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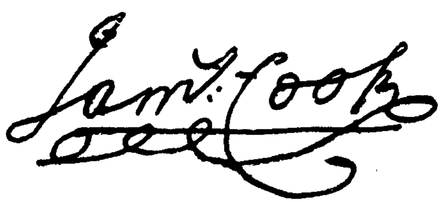

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CARL FELIX MOPPERT

Carl was born on 7th October, 1920, in Basel, Switzerland, the youngest of four children. His father was a Lutheran minister; his mother died when he was five years old. He went to school in Basel and to the University of Basel and the Eidgenossische Technische Hochschule, Zurich, specialising in mathematics, physics and chemistry. He gained his Dr.Phil. at Basel under the supervision of A. Ostrowski in 1949.

He worked as a school teacher in Basel until his appointment as Lecturer in the University of Tasmania in 1954. In 1958 he moved to Melbourne University as a Senior Lecturer and then in 1967 to Monash as a Senior Lecturer, where he was until his death on 16th September, 1984. He left a wife and six children.

At Monash he supervised the work of several candidates for higher degrees and he took great pleasure in the many seminar series which he organised and to which he contributed. As a mathematician, Carl Moppert was always more interested in ingenious and elegant arguments and less in powerful machinery. He loved to contemplate the facts and structures of mathematics, and was less interested in the processes. His interests ranged over many parts of classical mathematics, he made contributions to the theories of Riemann surfaces, Brownian motion, Euclidean and non-Euclidean geometry, linear algebra and the theory of fields. He detested the separation of mathematicians into camps, particularly the categorization of "pure" and "applied", and he moved easily between these two aspects of mathematics as his interests took him. His flair for invention produced the non-toppleable crane, the thermal pump and the sundial on the Monash University campus (see JCMN 24). Perhaps of all his creations the Foucault pendulum at Monash which he designed and installed with the help of Professor W. Bonwick best represents his spirit; it demonstrates in a clearly visible way a profound fact about the world, relying on elegant mathematics.

Thanks are due to Professor J.B. Miller of Monash University and to Carl's family for the material in the obituary note above.

CONTINUED FRACTIONS FOR FUNCTIONS

$$\tan x = \frac{x}{1-}\,\frac{x^2}{3-}\,\frac{x^2}{5-}\,\frac{x^2}{7-}\,\frac{x^2}{9-}\cdots$$

$$\tan^{-1} x = \frac{x}{1+}\,\frac{x^2}{3+}\,\frac{4x^2}{5+}\,\frac{9x^2}{7+}\,\frac{16x^2}{9+}\,\frac{25x^2}{11+}\cdots$$

$$\tanh x = \frac{x}{1+}\,\frac{x^2}{3+}\,\frac{x^2}{5+}\,\frac{x^2}{7+}\,\frac{x^2}{9+}\,\frac{x^2}{11+}\cdots$$

$$\tanh^{-1} x = \frac{x}{1-}\,\frac{x^2}{3-}\,\frac{4x^2}{5-}\,\frac{9x^2}{7-}\,\frac{16x^2}{9-}\,\frac{25x^2}{11-}\cdots$$

$$\exp x = 1+\frac{x}{1-}\,\frac{x}{2+}\,\frac{x}{3-}\,\frac{x}{2+}\,\frac{x}{5-}\,\frac{x}{2+}\,\frac{x}{7-}\cdots$$

$$= \frac{1}{1-}\,\frac{x}{1+}\,\frac{x}{2-}\,\frac{x}{3+}\,\frac{x}{2-}\,\frac{x}{5+}\,\frac{x}{2-}\,\frac{x}{7+}\,\frac{x}{2-}\cdots$$

$$= 1+\frac{x}{1-x+}\,\frac{x}{1+}\,\frac{1}{1+}\,\frac{x}{3-x+}\,\frac{x}{1+}\,\frac{1}{1+}\,\frac{x}{5-x+}\,\frac{x}{1+}\,\frac{1}{1+}\cdots$$

$$\log(1+x) = \frac{2x}{2+}\,\frac{x}{1+}\,\frac{x}{6+}\,\frac{2x}{1+}\,\frac{2x}{10+}\,\frac{3x}{1+}\,\frac{3x}{14+}\,\frac{4x}{1+}\,\frac{4x}{18+}\,\frac{5x}{1+}\cdots$$

$$= \frac{2x}{2+x-}\,\frac{x^2}{6+3x-}\,\frac{4x^2}{10+5x-}\,\frac{9x^2}{14+7x-}\,\frac{16x^2}{18+9x-}\cdots$$

MORE ABOUT TRIANGLE CENTRES IN THE COMPLEX PLANE

(JCMN 30, pp.3127 & 3133; 31, p.3182; 32, p.4017; 34, p.4061.

A.P. Guinand

Introduction If the centroid G and the orthocentre H of a triangle are given as distinct points, then the triangle cannot be equilateral and the line GH is its Euler line. The positions of the circumcentre O and nine-point centre N are also fixed since they lie on the Euler line and OG:GN:NH = 2:1:3.

In [1], [2], and earlier notes in JCMN it was shown that, of the four tritangent centres, the incentre always lies inside the circle on diameter GH (or critical circle), while all three excentres lie outside it. Furthermore there is a curiously shaped acentric lacuna, a closed region inside which no tritangent centre of either kind can lie.

In later notes in JCMN other writers showed how some results on triangle centres can be elegantly expressed in the setting of the complex plane. In the present note I show how properties of the critical circle and the acentric lacuna fit into the complex setting.

In the complex plane method the circumcentre O is at zero, and the vertices of the triangle are at points u^2, v^2, w^2, all of unit modulus. Thus the triangle has fixed unit circumradius, and u, v, w are determined only up to ambiguity of sign. The centroid G is $(u^2+v^2+w^2)/3$, and consequently N is $(u^2+v^2+w^2)/2$ and H is $u^2+v^2+w^2$. Owing to the ambiguities of signs the number $-vw-wu-uv$ can take four different values, and these correspond to the four tritangent centres. For convenience, take the sign of u as fixed; that still allows changes of signs of v and w to generate all four tritangent centres.

The critical circle. If I is a tritangent centre at $-vw-wu-uv$ then

$$OI = |-vw-wu-uv| = |uvw|.|\bar{u}+\bar{v}+\bar{w}| = |u+v+w|, \quad (1)$$

since $\bar{u} = 1/u$, etc. Also

$$IN = |\tfrac{1}{2}(u^2+v^2+w^2)+vw+wu+uv| = \tfrac{1}{2}|u+v+w|^2. \quad (2)$$

The critical circle, being on diameter GH, can also be regarded as an Apollonian circle, the locus of points I for which OI = 2.IN. That is $|u+v+w| = |u+v+w|^2$, but $u+v+w=0$ only occurs for equilateral triangles. Hence the critical circle goes through any $-vw-wu-uv$ for which $|u+v+w|=1$.

If the point I at $-vw-wu-uv$ is inside the critical circle, then $|u+v+w| < 1$, and I is the incentre. If $|u+v+w| > 1$, then I is an excentre, outside the critical circle.

If $|u+v+w| = 1$ then

$$\begin{aligned} 1 = |u+v+w|^2 &= (u+v+w)(\bar{u}+\bar{v}+\bar{w}) \\ &= 3+\bar{v}w+\bar{w}u+\bar{u}v+v\bar{w}+w\bar{u}+u\bar{v} \\ &= 3+(w/v)+(u/w)+(v/u)+(v/w)+w/u+(u/v). \quad (3) \end{aligned}$$

Clearing of fractions and factorizing, this yields

$$(v+w)(w+u)(u+v)=0.$$

Hence at least one pair from u^2, v^2, w^2 must be equal, corresponding to a degenerate triangle with two coincident vertices.

Since any non-degenerate triangle has one incentre and three excentres we arrive at the following result:

> If u^2, v^2, w^2 are three unequal complex numbers of unit modulus, then just one combination of signs in $|u \pm v \pm w|$ makes this quantity less than 1, while the other three combinations make it greater than 1.

More complicated inequalities of similar nature were proposed in the last two JCMN references given, but the present result is easily seen without reference to triangle geometry.

It is equivalent to the assertion that, in a unit-sided rhomboidal lattice as in Figure 1, any point 0 at unit distance from u is less than unit distance from one and only one of the four points $u \pm v \pm w$, provided no two of the directions of $\pm u$, $\pm v$, $\pm w$ coincide. Details are left to the reader.

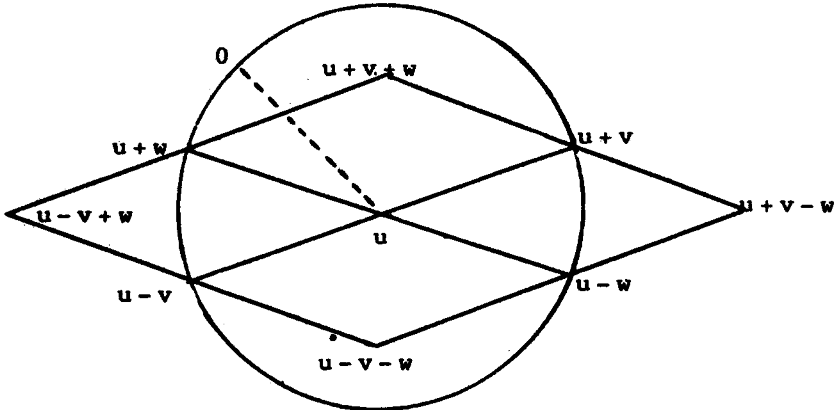


Figure 1.

Exercise: Derive similar results for the inequality

$$9|u^2+v^2+w^2+vw+wu+uv|^2+|u^2+v^2+w^2+3vw+3wu+3uv|^2 < 4|u^2+v^2+w^2|^2.$$

<u>The acentric lacuna</u>. If K is the reflection of H in 0 then K is $-u^2-v^2-w^2$, and

$$IK = |u^2+v^2+w^2-vw-wu-uv|. \qquad (4)$$

In [2] it is shown that the acentric lacuna can be characterized as the set of point I for which

$$9.OI^2 - 4.IN^2 \le 4.IN.IK \qquad (5)$$

By (1), (2) and (4) the reverse of this is

$$9|u+v+w|^2 - |u+v+w|^4 \ge 2|u+v+w|^2|u^2+v^2+w^2-vw-wu-uv|,$$

or, since $u+v+w \neq 0$ for non-equilateral triangles,

$$|u+v+w|^2 + 2|u^2+v^2+w^2-vw-wu-uv| \le 9. \qquad (6)$$

This suggests that complex methods of investigating the acentric lacuna should start as follows:

THEOREM. If u, v, w are complex numbers of unit modulus then the inequality (6) holds with equality if and only if two of u, v, w are equal.

Proof. If $\omega = \exp 2\pi i/3$ (= a cube root of 1) then

$$\omega + \omega^2 = -1,$$

and $u^2+v^2+w^2-vw-wu-uv = (u+\omega v+\omega^2 w)(u+\omega^2 v+\omega w)$.

Since $|u+v+w| \le 3$ it follows by rearranging and squaring that it is sufficient to prove that

$$4|u+\omega v+\omega^2 w|^2|u+\omega^2 v+\omega w|^2 \le [9-(u+v+w)^2]^2. \qquad (7)$$

Writing $Z = \bar{v}w + \bar{w}u + \bar{u}v$ in (3) it becomes

$$|u+v+w|^2 = 3 + Z + \bar{Z}.$$

Also

$$\begin{aligned}|u+\omega v+\omega^2 w|^2 &= (u+\omega v+\omega^2 w)(\bar{u}+\omega^2\bar{v}+\omega\bar{w})\\ &= u\bar{u}+v\bar{v}+w\bar{w}+\omega(\bar{v}w+\bar{w}u+\bar{u}v)+\omega^2(v\bar{w}+w\bar{u}+u\bar{v})\\ &= 3+\omega Z+\omega^2\bar{Z}.\end{aligned}$$

Similarly $|u+\omega^2 v+\omega w|^2 = 3+\omega^2 Z+\omega\bar{Z}$.

Hence (7) is equivalent to

$$4(3+\omega Z+\omega^2\bar{Z})(3+\omega^2 Z+\omega\bar{Z}) \le [6-Z-\bar{Z}]^2.$$

Multiplying out, rearranging, and dividing by 3, this reduces to

$$(Z-\bar{Z})^2 \le 0. \qquad (8)$$

Since $Z - \bar{Z}$ is either zero or a pure imaginary (8) is true, and hence so is (6). There is equality if and only if $Z - \bar{Z} = 0$, and then

$$\bar{v}w+\bar{w}u+\bar{u}v-v\bar{w}-w\bar{u}-u\bar{v} = 0.$$

Replacing conjugates by reciprocals and multiplying by uvw, this reduces to $(v-w)(w-u)(u-v) = 0$, so equality in (6) holds if two of u, v, w are equal. Q.E.D.

It can be confirmed that the inequality (5) does characterize some region free of tritangent centres by noting that it is quite clearly satisfied for I sufficiently near 0. The boundary of the region so defined consists of those points I for which equality holds in (5). In the complex setting this boundary therefore corresponds to those points $-vw-wu-uv$ for which two of u, v, w are equal. In order to fit this into a scenario where O, H, and the Euler line are fixed, begin by rotating the complex plane about O so that the equal elements are $v = w = 1$. Then I is at $-1-2u$, and H is at u^2+2. Then H can be fixed at the unit point on the real axis by dividing relevant numbers by u^2+2. Consequently the boundary of the acentric lacuna with respect to fixed OH is the locus of the points

$$z = \frac{-1-2u}{2+u^2} \qquad (9)$$

as u varies around the unit circle. Putting $u = e^{i\theta}$, and $z = x+iy$, (θ, x, y real) leads to parametric equations for the boundary

$$x = -\frac{2\cos^2\theta + 6\cos\theta + 1}{8\cos^2\theta + 1}, \qquad y = \frac{2\sin\theta(\cos\theta - 1)}{8\cos^2\theta + 1}.$$

These can be used to plot the curve, but there also exists a convenient ruler-and-compass construction for points on the curve. This construction was discovered using polar coordinates [2], but once it is known, it is easily proved by the complex method.

If u is replaced by $-u$ in (9) then

$$z' = \frac{-1+2u}{2+u^2}$$

corresponds to another point (I', say) on the boundary.

Then $$\frac{z+1}{z'+1} = \left\{\frac{1-u}{1+u}\right\}^2 = \left\{\frac{1-e^{i\theta}}{1+e^{i\theta}}\right\}^2 = -\tan^2\tfrac{1}{2}\theta\ .$$

That is, $z+1$ is a real multiple of $z'+1$. Hence the points corresponding to z, z', -1 are collinear; that is I, I' and K are collinear.

If M is the midpoint of II' and its complex affix is m, then

$$m = \tfrac{1}{2}(z+z') = -1/(2+u^2).$$

Let P be the point whose affix is $-(2/3)$. Then

$$PM = |m + (2/3)| = \left|\frac{2}{3} - \frac{1}{2+u^2}\right| = \frac{1}{3}\left|\frac{2u^2+1}{2+u^2}\right| = \frac{1}{3}$$

for all u of unit modulus. Hence the locus of M is a circle, centre at $-(2/3)$, radius $(1/3)$. That is the reflection of the critical circle in the circumcentre O.

Further

$$IM = I'M = |z - m| = \left|\frac{-1-2u}{2+u^2} + \frac{1}{2+u^2}\right|$$

$$= 2\left|\frac{1}{2+u^2}\right| = 2.OM$$

To sum up, we have

CONSTRUCTION. Draw the circle on diameter KL, where K and L are the reflections of the orthocentre H and the centroid G in the circumcentre O. Take any line through K, intersecting the circle again in M. Mark points I and I' at distances 2.OM each way along the line from M. I and I' lie on the boundary of the acentric lacuna, as in Figure 2.

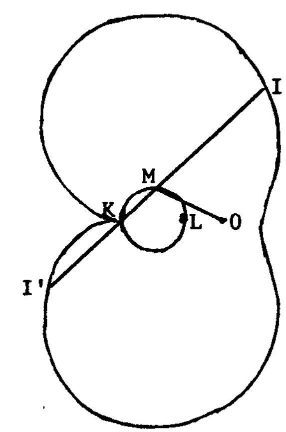


Figure 2.

Remark. It can also be shown that any point outside the acentric lacuna defined as above is a tritangent centre of some real triangle, possibly degenerate. That has not been attempted here. Cf. [1].

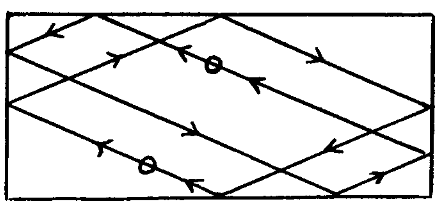
REFERENCES

[1] A.P.Guinand, Euler lines, tritangent centres, and their triangles, American Math. Monthly, 91 (1984), 290-300.

[2] ----------- Incentres and excentres viewed from the Euler line. Math. Magazine 58 (1985) (to appear)

REFLECTIONS IN THE BATHROOM

An advertisement by the local builders' merchant for "mirror tiles" set me thinking about how hard it is to see one's own back. If I tiled the walls of my bathroom with these mirror tiles could I look directly (not obliquely) at my back? In a rectangular room it would be possible by standing at any point of the rhombus whose vertices are the mid-points of the sides, or indeed by standing at any point not on either diagonal and facing in the direction of one of the diagonals.



But what if the bathroom were triangular?

FUNCTION FROM BLASIUS (JCMN 34, p.4053)

This problem was about the solution $f(x) = x^2/2! - x^5/5! + \dots$ of the differential equation $ff'' + f''' = 0$. What is the nature of the singularity where the circle of convergence cuts the negative real axis? A partial answer is that it is not a pole.

In fact the differential equation has a one-parameter family of solutions $f(x) = 3/(x+b)$, but no other solution has a pole, and the singularity that we are investigating is therefore not a pole. To prove this, suppose that some non-zero solution had a pole, then we may assume the pole to be at the origin because the differential equation is unchanged by translation. Let the pole be of order n. Then in some neighbourhood of the origin

$$f(x) = \sum_{r=0}^{\infty} a_r x^{r-n} \quad \text{where } a_0 \neq 0 \text{ and } n \geq 1.$$

Substituting in the differential equation and picking out the lowest powers of x, we find

$$n(n+1)a_0^2 x^{-2n-2} - n(n+1)(n+2)a_0 x^{-n-3} + \dots = 0$$

Since $a_0 \neq 0$ it follows that $n = 1$ and $a_0 = 3$. Now put

$$f(x) = 3/x + cx^m + \text{higher powers}$$

where $c \neq 0$ and $m \geq 0$. The differential equation gives

$$(3/x + cx^m)(6/x^3 + cm(m-1)x^{m-2}) - 18/x^4 + cm(m-1)(m-2)x^{m-3} + \text{higher powers} = 0.$$

Equating the coefficients of cx^{m-3} we find

$$6 + 3m(m-1) + m(m-1)(m-2) = 0$$

or $(m+2)(m^2 - 2m + 3) = 0.$

It follows that m cannot be a non-negative integer, and so we has disproved the existence of a pole.

MONOPOLY WITH TAXATION

G. F. D. Duff

Suppose that some corporation is given by the Government the monopoly on production of a certain commodity, i.e. nobody else is allowed to produce it. What will happen?

To set up a simple model we suppose that the demand (x things per year) is related to the selling price (y per thing) by a linear relation $a = bx + y$ and that the cost of producing x things in one year is $qx + c$. With no taxation the profit to the corporation is $x(a - bx) - qx - c$. This expression may be written (completing the square) as

$$(a-q)^2/(4b) - c - (2bx - a + q)^2/(4b)$$

and the corporation will obtain the best possible profit of $(a-q)^2/(4b) - c$ by choosing $x = (a-q)/(2b)$ and $y = (a+q)/2$.

Now suppose that the Government imposes a tax of t on each thing produced. For production x at price y the profit to the corporation is $x(a-bx) - tx - qx - c$ and by choosing $x = (a-q-t)/(2b)$ and $y = (a+q+t)/2$ the profit is maximized to the value $(a-q-t)^2/(4b) - c$. The revenue to the Government is $tx = t(a-q-t)/(2b)$, but this amount is forthcoming only if the corporation remains in business, i.e. continues to make a profit. The Government will therefore choose t to maximize the quadratic $t(a-q-t)/(2b)$ subject to the condition that $a - q - t > 2(bc)^{\frac{1}{2}}$. The solution is that t must be the smaller of $(a-q)/2$ and $a - q - 2(bc)^{\frac{1}{2}}$.

A Note from the Editor

Compare the model described above with the Queensland State Electricity Commission. Of course we cannot expect them to match well; one reason is that electricity is sold in Queensland according to an elaborate tariff charging different prices to different users. Also your Editor has not looked up the statistics on electricity pro-

duction which are probably published every year.

Take parameters $a = 15$, $b = 1/10$, $c = 350$ and $q = 1$, where (to avoid having very large or very small numbers) we have measured annual costs in units of one million dollars per year, annual production (such as x) in units of a hundred million k.w.h. per year, and unit costs (such as y) in units of one cent per k.w.h.

The Commission is constitutionally incapable of making a profit or a loss and is under no legal directive on its tariff of charges. Therefore, in order to provide ourselves with a well-defined and self-consistent model, we regard the Commission as a partnership of all its employees, and we ascribe to it the objective of maximizing the average of their wages. The parameter $c = 350$ may be regarded as consisting of the sum of three terms, 50 which is the interest on the loans that have provided the equipment, 50 which is the annual cost of having work done by contract, and 250 which is the cost of providing the minimal level of wages (below which the employees would leave to work elsewhere). The "profit" (as calculated in the article above) is then the amount that is added to the 250 to provide the actual wages.

The output x and sale price y are related by $15 = x/10 + y$, and the costs of production and distribution are $x + 350$. With no taxation the optimum price is $y = 8$, and the annual production is $x = 70$. The income is 560 and cost is 420 so that the profit is 140, i.e. there is available $140 + 250 = 390$ for distribution as wages. Since $(a-q)^2 < 16\,bc$ the Government's choice of the taxation rate t to maximize revenue is $t = a - q - 2(bc)^{\frac{1}{2}} = 14 - 2\sqrt{35}$ = approximately 2. This will lead to a price of $y = 9$ instead of 8 and an annual production of $x = 60$ instead of 70, and Government revenue from the tax will be $xt = 120$.

It is interesting to recall that among the ways used by Elizabeth I of England for raising revenue was the sale of monopolies to the highest bidder. Was that more productive than a tax on the product of the monopoly?

FROM CAPTAIN COOK'S JOURNAL

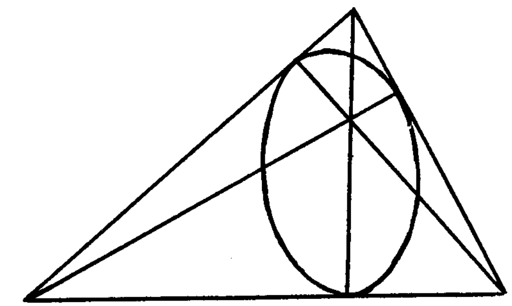
Wednesday 10th October 1770, according to our reckoning, but by the people here Thursday 11th....

(This was when the Endeavour had just arrived at Batavia on the north coast of Java in the Dutch East Indies. The International Date Line had not then been invented. On this date Captain Cook made the calendar adjustment necessitated by sailing Westward round the World.)

Friday 12th October 1770... About 9 o'clock in the Evening we had much rain, with some very heavy Claps of Thunder, one of which carried away a Dutch Indiaman's Main Mast by the Deck, and split it, the Maintopmast and Topgallantmast all to shivers. She had an Iron Spindle at the Maintopgallant Mast head which had first attracted the Lightning. The ship lay about 2 Cable lengths from us, and we were struck with the Thunder at the same time, and in all probability we should have shared the same fate as the Dutchman, had it not been for the Electrical Chain which we had but just before got up; this carried the Lightning or Electrical matter over the side clear of the Ship. This instance alone is sufficient to recommend these Chains to all Ships whatever, and that of the Dutchman ought to Caution people from having Iron Spindles at their Mast heads.

POINT IN A TRIANGLE

(JCMN 32, p.4008, 33, p.4030 and 34, p.4062)



The previous note was about the conic that can be inscribed in a triangle to touch the sides at the feet of the altitudes. The question was asked whether the principal axes of the conic were the principal axes of inertia of the system consisting of particles of mass a^2, b^2 and c^2 at the vertices A, B and C respectively. The answer is YES.

We already know (see the previous notes) that the centre of the conic is the centroid of the masses. Also it is easy to see that the moment of inertia of the masses about any side of the triangle is $\Delta^2/4$ where Δ is the area.

Lemma 1. Given any plane system of masses, and any constant K, the lines in the plane about which the system has moment of inertia K all touch a conic. As K varies the conics form a confocal system.

Proof. Take Cartesian coordinates with origin at the centroid of the system, and with axes the principal axes of inertia. Let the moment of inertia be A about the x-axis and B about the y-axis (so that $A = \int y^2 dm$ and $B = \int x^2 dm$).

The moment of inertia about any line $x\cos\alpha + y\sin\alpha + p = 0$ will be $\int (x\cos\alpha + y\sin\alpha + p)^2\,dm = B\cos^2\alpha + A\sin^2\alpha + p^2M$. If this is to equal K then we must have

$$B\cos^2\alpha + A\sin^4\alpha + p^2M - K(\cos^2\alpha + \sin^2\alpha) = 0$$

so that the condition for a line $lx + my + n = 0$ to have moment of inertia K about it is

$$l^2(B-K) + m^2(A-K) + n^2M = 0.$$

This is the line equation of a conic, and as K varies these conics form a confocal system. Q.E.D.

Now consider our triangle ABC with masses a^2, b^2 and c^2 at the vertices. The lines about which the moment of inertia is $\Delta^2/4$ envelope a conic, which touches all three sides. This conic has principal axes parallel to the principal axes of inertia, because they are parallel to the x- and y-axes of Lemma 1.

Lemma 2. This conic is the conic described in the previous note, i.e., the conic that touches the sides at the feet of the altitudes.

Proof. Let P be the point of contact of the conic on BC. Consider the tangent at a variable point Q of the conic. The moment of inertia about the tangent is constant, and so the contribution to it from the mass a^2 at A is a maximum when Q passes through P, because then the other two particles contribute nothing. The perpendicular distance from A to the tangent is a maximum when Q passes through P. This is possible only if AP is perpendicular to BC. A similar argument applies to the other points of contact, and so the three points of contact are the feet of the altitudes. Q.E.D.

ALGEBRAIC EQUATIONS

In Hall and Knight's Higher Algebra, page 454 we read "If two or more of the roots of an equation are connected by an assigned relation, then the properties proved in Art. 539 (i.e. the relations between the coefficients and the elementary symmetric functions of the roots) will sometimes enable us to obtain the complete solution." Problem - Clarify the "sometimes".

The two examples given by Hall and Knight are:
Example 1, solve $4x^3 - 24x^2 + 23x + 18 = 0$, having given that the roots are in arithmetic progression.
Example 2, solve $24x^3 - 14x^2 - 63x + 45 = 0$, one root being double another.

One might conjecture a proposition such as "Suppose that we have a polynomial f of degree n over a field A, and suppose also that we know an algebraic relation (also with coefficients in A) connecting the roots (not of course symmetric in the roots). Then the polynomial f is reducible over some field C of degree less than n (over A)."

EDITORIAL

This year in October we celebrate the tenth anniversary of JCMN. What would we like for a birthday present? More readers and more contributions! Please do what you can.

For its first eight years, 1975-1983, the JCMN was published by the Mathematics Department of the James Cook University, address:

> James Cook University of North Queensland,
> Post Office, James Cook, N.Q. 4811
> Australia.

The issues from this period have been reprinted as paperback volumes:

Volume 1	(Issues 1-17)	
Volume 2	(Issues 18-24)	(out of print)
Volume 3	(Issues 25-31)	

and they are available for $10 (Australian) from the Head of the Mathematics Department. Cheques for these should be payable to the James Cook University. I should explain that I am now Head of the Mathematics Department, but will retire at the end of 1985. My successor will no doubt continue to sell the old volumes.

Since Issue 32 (October 1983) I have continued to edit JCMN and have arranged the printing in Singapore. My address is either at the University (as above) or at home (see p. 4118) until early 1986, when my wife and I plan to leave Townsville. In the October issue I shall give the new address to which contributions may be sent.

Basil Rennie