On Optimal Solutions to Planetesimal Growth Models

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Introduction: The protoplanetary discs are believed to be the favoured sites of planet formation. Initially composed of gas mixed with interstellar dust grains of varying sizes, this material will eventually assemble itself into planets, satellites, comets, asteroids and other orbiting debris observed in our Solar System and in others. Planet formation theories rely crucially on the criteria which govern fragmentation or coagulation of planetesimals undergoing collisions (Safronov 1972, Weidenschilling 1993). This is true not just for core accretion theory (Pollack 1996), which requires planets to form in a bottom-up process directly from the collision of dust grains, but also for planet formation by gravitational instability, as the grains are expected to undergo potentially enhanced collisional evolution in the spiral structures induced by marginally unstable protostellar discs (Rice 2004, Clarke 2009).

Collisionally growing planetesimals typically encounter two obstacles in their evolution towards objects approaching planetary masses. One of these is fragmentation as an outcome of planetesimal-planetesimal collisions (Jones 1996). The probability of fragmentation typically increases with grain size until the planetesimals are of approximately decimetre size. Once past this bottleneck, the probability of fragmentation begins to decrease with increasing size, and the planetesimals can grow more efficiently. (Stewart 2009) developed a velocity-dependent model to describe this behaviour, which compares the reduced mass kinetic energy of the system (Q_R) with the catastrophic disruption criterion (Q^*_{RD}), where the latter quantity is fitted to a mixture of numerical and experimental data. By doing this, they identify a universal law for the mass of the largest remnant of two-body collisions, M_{lr}, where the number of bodies remaining after the collision is unknown.

If we are to understand the efficiency with which protostellar disc systems will form planets, it is useful to investigate to what extent planetesimal growth is optimised, and what distributions of mass and velocity are required to produce optimised growth. With a model that describes the efficiency of collisions in growing individual bodies, it seems it should be possible to apply a variety of optimisation approaches to discover these optimal distributions.

Objectives: It was recently observed by the first PI that a natural optimization formulation of the problem leads to a nonconvex quadratic program with a simplex constraint. The number of variables of this mathematical program grows linearly with the discretization level of a certain underlying probability distribution. We propose to carry these initial ideas to fruition by developing an optimization model for studying optimal planetesimal collisions. We will study the model, develop suitable exact and heuristic algorithms for solving it and then interpret the meaning of the computational results for the formation of planetary systems. We aim to finalize the results by the end of 2012 and submit the work to Monthly Notices of the Royal Astronomical Society. We hope that this collaboration, if funded, will be a lasting one, leading to further joint projects.

Budget: £4,000 with the following structure:

1. £1,500 investigator time for P.R.
2. £1,500 investigator time for D.F.
3. £1,000 conference travel

References: Stewart S.T., Leinhardt Z. M., Velocity-dependent catastrophic disruption criteria for planetesimals, ApJ, 2009.