$X_t: \quad \cancel{4}x_1^4+\dots+x_6^4 - t(x_1^2+\dots+x_6^2)^2=0$

$X_{1/2}$ Burkh, $X_{1/4}$ Igusa $(X_\infty)$ ; Aut $=S_6$ кроме $X_{1/2}$

$\Sigma_{30}: S_6\cdot(1\,\omega\,\omega^2\,1\,\omega\,\omega^2)$

$$\operatorname{Sing} X_t = \begin{cases} \Sigma_{30}\cup\Sigma_{15} \ (\Sigma_{15}) & t=\frac{1}{2} \ (t=\frac14) \quad x_1=x_2,\ x_3=x_4,\ x_5=x_6 \quad (1\ -1\ 0\ 0\ 0\ 0) \\ \Sigma_{30}\cup\Sigma_{10} & t=\frac16 \quad (1\ 1\ 1\ -1\ -1\ -1) \\ \Sigma_{30}\cup\Sigma_{6} & t=\frac{7}{10} \quad (-5:1:1:1:1:1) \\ \Sigma_{30} & \text{otherwise} \end{cases}$$

$\operatorname{Sing} X_{1/4} = C\mathcal{R} \cong (15_3)$

Beauville: $t \neq \frac12, \frac14, \frac16, \frac{7}{10}, \infty \Rightarrow X_t$ non-rational

Todd: $X_{1/2}$ rational (and determinantal!)

Folklore: $X_{1/4}$ rational (dual to Segre)

Todd': $X_{\frac{7}{10}}, X_{1/6}$ rational

$S_6: W_5 \qquad W_5\otimes \mathrm{sgn} \oplus I = \mathrm{Perm}$

$SU_4 \xrightarrow{2:1} SO_6 \Rightarrow 2.S_6: \mathcal{U}_4 \cong \mathbb{C}^4$

Observations: $\mathcal{U}_4$ irr., $\mathcal{U}_4|_{\mathfrak{A}_5^{st}} = 2+2$, $\mathcal{U}_4|_{\mathfrak{A}_4^{nst}} = 2+2$

$\mathfrak{A}_6$ irr $\equiv \qquad S_5^{nst}$-irr. $(5, 5')$

$\exists\ \mathfrak{A}_6$-eq. $\mathbb{P}^3 \dashrightarrow X_{\frac{7}{10}}$ contr.: lines int. 4 lines, tw. cubics int. 6 twice

($\mathfrak{A}_5^{nst}$-inv, cf. $SL_2(\mathbb{C})$)

$S_5^{nst}$-eq. $\mathbb{P}^3 \dashrightarrow X_{\frac16}$ contr.: $\langle L_i, L_j'\rangle \cap \langle L_i', L_j\rangle$, $(-2)$-curves over $L_i, L_i'$

$\mathfrak{A}_6$: symmetric link $\qquad S_5$: ?

Coble 4-fold

$\mathbb{P}(2, 1^6)$ $\quad 0 = x_1 + \dots + x_6 = x_0^2 - F_{4}$

Особо вуаль $C.R.$

$\mathrm{Aut} \cong S_6 \times \mathbb{Z}/2\mathbb{Z}$

$$(\sigma, \varepsilon) \cdot (x_0, \dots, x_6) = (\varepsilon \cdot \mathrm{sgn}(\sigma) x_0, x_{\sigma(1)}, \dots, x_{\sigma(6)})$$

Th.: 1. $\rho_{5,1} : \mathbb{P}_S(\mathcal{U}_3) \longrightarrow Y$ — $S_5^{\text{нет}}$-экв. малое разр. особ.

2. $\rho_{4,2} : Bl_{p_0, p_1, p_2, p_3}(\mathbb{P}^2 \times \mathbb{P}^2) \longrightarrow Y$ — $(S_4 \times S_2)^{\text{нет}}$-экв. . .

3. [illegible]

$$\begin{tikzcd}
\mathbb{P}_S(\mathcal{U}_3) \arrow[rr, dashed, "\text{flops}"] \arrow[dr, "\rho_{5,1}"'] \arrow[dd] & & Bl \arrow[dl, "\rho_{4,2}"] \arrow[dd] \\
& Y & \\
S \arrow[rr] & & \mathbb{P}^2
\end{tikzcd}$$

$S_4^{\text{нет}}$-eq.

$$t = \frac{\tau^2 + 1}{4}$$

$$s = \frac{\tau^3 - \tau}{5\tau^2 + 3}$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\tau$ | $0$ | $1$ | $-1$ | $\pm\frac{1}{\sqrt{5}}$ | $\mp\frac{3}{\sqrt{5}}$ | $\pm\frac{1}{\sqrt{-3}}$ | $\pm\frac{3}{\sqrt{-3}}$ | $\infty$ | $\pm\sqrt{-\frac{3}{5}}$ |
| $s$ | $0$ | | | $\mp\frac{1}{5\sqrt{5}}$ | | $\mp\frac{1}{\sqrt{-3}}$ | | $\infty$ | |
| $t$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{3}{10}$ | $\frac{7}{10}$ | $\frac{1}{6}$ | $-\frac{1}{2}$ | $\infty$ | $\frac{1}{10}$ |