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School of Mathematics

Using quizzes to deliver a course online

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May 2018

- New 20-credit course
- SQA Advanced Higher / Further Maths A-Level
- Ready for September
- Online
- Jointly with Richard Gratwick

Fundamentals of Algebra and Calculus

1	Vectors
2	Principles and techniques of differentiation
3	Polynomials and rational functions
4	Principles of integration
5	Functions and graphs
6	Further techniques and applications of differentiation
7	Complex numbers
8	Methods of integration
9	Sequences and series
10	Applications of integration

A typical week

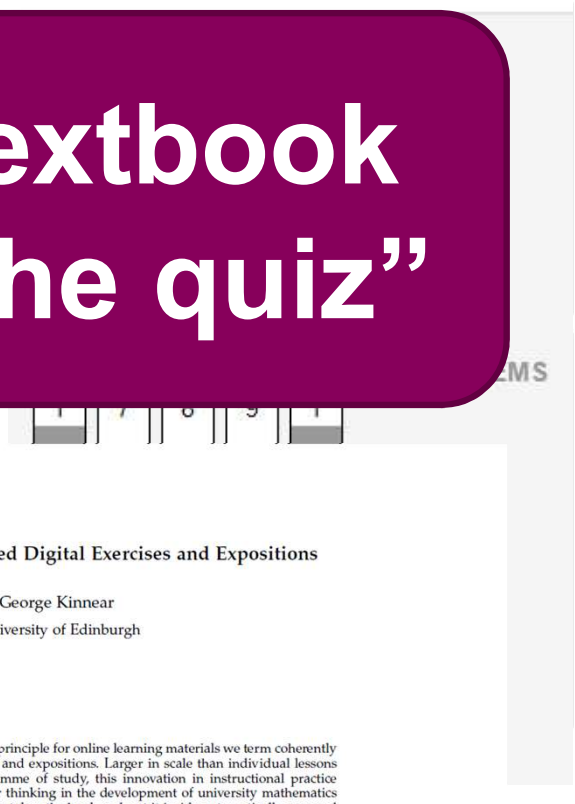
Week 3: Polynomials and rational functions

- Getting started
- 1. Polynomials
- 2. Graphs of polynomials
- 3. The Binomial Theorem
- 4. Rational functions
- Week 3 Practice Quiz
- Week 3 Final Test

Restricted Not available unless: The activity **Week 3 Practice Quiz** is complete and passed

A typical section

“Textbook in the quiz”



Coherently Organised Digital Exercises and Expositions

Christopher J. Sangwin, George Kinnear
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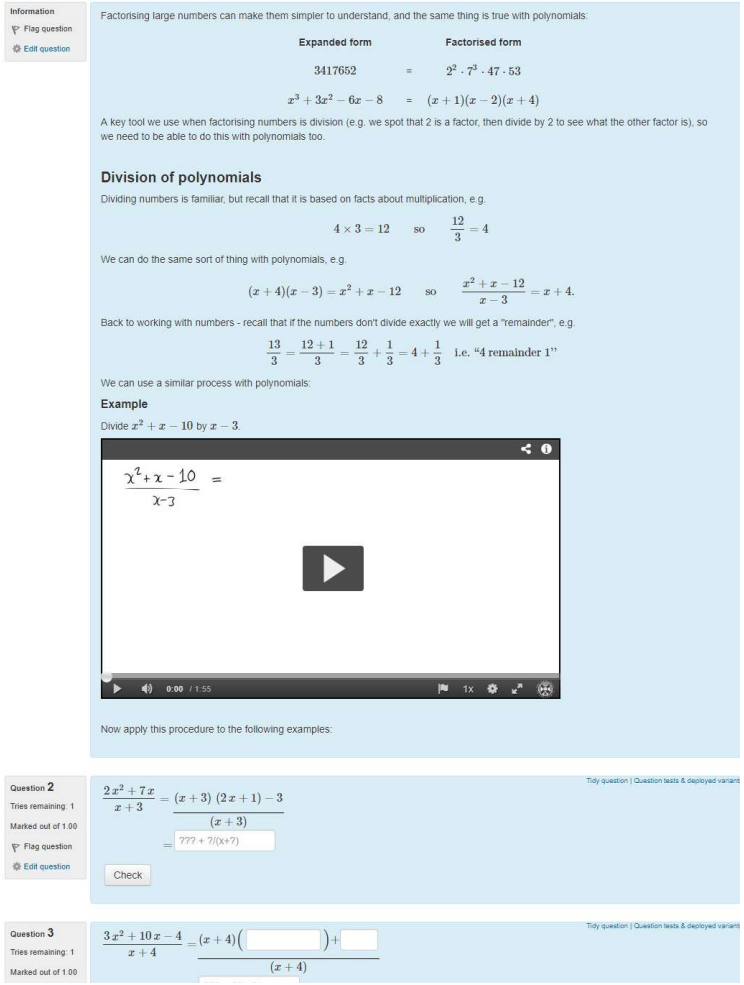
ARTICLE HISTORY
Compiled May 29, 2020

ABSTRACT
We describe an organising principle for online learning materials we term coherently organised digital exercises and expositions. Larger in scale than individual lessons but smaller than a programme of study, this innovation in instructional practice is increasingly guiding our thinking in the development of university mathematics courses. Essentially we have taken the book and put it inside automatically assessed online quizzes. In doing this we embrace the potential provided by new technology to implement evidence-based practices such as spaced retrieval practice. This paper discusses details of this innovation, and how we have implemented it. On the basis of these experiences, we believe this innovation has the potential to change the model of education for university mathematics courses in substantial and non-trivial ways.

KEYWORDS
Learning design; online assessment; university mathematics education; Online teaching and learning



Information
Flag question
Edit question

Factorising large numbers can make them simpler to understand, and the same thing is true with polynomials:

Expanded form		Factorised form
3417652	$=$	$2^2 \cdot 7^3 \cdot 47 \cdot 53$
$x^3 + 3x^2 - 6x - 8$	$=$	$(x+1)(x-2)(x+4)$

A key tool we use when factorising numbers is division (e.g. we spot that 2 is a factor, then divide by 2 to see what the other factor is), so we need to be able to do this with polynomials too.

Division of polynomials

Dividing numbers is familiar, but recall that it is based on facts about multiplication, e.g.

$$4 \times 3 = 12 \quad \text{so} \quad \frac{12}{3} = 4$$

We can do the same sort of thing with polynomials, e.g.

$$(x+4)(x-3) = x^2 + x - 12 \quad \text{so} \quad \frac{x^2+x-12}{x-3} = x+4.$$

Back to working with numbers - recall that if the numbers don't divide exactly we will get a "remainder", e.g.

$$\frac{13}{3} = \frac{12+1}{3} = \frac{12}{3} + \frac{1}{3} = 4 + \frac{1}{3} \quad \text{i.e. “4 remainder 1”}$$

We can use a similar process with polynomials:

Example

Divide $x^2 + x - 10$ by $x - 3$.

$$\frac{x^2+x-10}{x-3} =$$

0:00 / 1:55 1x

Now apply this procedure to the following examples:

Question 2
Tries remaining: 1
Marked out of 1.00
Flag question
Edit question

Tidy question | Question tests & deployed variants

$$\frac{2x^2+7x}{x+3} = \frac{(x+3)\,(2x+1) - 3}{(x+3)}$$

= ??? + ?/(x+?)

Check

Question 3
Tries remaining: 1
Marked out of 1.00

Tidy question | Question tests & deployed variants

$$\frac{3x^2+10x-4}{x+4} = (x+4)\left(\quad\right) + \quad$$

$$(x+4)$$

Exposition

Video worked examples

Problems

A typical question: STACK

Fully factorise the polynomial $p(x) = 3\,x^4 + 16\,x^3 + 3\,x^2 - 46\,x + 24$, given that $x = -3$ is a root.

$p(x) =$ (x^2+2*x-3)*(x+4)*(3

Your last answer was interpreted as follows:

$$(x^2 + 2\,x - 3)\ (x + 4)\ (3\,x - 2)$$

The variables found in your answer were: $[x]$

Check

Your answer is partially correct.

Your answer is not factored. You could still do some more work on the term $x^2 + 2\,x - 3$.

The factor $3\,x - 2$ is correct.

The factor $x + 4$ is correct.

Marks for this submission: 0.50/1.00.

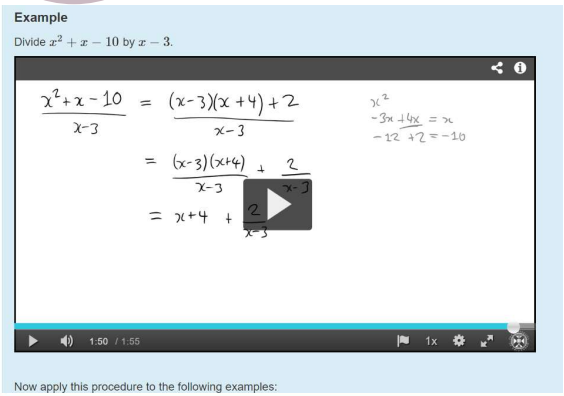
randomization

input validation

robust grading

Three strategies

1 Faded worked examples



$$\frac{2x^2+7x}{x+3} = \frac{(x+3)(2x+1)-3}{(x+3)} = \text{??? + ?/(x+?)}$$

$$\frac{3x^2+10x-4}{x+4} = \frac{(x+4)(\quad)+\quad}{(x+4)} = \text{??? + ?/(x-?)}$$

$$\frac{x^2-4}{x-1} = \text{??? + ?/(x-?)}$$

Check

Worked Example

↓

Worked Example with last step as a task

↓

Worked example with more steps to complete

↓

Problem solving

"The fading procedure fosters learning"

The Journal of Experimental Education, 2002, 70(4), 293–315

From Example Study to Problem Solving: Smooth Transitions Help Learning

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ABSTRACT. Research has shown that it is effective to combine example study and problem solving in the initial acquisition of cognitive skills. Present methods for combining these learning modes are static, however, and do not support a transition from example study in early stages of skill acquisition to later problem solving. Against this background, the authors proposed a successive integration of problem-solving elements into example study until the learners solved problems on their own (i.e., complete example → increasingly more incomplete examples → problem to-be-solved). The authors tested the effectiveness of such a fading procedure against the traditional method of using example–problem pairs. In a field experiment and in 2

2

“Give an example”

For each case below, type in a quadratic, e.g. `2x^2+3x+1`, whose graph has exactly the given number of intersections with $y = x^2$. If it is not possible, then enter `none`.

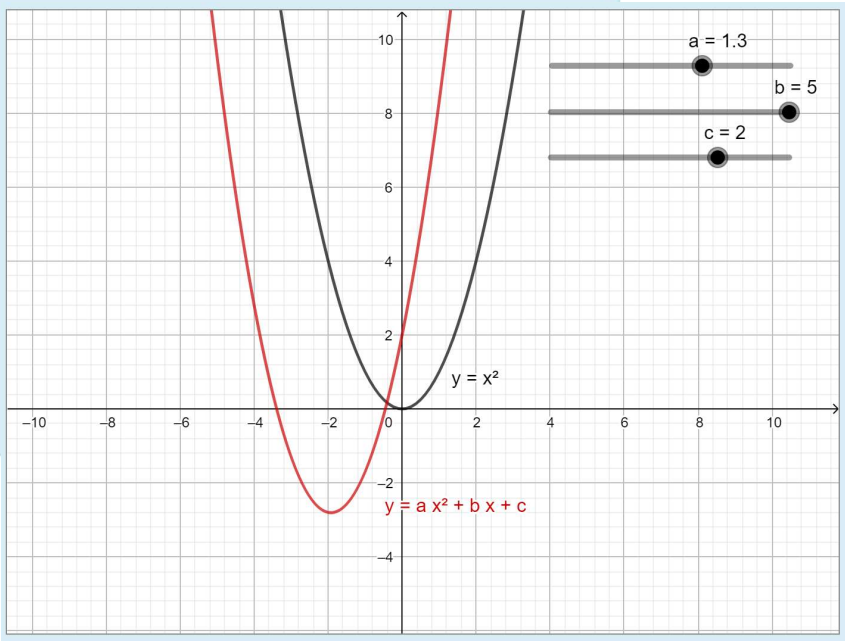
No intersection: $y =$

1 intersection: $y =$

2 intersections: $y =$

3 intersections: $y =$

Check



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3 Retrieval practice

*"**retrieval of information from memory** produces better retention than restudying …*

the testing effect"

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Review

Cell PRESS

The critical role of retrieval practice in long-term retention

Henry L. Roediger III[1] and Andrew C. Butler[2]

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Learning is usually thought to occur during episodes of studying, whereas retrieval of information on testing simply serves to assess what was learned. We review research that contradicts this traditional view by demonstrating that retrieval practice is actually a powerful mnemonic enhancer, often producing large gains in long-term retention relative to repeated studying. Retrieval practice is often effective even without feedback (i.e. giving the correct answer), but feedback enhances the benefits of testing. In addition, retrieval practice promotes the acquisition of knowledge that can be flexibly retrieved and transferred to different contexts. The power of retrieval practice in consolidating memories has important implications for both the study of memory and its application to educational practice.

Introduction

> A curious peculiarity of our memory is that things are impressed better by active than by passive repetition. I mean that in learning (by heart, for example), when we almost know the piece, it pays better to wait and recollect by an effort within, than to look at the book again. If we recover the words the former way, we shall probably know them the next time; if in the latter way, we shall likely need the book once more.
>
> William James [1]

Psychologists have often studied learning by alternating series of study and test trials. In other words, material is presented for study (S) and a test (T) is subsequently given to determine what was learned. After this procedure is repeated over numerous ST trials, performance (e.g. the number of items recalled) is plotted against trials to depict the rate of learning; the outcome is referred to as a learning curve and it is negatively accelerated and is fit by a power function. Thus, most learning occurs on early ST trials, and the amount of learning decreases with additional trials. The critical assumption is that learning occurs during the study phases of the ST ST ST. . . sequence, and the test phase is simply there to measure what has been learned during previous occasions of study. The test is usually considered a neutral event. For example, researchers in the 1960 s debated whether learn-

ignored the possibility that learning occurred during the retrieval tests [2–5]. Exactly the same assumption is built into our educational systems. Students are thought to learn via lectures, reading, highlighting, study groups, and so on; tests are given in the classroom to measure what has been learned from studying. Again, tests are considered assessments, gauging the knowledge that has been acquired without affecting it in any way.

In this article, we review evidence that turns this conventional wisdom on its head: retrieval practice (as occurs during testing) often produces greater learning and long-term retention than studying. We discuss research that elucidates the conditions under which retrieval practice is most effective, as well as evidence demonstrating that the mnemonic benefits of retrieval practice are transferrable to different contexts. We also describe current theories on the mechanisms underlying the beneficial effects of testing. Finally, we discuss educational implications of this research, arguing that more frequent retrieval practice in the classroom would increase long-term retention and transfer.

The testing effect and repeated retrieval

The finding that retrieval of information from memory produces better retention than restudying the same information for an equivalent amount of time has been termed the testing effect [6]. Although the phenomenon was first reported over 100 years ago [7], research on the testing effect has been sporadic at best until recently (but see Box 1 for some classic studies). In the last 10 years, much research has shown powerful mnemonic benefits of retrieval practice [8–10] . The data in Figure 1 come from a study in which two groups of students retrieved information several times

Glossary

Expanding retrieval schedule: testing of retention shortly after learning to make sure encoding is accurate, then waiting longer to retrieve again, then waiting still longer for a third retrieval and so on.
Feedback: providing information after a question. General (right or wrong) feedback is not very helpful if the correct answer is not provided. Correct answer feedback usually produces robust gains on a final criterion measure.
Negative suggestion effect: taking a test that provides subtly wrong answers (e.g. true or false, multiple choice) can lead students to select a wrong answer, believe it is right, and thus learn an error from taking the test.
Retrieval practice: act of calling information to mind rather than rereading it or hearing it. The idea is to produce 'an effort from within' to induce better retention.

3

Complete the following table of standard derivatives. Try to do this without checking your notes -- which of the most important standard derivatives can you remember?

Function	Derivative
x^n	

Check

At the start of integration (Week 4) – recall practice of differentiation (Week 2)

3

Complete the following table of standard derivatives. Try to do this without checking your notes -- which of the most important standard derivatives can you remember?

Function	Derivative
x^n	n*x^(n-1) ✔
sin(x)	cos(x) ✔
cos(x)	-sin(x) ✔
k	0 ✔
e^x	e^x ✔

✖

What about the derivatives of these functions?

$$\{\ln(x), \sec(x), \tan(x)\}$$

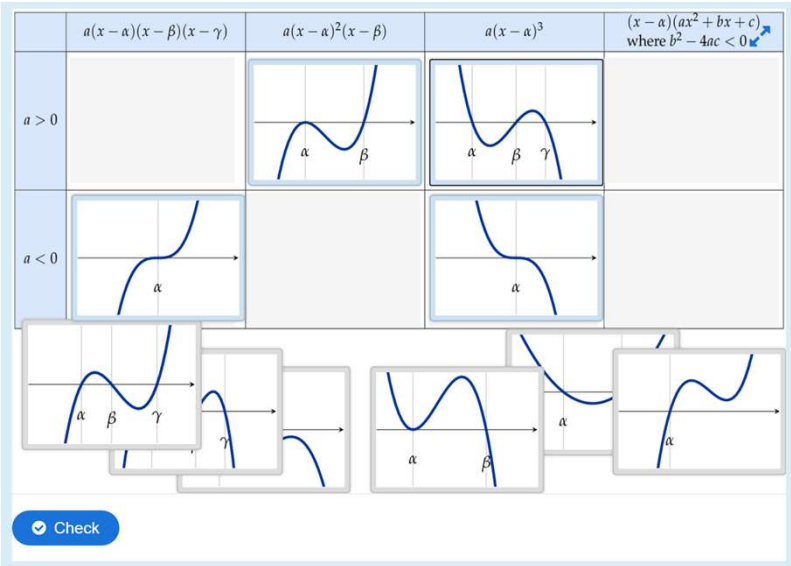
Further ideas

Multiple choice

Which of the following are antiderivatives of x^5?

- [ ] (a) $5\,x^4 + x$
- [ ] (b) $\dfrac{x^6}{6} + 5$
- [ ] (c) $\dfrac{x^6}{6} + C$
- [ ] (d) $\dfrac{x^6}{6}$
- [ ] (e) $5\,x^4$
- [ ] (f) $\dfrac{x^5}{6}$

Sorting/matching



Proof comprehension

Complete the following proof.

Let $P(n)$ be the statement

$\sum_{k=1}^{n} k^2 = \frac{n\cdot(n+1)\cdot(2\cdot n+1)}{6}$

Since $1^2 =$ ____

and $\frac{1\cdot(1+1)\cdot(2\cdot 1+1)}{6} =$ ____

it follows that $P(1)$ is true.

Assume that $P(n)$ is true.

$\sum_{k=1}^{n+1} k^2 =$ sum(?,k,1,n)+?

$= \frac{n\cdot(n+1)\cdot(2\cdot n+1)}{6} + (n+1)^2$ (No answer given)

$= \frac{(n+2)\cdot(2\cdot n+3)\cdot(n+1)}{6}$

$=$ ____

Since $P(1)$ and $P(n) \Rightarrow P(n+1)$ it follows that $P(n)$ is true for all $n \in \mathbb{N}$ by the principal of mathematical induction.

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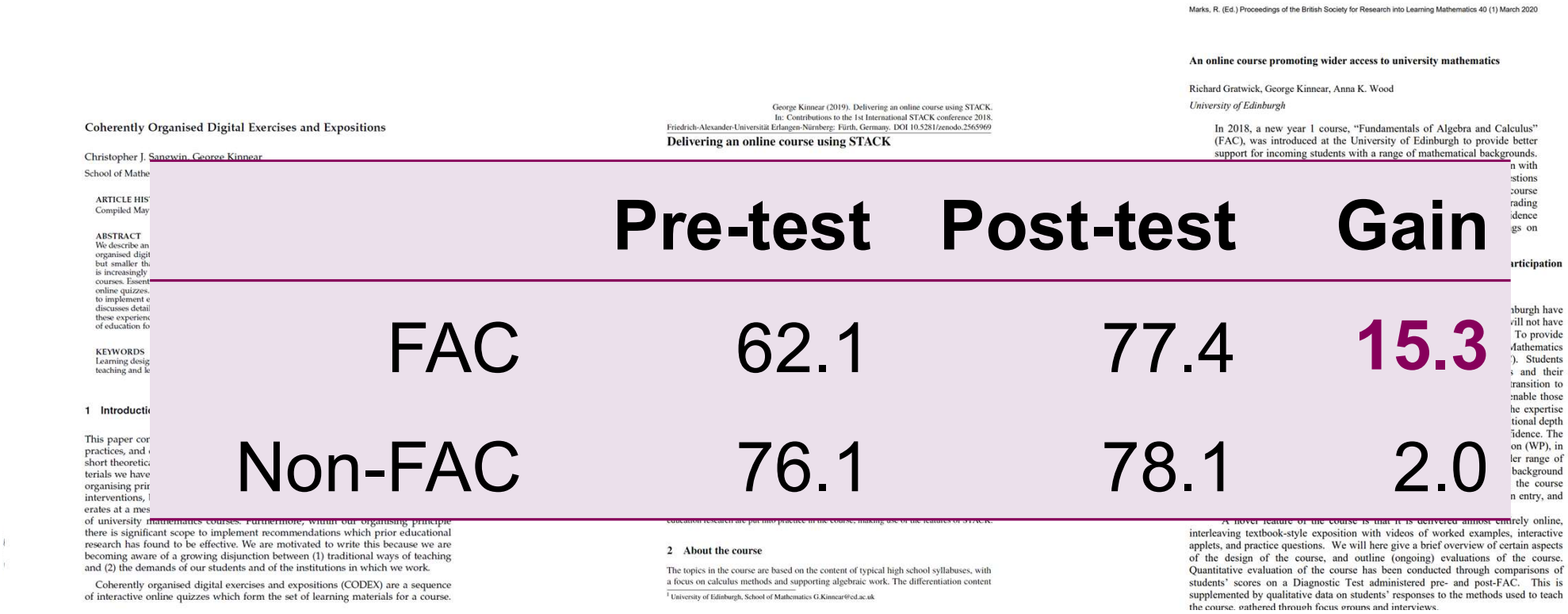
Bickerton, R. and Sangwin, C. J. (2020)

Next steps

Read more about FAC

www.maths.ed.ac.uk/gkinnear/research


Coherently Organised Digital Exercises and Expositions

Christopher J. Sangwin, George Kinnear

George Kinnear (2019). Delivering an online course using STACK.
In: Contributions to the 1st International STACK conference 2018.
Friedrich-Alexander-Universität Erlangen-Nürnberg: Fürth, Germany. DOI 10.5281/zenodo.2565969

Delivering an online course using STACK

Marks, R. (Ed.) Proceedings of the British Society for Research into Learning Mathematics 40 (1) March 2020

An online course promoting wider access to university mathematics

Richard Gratwick, George Kinnear, Anna K. Wood

University of Edinburgh

In 2018, a new year 1 course, "Fundamentals of Algebra and Calculus" (FAC), was introduced at the University of Edinburgh to provide better support for incoming students with a range of mathematical backgrounds.

	Pre-test	Post-test	Gain
FAC	62.1	77.4	**15.3**
Non-FAC	76.1	78.1	2.0

“Developing effective resources for online teaching and assessment of mathematics”

with Igor’ Kontorovich and Chris Sangwin

1. STACK server
2. Training
3. Content

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1. Indefinite integration

The technique known as **integration by parts** is used to integrate a product of two functions, such as in these two examples:

(i) $\int e^{2x} \sin 3x \, dx$ (ii) $\int_0^1 x^3 e^{-2x} \, dx$

Note that in the first example, the integrand is the product of the functions e^{2x} and $\sin 3x$, and in the second example the integrand is the product of the functions x^3 and e^{-2x}. Note also that we can change the order of the terms in the product if we wish and write

(i) $\int (\sin 3x) e^{2x} dx$ (ii) $\int_0^1 e^{-2x} x^3 \, dx$

What you must never do is integrate each term in the product separately and then multiply - the integral of a product is not the product of the separate integrals. However, it is often possible to find integrals involving products using the method of integration by parts - you can think of this as a *product rule* for integrals.
The integration by parts formula states:

Key Point 5

Integration by Parts for Indefinite Integrals

For indefinite integrals, given functions $f(x)$ and $g(x)$:

$$\int f \cdot g \, dx = f \cdot \int g \, dx - \int \left(\frac{df}{dx} \cdot \int g dx \right) dx$$

Alternatively, given functions u and v:

$$\int u \frac{dv}{dx} dx = u.v - \int v \frac{du}{dx} dx$$

Study the formula carefully and note the following observations. Firstly, to apply the formula we must be able to differentiate the function f to find $\frac{df}{dx}$, and we must be able to integrate the function, g. Secondly the formula replaces one integral, the one on the left, with a different integral, that on the far right. The intention is that the latter, whilst it may look more complicated in the formula above, is simpler to evaluate. Consider the following Example:

Example 15

Find the integral of the product of x with $\sin x$; that is, find $\int x \sin x \, dx$.

Solution

Compare the required integral with the formula for integration by parts: we choose

$f = x$ and $g = \sin x$

It follows that

$\frac{df}{dx} = 1$ and $\int g \, dx = \int \sin x \, dx = -\cos x$

(When integrating g there is no need to worry about a constant of integration. When you become confident with the method, you may like to think about why this is the case.)

Applying the formula we obtain

$$\begin{aligned} \int x \sin x \, dx &= f \cdot \int g \, dx - \int \left(\frac{df}{dx} \cdot \int g dx \right) dx \\ &= x(-\cos x) - \int 1(-\cos x) \, dx \\ &= -x \cos x + \int \cos x \, dx = -x \cos x + \sin x + c \end{aligned}$$

Task

Find $\int (5x+1) \cos 2x \, dx$.

Let $f = 5x + 1$ and $g = \cos 2x$. Now calculate $\frac{df}{dx}$ and $\int g \, dx$:

Your solution

Answer

$\frac{df}{dx} = 5$ and $\int \cos 2x \, dx = \frac{1}{2} \sin 2x$.

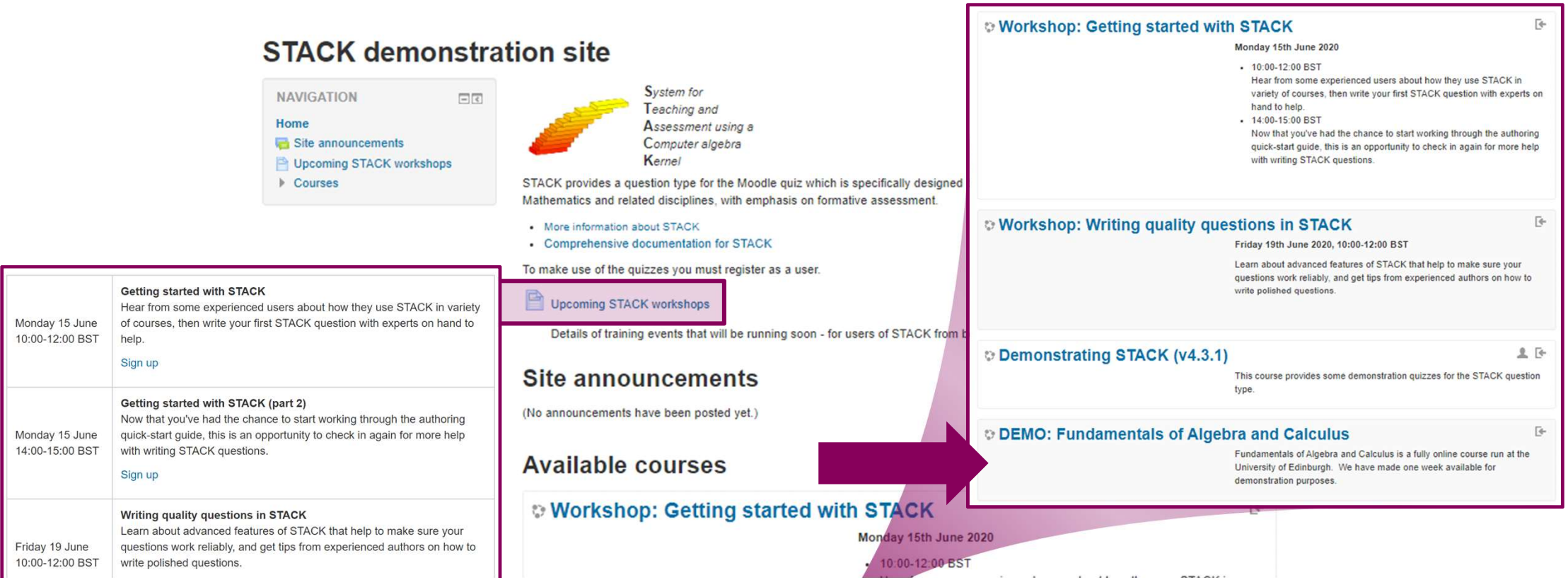
Substitute these results into the formula for integration by parts and complete the Task:

Your solution

Contact Chris: c.j.sangwin@ed.ac.uk

STACK demo

tinyurl.com/stack-demo-site



Thank you!

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