Problems Sessions

① $X = \mathbb{F}_1 \xrightarrow{\pi:\ \mathrm{Bl}_{pt}\, p \in \mathbb{P}^2} \mathbb{P}^2$

$E \longmapsto p$

Ques: Largest $\frac{p}{q}$, $p > 0$

$D \in \left| n\left(\pi^*(-K) - \frac{p}{q} E\right) \right| \neq \emptyset.$

D has (X, D) K-ss.

Note: $p < q$

(Jesus idea: $(X, (1-\beta)D)$ $\quad D \in |-K|$

$\alpha(X, (1-\beta)D) = \sup\{ t \mid (X, (1-\beta)D + t\beta D') \text{ lc } \quad D' \in |-K| \}.$

② 105 Fano 3folds $\supset$ ① General element is K-st

② $\dim(\text{Moduli}) > 1$.

$\|$

1.

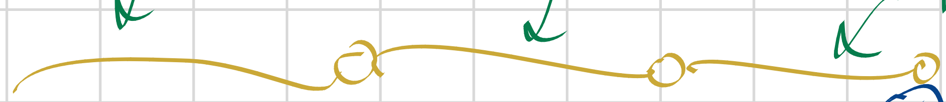
Find a compactification $\overline{M}^{GIT}$

$\uparrow$

$\overline{M}^{K}$

If $\dim = 1$, then $\overline{M}^{GIT} = \mathbb{P}^1$ or $\mathbb{P}^1 \setminus \{pts\}$

K-st K-st K-st



Method 2: Find the K-ps of these explicitly

Ques:

If K-st. $\Rightarrow$ it shows

Method 1: Find the GIT st. of the memb & get a map $\overline{M}^{GIT} \longrightarrow \overline{M}^{K}$

(3) Spotti - MG - :

$X_d \subseteq \mathbb{P}^n$ List of all known GIT compactifications.

Ques: $\Rightarrow$ What sing can it have if GIT stable?

(2) Eg: Family 2.22.

$$D_{1,2} \subseteq \mathbb{P}^2_{xyz} \times \mathbb{P}^2_{uvw} \xrightarrow{pr_1} \mathbb{P}^2$$

Trivial conic = $\mathbb{P}^1$-bundle

Real conic bundle $\downarrow pr_2$ $\mathbb{P}^2$

$$X_\mu = \left\{ x u^2 + y v^2 + z w^2 + \mu(xvw + yuw + zuv) = 0 \right\}$$

$\mu \in \mathbb{C}$; $\mu^3 \neq -1$.

X_μ : K-ps $\forall \mu$.

$\mu^3 = -1$ Gorenstein canonical sing.

Prediction: K-ps.

$$Y_1 = \left\{ (u^2 + t vw) x + (t uw + v^2) y + w^2 = 0 \right\}$$ Y_1 : K-strictly ss.

't': shows degeneration to X_μ.

$t \neq 0 : Y_1$ $t = 0$ Y_1

$$Y_2 = \{(u^2 + t\,vw)x + v^2 y + w^2 z = 0\}.$$

Y_2 : K-strictly ss.

Ques: $G \curvearrowright M_\mu \simeq \mathbb{P}^1$.

Can you find a group $G \curvearrowright M_\mu$ s.t you know when 2 elements in the family are isomorphic?

Gives the K-moduli?

(A) $X_d \subseteq \mathbb{P}^n$

$\mathcal{H} = \mathbb{P}(H^0(\mathbb{P}^n, \mathcal{O}(d))) \curvearrowleft G = PGL(n+1)$.

Consider GIT quotient: $\mathcal{H}/G$. Describe this.

Check if $0 \in \mathrm{Int}$ of the convex hull to know stability.

Eg: $X_{2a} \subset \mathbb{P}(1,1,a,a)$ (Petracci-Liu)

Gives complete answer of K-moduli.

When $\left.\begin{array}{l} a = 2 \\ a = 4 \end{array}\right\}$ General element smoothable but not smooth.

Ques: Compare Non Red GIT moduli & Red. GIT moduli of X_{2a}.

(5) $G \curvearrowright \mathbb{P}^1$ Popov Appendix (Computation Invariant Theory).

Non-reductive conjecture.

(6) Quartic 3-folds w/ terminal / canonical sing.

Which ones are K-ps?

" " GIT stab.?

$$X \xrightarrow{2:1} Q \quad \text{Ramified in } C$$

$$C = \underset{\text{quartic hyp}}{S_4} \cap \underset{\text{quadric}}{Q} \subset Q \subseteq \mathbb{P}^4$$

quartic 3 folds

hypersurfaces

$$X_{4,2} \hookrightarrow \mathbb{P}(1^5, 2) \quad x_5$$

- Generic case is when $x_5 = 0$

Ques: K-st of all quartic 3-folds?

(7) Curve of genus $g >> 0$;
Moduli of VB $\Rightarrow$ Fano
Is it K-stable?
} Andrea.

(Int of 2 quadrics is known.)

(8) Unstable Fano Eg: $\mathbb{F}_1 \times \mathbb{P}^1$.

Conjecture: If $\mathrm{Pic}(X) = \mathbb{Z}$, X terminal $\mathbb{Q}$-fact.

If X is Bir. Super Rigid $\Rightarrow$ K-stable.

$\Downarrow$

Very Non-Rational.

For smooth Fano 3-folds, If irrational $\Longrightarrow$ K-stable.

$$Y \subset X_d \subseteq \mathbb{P}^n$$

$Y = X_d \cap \mathcal{O}(a) \cap \mathcal{O}(b)$ with $a \leq b$

(Non-rationality of these X_d is well studied)

$$\begin{array}{c} \tilde{X} \\ \pi : Bl_Y \downarrow \\ Y \subset X_d \end{array}$$

Ques: Compute $\beta(\pi^*(\mathcal{O}(a)))$. (Many times $\beta < 0$).

(9) Can we find more examples of K-unstable Fanos with $\mathrm{Pic}\, X = \mathbb{Z}[-K_X]$.

$Gr(2,5)$: K-ps

$Gr(2,5) \cap H_1$: K-unstable

$Gr(2,5) \cap H_1 \cap H_2$: "

Ex. of K-unst. Fanos w/ Pic $= \mathbb{Z}$.

$Gr(2,5) \cap H_1 \cap H_2 \cap H_3$: K-ps.

(10) Anna-Maria Cast. counter example - Compute β.
(Vanya)