Maximaths project

Instabilities of dense granular flows

Jin Sun (School of Engineering) and Jacques Vanneste (School of Mathematics)

Background. Dense granular flows are ubiquitous in nature and occur, for instance, in snow avalanches and in landslides; they are also widely encountered in industrial processes, including hopper discharges, chute flows, etc. Modelling such flows is therefore essential for many applications, natural-hazard mitigation and industrial operations in particular; but it is also highly challenging.

One approach models the granular medium as a continuum. In this approach, the equations governing time-dependent granular flows resemble the Navier–Stokes equations for a viscous flow. The constitutive law for the internal stress is however much more complex and typically involve tensorial fields which represent microscopic properties and are governed by additional evolution equations. One consequence is the existence of instability mechanisms very different from those known for standard viscous flows, and which depend strongly on the constitutive behaviour of the granular material. It is important to understand these instabilities, because of their physical consequences but also because this provides a way of testing proposed continuum models.

Project. In this project, we focus on the stability analysis of rate-independent dense granular flows. This is relatively unexplored area due to the difficulty of obtaining reliable constitutive models for such flows. For example, the stress is undetermined in this regime when the kinetic theory for granular flows is used. To overcome this difficulty, a new constitutive model has recently been developed by Sun and Sundaresan (2011). This model has proved successful in reproducing the steady and unsteady shear behaviour of dense granular flows, but so far, the instability mechanisms that it describes have not been investigated. The main objective of the project, therefore, is to carry out a stability analysis of simple steady flows governed by this model. The collaboration combines Sun and Vanneste's complementary expertise in rheological modelling and stability analysis. It will lead to new results and (hopefully) deeper insights into the mathematical description of instabilities of granular flows.

We plan to begin the study with the simplest possible basic state and perturbations to the flow. The results will then be compared to those from discrete-element simulations and experiments performed in Suns group, and to previous results using different constitutive models. The results will be used to improve theoretical development and may have direct implications on industrial operations involving dense granular flows. The tasks planned can be accomplished within a few months; they are expected to lead to further collaborative investigations of more complex flows. Therefore, we view a joint EPSRC application on the subject as a realistic outcome of the project.

Work plan. The main task to be carried out is the linear stability analysis of a rather complex set of PDEs. This can be carried out efficiently by a researcher with a strong experience of the type of calculations needed. Fortunately, Dr Eric Danioux, a former postdoc with Vanneste, is available and willing to undertake this task. We therefore request, as sole resource for the project, the funding of him for 1 month. This should be enough time to make the mathematical progress required for the numerical and experimental parts of the project. Further mathematical analysis will be carried out by PhD students in Sun's group advised by Vanneste.

Jin Sun, Jacques Vanneste, 10/05/2012