not only in Spain but also in many other countries where natural gas has massively replaced old coal and oil thermal units. However, as important as the objective function is the production of the different units and this may substantially change from one scenario to another. So, the model results are much more useful by providing the output of the thermal and hydro units and the spillage of hydro reservoirs under each scenario. This is the value of a stochastic programming model for electricity production planning.

7.6 Conclusions

In this chapter we have presented a medium-term stochastic hydrothermal coordination model for complex multireservoir and multiple cascaded hydro subsystems. Nonlinear water head effects are modeled for important large reservoirs. Stochasticity of natural hydro inflows is considered.

The optimization problem is stated as a stochastic nonlinear optimization problem solved directed by a general-purpose nonlinear solver giving a close initial solution provided by a linear solver.

A case study of a complex and large-scale electric system with a 2-year time scope with weekly detail has been tested and thorough results were presented and discussed. In particular it is shown the importance of considering the nonlinear modeling in obtaining realistic hydro reservoir operations and the value of the stochasticity for this case.

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