NATCOR - Xpress case study

Margaret Oil produces three products: gasoline, jet fuel, and heating oil. The average octane levels must be at least 8.5 for gasoline, 7 for jet fuel, and 4.5 for heating oil. To produce these products, Margaret purchases crude oil at a price of £11 per barrel. Each day, at most 15000 barrels can be purchased.

Before crude can be used to produce products for sale, it must be distilled. It costs £0.10 to distill a barrel of oil and the result of the distillation is 0.25 barrels of distilled 1, 0.25 barrels of distilled 2, and 0.5 barrels of naphtha. Distilled naphtha can be used only to produce gasoline or jet fuel. Distilled oil can be used to produce all three products.

The octane level of each type of oil is as follows: distilled 1, 9; distilled 2, 4; naphtha, 8. All gasoline produced can be sold at £18 per barrel, all jet fuel produced, £16 per barrel; and all heating oil produced, £14 per barrel. Marketing considerations dictate that at least 3000 barrels of each product must be produced daily. How can Margaret Oils maximize its daily profit?

	Gasoline	Jet fuel	Heating oil
Minimum octane	8.5	7	4.5
Price (£)	18	16	14

	Distilled 1	Distilled 2	Naphtha
Distill (barrels)	0.25	0.25	0.5
Octane	9	4	8

Solution:

Let us define “product 1” as gasoline, “product 2” as jet fuel, and “product 3” as heating oil.

The only difficulty of the model is the high number of variables that are involved. But, if we “follow the natural direction of the chain”, we will see that the model is quite straightforward.

First, we define x as the number of crude oil barrels purchased. There is an obvious constraint limiting how many we can purchase daily:

$$x \leq 15000.$$

Besides, each barrel of crude that we buy will be distilled (because we have no capacities). Thus, in the objective function we will have the term

$$-(11 + 0.1)x = -11.1x.$$

Next, we define y_1 as the number of barrels of distilled 1, y_2 as the number of barrels of distilled 2, and y_3 as the number of barrels of naphtha. The number of these barrels that we have is given by the following constraints:

$$y_1 = 0.25x,$$
$$y_2 = 0.25x,$$
$$y_3 = 0.5x.$$

Now, we need to decide which barrels of these materials are used for each of the final products. Thus, it seems natural to define z_{1i} as the number of barrels of distilled 1 used to produce product i, z_{i2} as the number of barrels of distilled 2 used to produce product i, and z_{i3} as the number of barrels of naphtha used to produce product i, $i = 1, 2, 3$.

The correct definition between variables y and z is given by the following constraints:

$$\begin{aligned} z_{11} + z_{12} + z_{13} &\leq y_1, \\ z_{21} + z_{22} + z_{23} &\leq y_2, \\ z_{31} + z_{32} + z_{33} &\leq y_3. \end{aligned}$$

Let us not forget that naphtha cannot be used to produce heating oil:

$$z_{33} = 0.$$

Finally, we need to track how many barrels of each product we have produced. Let us define w_i as the number of barrels of product i produced, $i = 1, 2, 3$. then we have that

$$\begin{aligned} w_1 &= z_{11} + z_{21} + z_{31}, \\ w_2 &= z_{12} + z_{22} + z_{32}, \\ w_3 &= z_{13} + z_{23} + z_{33}. \end{aligned}$$

Now, these w_i variables are involved in several expressions:

- Compulsory minimum production:

$$w_i \geq 3000, \quad i = 1, 2, 3.$$

- Minimum average octane of each product:

$$\begin{aligned} \tfrac{9z_{11}+4z_{21}+8z_{31}}{w_1} &\geq 8.5, \\ \tfrac{9z_{12}+4z_{22}+8z_{32}}{w_2} &\geq 7, \\ \tfrac{9z_{12}+4z_{23}}{w_3} &\geq 4.5. \end{aligned}$$

- Revenue in the objective function:

$$18w_1 + 16w_2 + 14w_3.$$

Notice that, although the constraints on the minimum average octane are nonlinear, it is very easy to linearize them if we multiply them by w_i.

In the optimal solution, we have that:

- $x^* = 15000$.
- $y^* = (3750, 3750, 7500)$.
- $z^* = (3450, 0, 300, 0, 1050, 2700, 3450, 4050, 0)$,
- $w^* = (6900, 5100, 3000)$.

The optimal profit is £81300. □