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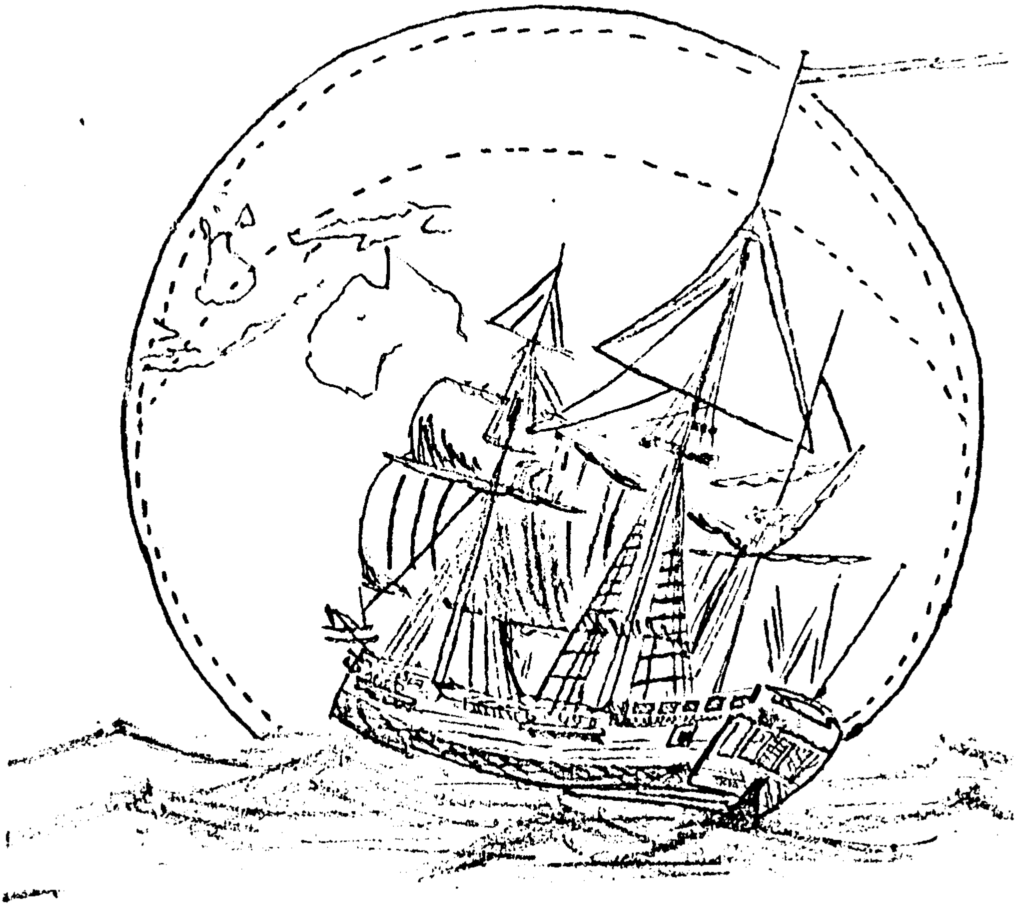


CONTENTS OF JCMN 31

OBITUARY

With regret we record the death on 20th January of H. Kestelman. Below is reprinted the obituary notice written by Prof. Larman of University College London, and the last of Kes's many contributions to JCMN, posted from London a week before he died.

HYMAN KESTELMAN 1908-1983

D.G. Larman

Many members of University College, past and present, will have learned with sadness of the death on 20 January 1983 of Mr. Hyman Kestelman, member of the Mathematics Department for 55 years.

After taking his intermediate exams at Birkbeck College, Kes joined the Mathematics Department as an undergraduate in 1928. During 1931-32 he studied for an M.Sc. degree and, at that time, met his future wife Shep, who was an undergraduate in Mathematics. Between 1931 and 1949 Kes was a lecturer although he did not go to Bangor when the department was evacuated during the war. A natural pacifist, he applied to join the fire service and was frustrated by being rejected on the grounds that his parents were not British. He contented himself with teaching at Birkbeck College during this period.

In 1949 he was made a Reader in Mathematics and in 1951 he became Tutor, a post which he enjoyed for 19 years. This involved organising examinations and admissions as well as carrying out the normal welfare duties of a Tutor! He was a true father figure to hundreds of students who passed through the department. These included many of the weaker students who were able, by his encouragement, to make valuable contributions to the mathematical life of society but who would, without doubt, have been discouraged into failure without his sympathy and understanding. Despite this arduous task he managed to find time to be Chairman of the Chamber Music Society, President of the Maths and Physics Soc., and Auditor for the Common Room and for the AUT. In 1973 the College recognised his unique contributions by making him a Fellow.

Within the department his skills as a lecturer and his breadth of mathematical knowledge were legendary. His book, with Sir Harrie Massey, on Ancillary Mathematics was the standard work for physics students and others throughout the college. At a more specialist level his book on modern theories of integration is a standard work in advanced analysis.

Kes leaves a galaxy of former students who are well aware of their debt to him. These include two Fields Medalists, six Fellows of The Royal Society and many Professors of Mathematics. His teaching ability ranged from interesting and encouraging first-year students who had to study mathematics although it was not their primary interest, to stimulating and helping the most eminent visiting professor. One of his outstanding qualities was his willingness to give very generously of time and effort in assisting with mathematical problems that arose in other sciences such as statistics, physics and biometry.

From 1975 he enjoyed a very active retirement. As an Honorary Research Fellow he came into College regularly to discuss mathematics and continued to contribute to several journals. A further outlet for his energies was his election in 1980 as the Honorary Librarian of the London Mathematical Society.

Kes will be remembered with great affection by past and present colleagues and by many generations of past students. We shall miss him greatly.

GENERALIZED HADAMARD INEQUALITY

H. Kestelman

Let A be a positive definite Hermitean matrix, written in block notation as $A = \begin{pmatrix} P & N \\ N^* & Q \end{pmatrix}$

where P and Q are square and N has Hermitean conjugate N^*. Prove that $\det A \le \det P \det Q$ with equality if and only if $N = 0$.

EXERCISE IN ANALYSIS

A sequence is defined by $s_{n+1} = \sqrt{(s_n + 2)}$ and $s_1 = b > 0$. Find a compact formula for s_n in terms of n.

The genesis of this problem may be traced back to page 35 of J.C. Burkill's book "A First Course in Mathematical Analysis" (C.U.P., 1964).

GROUP TABLE MATRICES

C.J. Smyth

Let G be an abelian group with n elements $g_1, \dots, g_n$. Then its group table is an $n \times n$ matrix with entries in G, with (i, j)th element $g_i g_j$, which equals g_k for some k. Given complex numbers $c_1, c_2, \dots, c_n$, define the matrix C by replacing g_i in the group table matrix by c_i, for all i. C is a generalised circulant, the usual circulant being obtained by taking G as a cyclic group.

Let $\chi_1, \chi_2, \dots, \chi_n$ be the characters of G, and $X = \frac{1}{\sqrt{n}}(\chi_j(g_i))$.

Then $CX = \bar{X}D$, $C = \bar{X}D\bar{X}^T$, where $D = \text{diag}(d_1, \dots, d_n)$ and $d_i = \sum_{\ell} c_\ell \chi_i(g_\ell)$.

Since $X\bar{X}^T = I$, $|\det X| = 1$, and $\det C = \det D \times (\det \bar{X})^2$. To calculate $(\det \bar{X})^2$, choose a particular C, C^* say, with $c_1 = 1$, $c_2 = c_3 = \dots = c_n = 0$, where we assume g_1 is the group identity. Then C^* is a permutation matrix, with the (i, j)th entry 0 unless $g_j = g_i^{-1}$. Here $D = I$, so $(\det \bar{X})^2 = \det C^* = \pm 1$. To work out which sign, we note that C^* corresponds to a permutation of the group elements $g_1, g_2, \dots g_n$ defined by mapping g_i into g_i^{-1}. This permutation is a product of transpositions $(g_i\ g_k)$ where $g_k = g_i^{-1}$, and $i \neq k$. Hence $\det C^* = (-1)^{\frac{1}{2}(n - \#\text{ of involutions} - 1)}$, the -1 being for the identity. Hence if n is odd, $\det C^* = (-1)^{\frac{1}{2}(n-1)}$.

If n is even, write G as a direct product of cyclic groups of prime power order, and let $N(G) = \#$ of such cyclic groups of order a power of 2 in this product.

Then G contains $2^{N(G)} - 1$ involutions, and

$$\det C^* = (-1)^{n/2 - 2^{N(G)} - 1}$$

Hence

$$\det C^* = \begin{cases} (-1)^{n/2 - 1} & \text{if } N(G) = 1 \\ 1 & \text{if } N(G) \geq 2 \end{cases}$$

since in the latter case $4|n$. Finally

$$\det C^* = \begin{cases} -1 & \text{if } G \cong C_R \times G_1,\ C_R \text{ cyclic of order } R = 2^r,\ r \geq 2, \\ & G_1 \text{ a group of odd order} \\ 1 & \text{otherwise for even } n \\ (-1)^{\frac{1}{2}(n-1)} & n \text{ odd} \end{cases}$$

so that

$$\det C = \det C^* \prod_i \left(\sum_\ell c_\ell \chi_i(g_\ell)\right).$$

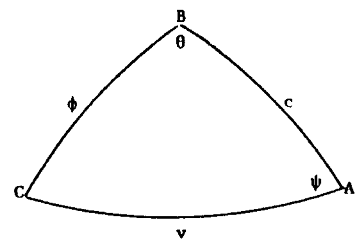
My questions are (1) Does anyone know of a reference to the final result in the literature? (2) Is there a corresponding formula for $\det C$, arising from the group table of a non-abelian group? (3) We have calculated $(\det \bar{X})^2$. but what is $\det \bar{X}$? To make this question meaningful, one has to have a specified order for $g_1, g_2, \dots, g_n$ and for $\chi_1, \chi_2, \dots, \chi_n$.

GEOMETRICAL EXERCISE (JCMN 30, p. 3142)

J.B. Parker

This problem (by H. Kestelman) was, given b positive, to describe the set of complex numbers $\cos\phi + b\sin\phi\exp(i\theta)$.

On the unit sphere take fixed points A and B, distance apart $= c = \pi/2 + \tan^{-1} b$. Now consider the spherical triangle ABC.



As θ and φ vary the point C moves all over the sphere, but without loss of generality we may suppose both θ and ψ to be between 0 and π (for if not we may replace θ by $2\pi - \theta$ and ψ by $2\pi - \psi$).

The cosine rule gives the two equations

$$\cos v = \cos\phi\cos c + \sin\phi\sin c\cos\theta$$
$$\cos\phi = \cos v\cos c + \sin v\sin c\cos\psi$$

Multiply the first by $\cos c$ and add. Then divide by $\sin^2 c$.

$$\cos\phi = \cot c\sin\phi\cos\theta + \operatorname{cosec} c\sin v\cos\psi$$

Expressing c in terms of b, this becomes

$$\cos\phi + b\sin\phi\cos\theta = \sqrt{1+b^2}\sin v\cos\psi$$

The sine rule gives

$$b\sin\phi\sin\theta = b\sin v\sin\psi$$

If the given complex number is $x + iy$ then the equations above express x and y parametrically as

$$x = \rho\sqrt{1+b^2}\cos\psi \quad\text{and}\quad y = \rho b\sin\psi$$

where ρ ($= \sin v$) takes any value between 0 and 1 and where ψ is any angle (recall that if θ is between 0 and π then so is ψ and similarly for the interval from π to 2π). The required locus is therefore the inside and boundary of the ellipse $x^2/(1+b^2) + y^2/b^2 = 1$.

NON-HARMONIC SERIES

P. Erdös

Let $a_1 < a_2 < \ldots$ be an increasing sequence of positive integers such that $\Sigma\, 1/a_k$ converges. Put $f(n) = \Sigma\, 1/|n - a_k|$ for integer n, with f(n) undefined when n is one of the a_k. Prove that $\liminf f(n) = 0$. This is not hard, but not quite trivial.

A more difficult problem is as follows. Would the further assumption that $\limsup f(n) = \infty$ imply that the set of values of f was everywhere dense in $(0, \infty)$?

KING ARTHUR AND THE M FAT KNIGHTS

M. Sved

Camelot was bristling with excitement, expecting the visit of a party of knights from Scotland. In the party there were to be the m Fabulous knights, each of them valiant, invincible, and larger than life-size. While the knights of Camelot were busy arranging the various diversions, jousts, hunts and competitions, Queen Guinevere planned the feasting and drinking. She looked at the Round Table and sighed:

– The Fabulous knights will be too big for our seats, and we shall have to allocate two adjacent seats for each –

Sir Gawain added: – We do not even know how many other knights will accompany the Fabulous m. Shall we be able to seat all our guests? – He looked to King Arthur.

– Chivalry demands that we give preference to our guests, even if they crowd out every one of us. If there are s other knights with the fabulous m, then of the n places at the Round Table only $n - 2m - s$ will be available for us. When our visitors arrive a suitable number of knights from Camelot will find themselves obliged to travel away on a lengthy Quest – said King Arthur in a tone of finality. – The Queen will make all the necessary arrangements. But first let someone tell us in how many ways we can arrange m double seats round our table and then divide the others, some for Scots and some for our own knights. –

– Let us explore all the possibilities – said Queen Guinevere. – Suppose that I take any even number 2j, not less than 2m and not more than n. Then I can pick 2j roses, half of them red and half white, from the castle gardens, and put them round the table, red and white alternately. I can do this in $2\binom{n}{2j}$ ways. – The knights nodded in agreement. They knew that the n seats at the Round Table were in fact distinguishable one from another, though the King insisted that all were of equal status.

– Then, – continued the Queen – there are $\binom{j}{m}$ ways to arrange the seats. From the j seats with red roses I choose m, and allocate each as the left-hand seat of the pair for a Fabulous knight. I can do this because to the right of a red rose is either a white rose or none. Having arranged for the Fabulous m, the other seats are divided, those with roses for the Scots and those without for our own knights. –

At this stage Merlin, the court magician, interjected – If you will pardon me, your Majesty, you are going about this ... –

– The wrong way? –

– Of course not, but the long way. I suggest first choosing the m pairs of seats for the Fabulous m, which can be done in $\frac{n}{n-m}\binom{n-m}{m}$ ways. –

– Is that obvious, Merlin, or one of those strange facts that you remember from the future? –

- It is obvious, your Majesty. First observe that $m!\binom{n-m}{m} / (n-m)$ is the number of ways of seating m named ordinary people round a perfectly symmetric table of (n-m) places, which is the number of ways of seating m named Fabulous knights round a perfectly symmetric table of n places. Our Round Table is not perfectly symmetrical, and so if we knew the names of the Fabulous m we could seat them in $m!\,\frac{n}{n-m}\binom{n-m}{m}$ ways. So far we do not know one from another, and so we can allocate their seats in $\frac{n}{n-m}\binom{n-m}{m}$ ways. The other $n-2m$ places may be divided between Scots and English in 2^{n-2m} ways, and the number that the King seeks is $2^{n-2m}\frac{n}{n-m}\binom{n-m}{m}$, is it not? -

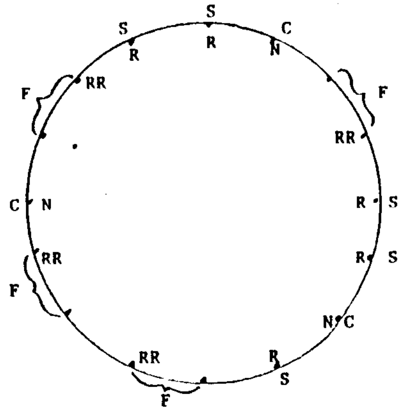
- That is clear - said Sir Gawain - and as Queen Guinevere has just explained why the number must be $\sum_{j=m}^{[n/2]} 2\binom{j}{m}\binom{n}{2j}$ the rules of chivalry have led to a Binomial Identity. -

Sir Mordred was a frustrated knight with many grudges, and he liked to make things difficult for the Queen. Now he said:

- The Queen's number may be smaller than Merlin's, for I cannot see that the method of red and white roses exhausts all the possibilities. How do we know that every one of the arrangements counted by Merlin can be arrived at by the Queen's method? -

Sir Lancelot, who worshipped the Queen, jumped to her defence.

With his sword he drew a big circle in the sand.



- Take any possible arrangement. - said Sir Lancelot. He marked at random F for Fabulous, S for Scot and C for Camelot round the circle.

- We try to work out how this arrangement would come from one of the flower patterns of the Queen. I mark RR for the red rose that must have been at the left-hand place of the two seats for a Fabulous knight, and R for rose at each place for one of the other Scots knights, and N for no rose at the place for a Camelot knight. Now there is one and only one way to find the positions and colours of all the roses, because between two red roses there must be an odd number of roses, and from the positions of the roses we know their colours because red and white alternate. -

In the silence that followed, King Arthur drew his sword, scratched in the sand Q.E.D., and stalked away. The others looked expectantly at Merlin.

– I think – he said – that His Majesty's inscription stands for Queen Eloquently Defended. And the Binomial Identity is the one that I remember seeing in the James Cook Mathematical Notes on page 3111 and 3152. –

Editor's note: The Round Table is in the Great Hall of Winchester Castle. Perhaps one of our readers in England will verify that the table has no rotational symmetry.

ANOTHER RECURRENCE

C.J. Smyth

Suppose that $a_0 \geq 0$ and $a_{n+1} = f(a_n)$ where

$$f(x) = \begin{cases} (x-1)/2 & \text{if } x \geq 1 \\ 2x/(1-x) & \text{if } 0 \leq x < 1 \end{cases}$$

Show that when a_0 is rational the sequence is ultimately periodic. In particular, if $a = p/q$ with p and q relatively prime and $p+q$ odd, show that the sequence is truly periodic (i.e. $a_m = a_0$ for some $m > 0$) and find the period.

BINOMIAL IDENTITY 15 (JCMN 30, p. 3141)

J.B. Parker

The identity suggested was

$$\sum_{j=k}^{n} \frac{2n}{n+j} \binom{n+j}{n-j} \binom{2j}{j-k} (-1)^{j-k} = 0 \quad \text{for } 0 \leq k < n,$$

Consider the function $(1+u)^{k-n} (1+1/u)^{n-k}$ which $= u^{k-n}$.

When expanded in positive powers and (finitely many) negative powers of u, the coefficient of u^{2k} is zero (because $2k \geq 0 > k-n$).

$$0 = \sum_{s=0}^{n-k} \text{Coefficient of } u^{2k+s} \text{ in } (1+u)^{k-n} \times \text{Coefficient of } u^{-s} \text{ in } (1+1/u)^{n-k}$$

$$= \sum_{s=0}^{n-k} \binom{k-n}{2k+s} \binom{n-k}{s}$$

Use the identity $\binom{-a}{b} = (-1)^b \binom{a+b-1}{a-1}$ valid for $a \geq 1$.

$$0 = \sum_{s=0}^{n-k} (-1)^s \binom{n+k+s-1}{n-k-1} \binom{n-k}{s}$$

$$= \sum_{s=0}^{n-k} (-1)^s \frac{n-k}{n+k+s} \binom{n+k+s}{n-k} \binom{n-k}{s}$$

Now use the multiplication identity $\binom{a}{b}\binom{b}{c} = \binom{a}{a-b+c}\binom{a-b+c}{c}$

$$0 = \sum_{s=0}^{n-k} (-1)^s \frac{n-k}{n+k+s} \binom{n+k+s}{2k+2s} \binom{2k+2s}{s}$$

Change the variable of summation from s to $j = k+s$.

$$0 = \sum_{j=k}^{n} (-1)^{j+k} \frac{n-k}{n+j} \binom{n+j}{n-j} \binom{2j}{j-k}$$

which is the original identity apart from the factor $2n/(n-k)$.

WRECK OF H.M.S. ENDEAVOUR (JCMN 30, p. 3147)

J.B. Douglas

At the "Captain Cook Landing Place and Museum" at Botany Bay near Sydney there is a piece of oak, 14 inches long, 9 inches in circumference, and weighing 1 pound; it was a gift from the Newport Historical Society (of Newport, Rhode Island, U.S.A.) and was cut from a larger piece. The large piece was taken out of Newport harbour from the abandoned hulk of the whaling bark La Liberte, formerly H.M. Bark "Endeavour" commanded by Captain James Cook in his Pacific exploration of 1768-1771.

BIGGEST PRIME

C.J. Smyth

It has been found by a Cray computer that $2^{86243} - 1$ is prime, this is nearly as big as the square of the largest previously known prime, $2^{44497} - 1$.

The Cray can do arithmetical operations on vectors of 80 real numbers as fast as more mundane machines can do the operations on single real numbers. (I do not know who perpetrated this new prime).

BINOMIAL IDENTITY 16

M. Sved

Prove

$$\sum_{k=2}^{n} \binom{k}{2}^2 = 6\binom{n+1}{5} + 6\binom{n+1}{4} + \binom{n+1}{3} = 6\binom{n+2}{5} + \binom{n+1}{3} .$$

More generally

$$\sum_{k=p}^{n} \binom{k}{p}^2 = \sum_{j=0}^{p} \binom{2p-j}{j}\binom{2p-2j}{p-j}\binom{n+1}{2p-j+1} .$$

EIGENVALUE CALCULATION

R. Renner

A matrix has all diagonal elements = a and all off-diagonal elements = b. What are the eigenvalues?

NAMES

(JCMN 24, p. 144 and JCMN 25, p. 3006)

In the yellow pages of the Townsville Telephone Directory under the heading of "Speed Variation Equipment" there used to be found:-

CHARLES & HUNTING (AUST)

SPECIAL FUNCTIONS

(JCMN 29, p. 3103 and 30, p. 3142)

A. Brown

H. Kestelman has given a neat solution to a problem posed by J.P. Parker concerning the function $\phi(x) = \int_{-\infty}^{x} \exp(-t^2/2)\,dt$ and its derivative $\phi'(x) = \exp(-x^2/2)$. Parker noted that ϕ'/ϕ is monotonic decreasing and asked for a proof that $x + (\phi'/\phi)$ is positive and increasing for all real x. If we write $g = x\phi + \phi'$ and $h = g/\phi = x + (\phi'/\phi)$, Kestelman uses the result $\phi'' = -x\phi'$ to show that $g' = \phi > 0$. Since $g(-\infty) = 0$, g and in turn h must be positive. Indeed this is implicit in Parker's statement that (ϕ'/ϕ) is monotonic decreasing, since it can be shown that $(\phi'/\phi)' = -g\phi'/\phi^2$. We shall use this result in the subsequent differentiation.

From $h = x + (\phi'/\phi)$, $h' = 1 - (g\phi'/\phi^2)$ and $k = h'\phi = \phi - (g\phi'/\phi)$, with $k(-\infty) = 0$, Using $g' = \phi$ and $(\phi'/\phi)' = -g\phi'/\phi^2$, we have $k' = \phi' - \phi(\phi'/\phi) + g^2\,(\phi'/\phi^2) = \phi'\,(g/\phi)^2 > 0$. Hence k is positive and consequently $h' = k/\phi > 0$, which completes the proof that h is positive and increasing for all finite x.

The proof above is not exactly the same as Kestelman's although it is in the same spirit. One drawback about both proofs is that they are rather formal and do not readily indicate what h(x) looks like, which is regrettable since ϕ and ϕ' are important functions in applications, in that they appear in most books on statistics and, with minor modification, as the error function or the Maxwellian velocity distribution in physical problems. Apart from a constant factor, they

are tabulated in considerable detail in the "Handbook of Mathematical Functions" by Abramowitz and Stegun [Table 26.1] and it is easy to obtain the following table of values:

x	-5	-4	-3	-2	-1	0	1	2
ϕ'/ϕ	5.19	4.23	3.28	2.37	1.53	0.80	0.29	0.06
h	0.19	0.23	0.28	0.37	0.53	0.80	1.29	2.06

For $x \geq 3$, ϕ'/ϕ is zero to two decimal places and the table would show $h = x$ to this degree of accuracy. As $x \to \infty$, ϕ'/ϕ tends to zero and h behaves like x, so no difficulty arises. On the other hand, ϕ' and ϕ both tend to zero as $x \to -\infty$ and indeed they are both of order 10^{-6} at $x = -5$. To discuss the behaviour of h when x is less than -5, we can use the asymptotic expansion (as $x \to -\infty$) $\phi(x) \sim \exp(-x^2/2) \times (-u + u^3 - 3u^5 + 15u^7 - 3.5.7\,u^9 + \ldots.)$, where $u = 1/x$ and $x < 0$. This gives

$$\phi'/\phi = -x\,\{1 + u^2 - 2u^4 + 10u^6 - 74u^8 + O(u^{10})\} \quad ,$$

$$h = -u\,\{1 - 2u^2 + 10u^4 - 74u^6 + O(u^8)\} \quad ,$$

$$h' = u^2\,\{1 - 6u^2 + 50u^4 - 518u^6 + O(u^8)\} \quad ,$$

and thus h behaves like $-1/x$, h' behaves like $1/x^2$, as $x \to -\infty$. In a sense, these results merely confirm the theory but they give a better picture of the behaviour of h as x varies.

REAL FUNCTIONS

Marta Sved

I have come across a neat little problem in the Hungarian school journal. Find a real-valued function $f(x)$ defined for all real x such that each value in the range of $f(x)$ occurs exactly twice.

There are of course many solutions, not trivial on first sight. One solution is

$$f(x) = (2x - 2[x] - 1)^2 + [x]$$

where $[x]$ means the integer part of x.

Arising out of this is the following conjecture.

Suppose that there is a real $f(x)$ for all real x, and an integer $k > 1$, and each value $f(x)$ in the range of f occurs exactly k times. Then the function has infinitely many discontinuities.

QUOTATION CORNER (15)

Computer manuals were now so simple that the average person could understand them, he said.

- Report in the Weekend Australian (15 Jan 1983) on comments by Mr. Kevin Fitzgerald, of the Computer Abuse Research Bureau in Melbourne, about the prevalence of computer fraud.

A CIRCLE AND A TRIANGLE

(JCMN 24, p. 141 & 25, p. 3009)

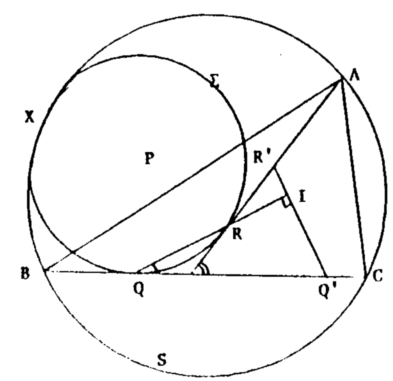
S.N. Collings

In JCMN 24, Clive Davis stated a result concerning a circle touching two sides of a triangle and also its circumcircle. This has now been extended using a trigonometric lemma.

Lemma. ABCD is a quadrilateral with Q a variable point on AB. CQ makes angle θ with AB, and a line m through D makes angle 2θ. If m meets CQ at R, the perpendiculars at Q to AB and at R to m meet on a fixed parabola.

This provides another proof of Davis's original property, and also leads to the following results.

1. ABC is a triangle, in-centre I, having circumcircle S. Σ is a circle touching S, touching BC at Q, and having AR as a tangent from A as shown. Then QR passes through I.



2. If $IQ' \perp IQ$, the circle (centre P') touching BC at Q' and AR at R' touches S also.

3. P, I, P' are collinear. Also XX' passes through a fixed point.

4. ΔABC has mid-pt. triangle $A'B'C'$, pedal triangle DEF and intriangle XYZ (where the incircle touches). Then YZ contains the incentre of $\Delta FB'E$ or of $\Delta C'B'E$; also of $\Delta B'C'F$ or of $\Delta EC'F$ & etc.

TRIANGLES FROM CENTRES OUT (JCMN 30, p. 3127)

S.N. Collings

An alternative proof of Theorem 1 (p. 3128) is possible as follows. Let O, G, N, H be circumcentre, centroid, nine-point centre and orthocentre respectively. They are all on the Euler axis, in the given order, and $OH = 2ON = 3OG$ (p. 3133). Write I for the incentre and use $OI^2 = R(R - 2r)$ (equation (4) on p. 3128) and Feuerbach's theorem (the nine-point circle touches the incircle). Then:

$$NI = \text{distance between centres} = \text{difference of radii} = R/2 - r$$
$$= OI^2/(2R)$$

The incircle is inside the circumcircle and so $OI < R$ and $NI < OI/2$. The locus of points X for which $2NX < OX$ is the interior of the circle where $2NX = OX$, which is the circle on GH as diameter. This gives Guinand's Theorem 1 - For all non-degenerate, non-equilateral triangles the incentre I lies inside the circle on diameter GH.

The equation $NI = OI^2/(2R)$ above may also be obtained by the methods of "Geometry by Numbers", page 3133. Take u, v and w, all of unit modulus, such that the vertices of the triangle ABC are u^2, v^2 and w^2, and the incentre is $-uv - vw - wu$. Then it is easy to verify that $NI = |u + v + w|^2/2$ and $OI = |u + v + w|$.

The problem of constructing the triangle ABC from the given positions for O, I and N is essentially that of Problem EFG28 in my Puzzle Corner in the Bulletin of the Institute of Mathematics and its Applications, 1981, page 125. EFG28 was as follows:-

O, I, J, H are points in a plane with I being the midpoint of OJ. If $OI > JH$, show that there exists a unique triangle having O as circumcentre, H as orthocentre and I as incentre.

REAL AND COMPLEX MATRICES

H. Kestelman

Let M be a square symmetric matrix with complex elements. Show that there exists a non-zero column matrix $\underline{v}$ and a real number λ such that $M\underline{v} = \lambda\bar{\underline{v}}$. Show also that the condition of symmetry cannot be omitted from this theorem.

NEWS FOR ALL PAST CONTRIBUTORS

In response to an enquiry from Riga, the Editor, on your behalf, has given permission for the translation into Latvian and for the publishing there of the first two volumes of JCMN (Issues 1-24). It is not yet known whether the project will go ahead; the appropriate authorities in Latvia are considering the recommendation.

BOUND VOLUMES

Reprints of earlier issues are available for sale, bound as paper-back volumes, Volume 1 (Issues 1-17) for \$10 and Volume 2 (Issues 18-24) for \$5. These prices are in Australian currency and include sea-mail postage. We plan to print Volume 3 (Issues 27-31) similarly.

FINAL EDITORIAL

This issue concludes the publication of the James Cook Mathematical Notes by the James Cook University of North Queensland. On behalf of all readers we record our thanks for the costs of printing and postage provided from the funds of the Mathematics Department.

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