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**MR1697595 (2000b:01029)**
Monk, David
Obituary: Professor W. L. Edge: 1904–1997.
*Proc. Edinburgh Math. Soc. (2)* 41 (1998), no. 3,
631–641.
01A70

| Citations |
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| From References: 0 |
| From Reviews: 0 |

W. L. Edge (1904–1997) was a lecturer at the University of Edinburgh from 1932 until his retirement in 1975, and was associated with some of the most famous mathematicians of his time, including E. T. Whittaker, H. W. Turnbull, and H. S. M. Coxeter. Just two years into his career at Edinburgh he was elected a Fellow of the Royal Society of Edinburgh. He carried out extensive research on the classification of surfaces, and this work is nicely summarized in the present obituary. Most amazing of all is Edge's continuing productivity, even with failing eyesight, into his 90th year. Between 1932 and 1994 he produced 93 research papers, all, according to the author, written in a visual geometric style.
Reviewed by R. L. Cooke

**MR1298589 (95j:14075)**
Edge, W. L.
$28$ real bitangents. (English summary)
*Proc. Roy. Soc. Edinburgh Sect. A* 124 (1994), no. 4,
729–736.
14P05 (14H99 14N10)

| Citations |
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| From References: 1 |
| From Reviews: 0 |

It is classically known that the number of real bitangent lines of a smooth real plane quartic is related to the number of ovals of the quartic and their nesting (for a modern treatment, see B. H. Gross and J. D. Harris [Ann. Sci. École Norm. Sup. (4) 14 (1981), no. 2, 157--182; MR0631748 (83a:14028)]). In particular it is possible to have 28 real bitangent lines if and only if the plane quartic consists of four mutually external ovals. Furthermore Plücker discovered that it can be the case that such a quartic has 28 real bitangent lines with their 56 contact points also real. The present note is devoted to presenting an explicit example in which this happens and the equations of the 28 bitangent lines can be easily calculated.

In more generality, one can consider the analogous problem for a real canonical curve of genus $g \geq 3$. First of all there is the question of computing the number of odd real theta-characteristic on a real algebraic curve and of studying their configuration. These questions were classically considered in their generality by A. Comessatti in a series of papers in the 1920's. His results have been rediscovered in recent times by Gross and Harris (see the reference above). In this setting one may specifically ask if there is a real canonical curve of genus $g \geq 3$ with $g+1$ ovals, possessing $2^{g-1}(2^g-1)$ hyperplanes each tangent at $g-1$ distinct points of the curves, each of which is real. The present note can be seen as a resolution of this problem for the first case, $g=3$.
Reviewed by Ciro Ciliberto

**MR1169251 (93e:01021)**
Edge, W. L.
Obituary: Robert Schlapp, M.A., Ph.D., F.R.S.E.

| Citations |
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| From References: 0 |

(1899–1991).
Proc. Edinburgh Math. Soc. (2) 35 (1992), no. 2, 329–334.
01A70

From Reviews: 0

This obituary contains a biography (and a photo) of R. Schlapp, who spent most of his professional life at Edinburgh University and who continued many of his various activities well beyond his retirement in 1969.

**MR1121664 (93b:14052)**
Edge, W. L.
A plane sextic and its five cusps.
Proc. Roy. Soc. Edinburgh Sect. A 118 (1991), no. 3-4, 209–223.
14H45 (14H20)

**Citations**
From References: 1
From Reviews: 0

G. Humbert [J. École Polytech. 64 (1894), 123--149; Jbuch 25, 1252] discovered a plane sextic curve $H$ of genus $5$, and having five cusps for its singular points. These five cusps and $H$ have pretty geometrical properties: the two residual intersections of $H$ with the conic $\Gamma$ through the cusps coalesce at a single contact $T$; all five cuspidal tangents pass through $T$; the two residual intersections of $H$ with the join of a pair of cusps coalesce at a single contact; the contacts with joins of any disjoint pairs of cusps are collinear with the fifth cusp. Let $\mathrm{C}$ be the special canonical curve of order 8 and genus 5 in projective $4$-space that is the intersection of three quadrics with a common self-polar simplex $\Sigma$. The author [Proc. Cambridge Philos. Soc. 47 (1951), 483--495; MR0042154 (13,62c)] showed that $H$ is the projection of $\mathrm{C}$ from any one of its tangents that passes through a vertex of $\Sigma$, and that this projection accounts for the geometric properties of the cusps listed above. Here he shows that $H$ and all these cuspidal properties can be displayed in the Euclidean plane. Computer-drawn pictures of a particular $H$ are given: $H$ is quadripartite, consisting of a helmet, a pear, and pair of sausages! Humbert showed that any pentad on $\Gamma$ can serve for the cusps and the position of $T$ on $\Gamma$ can be freely chosen also. The author takes $\Gamma$ to be the ellipse $x^2+3y^2=4$ , the cusps to be the vertices of the inscribed square of $\Gamma$ together with one end of the major axis, and $T$ to be the other end of the major axis. With this choice all algebraic computations are neat and expeditious. $H$ is produced as one member of the pencil defined by two composite sextic curves, namely $(\alpha)\,\Gamma$ , the sides of the square parallel to the minor axis and the repeated tangent at the fifth cusp, and $(\beta)$ the other two sides and the two diagonals of the square and the repeated major axis. Another particular sextic of the pencil has two tacnodes and is also drawn. The generation of $H$ by a $(2,2)$ correspondence between the lines of the pencil through one of its cusps and the conics through the other four is discussed, and so are the adjoint cubics of $H$. Unlike a general plane curve of genus $5$ which has 496 contact adjoints, $H$ has $10$ singly infinite systems of contact adjoints.
Reviewed by R. H. Dye

**MR1007528 (91b:14045)**
Edge, W. L.
The quartic projections of Castelnuovo's normal surface with hyperelliptic prime sections.
Proc. Roy. Soc. Edinburgh Sect. A 111 (1989), no. 3-4, 315–324.
14J25 (14N05)

**Citations**
From References: 0
From Reviews: 0

A century ago Castelnuovo proved that a nonruled surface whose prime sections have genus 2 is the projection of a nonsingular rational surface of order 12 in

projective 11-space. One of his two parent duodecimics is the surface $\Phi$ that is represented in a plane $\pi$ by quintics with a triple point $X$ and a double point $Y$, say $C^5(X^3, Y^2)$. The motivation of the present paper is to explicitly examine one such projection in action; namely for the quartic surface $F$ in 3-space $\Sigma$ that has a double line $\lambda$. Full algebraic details are worked out for the case when $\lambda$ is a cuspidal line on $F$. $\Phi$ contains a pencil of conics $\gamma$ and a pencil of cubics $\delta$, mapped, respectively, by the lines of $\pi$ through $X$ and those through $Y$. To project $\Phi$ to $F$ one must take as vertex of projection $V$ a 7-space spanned by eight points $A_1, A_2, \cdots, A_8$ on $\Phi$. $V$ must not contain a tangent plane to $\Phi$ nor contain infinitely many points of $\Phi$. There is in $\pi$ a unique quartic $C^4(X^2 Y^2)$ containing the map $a_j$ of the $A_j$. There corresponds on $\Phi$ an elliptic decimic curve $q$ bisecant to every $\gamma$ and $\delta$, and forming a prime section of $\Phi$ with any $\gamma$. This $q$ lies in a 9-space through $V$ which meets a solid $\Sigma$ skew to $V$ in a line $\lambda$. The projection from $V$ of $\Phi$ is the required quartic surface $F$ with $\lambda$ for double line, and having four pinch points on $\lambda$. Details of this mapping are nicely explored: for example, corresponding to the 56 different partitions of the octad $A_1, \cdots, A_8$ into a triad and a pentad there arise 56 planes meeting $F$ in pairs of conics. A special case occurs when $C^4(X^2, Y^2)$ consists of a repeated conic $C^2(X, Y)$. Then $q$ would consist of a repeated quintic $\chi$. Using the explicit parametrisation of $\Phi$ and a particular $C^2(X, Y)$ he constructs a $V$ such that the corresponding $\lambda$ is cuspidal on $F$. His $V$ is spanned by the tangents to four $\gamma$ at their points of intersection with $\chi$. The algebra produces an equation for $F$, and unravels the geometry of the projection.
Reviewed by R. H. Dye

**MR0918894 (89f:14058)**

| Citations |
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| From References: 0 |
| From Reviews: 0 |

Edge, W. L.
Geometry related to the key del Pezzo surface and the associated mapping of plane cubics.
*Proc. Roy. Soc. Edinburgh Sect. A* 107 (1987), no. 1-2, 75–86.
14N05 (14J26)

By taking their 10 coefficients as coordinates, plane cubic curves may be represented by the points of a projective 9-space $\Sigma$. Those cubics consisting of a single line counted thrice are represented by a surface $F$ of order 9; $F$ is the parent del Pezzo surface. This mapping is the analogue of the well-explored representation of the conics in a plane by the points of 5-space, where the repeated lines correspond to the Veronese surface $V$. The author discovers some elegant geometry related to $F$ and compares and contrasts this with the familiar results for $V$. His key tools are the tangent planes $\Omega_2$ and the second osculating spaces $\Omega_5$ to $F$: at a point $P$ on $F$ the osculating planes to all curves on $F$ through $P$ lie in the osculating space $\Omega_5(P)$. The author had earlier shown by elementary methods [J. London Math. Soc. (2) 27 (1983), no. 3, 402--412; MR0697133 (85b:14070)] that the 4-fold $T_4$ generated by the $\Omega_2$ has order 24. A similar discussion shows that the 7-fold $U_7$ generated by the $\Omega_5$ has order 21. Two 5-subspaces of $\Sigma$ generally meet in a line. Here the striking fact is that any two $\Omega_5$ meet in a plane: compare this with the fact that any two tangent planes of $V$ meet in a point. Moreover any three $\Omega_5$ have a common point. Such a point represents a cubic consisting of three sides of a triangle, and the locus of these points is a 6-fold $\Delta_6$ which is a triple submanifold of $U_7$. To the cubics consisting of the triply-repeated lines of a pencil correspond the points of a twisted cubic $\gamma$ on $F$, lying in a solid $[\gamma]$. The common point of the $\Omega_5$ at three points on such a $\gamma$ is that common to the osculating planes to $\gamma$ at these points. Through it passes a unique chord of $\gamma$. Almost instantly it follows that the 5-fold $C_5$ generated by the chords of $F$ is generated by the solids containing the twisted cubics on $F$ and has order 15, $C_5$ is a submanifold of $\Delta_6$ corresponding to degenerate triangles with concurrent sides. The $\Omega_5$ at the points of the same $\gamma$ all lie in the same hyperplane $[\Gamma]$, which meets $F$ in $\gamma$ counted thrice. The points of $[\Gamma]$ map cubics with a double point. The primal $N_8$ generated by these $[\Gamma]$ has order 12. The simplex of reference in $\Sigma$ can be expeditiously employed to yield such results as: the intersections of a fixed $\Omega_5$ with all the $\Omega_2$ trace a Veronese surface $V$, and the planes in which the other $\Omega_5$ meet the fixed $\Omega_5$ are the tangent planes of $V$. And there is more. For example, the trisecant planes of $F$ generate a quartic primal $S_8^4$. The geometry of this provides an elucidation of an old result of classical invariant theory about ternary cubics that are the sum of three linear cubes. The author's arguments have not, so far, yielded the order of $\Delta_6$. \n J. Yerushalmy\en [Amer. J. Math. 54 (1932), 129--144; Jbuch 58, 696] gave an argument to show that this order is 15, using an enumerative formula of Caporali. The author became aware of Yerushalmy's discussion of $F$ after publication of his paper. The two treatments are very different: Yerushalmy mentions the $\Omega_2$ rather late, and Zariski seems to have asked some questions about the conclusiveness of a proof.

Reviewed by R. H. Dye

**MR0892700 (88k:14033)**

Edge, W. L.

Geometry relevant to the binary quintic.

*Proc. Edinburgh Math. Soc. (2)* 30 (1987), no. 2, 311–321.

14N05 (14H45)

| Citations |
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| From References: 0 |
| From Reviews: 0 |

Let $F$ be a binary quintic form. The symbol ${}_dC_n$ denotes a covariant of order $n$ in the binary variables and degree $d$ in the coefficients of $F$. The author gave elsewhere a geometrical interpretation of a quadratic covariant ${}_2C_2$ and an invariant ${}_4C_0$ [same journal (2) 29 (1986), no. 1, 133--140; MR0829189 (87c:14067)]. Now he shows that the same geometrical setting quickly trawls many other interesting covariants. By taking its coefficients, $F$ is represented by a point $P$ of projective 5-space. Via this representation the fifth powers of linear forms correspond to the points on a rational normal quintic curve $C$. The osculating solids of $C$ generate an octavic primal $\Omega_4^8$: its points map those $F$ having a repeated zero. Thus, if, in the form for $\Omega_4^8$, the coordinates of $P$ are substituted, one obtains the discriminant ${}_0C_8 \equiv I_8$ of $F$. The tangent lines to $C$ generate a surface $\Omega_2^8$. The author's earlier paper [op. cit.] established that $\Omega_2^8$ was the base surface of a net $N$ of quadrics, and that through $P$ pass two point-cones of $N$ whose vertices are two points $A$, $B$ on $C$: they correspond to the zeros of ${}_2C_2$. However, there is a unique point $J$ that is the residual intersect of $C$ with the 4-subspace spanned by $P$ and the tangent lines to $C$ at $A$ and $B$. To $J$ corresponds a linear covariant ${}_5C_1$ of $F$. The harmonic conjugate $J'$ of $J$ on $C$ with respect to $A$ and $B$ immediately produces another, a ${}_7C_1$ (the Jacobian of ${}_2C_2$ and ${}_5C_1$). Through a general $P$ passes a unique trisecant plane of $C$: it maps a covariant $j \equiv {}_3C_3$ . This is the covariant whose three linear factors provide the three summands when $F$ is expressed as the sum of three fifth powers. The discriminant of $j$ is an invariant ${}_{12}C_0 \equiv I_{12}$ . The locus $\pi$ of those $P$ satisfying $I_{12} = 0$ has various geometric descriptions. It is generated by the cubic line-cones projecting $C$ from its own tangents; it is generated by the sextic point-cones projecting $\Omega_2^8$ from the points of $C$; it consists of the $P$ such that two of the three intersections of $C$ with its trisecant plane through $P$ coalesce; it consists of the $P$ such that two of the four bitangent solids of $C$ through $P$ coalesce. Other covariants cannot be prevented from emerging: for example, the quadratic mapping the pair of points harmonic on $C$ with respect to both $A$, $B$ and $J$, $J'$; the skew invariant ${}_0C_{18} \equiv I_{18}$ whose vanishing corresponds to those $P$ that lie in the osculating prime at one of the three intersections of $C$ with its trisecant plane through $P$. Not only does the geometry readily produce covariants, it produces simple and readily written down algebraic forms for them in terms of the coefficients of $F$. \n A. Cayley\en [Collected mathematical papers, Vol. II, see pp. 282--309, Cambridge Univ. Press, Cambridge, 1889] had produced algebraically the covariants, but his algebraic expressions for them had many terms: that for $I_{12}$ had 252! In contrast, $I_{12}$ is given here as a very simple quartic in four quantities which are themselves simple $3 \times 3$ determinants, each of whose entries is a coefficient of $F$.

Reviewed by R. H. Dye

**MR0829189 (87c:14067)**

| Citations |
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| From References: 0 |
| From Reviews: 1 |

Edge, W. L.

An aspect of the invariant of degree 4 of the binary quintic.

*Proc. Edinburgh Math. Soc. (2)* 29 (1986), no. 1, 133–140.

14N05 (14H45)

Let $f$ be a binary form of odd degree $n = 2m + 1$ . Classical invariant theory produces a quadratic covariant $\Gamma$ of $f$, and the discriminant of $\Gamma$ is an invariant $\Delta$ of $f$. In this paper the author gives a geometric interpretation of $\Gamma$ and $\Delta$. In doing so he discloses some pretty geometry of the rational normal curve. The features of the general case are amply demonstrated in the case $m = 2$ , $n = 5$ ; the paper concentrates on this case. Via its coefficients a binary quintic can be represented as a point of projective 5-space. To the forms that are perfect 5th powers of linear forms correspond the points of a rational normal quintic curve $C$. It is known that the tangent lines to $C$ generate a scroll $\Omega_2^8$. By the representation $C$ comes equipped with its simple canonical parametric form: it is thus easy to give two points which span a tangent line. The author uses these and some expeditious algebra to prove that the quadrics containing $\Omega_2^8$ are those of a net $N$, whose explicit equation is found. From the matrix of a general member of $N$ it is easily seen that the singular members of $N$ are all point cones having their vertices on $C$. Through a point $P$ pass two of these cones: if $P$ corresponds to $f$ their vertices have for parameters on $C$ the zeros of $\Gamma$. The envelope $K$ of the family of cones is a quartic primal. Its points correspond to the binary quintics having $\Delta = 0$ . $\Omega_2^8$ is a double surface on $K$; $K$ also contains the 3-fold $\Omega_3^9$ generated by the osculating planes to $C$. More can be said. The chords of $C$ generate a sextic 3-fold $M_3^6$ which meets $K$ in $\Omega_2^8$ counted thrice. Each quadric of $N$ contains two osculating planes which coincide if it is a cone. The natural generalisation concerns the net of quadrics in $(2m+1)$ -space containing the osculating $(m-1)$ -spaces of the rational $(2m+1)$ -ic curve. Now in his ninth decade, the author displays his customary mastery.

Reviewed by R. H. Dye

**MR0812999 (87d:51013)**

| Citations |
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| From References: 0 |
| From Reviews: 0 |

Edge, W. L.

$\mathrm{PGL}(2,11)$ and $\mathrm{PSL}(2,11)$ .

*J. Algebra* 97 (1985), no. 2, 492–504.

51E15 (05B25 20B25 20G40)

The author takes up a theme from an earlier paper [the author, Canad. J. Math. 8 (1956), 362--382; MR0080311 (18,227d)] to give an elegant exposition of the largest order case in which $\mathrm{PSL}(2,q)$ has a transitive permutation representation of order $q$. On $\mathrm{PG}(1,11)$ it is shown that there are 22 ways of partitioning the line into 6 pairs, every two of which have a cross-ratio of 2 or $\frac{1}{2}$; that is, no two pairs are harmonic conjugates, but they are the union of harmonic conjugate pairs. The 22

hexads of pairs fall into two sets of 11; each set of 11 provides a representation for either group of the title. The geometry is more interesting when a conic $P_2$ is considered instead of $PG(1,11)$. Then a hexad defines a hexagram in the plane which has the Pascal property in 10 ways; that is, there are 10 ways to partition the six sides of the hexagram into three pairs such that the intersections of the pairs are collinear.
Reviewed by J. W. P. Hirschfeld

**MR0745911 (85m:20067)**
Edge, W. L.
$PGL(2,7)$ and $PSL(2,7)$.
*Mitt. Math. Sem. Giessen* No. 164 (1984), 137–150.
20G40 (51A20 51E30)

| Citations |
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| From References: 1 |
| From Reviews: 0 |

The author aims to shed further light on some known peculiar features of the groups $PSL(2,7)$ and $PGL(2,7)$, by looking at geometric configurations imposed on the projective line $PG(1,7)$ and the projective plane $PG(2,7)$, respectively. In particular, the well-known permutation representation of degree 7 of $PSL(2,7)$ is exhibited (the seven objects being certain "quartets of duads'' in $PG(1,7)$), and the isomorphism between $PGL(2,7)$ and $SO(3,7)$ is illustrated by a very detailed study of the geometry of the 8-point conic $x^2+y^2+z^2=0$ in $PG(2,7)$ (examined by the author in an earlier paper [Canad. J. Math. 8 (1956), 362--382; MR0080311 (18,227d)]). Several nice remarks can be viewed as a clarification and a commentary on a paper of H. S. M. Coxeter [Proc. London Math. Soc. (3) 46 (1983), no. 1, 117--136; MR0684825 (84e:05059)], as well as on a few points of J. H. Conway's paper [Finite simple groups (Oxford, 1969), 215--247, Academic Press, London, 1971; MR0338152 (49 #2918)].
Reviewed by Lino Di Martino

**MR0738600 (85m:14069)**
Edge, W. L.
Fricke's octavic curve.
*Proc. Edinburgh Math. Soc. (2)* 27 (1984), no. 1, 91–101.
14N10 (14H45)

| Citations |
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| From References: 0 |
| From Reviews: 0 |

In two preceding papers the author [see J. London Math. Soc. (2) 18 (1978), no. 3, 539--545; MR0518240 (80a:14014); ibid. (2) 23 (1981), no. 2, 215--222; MR0609101 (83h:14023)] studied Bring's sextic curve $B$ in the complex projective space $\mathbf{P}_3$. This $B$ is defined by means of 5 supernumerary coordinates $(x,y,z,t,u)$, satisfying the relation $S_1\equiv x+y+z+t+u=0$ as the intersection $B=Q\cap D$, where $Q{:}S_2\equiv x^2+y^2+z^2+t^2+u^2=0$ and $D{:}S_3\equiv x^3+y^3+z^3+t^3+u^3=0$. In the present paper the author studies Fricke's octavic curve $F$ in $\mathbf{P}_3$, defined as the intersection $F=Q\cap\Phi$ with $\Phi{:}S_4\equiv x^4+y^4+z^4+t^4+u^4=0$. The name of this curve goes back to R. Fricke , who introduced it in a paper of his [Acta Math. 17 (1893), 564--594; Jbuch 25, 726]. The author especially extends Fricke's research by giving some additional properties of the curve $F$.

First, it is seen at once that the curves $B$ and $F$ are both transformed into themselves by the group of 5! projectivities defined by all permutations of $(x,y,z,t,u)$. The coordinates $(x,y,z,t,u)$ in $\mathbf{P}_3$ define a reference pentahedron, called $P$. The faces of this $P$ meet by pairs in 10 edges $e_{ij}$, by threes in 10 vertices $V_{ij}$, where the notation is to be understood as follows: the point $V_{ij}$ $(i\neq j)$ is opposite to that edge common to those 2 faces of $P$ not containing $V_{ij}$; joining $V_{ij}$ with this edge, we get a so-called diagonal plane called $d_{ij}$. The join of 2 vertices whose binary suffixes are disjoint is a diagonal line of $P$; there are 15 of these lines, lying all on the cubic surface $D$.

Each diagonal plane $d_{ij}$ meets $\Phi$ in a pair of conics with double contact. The lines joining the 2 contact points of the conics in $d_{ij}$ are edges of $P$, and it results that all edges of $P$ are bitangents of $\Phi$ and chords of $F$. So the 10 edges of $P$ have 20 intersections with $F$, and these points are called $\Omega$-points. These points appear in pairs, and a plane $d_{ij}$ has a 4-point intersection with $F$ at both points of a certain $\Omega$-pair.

Another kind of important set of points are the 24 points common to $B$ and $F$, called by the author the set $I$. The tangents to $F$ in the points of this set are inflexional.

In Section 4 the author considers a vertex point $V_{ij}$ and its polar plane $\pi_{ij}$ with respect to $Q$. All 8 intersection points of $\pi_{ij}$ with $F$ are so-called stalls, i.e., points with a stationary osculating plane. Owing to the 10 polar planes $\pi_{ij}$, the total number of these stalls for $F$ is 80. Among these are the 20 $\Omega$-points already treated; the other 60 are called $\Sigma$-points. In Section 5 the author treats the 10 quartic cones $q_{ij}$ generated by all those chords of $F$ passing through one of the 10 points $V_{ij}$.

In Sections 7--10 of the paper he studies the tritangent planes of $F$. First, from Section 7 he deduces the existence of 60 tritangent planes whose contacts are all points $\Omega$; there are to be added another 120 tritangent planes of $F$, where only one of the contacts is a stall. $F$ is a curve of genus $p=9$.

In the final Section 11, he demonstrates the following interesting fact for $F$: Every line on $Q$ is cut equiharmonically by $F$.

Reviewed by W. Burau

**MR0733711 (85b:14047)**

Edge, W. L.

Tangent spaces of a normal surface with hyperelliptic sections.

*Canad. J. Math.* 36 (1984), no. 1, 131–143.

14J26 (14M07 14N05)

**Citations**
From References: 0
From Reviews: 0

This paper concerns the geometry of the algebraic surface $\Phi \subset \mathbf{P}^{3p+5}$ of degree $4p+4$ with hyperelliptic sections of genus $p$. $\Phi$ is mapped onto a plane $\pi$ by curves of degree $p+3$ with two fixed multiple base points of multiplicity 2 and $p+1$, so it contains $\infty^1$ conics $\gamma$ and $\infty^1$ rational curves $\delta$ of degree $p+1$. At each point $P$ of $\Phi$ there is a nest of tangent spaces $P \equiv \Omega_0 \subset \Omega_2 \subset \Omega_5 \subset \cdots$, of respective dimensions $0, 2, 5, \cdots, 3p+2, 3p+8$, where the dimension increases by three at each step save the first and last. The author studies the primal $M$ of order $8p+4$ generated by the $\Omega_{3p+2}$'s as $P$ varies over $\Phi$, the singularities and some remarkable subvarieties of $M$. The case $p=2$ is treated with particular care. The study of the singularities of $M$ is made easier since $M$ has a determinantal equation. The planes of the conics $\gamma$ and the solids of the cubics $\delta$ generate two rational varieties of degree 9 and 8, respectively, whose multiplicities for $M$ are 5 and 10; moreover the fivefold of chords, the fourfold of tangent planes $\Omega_2$ and the manifold generated by $\Omega_5$ are multiple subvarieties of $M$, with multiplicities 4, 5 and 3, respectively.

Reviewed by D. Gallarati

**MR0705272 (84j:01063)**

Edge, W. L.

Obituary: F. Bath.

*Proc. Edinburgh Math. Soc. (2)* **26** (1983), no. 2, 279–281.

01A70

**Citations**
From References: 1
From Reviews: 0

Bibliography of seven items (algebraic geometry).

**MR0726667 (86a:14030)**

Coxeter, H. S. M.(3-TRNT); Edge, W. L.

The simple groups $\mathrm{PSL}(2, 7)$ and $\mathrm{PSL}(2, 11)$ .

*C. R. Math. Rep. Acad. Sci. Canada* 5 (1983), no. 5, 201–206.

14H45 (20D06 51M99 51N35)

| Citations |
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| From References: 0 |
| From Reviews: 0 |

The authors begin with the well-known plane quartic curve $C$, first treated by F. Klein and R. Fricke [Theory of elliptic modular functions, I (German), see p. 712, Teubner, Leipzig, 1890; Jbuch 13, 455]. $C$ has genus $p=3$ and is transformed into itself by the collineations of a group isomorphic to the simple group $G_{168}=\mathrm{PSL}(2,7)$ . E. Ciani [Rend. Circ. Mat. Palermo 13 (1899), 347--373; Jbuch 30, 527] gave the equation of $C$ in the form $x^4+y^4+z^4+\bar{c}(y^2z^2+z^2x^2+x^2y^2)=0$ with $c=(-1+i\sqrt{7})/2$ . Now in the plane of $C$ there are 2 sets of 7 conics, each intersecting $C$ in 8 points, which are contact points of 4 bitangents, so that we get twice the total number of 28 bitangents. In the present paper the authors give the equations of the 14 conics based on Ciani's equation. For a better understanding of the whole configuration connected with it the authors draw a graph with 14 points and 21 lines where the points correspond to the 14 conics; this graph coincides with Levi's graph of the Fano plane $\mathrm{PG}(2,2)$ and has already been described in a book by Coxeter [Twelve geometric essays, see p. 118, Southern Illinois Univ. Press, Carbondale, Ill., 1968].

In the second part of the paper another curve $D$ of the complex $\mathbf{P}$ is treated in a similar way. This curve $D$ has order 20, genus 26 and is transformed into itself by the collineations of a group isomorphic to the simple group $G_{660}=\mathrm{PSL}(2,11)$ . The first treatment of this curve and its self-collineations occurred in Klein and Fricke's book [Theory of elliptic modular functions, II (German), see p. 438, Teubner, Leipzig, 1892, Jbuch 24, 412]. The analogy with Klein's plane quartic of genus 3 in $\mathbf{P}_2$ consists in the following facts: $G_{660}$ has 2 conjugate sets of 11 icosahedral subgroups. Each of these subgroups permutes the 40 points of intersection of the curve $D$ with a certain quadric. The authors forego giving the more complicated equations of these quadrics. But for a better interpretation they draw two graphs, one with 22 points representing the 22 quadrics and 55 lines, and the other with 22 points and 66 lines.

Reviewed by W. Burau

**MR0722570**

Edge, W. L.

Obituary: Geoffrey Timms.

*Proc. Edinburgh Math. Soc. (2)* 26 (1983), no. 3, 393–394.

01A70

| Citations |
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| From References: 0 |
| From Reviews: 0 |

{There will be no review of this item.}

**MR0697133 (85b:14070)**

Edge, W. L.

The pairing of del Pezzo quintics.

*J. London Math. Soc. (2)* 27 (1983), no. 3, 402–412.

14N05 (14J26)

| Citations |
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| From References: 0 |
| From Reviews: 1 |

In what follows, the Veronesian $V_2^3$ in $\mathbf{P}_9$ is the surface parametrised by all cubic monomials in three homogeneous variables. Then, by projection of $V_2^3$ from the $\mathbf{P}_3$ spanned by 4 points $A,B,C,D$ in general position, lying on $V_2^3$, we get the del Pezzo

quintic surface $F_2^5$ in $\mathbf{P}_5$. On this $F_2^5$ lie 10 straight lines, labelled 01, 02, 03, 04, 12, 13, 14, 23, 24, 34, where 01, 02, 03, 04 arise by dilatation of the points $A,B,C,D$ on $V_2^3$ and $12, 13, \cdots, 34$ by projection of the 6 rational cubics $V_1^3$ on $V_2^3$ passing through any two of the points $A,B,C,D$. It follows very simply, that each of the 10 lines on $F_2^5$ is intersected by exactly 3 other ones; for example, 01 by 23, 24, 34 and 12 by 03, 04, 34. It is possible to form 12 pentagons from the 10 lines; each of these pentagons $p$ is completed by another $p'$, so that $p$ and $p'$ are the totality of the 10 lines; for example, $p = (02, 13, 24, 34, 14)$ and $p' = (01, 34, 14, 04, 23)$. On the other hand, $F_2^5$ possesses 5 pencils of conics, labelled (0), (1), (2), (3), (4), where the pencils (1), (2), (3), (4) arise by projection from the rational normal curves $V_1^3 \subset V_2^3$, passing through one of the points $A,B,C,D$, and the conics of (0) from the $V_1^6 \subset V_2^3$ containing all 4 points $A,B,C,D$. In the space $P_5$ spanned by $F_2^5$ there exists a linear system $L_{15}$ of quadrics, and $L_{15}$ contains a special pencil $L_1$ of quadrics. The quadrics of $L_1$ intersect $F_2^5$ in curves of a system $P$ of particular importance. The curves of this set $P$ are generally irreducible curves $C_1^{10}$ of order 10 and genus 6. However, special curves of $P$ are the following ones: (1) the 10 straight lines on $F_2^5$, (2) two sets of 5 conics; each set contains one conic in one of the pencils $(0), (1), (2), \cdots, (4)$, and (3) two rational curves, $R$, $R'$, each having 6 nodes. The curves of $P$ have 20 common base points, two of them on each of the 10 lines. $g_1$ may be one of the 10 lines, intersected by 3 other lines of the configuration in the points $G_0, G_0', G_0''$; then the base points of the pencil $P$ lying on $g_1$ are the points $H_0, H_0'$ of the so-called Hessian dyad, determined by $G_0, G_0', G_0''$. Now an essential fact for the geometry of $F_2^5$ is the following: $F_2^5$ is transformed into itself by a group $S$ of 5! autocollineations, isomorphic to the symmetric group of all 5! permutations of 5 symbols. The corresponding collineations of $F_2^5$ effect all 5! permutations of the 5 pencils $(0), (1), \cdots, (4)$ of conics on $F_2^5$. But a generic curve $C_1$ of $P$ is only transformed into itself by the alternating subgroup $S^+ \subseteq S$, and so each curve $C_1 \subset P$ has in general a companion $\bar{C}_1$, such that the set $\{C_1, \bar{C}_1\}$ is transformed into itself by the whole group $S$. Such a property is only possessed in a trivial manner by the reducible curve consisting of the 10 lines and another distinguished curve $W$. In the plane $X_2$ which is the bijective image of $V_2^3$ and therefore birationally equivalent to $F_2^5$, we find another model for $W$, that is, a plane curve $w$ of order 6 with 4 nodes. $w$ is also transformed into itself by a plane group of 5! elements, consisting of collineations and Cremona transformations. This curve $w$ has already been discovered in another way in a classical paper of A. Wiman [Math. Ann. 48 (1896), 195--240; Jbuch 30, 600]. The present paper is connected with two others of the author [Math. Proc. Cambridge Philos. Soc. 89 (1981), no. 3, 413--421; MR0602296 (82f:14053); ibid. 90 (1981), no. 2, 239--249; MR0620733 (82m:14013)], where many of the facts mentioned on $F_2^5$ already have been treated. But essentially new in the present paper is the following discovery: In the above-mentioned pencil $L_1$ of quadrics of $P_5$ there are two cones $K^1$ and $K^2$; these cones have the singular planes $k^1$ and $k^2$, skew to one another, so that the harmonic inversion, defined by $k^1$ and $k^2$, transforms $F_2^5$ into a companion surface ${}'F_2^5$. These surfaces $F$ and ${}'F$ are symmetrically related to one another, and the pencils $P$ on $F$ and $P'$ on ${}'F$ are cut out by the same pencil of quadrics in $\mathbf{P}_5$. The author proves all the facts mentioned by coordinate calculation; but we must omit giving the corresponding formulas here.
Reviewed by W. Burau

**MR0661790 (83j:14037)**

Edge, W. L.

Algebraic surfaces with hyperelliptic sections. *The geometric vein,* pp. 335–344, *Springer, New York-Berlin,*

**Citations**
From References: 0
From Reviews: 0

1981.
14J25

In 1890 [Rend. Circ. Mat. Palermo 4 (1890), 73--88; Jbuch 22, 788] G. Castelnuovo had studied and classified algebraic surfaces with hyperelliptic prime sections. If these sections have genus $p=2$ the first type of them resulted as projections of a normal surface $\Phi$ in $S_{3p+3}$ where $\Phi$ has the order $4p+4$. In the present paper these investigations of Castelnuovo are continued and completed. In a more detailed manner the author treats the case $p=2$; there $\Phi$ is immersed in $S_{11}$ and has order 12. $\Phi$ is generated by projection of the Veronesian $V_2^5 \subset P_{20}$ (in the reviewer's notation $V_2^5$ is the surface-image for the totality of quintic curves in the plane $\pi$). We get $\Phi$ by projection of $V_2^5$ from a center $Z_8 = T_2(P_0)+T_5(Q_0)$ , where $T_2(P_0)$ is the tangent plane of $V_2^5$ in $P_0$ and $T_5(Q_0)$ the first osculation space of $V_2^5$ in $Q_0$. $\Phi$ is doubly fibrated by conic sections $V_1^2$ and cubic normal curves $V_1^3$ and may be considered as the product-variety $V_1^2 \times V_1^3$ in $S_{11}$. An essential property of $\Phi$ is: In the point $\Omega_0 \in \Phi$there exists the following nest of osculating spaces: $\Omega_0 \subset \Omega_2 \subset \Omega_5 \subset \Omega_8 \subset \Omega_{10}$ with the peculiarity: The osculating space of the second step has 8 dimensions instead of 9, as occurs for general surfaces in $S_{11}$. By projection from an osculating space $\Omega_8$ as center we get a mapping of $\Phi$ on a plane $\pi$, the starting point for Castelnuovo's considerations. Another essential fact is the following: one can find a quadruple infinity of splittings: $S_{11} = H_5 + H_5{}'$ , so that we have the facts: (1) $\Phi$ is invariant under the harmonic inversion with $H_5$ and $H_5{}'$ as spaces of fixed points. (2) By the projection of $S_{11}$ from $H_5$ to $H_5{}'$ or likewise from $H_5{}'$ to $H_5$ the surface $\Phi$ is transformed onto a doubly covered quintic surface of del Pezzo with 2 nodal points. At the end of his paper the author indicates the situation for $p>2$. There the nest of tangent spaces in a point of $\Phi$ is:

$$\Omega_0 \subset \Omega_2 \subset \Omega_5 \subset \Omega_8 \subset \cdots \subset \Omega_{3p-1} \subset \Omega_{3p+2} \subset \Omega_{3p+4}.$$

These spaces are paired in a involutory correlation, resulting in a null polarity for $p$ even and an involution in a quadric, if $p$ is odd. The mentioned splittings of $S_{11}$ are now generalised in the following manner: $S_{6m+5} = H_{3m+2} + H_{3m+2}{}'$ for $p=2m$ , and $S_{6m+8} = H_{3m+4} + H_{3m+3}{}'$ for $p=2m+1$ . The generalised surface $\Phi$ is now the product-variety $V_1^2 \times V_1^{p+3} \subset S_{3p+5}$ , so that for $p=3$, $\Phi$ is a product-surface $V_1^2 \times V_1^6$ immersed in $S_{14}$. In this case there exist projections of $S_{14}$ from an $H_7$ onto an $H_6{}'$, with the effect that $\Phi$ is transformed in a doubly covered sextic surface of del Pezzo in $H_6{}'$, possessing a skew hexagon of lines.

{For the entire collection see MR0661767 (83e:51003).} Reviewed by W. Burau

**MR0609101 (83h:14023)**
Edge, W. L.
Tritangent planes of Bring's curve.
*J. London Math. Soc. (2)* **23** (1981), no. 2, 215–222.
14H45

| Citations |
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| From References: 1 |
| From Reviews: 1 |

Bring's curve $B$ of the complex projective space $P_3$ is a space curve of order 6 and genus $p=4$ which can be represented as a complete intersection of a quadric $Q_2$ and a diagonal cubic surface $D_2$. It is useful to define $Q_2$ and $D_2$ by $x^2+y^2+z^2+t^2+u^2=0$ and $x^3+y^3+z^3+t^3+u^3=0$ where the five coordinates are subject to the relation $x+y+z+t+u=0$ . This curve $B$ is then clearly preserved under the symmetric group $S_5$ of all 120 permutations of the five coordinates. $B$ is singularity-free and has 60 hyperosculating points, called "stalls'', i.e. points where the osculating plane has 4 points of contact with $B$. One of these stalls, for example, is the point $(1,1,\alpha,\beta,\gamma)$ where $(\alpha,\beta,\gamma)$ are the roots of $\tau^3+2\tau^2+3\tau+4=0$. The remaining 59 stalls result from this through permutation of the coordinates. The stalls lie in groups of six in one of ten planes, all of which are described each time by two coordinates equalling zero. The tangents to $B$ at six such coplanar points then each pass through one point each. For the plane $x=y=0$, for example, this is the point $(1,-1,0,0,0)$. This yields the vertices of 10 cubic cones on which $B$ is located. Furthermore, through each tangent in a stall there pass 3 planes which touch $B$ in two other stalls. For the tangent (1) $z/\alpha=t/\beta=u/\gamma$ the three planes are: (2) $z/\alpha=t/\beta$, $z/\alpha=u/\gamma$, $t/\beta=u/\gamma$. We thus obtain 60 tritangent planes to $B$ of this kind. However, through each "stall tangent'' there passes a tritangent plane whose further contact points are not "stalls''. For the tangent (1) this is the plane

$$(\alpha-1)(\alpha+4)z+(\beta-1)(\beta+4)t+(\gamma-1)(\gamma+4)u=0,$$

as can be seen from some calculations.

In all, $B$ thus has 120 tritangent planes. For the general canonical curve $C_1^6$ for $p=4$ these had been discovered by A. Clebsch in his classical work [J. Reine Angew. Math. 63 (1864), 189--243] via his transcendental methods. Here, Clebsch found that the 360 contacts of the 120 tritangent planes which the general $C_1^6$ also has, lie on a surface $f_2^{60}$ of degree 60 composed of 30 quadrics. In the case of Bring's curve $B$---not considered by Clebsch---$f_2^{60}$ is composed of 3 surfaces of degree 20 belonging to a pencil; two of those are composed of the above-mentioned 6-point planes (all counted with multiplicity two) so that one pencil surface remains, $f_2^{20}$, which cuts the still uncounted contact points on $B$ which are not "stalls''. Bring's curves have been studied before by the author [cf. J. London Math. Soc. (2) 18 (1978), no. 3, 539--545; MR0518240 (80a:14014)], also by A. B. Coble [Algebraic geometry and theta functions, Amer. Math. Soc., New York, 1929; Jbuch 55, 808] and by A. Emch [Comment. Math. Helv. 7 (1934), 1--13; Zbl 10, 35], who started with the ten cubic cones through $B$ mentioned earlier; H. S. M. Coxeter [Proc. London Math. Soc. (2) 34 (1932), 126--189; Zbl 5, 76] encountered a specialization of $B$ whose tritangent planes correspond to the 120 diagonals of a polytope in the $R_7$. The two systems of 60 tritangent planes each were discovered by the author in collaboration with Du Val in 1979.

Reviewed by W. Burau

**MR0634146 (83b:51022)**

Edge, W. L.

A specialised net of quadrics having self-polar polyhedra, with details of the five-dimensional example.

*Canad. J. Math.* **33** (1981), no. 4, 885–892.

51M20 (14C21)

| Citations |
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| From References: 0 |
| From Reviews: 0 |

Let $C$ be the rational normal curve in an $n$-dimensional projective space given in the parametric form $x_r=(-1)^r\binom{n}{r}\theta^{n-r}$. Then the osculating hyperplanes at $n+2$ distinct points of $C$ for which $\theta=\eta^j\zeta$, where $\zeta$ is any complex number and $\eta=\exp(2\pi i/(n+2))$, bound an $(n+2)$-hedron $H$. $H$ is polar for all the quadratics $\sum_{j=0}^{n+1}\alpha_j(x_0+x_1\eta^j\zeta+\cdots+x_n\eta^{n\cdot j}\zeta^n)^2=0$. The author studies the special case where the $\alpha_j$ are chosen such that the equation does not depend on $\zeta$. The 5-dimensional case is discussed in more detail.

Reviewed by Jörn Zeuge

**MR0620733 (82m:14013)**

Edge, W. L.

| Citations |
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| From References: 4 |

A pencil of specialized canonical curves.

From Reviews: 1

*Math. Proc. Cambridge Philos. Soc.* **90** (1981), no. 2, 239–249.
14H10

This paper continues the investigations of the author [same journal 89 (1981), no. 3, 413--421; MR0602296 (82f:14053)] of a pencil $P$ of canonical curves of genus 6 on a del Pezzo quintic surface $F$ in a 5-dimensional projective space. The author investigates the group $G$ of self-projectivities of $F$, which is isomorphic to the symmetric group $S_5$. After introducing a coordinate system, $G$ acts as a group of linear substitutions thus realizing an irreducible representation of degree 6 of $S_5$. The intersection $\Gamma$ of the single quadratic invariant $Q$ of this representation and $F$ is a canonical model of Wiman's four-nodal plane sextic. Further invariants of the group $G$ and its subgroup $G^T$ are studied.
Reviewed by Jörn Zeuge

**MR0602296 (82f:14053)**

Citations
From References: 5
From Reviews: 3

Edge, W. L.
A pencil of four-nodal plane sextics.
*Math. Proc. Cambridge Philos. Soc.* **89** (1981), no. 3, 413–421.
14N05 (14D25 14E05)

In einer klassischen Arbeit von A. Wiman [Math. Ann. 48 (1896), 195--240; Jbuch 30, 600] wird auch eine ebene Sextik $W$ vom Geschlecht $p=6$ mit 4 singulären Punkten behandelt. Diese $W$ gestattet eine zur symmetrischen Gruppe $\mathrm{S}_5$ isomorphe Gruppe $G$ von birationalen Transformationen in sich. Davon sind 24 Transformationen Autokollineationen, die übrigen 96 quadratische Cremona-Transformationen. In der vorliegenden Arbeit beschreibt der Verfasser nun in Ergänzung zu den Wimanschen Untersuchungen die Kurve $W$ erstmalig rein elementar. Der Ausgangspunkt sind 4 Punkte allgemeiner Lage $A$, $B$, $C$, $D$ der Ebene. Durch sie sind 4 Geradenbüschel $\alpha$, $\beta$, $\gamma$, $\delta$ sowie ein Kegelschnittbüschel $\varepsilon$ bestimmt. Die Gruppe $\mathrm{S}_5$ wird dann in der Ebene durch eine Gruppe $G$ dargestellt, $G$ besteht aus 24 Autokollineationen, die nach dem Hauptsatz der projektiven Geometrie jeweils durch eine Permutation der Punkte $A$, $B$, $C$, $D$ festgelegt sind. Dann gehören zu $G$ noch 96 quadratische Cremona-Transformationen, die sich auch leicht angeben lassen. Sie führen jeweils die Gerarden der Büschel $\alpha$, $\beta$, $\gamma$, $\delta$ in die Kegelschnitte von $\varepsilon$ über. Die gesuchte Sextik soll in $A$, $B$, $C$, $D$ Doppelpunkte haben. Davon sind bereits die Doppelpunktstangenten festgelegt. In $D$ sind dies z.B. die Gerarden $d$, $d'$, die das zu $DA$, $DB$, $DC$ zugeordnete Hessesche Paar sind. Analog bestimmt man die Paare $a$, $a'$; $b$, $b'$ und $c$, $c'$ von Gerarden je durch $A$, $B$, $C$. Ähnlich ergeben sich die Schnittpunkte von $W$ mit den 6 Gerarden $AB, AC, \cdots, CD$ außerhalb der Punkte $A$, $B$, $C$, $D$. In dem Büschel $\varepsilon$ sind weiterhin die beiden Kegelschnitte $\Omega$ und $\Omega'$ ausgezeichnet. $\Omega$ ist dabei dadurch bestimmt, daß dieser Kegelschnitt die Geraden $a'$, $b'$, $c'$, $d'$ und $\Omega'$ dadurch daß er die Geraden $a$, $b$, $c$, $d$ berührt. Zunächst ist die gesuchte Kurve $W$ auch noch nicht bestimmt, wohl aber ein Büschel von Sextiken, dem sie angehört. Dieses Büschel $P$ wird durch die beiden zerfallenden Kurven $\Sigma = \Omega a'b'c'd'$, $\Sigma' = \Omega' abcd$ aufgespannt. Ihm gehört außerdem noch das Produkt $\Pi$ der 6 Verbindungsgeraden der Punkte $A$, $B$, $C$, $D$ an. Es ergibt sich ferner, daß die Gruppe $\mathrm{S}_5$ eine Involution im Büschel $P$ induziert, sodaß bei jeder ungeraden Permutation $\Sigma$ und $\Sigma'$ einander zugeordnet sind. Bei dieser Büschelinvolution ist $\Pi$ das eine Fixelement, das andere ist die gesuchte Kurve $W$. In $P$ gibt es außerdem noch Paare von rationalen Sextiken, welche außer in $A$, $B$, $C$, $D$ noch in je einem weiteren Punkt auf den 6 Geraden $AB, \cdots, CD$ singulär sind. Wesentlich ist nun noch

folgende hyperrämliche Deutung der ganzen Fragestellugen: Bildet man die Ebene auf die Veronesesche $V_2^3$ des $S_9$ (in der Bezeichnung des Referenten) ab und projiziert diese $V_2^3$ aus dem $Z_3$, der durch die Bildpunkte der Punkte $A$, $B$, $C$, $D$ aufgespannt wird, so entsteht (in der Bezeichnung vom Verfasser) aus $V_2^3$ eine del-Pezzo-Fläche $F$ der Ordnung 5 des $P_5$. Auf dieser $F$ liegen 10 Geraden und 5 Kegelschnittscharen, welche Bilder der zu Beginn genannten 6 Büschel $\alpha, \cdots, \varepsilon$ sind. Die Bilder von $\Sigma$ und $\Sigma'$ sind Kurven 10. Grades auf $F$ geworden, die einen Kegelschnitt aus jeder der 5 Scharen enthalten. Die Gruppe $\mathrm{S}_5$ wird dann durch lauter Autokillineationen von $F$ dargestellt. Eine wohlbestimmte Quadrik schneidet jetzt aus $F$ eine Kurve 10. Grades aus, das Bild von $W$.
Reviewed by W. Burau

**MR0581668 (82e:14043)**
Edge, W. L.
The discriminant of a cubic surface.
*Proc. Roy. Irish Acad. Sect. A* **80** (1980), no. 1, 75–78.
14J25 (14M20)

| Citations |
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| From References: 0 |
| From Reviews: 0 |

Durch Nullsetzen einer quaternären Form $F$ vom Grade $n$ definiert man bekanntlich eine Fläche $n$-ten Grades des projektiven $P_3$. Die Diskriminante $\Delta$ von $F$ erweist sich dann als Invariante des Grades $4(n-1)^3$ in den Koeffizienten von $F$, hat also bei $n=3$ den Grad 32, ist mithin als Invariante bereits vom höheren Grad. Im Jahre 1860 hat nun Salmon $\Delta$ in folgender Weise durch gewisse einfachere Grundinvarianten $A$, $B$, $C$, $D$ ausgedrückt:

$$\Delta = (A^2 - 64B)^2 - 2^{14}(D + 2AC).$$

In der vorliegenden Arbeit wird jetzt nachgewiesen, daß hierin $2AC$ durch $\frac{1}{8}AC$ zu ersetzen ist. Dieser Fehler ist lange Zeit nicht bemerkt worden, und die genannte Formel ist in allen Auflagen des Standardwerks von G. Salmon, A treatise on the analytic geometry of 3 dimensions (1862--1915) [Vol. I, sixth edition, Longmans, Green, London, 1914; Vol. II, fifth edition, 1915] falsch wiedergegeben worden. Der Verfasser entdeckte den Fehler am Beispiel einer einfachen, mit Singularität behafteten kubischen Fläche, für deren kubische Form die Diskriminante nur in der richtigen Form verschwindet, wie es sein muß.
Reviewed by W. Burau

**MR0579807 (81k:14034)**
Edge, W. L.
A special polyhedral net of quadrics.
*J. London Math. Soc. (2)* **22** (1980), no. 1, 46–56.
14J99

| Citations |
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| From References: 0 |
| From Reviews: 0 |

Der Verfasser nimmt in der vorliegenden Arbeit Untersuchungen wieder auf, mit denen er sich bereits vor längerer Zeit beschäftigt hatte [Acta Math. 64 (1935), 185--242; Zbl 11, 415; ibid. 66 (1936), 253--332; Zbl 13, 361; Proc. Edinburgh Math. Soc. (2) 4 (1936), 185--209; Zbl 14, 37; Proc. London Math. Soc. (2) 47 (1942), 455--480; MR0008170 (4,254a)]. Bereits Reye und Schur hatten entdeckt, daß nicht jedes Netz von Quadriken des $P_3$ ein solches von Polaren bezüglich einer kubischen Fläche $f_2^3$ ist, dann hatte der Verfasser in den zitierten Arbeiten gefunden, daß es zweckmäßiger ist, die speziellen Netze nicht von der $f_2^3$ aus zu erklären, sondern auf folgende Weise gleich für beliebiges $n$: Man gebe sich auf einer rationalen Normkurve $C \subset P_n$ eine Punktgruppenschar $g_{n+2}^1$ vor. Die $n+2$ Schmieg-$P_{n-1}$ an $C$ in den $n+2$ Punkten einer Punktgruppe von $g_{n+2}^1$ bilden dann

einen Polytop, dessen Ecken eine Kurve $\vartheta$ der Ordnung $\binom{n+1}{2}$, die Jakobische Kurve des fraglichen Netzes, das aus $\vartheta$ dann bis auf endlich viele Möglichkeiten, d.h. fast eindeutig, bestimmt ist. In der dritten oben zitierten Arbeit hatte der Autor bereits bei $n=3$ durch eine kubische Normkurve $C$ und eine $g_5^1$ ein sog. polyedrales Netz von Quadriken des $P_3$ definiert und im einzelnen beschrieben. In der vorliegenden Arbeit werden nun polyedrale Quadrikennetze zwar für beliebiges $n\geq 3$ genau beschrieben, aber mithilfe von Scharen $g_{n+2}^1$ auf der rationalen Normkurve $C$ des $P_n$ von recht spezieller Art. Die Punktgruppen von $g_{n+2}^1$ auf $C$ sind in einer Parametrisierung von $C$ als Nullstellen aller Polynome $n+2$ -ten Grades erklärt, die sich als Linearkombinationen von 2 festen Polynomen des Grades $n+2$ ergeben. Als diese festen Polynome nimmt der Autor $\vartheta^{n+2}-\alpha^{n+2}$ und $\vartheta^{n+2}-\beta^{n+2}$. Dies führt dann zu einem Quadrikennetz des $P_n$, das genauer studiert wird. Bei $n=4$ wurde in der zweitzitierten Arbeit schon gezeigt, daß bei allgemeinen Netzen die Jakobikurve 20 Trisekanten besitzt, und daß diese Trisekanten bei den speziellen Fällen eine Regelfläche $R_2^{20}$ bilden. In dem noch spezielleren Fall, den der Autor untersucht, zerfällt diese $R_2^{20}$ in 3 Teile, was im einzelnen untersucht wird.
Reviewed by W. Burau

**MR0549866 (82a:14014)**
Edge, W. L.
Nodal cubic surfaces.
*Proc. Roy. Soc. Edinburgh Sect. A* **83** (1979), no. 3-4, 333–346.
14J17

| Citations |
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| From References: 0 |
| From Reviews: 0 |

In der Mitte des 19. Jahrhunderts herrschte ein lebhaftes Interesse für Einzeluntersuchungen über Flächen 3. und 4. Grades, was in unserem Jahrhundert merklich nachgelassen hat. Es ist daher zu begrüßen, daß der Verfasser der vorliegenden Arbeit solche Fragen auf dem Gebiet der Singularitätentheorie von Flächen 3. Grades wieder aufgreift. Er zitiert an klassischen Autoren hierin Schläfli und Cayley, dem man jedoch noch Arbeiten von Clebsch, Sturm und Cremona hinzurechnen muß. Alle diese Autoren schrieben in den 60er Jahren des vorigen Jahrhunderts. Sie entdeckten insbesondere, daß alle kubischen Flächen $f_2^3$ mit Ausnahme des elliptischen Kegels rational sind und durch Polynome 3. Grades parametrisiert werden können. Von del Pezzo stammt nun die Einsicht, daß dies geometrisch auf folgendes hinausläuft: Alle $f_2^3$ mit Ausnahme des elliptischen Kegels sind wie auch alle Flächen der Ordnung $n$ des $P_n$ $(n\leq 9)$ Projektionen einer Fläche 9. Grades des $P_9$ [see P. del Pezzo, Rend. Circ. Mat. Palermo 1 (1887), 241--271; Jbuch 19, 841]. Diese Fläche pflegt der Referent mit Veronesescher $V_2^3$ zu bezeichnen, der Verfasser nennt sie nach del Pezzo. Cayley und Schläfli haben nun 7 Typen von $f_2^3$ mit einem singulären Punkt $S_0$ festgestellt, einem, wo $S_0$ konisch ist, 4 Typen mit biplanarem $S_0$ und 3 mit uniplanarem $S_0$. Die Typen unterscheiden sich vor allem durch die Klasse der Fläche, welche eine Zahl zwischen 4 und 10 sein kann und auch durch Schläfli angegeben wurde. Hierzu kommen noch einige weitere Typen mit nur isolierten singulären Punkten, deren Zahl $\leq 4$ ist. Es entsteht nun die Aufgabe, alle diese Typen durch Projektion aus einer $V_2^3$ entstehen zu lassen, was der Verfasser der vorliegenden Arbeit durchführt. Vor ihm hatte dies bereits G. Timms getan [see Proc. Roy. Soc. London Ser. A 119 (1928), 213--248; Jbuch 54, 709]. Aber in allen genannten Arbeiten findet der Verfasser nun gewisse Unvollkommenheiten und Lücken, welche er in seiner Arbeit verbessert. Die Grundtatsache ist zunächst die folgende: Bei der Projektion der $V_2^3$ aus 6 Punkten allgemeiner Lage entsteht eine singularitätenfreie $f_2^3$. Unter allgemein versteht man hierbei, daß die Urbilder dieser 6 Punkte auf der Parameterebene $X_2$ der $V_2^3$ nicht auf einem Kegelschnitt liegen und daß niemals 3 davon kollinear sind. Alle speziellen Typen von $f_2^3$ entstehen nun dann, wenn man diese Voraussetzungen verletzt, d.h., dies kann in zweifacher Weise geschehen: (a) Man läßt die Punkte zwar noch verschieden liegen, aber nicht mehr allgemein; (b) man läßt sie teilweise in bestimmter Weise infinitesimal zusammenfallen. Wir erwähnen jetzt nur folgende Einzelfälle der Diskussion: Eine $f_2^3$ mit einem konischen Doppelpunkt entsteht, wenn die 6 Punkte verschieden sind, aber auf einem regulären Kegelschnitt $K$ liegen (wir sprechen immer von Punkten der Ebene, meinen jedoch ihre Bildpunkte auf $V_2^3$); aber eine $f_2^3$ mit konischem Punkt entsteht auch, wenn 3 der Punkte kollinear sind, aber die übrigen allgemein liegen. Verteilen sich aber die 6 verschiedenen Punkte auf 2 Geraden, so entsteht eine $f_2^3$ mit biplanarem Punkt des allgemeinsten, von Cayley mit $B_3$ benannten Typs. Die $f_2^3$ mit einem biplanaren Punkt von einem der weiteren Typen $B_4$, $B_5$, $B_6$ entstehen der Reihe nach wenn die 6 Punkte auf $K_1$ entweder zu 4 und 2 zusammenfallen oder zu 5 und 1 und schließlich, wenn alle 6 zusammenfallen. Die $F_2^3$ mit einem uniplanaren Punkt entstehen, wenn für das Projektionszentrum $Z_5$ gilt, $Z_5 \subset T_6(V_1^3)$, wo $T_6(V_1^3)$ der Schmiegraum der $V_2^3$ längs einer $V_1^3$ ist. Die 3 Typen $U_6$, $U_7$, $U_8$ unterscheiden sich dann dadurch, daß $Z_5$ die $V_1^3$ entweder in 3, in 2 oder in einem Punkt schneidet. Von weiteren Einzelheiten sei nur noch erwähnt, daß eine $f_2^3$ mit 4 Doppelpunkten wieder auf 2 Weisen entstehen kann: Dadurch (a) daß die 6 Punkte in der Ebene die Ecken eines vollständigen Vierecks sind oder (b) daß sie 3 Mal zu je 2 auf dem Kegelschnitt $K$ zusammenfallen.

Reviewed by W. Burau

**MR0530803 (80h:14016)**

Edge, W. L.

The scroll of tangents of an elliptic quartic curve.

*Math. Proc. Cambridge Philos. Soc.* **86** (1979), no. 1, 1–11.

14J25

| Citations |
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| From References: 0 |
| From Reviews: 0 |

The equation of the scroll $R$ of tangents of the common curve $C$ of two quadrics in general position was found by Cayley in 1850, and reproduced by G. Salmon in his book [A treatise on the analytical geometry of three dimensions, fourth edition, especially p. 191, Hodges and Figgis, Dublin, 1882], providing an equation in covariant form. The author's procedure for finding this equation takes advantage of the invariance of every quadric through $C$ under an elementary abelian group $E$ of eight projectivities. The basic fact is that all points not in any face of the common self-polar tetrahedron $U$ of the quadrics are distributed in octads, handled as entities.

It is shown how the equation can be written so as to disclose $R$ as the envelope of a singly infinite system, of index 2, of quartic surfaces $Q_i$, each having nodes at the vertices of $U$ and at an octad on $C$ that varies with $i$. Each $Q_i$ meets $R$ in $C$ (reckoned twice) and touches $R$ along an elliptic curve of order twelve. In the second part of the paper, $R$ is placed among the combinants of the pencil of quadrics through $C$. The quartic surface $F$, whose covariant relation to the quadrics was discovered by J. A. Todd [Ann. Mat. Pura Appl. (4) 28 (1949), 189--193; MR0036539 (12,125f)] and explained by R. H. Dye [Proc. London Math. Soc. (3) 34 (1977), no. 3, 459--478; MR0437546 (55 #10470)] is placed in the present context.

In the third part, $R$ is transformed into a quartic surface $W$, having a cuspidal line $c$ and 4 nodes, forming a tetrahedron $u$, discovered long ago by Plücker in his pioneering research in line geometry but perhaps more easily studied in the present, less elaborate, setting. It can be generated by more than one quite elementary geometric process [cf. the author, Proc. London Math. Soc. (2) 41 (1936),

336--360; Zbl 14, 172]; one such, involving twisted cubics, is used to obtain a mapping of $W$ on a plane. $W$ proves to be its own polar reciprocal with respect to a quadric containing the line $c$ and for which $u$ is self-polar.
Reviewed by S. R. Mandan

**MR0519532 (80b:14017)**
Edge, W. L.
A canonical curve of genus 6.
*C. R. Math. Rep. Acad. Sci. Canada* **1** (1978/79), no. 2, 95–97.
14H45

| Citations |
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| From References: 0 |
| From Reviews: 0 |

Del Pezzo's quintic surface $F$ in 5-space contains 10 lines and admits a group $G$ of self-projectivities isomorphic to the symmetric group $S_5$. It is surprising that Baker, having come close to it in 1931, missed it.

Each line on $F$ meets 3 others such that the 10 Hessian duads of such triads of meets determine a pencil $P$ of curves of order 10 on $F$ containing 3 reducible members $A, B, B'$. Every curve of $P$ is fixed under the projectivities of the subgroup $A_5$ of $S_5$, and every one in its coset is an involution transposing $B$ with $B'$ and leaving fixed $A$ and an irreducible curve $W$, canonical of genus 6, unique on $F$ as its intersection with the fixed quadric of this irreducible representation of $S_5$. While a general curve of genus 6 has 15 moduli, $W$ has none, and properties of the specialisation may be worth investigating. $W$ is the canonical model of the 4-nodal plane sextic, shown by Wiman in 1897 to admit a group of 120 Cremona self-transformations.

All projectivities of any one of the 6 cyclic groups of order 5 of $S$ leave fixed 4 isolated points and all points of a chord of $F$ giving rise to 6 such chords whose 12 meets with $F$ form 2 hexads as nodes on 2 rational curves $R$, $R'$ in $P$. Every curve in $P$ other than $A, B, B', R, R'$ is nonsingular.
Reviewed by S. R. Mandan

**MR0518240 (80a:14014)**
Edge, W. L.
Bring's curve.
*J. London Math. Soc. (2)* **18** (1978), no. 3, 539–545.
14H45

| Citations |
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| From References: 5 |
| From Reviews: 2 |

Bring and Bring's curve $B$ were first noticed in an 1884 book by F. Klein in connection with an endeavour to reduce the general quintic equation to the form $x^5+ex+f=0$, so that $S_1=0=S_2=S_3$, where $S_n=\sum x_i{}^n$ and $x_i$ $(i=0,1,2,3,4)$ are the roots. Taking $x_i$ as supernumerary coordinates referred to a pentahedron $P$ with $S_1=0$, $B$ was defined as the intersection of the diagonal surface $D{:}S_3=0$ (containing the 15 diagonals of $P$) and the quadric $Q{:}S_2=0$. $B$ was noticed again in a paper of Wiman from 1895 as its own transform in the 10 harmonic inversions in the 10 vertices $V_{ij}$ (with $x_i+x_j=0=x_k$) of $P$ and the 10 planes $p_{ij}{:}x_i=x_j$ such that the meets of $B$ with these planes are its 60 stalls where its osculating planes have 4-point contact. The author shows the square of the product of these planes as $S_4^5=20S_5^4$, a member of the pencil of surfaces: $S_4^5=kS_5^4$, such that the 246 points of the Jacobian of a $g^1_{120}$ on $B$ cut by the pencil contain the 60 stalls.

It is further shown that the 15 diagonals of $P$ meet $B$ in 15 pairs of points on $S_5=0$ where its tangent planes are osculating planes of $B$. The 24 meets of $B$ with $S_4=0$ form 6 skew quadrilaterals $(q)$ on $Q$ whose sides are its tangents there and 6 pairs of diagonals lie on $D$ as a double-six $d$ residual of the 15 diagonals of $P$ making $Q$ its Schur quadric such that the last 15 lines can be labelled consistently by the Schläfli

symbols $c_{ij}$ indicating an alternative geometric definition of $B$ by taking $P$, defining $D$ thereby, $d$ on $D$ and its Schur quadric $Q$. The osculating planes of $B$ at the vertices of $(q)$ are tangent planes of $Q$; the tangents of $B$ generate a scroll of order 18 whose nodal curve as locus of their meets (each tangent meeting 14 others) is of order 126, split into 11 parts, 10 of order 9 in the planes $p_{ij}$ and the 11th of order 36 through the vertices of $(q)$. Finally the author merely alludes to the 120 tritangent planes of $B$.
Reviewed by S. R. Mandan

**MR0491743 (58 #10944)**

**Citations**
From References: 0
From Reviews: 0

Edge, W. L.
Cubic primals in [4] with polar heptahedra.
*Proc. Roy. Soc. Edinburgh Sect. A* **77** (1977), no. 1-2, 151–162.
14N05

Author's summary: "This note starts from the observation that it is not, in general, possible to express a homogeneous cubic polynomial in five variables as a sum of cubes of seven linear forms. Some of the geometry to which particular cubics which happen to be so expressible give rise is described. Further particularisations are mentioned, and one such cubic is investigated in some detail.''

**MR0469926 (57 #9706)**

**Citations**
From References: 0
From Reviews: 0

Edge, W. L.
The common curve of quadrics sharing a self-polar simplex.
*Ann. Mat. Pura Appl. (4)* **114** (1977), 241–270.
14H45

Author's summary: "When $n-1$ quadrics in projective space of $n$ dimensions have a common self-polar simplex, their common curve $\Gamma$ admits a group of $2^n$ self-projectivities. The consequent properties of $\Gamma$ are investigated, and further specializations are imposed which amplify the group and endow $\Gamma$ with further properties. There is some reference to the osculating spaces and principal chords of $\Gamma$, and some properties of particular curves in four and five dimensions are described.'' See also the author's earlier papers [Proc. Cambridge Philos. Soc. 75 (1974), 331--344; MR0349695 (50 #2188); Proc. Edinburgh Math. Soc. (2) 19 (1974/75), 39--44; MR0349696 (50 #2189); Proc. Roy. Soc. Edinburgh Sect. A 71 (1974), part 4, 337--343; MR0354696 (50 #7173)].
Reviewed by B. Orban

**MR0444675 (56 #3025)**

**Citations**
From References: 0
From Reviews: 0

Edge, W. L.
The flecnodal curve of a ruled surface.
*J. London Math. Soc. (2)* **15** (1977), no. 3, 534–540.
14J25

In 1849 Salmon proved that the points on an algebraic surface $F$, of order $n$ in [3], at which one of the two inflectional tangents has a 4-point intersection with $F$, are those of the common curve F of $F$ and a covariant surface of order $11n-24$. The tangent plane at any point cuts $F$ in a curve with a node at the contact, and the 2 nodal tangents to this plane curve are inflectional tangents to $F$; should either of these have a 4-point intersection with $F$, the node is called a flecnode. F is of order $n(11n-24)$, and is called the flecnodal curve of $F$.

If $F$ is a scroll $R$, one of the 2 inflectional tangents at a point $X$ is the generator $g$ through $X$, and the question arises whether the other inflectional tangent can have a

4-point intersection with $R$ at $X$. The locus of such $X$ is the flecnodal curve $\mathrm{F}$ of $R$.
In 1874 Voss used line geometry to investigate the geometry of $R$. He used Klein's mapping of the lines of [3] by the points of a quadratic $\Omega$ in [5]. $R$ is then mapped by a curve $C$, of the same order $n$ and genus $p$ as $R$, on $\Omega$. The order of $\mathrm{F}$ is $5n+12(p-1)$. The order of $\varphi$, the scroll generated by the tangents having 4-point intersection, is $8(n+3p-3)$.
In 1882 Veronese studied algebraic curves in higher spaces. He introduced their ranks---orders of manifolds generated by osculating spaces. In 1894 Segre found that the order of the manifold generated by the osculating $[k]$'s of $C$ is $(k+1)(n+pk-k)$. As these manifolds are generated by linear spaces, the order of each is the same as that of its polar reciprocal in a quadric.
The osculating solids of $C$ generate a 4-fold of order $4(n+3p-3)$, so that their polar lines generate a scroll $\Sigma$ of this same order. Since $\Sigma$ meets $\Omega$ in a curve $\Gamma$ of order $8(n+3p-3)$, this is the order of $\varphi$, the scroll of tangents of $R$ having 4-point intersection. The number of 5-point tangents of $R$ is $10(n+4p-4)$. The tangent planes to $R$ at the points of $\mathrm{F}$ form a developable of class $5n+12(p-1)$. The genus $\pi$ of $\Gamma$ is $4n+12p-13$. The simplest scroll having a proper flecnodal curve is the rational quartic scroll $R^4$, with $\mathrm{F}$ an octavic, of genus 3, touching 8 generators. The 4-point tangents generate an octavic scroll $\varphi^8$. The common curve of $R^4$ and $\varphi^8$ is $\mathrm{F}$ counted 4 times. The 4-point tangents of $R$ belong to the envelope of its osculating quadrics. The envelope will also include $R$ multiply.
$R^4$ has a cubic $\gamma$ for its nodal curve. There are 6 osculating quadrics of $R^4$ through an arbitrary point. A general scroll, of order $n$ and genus $p$, has $3(n+2p-2)$ of its inflectional tangents through an arbitrary point. The envelope of $R^4$ consists of $R^4$ itself counted thrice, and the octavic scroll $\varphi^8$ of 4-point tangents.
Reviewed by Dov Avishalom

**MR0429914 (55 #2923)**
Edge, W. L.
Non-singular models of specialized Weddle surfaces.
*Math. Proc. Cambridge Philos. Soc.* **80** (1976), no. 3, 399–418.
14J25 (14M20)

| Citations |
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| From References: 0 |
| From Reviews: 0 |

Nello spazio proiettivo $\mathbf{P}^5(\mathbf{C})$ si considerino tre quadriche passanti per una data retta $r$ ed aventi a comune un dato simplesso autopolare. Proiettando dalla retta $r$ la superficie $\mathrm{F}$ base della rete $\Phi$ di quadriche $\sum\lambda_i Q_i=0$ sopra uno spazio proiettivo a tre dimensioni si ottiene la superficie quartica di Weddle, della quale $F$ è un modello non singolare. Specializzando $\mathrm{F}$ e quindi la rete $\Phi$ si ottengono le superficie di Weddle specializzate le quali posseggono gruppi di trasformazioni proiettive in sè. La descrizione delle superficie $\mathrm{F}$ specializzate e delle relative reti $\Phi$ è lo scopo principale di questo lavoro nel quale ciascuna delle possibili situazioni è oggetto di accurata analisi.
Reviewed by D. Gallarati

**MR0364273 (51 #528)**
Edge, W. L.
A footnote on the mystic hexagram.
*Math. Proc. Cambridge Philos. Soc.* **77** (1975), 29–42.
14N05

| Citations |
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| From References: 0 |
| From Reviews: 0 |

This paper deals with those properties of Pascal's mystic hexagram called the Veronese properties. The author specializes the hexad on a conic $C$ from which the Veronese properties are obtained by making use of the intersections of $C$ with the sides of a self-polar triangle. Coincidences among different Pascal lines are shown, and equations of the 15 tritangent planes of an appropriate nodal cubic surface $D$ are given explicitly. The space figure chosen is such that the specialized plane figure is a projection of it. The 15 lines (other than those through its node) on $D$ are also projected, from a point on $D$ other than its node, into 15 of the 16 bitangents of a nodal plane quartic. After treating the process in general, the author discusses the particular case in which the center of projection is $(1,1,1,-\frac{2}{3}\lambda)$ and the resulting plane quartic is $12(x+y+z)xyz=\{3(yz+zx+xy)-(x^2+y^2+z^2)\}^2$ and lists explicit equations for the 15 relevant bitangents. The verification of the Veronese properties when Pascal lines coincide follows. The analogous figure over the Galois field GF(5) involving Clebsch's diagonal surface is also taken into consideration. Conditions are obtained under which the diagonal surface has a node, and the tritangent planes of the nodal diagonal surface over this field are found.

Reviewed by J. Verdina

**MR0354696 (50 #7173)**

Edge, W. L.

The chord locus of a certain curve in $[n]$.

*Proc. Roy. Soc. Edinburgh Sect. A* **71,** no. 4, 337–343. (1974).

14M10 (14H45)

| Citations |
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| From References: 0 |
| From Reviews: 1 |

Si consideri in $\mathbf{P}^n(\mathbf{C})$ la curva $\Gamma_n$ completa intersezione di $n-1$ quadriche aventi un comune $(n+1)$ -edro autopolare $T$, e quindi invariante per un gruppo abeliano di omografie (di ordine finito $2^n$). $\Gamma_n$ ha genere $1+2^{n-2}(n-3)$ e le ipersuperficie di ordine $n-3$ segano su di essa divisori canonici. Le corde di $\Gamma_n$ riempiono una varietà tridimensionale $M_n$ di ordine $2^{n-2}(2^{n-1}-n)$ che possiede certe singolarità che vengono qui esaminate. Sono singolari per $M_n$ la superficie delle tangenti di $\Gamma_n$, la stessa curva $\Gamma_n$, i vertici e gli spigoli di $T$, etc. Ad esempio, se $n=4$ e quindi $\Gamma_4$ e la curva canonica di genere 5 ed $M_4$ è una ipersuperficie d'ordine 16, $M_4$ ha molteplicità 8 in ogni vertice di $T$ ed ogni spigolo di $T$ a retta quadrupla per $M_4$. Inoltre $\Gamma_4$ è curva di molteplicità 6 per $M_4$, etc.

Reviewed by D. Gallarati

**MR0349696 (50 #2189)**

Edge, W. L.

The osculating spaces of a certain curve in $[n]$.

*Proc. Edinburgh Math. Soc. (2)* **19** (1974/75), 39–44.

14N05 (14H45)

| Citations |
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| From References: 1 |
| From Reviews: 1 |

The author treats the non-singular intersection $\Gamma$ of the $n-1$ quadric primals $\sum_{j=0}^{n} a_j^{\,k} x_j^{\,2}=0$, $k=0,1,\cdots,n-2$ , where no two of the numbers $a_j$ are equal. He shows that the osculating prime $\Gamma$ at $x=\xi$ is $\sum\{f'(a_j)\}^{n-2}\xi_j^{\,2n-3}x_j=0$ . Equations are given for all the osculating spaces $[s]$. The number of spaces $[s]$ that osculate $\Gamma$ and meet a given $[n-s-1]$ is shown to be $R_s=2^{n-1}(n-s)(2_s-1)$ . In particular, the number of the osculating primes of $\Gamma$ passing through an arbitrary point is $R_{n-1}=2^{n-1}(2n-3)$ ; this formula verifies the classical result for $n=3$ (there are 12 osculating planes of an elliptic quartic through an arbitrary point in a three-dimensional space [3]) and for $n=4$. Also, the order of the primal generated by the osculating space $[n-2]$ 's of $\Gamma$ is $R_{n-2}=2^n(2n-5)$ which checks with the classical result for $n=3$ (the tangents of an elliptic quartic generate a scroll of order 8). A more elaborate method reaching the same conclusion is also presented. However, one can apply such an alternate procedure only if one knows (1) the genus of $\Gamma$, (2)

a formula giving the number of points in the sets of a linear series $g$ on $\Gamma$, of multiplicity exceeding the freedom $r$ of $g$, and (3) a precise rule for calculating the number of times a multiple point of specified singularity has to be counted.
Reviewed by J. Verdina

**MR0349695 (50 #2188)**
Edge, W. L.
Osculatory properties of a certain curve in $[n]$.
*Proc. Cambridge Philos. Soc.* **75** (1974), 331–344.
14N05 (14H45)

| Citations |
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| From References: 0 |
| From Reviews: 1 |

The author makes use of quadrics obtained from an $n$-dimensional projective space $[n]$ given by $\Omega_k \equiv \sum_{j=0}^{n} a_j{}^k x_j{}^2 = 0\, (k = 0, 1, \cdots, n)$ , where the simplex of reference $S$ is self-polar for all quadrics. The $n-1$ quadrics $\Omega_k = 0\, (k = 0, 1, \cdots, n-2)$ intersect in a non-singular curve $\Gamma_n$ of order $2^{n-1}$ that is invariant under $n+1$ mutually commutative central harmonic inversions $h_j$. The centers $X_j$ of these inversions are the vertices of $S$, and the product $h_0 h_1 \cdots h_n$ is the identity projectivity. The $h_j$ generate an abelian group $E$ of $2^n$ projectivities. The author deals mainly with the relations of $\Gamma_n$ and its osculating spaces to the quadrics $\sum \alpha_j x_j{}^2 = 0$ that are invariant under $E$. These relations are treated after it is shown that each point of $\Gamma_n$ belongs, under a certain condition, to a batch of $2^n$ points on $\Gamma_n$ invariant under $E$. Use is made of the singly infinite system of quadrics $\sum (t + a_j)^{n-2} x_j{}^2 = 0$ , which contain $\Gamma_n$ and are described as osculants. After a discussion of the cases when $n$ is odd and $n$ is even, detailed treatments are given for the special cases $n = 3$ , $n > 3$ , $n = 4$ , $n > 4$ , and $n > 5$ .

When $n = 3$ , the author finds that $\sum \alpha_j x_j{}^2$ is touched by the osculating planes at three batches, but when this quadric is an osculant, the only osculating planes of $\Gamma_3$ that touch the osculant are those at the batch itself. When $n = 4$ , there are five batches on $\Gamma_4$ at which its osculating solids touch $\sum \alpha_j x_j{}^2 = 0$ . Also, if $\beta$ is a batch and $\beta'$ is a satellite of $\beta$ (i.e., $\beta'$ is the unique other batch at which the osculating solids of $\Gamma_4$ touch the osculant for $\beta$), and if $\beta''$ is a satellite of $\beta'$, it is obtained that there are 12 batches $\beta$ that coincide with $\beta''$. When $n = 5$ , the osculants of $\Gamma_5$ at a batch $\beta$ are touched by osculating primes of $\Gamma_5$ at two other batches; the author finds the condition under which these two batches coincide. He also shows that, in general, the vanishing of a certain determinant is the condition for an osculating space $[s]$ of $\Gamma_n$ to touch $\sum \alpha_j x_j{}^2 = 0$ .
Reviewed by J. Verdina

**MR0374162 (51 #10362)**
Edge, W. L.
Binary forms and pencils of quadrics.
*Proc. Cambridge Philos. Soc.* **73** (1973), 417–429.
14N10 (15A72)

| Citations |
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| From References: 0 |
| From Reviews: 0 |

L'autore considera in $\mathbf{P}^n(\mathbf{C})$ il fascio $\Sigma$ di quadriche $\theta\Omega_0 + \Omega_1 = 0$ , ove $\Omega_0 = \sum_{i=0}^{n} X_i{}^2$ e $\Omega_1 = \sum_{i=0}^{n} a_i X_1{}^2$ . Il discriminante della forma quadratica $p(\lambda\Omega_0 + \Omega_1) + q(\mu\Omega_0 + \Omega_1)$ può essere scritto così: $f(\lambda)p^{n+1} + \sum_0^{n-1} A_j(\lambda,\mu) p^{j+1} q^{n-j} + f(\mu) q^{n+1}$ , ove $f(\theta) = (\theta + a_0) \times (\theta + a_1) \cdots (\theta + a_n)$ è il discriminante di $\theta\Omega_0 + \Omega_1$ ed $A_j(\lambda,\mu) = A_{n-1-j}(\mu,\lambda)\, (j = 0, 1, \cdots, n-1)$ . Una coppia $\lambda, \mu$ per la quale risulti, per qualche $j$, $A_j(\lambda,\mu) = A_{n-1-j}(\lambda,\mu) = 0$ , $\lambda \neq \mu$ , fornisce una speciale coppia di quadriche di $\Sigma$. L'autore si occupa di queste interessanti coppie di quadriche di $\Sigma$ cercando di assegnarne una caratterizzazione geometrica. In particolare, per $j = 1$ is hanno le $\frac{1}{2}n(n1)$ coppie di quadriche "simmetricamente apolari''.

Reviewed by D. Gallarati

**MR0333948 (48 #12267)**
Edge, W. L.
An operand for a group of order 1512.
*J. London Math. Soc. (2)* **7** (1973), 101–110.
50D30

| Citations |
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| From References: 0 |
| From Reviews: 0 |

The author exhibits a tableau, built with the digits $0,1,\cdots,8$ , which is constructed with the help of the geometry of Study's quadric in $\mathrm{PG}(7,2)$ . The tableau consists of nine sets $\Omega_i$ of tetrads (each $\Omega_i$ containing fourteen tetrads), and is considered in connection with certain groups of permutations that leave the tableau, as a whole, invariant. These groups were already known in connection with other interesting geometrical questions [see R. H. Dye, same J. (2) 2 (1970), 746--748; MR0271218 (42 #6101); the author, Proc. London Math. Soc. (3) 4 (1954), 317--342; MR0064050 (16,218c)].

Reviewed by A. Barlotti

**MR0330170 (48 #8508)**
Edge, W. L.
The principal chords of an elliptic quartic.
*Proc. Roy. Soc. Edinburgh Sect. A* **71,** part 1, 43–50. (1972).
14H45

| Citations |
|---|
| From References: 0 |
| From Reviews: 0 |

Author's summary: "The curve $\Gamma$ common to two quadric surfaces has 24 principal chords; they are the sides of six skew quadrilaterals each of which has for its two diagonals a pair of opposite edges of that tetrahedron $S$ which is self-polar for both quadrics. These three pairs of quadrilaterals serve to identify the three pairs of quadrics through $\Gamma$ that are mutually apolar. The vertices of the quadrilaterals lie four on each side of $S$. Both nodes of the plane projection of $\Gamma$ from such a vertex are biflecnodes.''

These geometrical properties, in relation with the work of Severi, are proved using elementary techniques. Some of these properties are also proved with the help of the standard parametric representation of $\Gamma$ by elliptic functions.

Reviewed by Lieven Vanhecke

**MR0309934 (46 #9038)**
Edge, W. L.
Klein's encounter with the simple group of order 660.
*Proc. London Math. Soc. (3)* **24** (1972), 647–668.
20H15 (50A20)

| Citations |
|---|
| From References: 3 |
| From Reviews: 0 |

Wir versuchen, in Stichworten den außergewöhnlich reichhaltigen Inhalt der Arbeit anzudeuten. §§ 1--2 handeln von den bekannten Eigenschaften der einfachen Gruppen $g$ bzw. $G$ der Ordnung 168, bzw. 660, die auch als linear gebrochene Gruppen $\mathrm{LF}(2,7)$ bzw. $\mathrm{LF}(2,11)$ bezeichnet werden. $g$ läßt eine Kurve vierter Ordnung invariant, deren Eigenschaften insbesondere Felix Klein untersucht hat. Der wesentliche Inhalt vorliegender Abhandlung ist die analoge Untersuchung der unter $G$ invarianten Kurve $C$ der Ordnung 20, die bereits Klein bekannt war. Entsprechend den drei Punktgruppen $\mathrm{auf} k$ , die zu zyklischen Untergruppen $C_7$, $C_3$ und $C_2$ von $g$ gehören, gibt es $\mathrm{auf} C$ drei invariante Punktgruppen, die bei zyklischen Untergruppen $C_{11}$ , $C_3$ und $C_2$ von $G$ invariant bleiben. Den acht Wende-dreiecken von $k$ entsprechen 12 Wendepentagone von $C$, etc. § 3: $g$ besitzt zwei konjugierte Systeme von sieben Oktaederuntergruppen, entsprechend besitzt $G$ zwei konjugierte Systeme von 11

Ikosaederuntergruppen. Deren Eigenschaften werden untersucht. Ihnen entsprechen 22 Quadriken in einem $R^4$. In § 4 wird die von Klein entdeckte kubische Hyperfläche $K$, die unter den Kollineationen von $G$ invariant ist, untersucht. § 5: Das Geschlecht $p$ von $C$ wird mittels Abschätzung nach oben und unten zu $p = 26$ bestimmt und daraus Folgerungen gezogen. §§ 6--7 schlie-ßen an Untersuchungen von L. Seifert an [Publ. Fac. Sci. Univ. Masaryk No. 233 (1937), 1--15, especially §§ 6--7; Zbl 16, 370; ibid. No. 235 (1937), 1--9; Zbl 16, 370]. Eigen-schaften der Hesseschen Fläche $H$ einer allgemeinen kubischen Hyperfläche $F$. Falls ein Punkt $A$ auf der Doppelkurve $C$ von $H$ liegt, so liegt die Spitze seiner Polarquadrik auf einer Geraden $q$ von H. Folgerungen hieraus und Vergleich mit den Resultaten von Klein und L. Seifert. § 8: Tangenteneigenschaften der Geraden $q$. § 9: Die Beziehungen zwischen Hyperflächen $R$ der vierten Ordnung durch $C$ und Flächen $Q$ zweiter Ordnung ergeben Relationen zwischen den Ordnungen $n$ und $n'$ der zuge-ordneten Kurven und der Anzahl ihrer Schnitte mit $C$. § 10 behandelt den Spezialfall, wo $Q$ ein Paar von Polyedern ist. § 11: Zusammenhang obiger Beziehungen mit den von O. Hesse gefundenen. § 12: Nach C. Segre gibt es drei Arten von Bündeln quadratischer Kegel im $R^4$. Dies wird auf vorliegendes Problem angewendet. §§ 14--15: Bereits aus den Arbeiten von Klein können die Arten der Projektivitäten hergeleitet werden, bei denen $K$ invariant bleibt. Schneller erhält der Verfasser diese durch Betrach-tung der Charaktere der Gruppe $G$. Je größer die Anzahl der Projektivitäten ist, bei denen eine nicht singuläre kubische Hyperfläche $F$ invariant ist, je mehr geometrisch bedeutsame Eigenschaften besitzt sie. Dies wird näher ausgeführt. §§ 16--17: Folgerungen aus der Tatsache, daß die 55 Involutionen der Gruppe $G$ einer einzigen Klasse von konjugierten Elementen angehören. § 18: Drei der Elemente von $G$ erzeugen die Vierergruppe. Diese wird in eine Tetraedergruppe eingebettet: Geometrische Bedeutung hiervon. Dies führt in § 19 zu einer kubischen Hyper-fläche $F_3$ mit der Untergruppe $C_3$ der Tetraedergruppe. § 20: Betrachtung der zugehörigen Doppelkurve $C_3$. §§ 21–22: Die Tetraedergruppe kann zu einer Ikosaedergruppe erweitert werden; Folgerungen hieraus.

Reviewed by J. J. Burckhardt

**MR0447392 (56 #5704)**

Edge, W. L.

Permutation representations of a group of order 9, 196, 830, 720.

*J. London Math. Soc. (2)* **2** (1970), 753–762.

20C30

| Citations |
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| From References: 1 |
| From Reviews: 0 |

There will be no review of this item.

**MR0306210 (46 #5337)**

Edge, W. L.

The osculating solid of a certain curve in [4].

*Proc. Edinburgh Math. Soc. (2)* **17** (1970/71), 277–280.

14H99 (53A20)

| Citations |
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| From References: 1 |
| From Reviews: 0 |

L. O. Hesse [J. Reine Angew. Math. 41 (1851), 272--284; reprinted in Gesamelte Werke, pp. 263--278, Munich, 1897] found the equation of the osculating plane at the general point of a curve in $S_3$, given as the complete intersection of two surfaces, as a linear combination of the tangent planes to the two surfaces. The corresponding problem for the osculating hyperplane of the complete curve of intersection of three hypersurfaces in $S_4$ does not yet seem to have been approached. The present author makes a beginning by considering the particular case of the curve of intersection of three quadrics with a common self polar simplex, but otherwise general. Taking these in the form (1) $\sum_j x_j{}^2 = 0$ , $\sum_j a_j x_j{}^2 = 0$ , $\sum_j a_j{}^2 x_j{}^2 = 0$ , with $a_0, \cdots, a_4$ all different, he finds the osculating hyperplane at the point $(\xi_0, \cdots, \xi_4)$ to be (2) $\sum_k \{\sum_j a_j{}^4 \xi_j{}^2 - (\sigma - a_k)\sum_j a_j{}^3 \xi_j{}^2\}^2 \xi_k x_k = 0$ , where $\sigma = \sum_j a_j$ . For given $(\xi_0, \cdots, \xi_4)$ satisfying (1), (2) is the equation of the osculating hyperplane; whereas for given general $(x_0, \cdots, x_4)$ , (2) with $(\xi_0, \cdots, \xi_4)$ as variable coordinates is the equation of a quintic hypersurface that cuts the curve in the point of osculation of osculating hyperplanes through $(x_0, \cdots, x_4)$ , which are thus 40 in number.

For the intersection of three hypersurfaces more general than this, the methods used here are only partly applicable, and the problem is still entirely open.

Reviewed by P. Du Val

**MR0262234 (41 #6844)**

Edge, W. L.

The tacnodal form of Humbert's sextic.

*Proc. Roy. Soc. Edinburgh Sect. A* **68** 1970 257–269.

14.01

| Citations |
|---|
| From References: 0 |
| From Reviews: 0 |

The plane sextic curve in question is of genus 5, having (generally) five nodes, and is special in that its canonical model $C$ is the intersection of three quadrics in $S_4$ having a common self polar simplex. The plane curve is the projection of $C$ from a general chord. It is shown that this chord can be so chosen that the singularities of the projected curve are two tacnodes and a biflecnode. $C$ has an obvious group of 16 projective transformations into itself, and the corresponding birational self-transformations of the tacnodal sextic are studied in some detail.

The equations of $C$ can be taken in the form $\sum_{i=0}^4 x_i{}^2 = \sum_{i=0}^4 a_i x_i{}^2 = \sum_{i=0}^4 a_i{}^2 x_i{}^2 = 0$ , where $a_0, \cdots, a_4$ are five distinct constants. It is shown that if $a_0, \cdots, a_4$ are (i) two pairs and one focus (jacobian point) of an involution, (ii) a general triad and its hessian pair, or (iii) permuted cyclically by a homography, $C$ has a larger group of self-transformations, and is to be identified in these cases with curves found by A. Wiman [Stockh. Akad. Bihang 21 (1) (1895)].

Reviewed by P. Du Val

**MR0249435 (40 #2680)**

Edge, W. L.

Three plane sextics and their automorphisms.

*Canad. J. Math.* **21** 1969 1263–1278.

14.20

| Citations |
|---|
| From References: 0 |
| From Reviews: 0 |

The paper treats three special curves ("Wiman-curves'') in projective 4-space in reference to their self-projectivities and, furthermore, projections of Wiman's curves from one of their tangents into five-cusped plane sextics ("Humbert-sextics'').

Reviewed by Manfred Herrmann

**MR0234796 (38 #3110)**

Edge, W. L.; Ruse, H. S.

William Proctor Milne.

*J. London Math. Soc.* **44** 1969 565–570.

01.50

| Citations |
|---|
| From References: 1 |
| From Reviews: 0 |

Obituary with a bibliography of 60 titles.

**MR0218350 (36 #1437)**

Edge, W. L.

A new look at the Kummer surface.

*Canad. J. Math.* **19** 1967 952–967.

14.01

| Citations |
|---|
| From References: 0 |
| From Reviews: 1 |

The surface $\Phi$ of intersection of three quadrics in [5] (five-dimensional complex projective space) that have a unique common self-polar simplex $\Sigma$, which is throughout taken as simplex of reference, is invariant under the group $G$ of order 32, generated by the harmonic homologies $h_0, \cdots, h_5$, where $h_i$ changes the sign of $x_i$ only. $G$ has a subgroup $G^+$ consisting of identity with the 15 products $h_i h_j$. If $\Phi$ is of the form , where and are all different, it is denoted by , and it is this surface that is the topic of this paper. A necessary and sufficient condition for to be is that it contains a line in general position (i.e., not meeting for any ), when it contains 32 lines, permuted transitively by ; each of these meets six others, its images under . The net of quadrics through contains the subsystem , which are pairwise reciprocal with respect to the other quadrics of the net; and another necessary and sufficient condition for to be is that the net contains three linearly independent quadrics of which two are reciprocal with respect to the third.

The tangent plane to at a general point lies on one quadric of the net, and the rest cut it in line pairs of an involution. The double lines of this involution define "asymptotic'' directions at the point of contact, and these in turn define a singly infinite family of curves on , of index 2, whose tangents everywhere are asymptotic. The net is of the same nature as , and the subsystem of the former corresponding to in the latter traces the asymptotic curves on . The envelope of the asymptotic curves consists of the 32 lines on .

The equations were first studied by F. Klein, as those of the congruence of singular lines in a general quadratic complex [Math. Ann. 2 (1870), 198--226; Gesammelte mathematische Werke, Band I, pp. 53--80, Springer, Berlin, 1921]. He recognised in this connexion that is a birational image of the Kummer surface ; and this relation was further developed by H. F. Baker [Principles of geometry, Vol. IV: Higher geometry, Chapter 7, Cambridge Univ. Press, London, 1925]. It is here studied in detail, making use of the fact, pointed out some time ago by the author [J. London Math. Soc. 32 (1957), 463--466; MR0089475 (19,681f)], that the projection of from any line on it is the Weddle surface ; the six nodes of are the images of the six lines on meeting , and that of is the twisted cubic through the six nodes. The 16 nodes of are the images of and its images under ; the 16 conics on are the images of the remaining 16 lines on . is the projective model of the system of octavic curves on passing simply (or an odd number of times) through all its nodes, residual sections of by quartic surfaces through a Rosenhain tetrad of conics. The conics and nodes of a Rosenhain tetrad correspond to a double-four of lines on ; and eight lines on are a double four (and correspond to a Rosenhain tetrad on ) if and only if they are a prime section of . The specialisation of when is a tetrahedroid (wave surface) is also studied.

Reviewed by P. Du Val

**MR0208460 (34 #8270)**

Edge, W. L.

A canonical curve of genus .

*Proc. London Math. Soc. (3)* **17** 1967 207–225.

14.20

| Citations |
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| From References: 0 |
| From Reviews: 0 |

This paper gives a purely geometric description of the canonical curve of genus 7 with a group of 504 automorphisms. The construction begins with a scroll obtained by joining points on a Schur sextic in a solid to the corresponding points on a birationally equivalent canonical quartic in a plane in [6] skew to the solid. By this means, a canonical curve of genus 7 invariant under a harmonic inversion is constructed, and, by specializing the position, it is made invariant under an elementary abelian group of order 8 consisting of harmonic inversions. Further sevenfold symmetry is introduced and a final specialization produces invariance under a transformation of period 3. The elementary abelian group, the cyclic group of order 7 and the final transformation generate the group of order 504.

The geometrical attack on the problem naturally enriches our understanding of the canonical curve by associating other loci with it. Thus there are the Schur sextics and canonical quartics in correspondence with the curve and there are 28 different correspondences of with elliptic curves.

The points with non-trivial stabilizer under the group of order 504 are characterized as the intersections of with chords, planes and the solids containing the Schur sextics. Particularly interesting are those whose stabilizer is of order 3, which are shown to be the Weierstrassian points, each counted with multiplicity 2.

Reviewed by A. M. Macbeath

**MR0175023 (30 #5209)**

Edge, W. L.

Some implications of the geometry of the -point plane.

*Math. Z.* **87** 1965 348–362.

50.70

| Citations |
|---|
| From References: 3 |
| From Reviews: 2 |

The author examines with great thoroughness the geometry of the 21 point projective plane. A large number of configuration theorems are given, and a good look is taken at the group from the geometric point of view. The work is carried out in the classical geometric tradition.

**MR0161204 (28 #4412)**

Edge, W. L.

Fundamental figures, in four and six dimensions, over .

*Proc. Cambridge Philos. Soc.* **60** 1964 183–195.

50.60

| Citations |
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| From References: 0 |
| From Reviews: 0 |

In projective space of dimensions any points, no of which lie in a space , can be taken as basis (vertices of the simplex of reference) and unit point. Over , if , , is such a set, the relation of linear dependence among them is .

The points determine the others (say ); these lie in a space and form a symmetrical system of interlocking polygons, each consisting of the points having one index fixed. The points determine a null polarity in the in which the polar of is the containing the points .

The paper discusses the Richmond-type configurations consisting of the points and the Kummer-type configurations consisting of the points not in the hyper-plane spanned by the points .

Reviewed by T. G. Room

**MR0159877 (28 #3093)**

Edge, W. L.

An orthogonal group of order .

*Ann. Mat. Pura Appl. (4)* **61** 1963 1–95.

20.75 (50.60)

| Citations |
|---|
| From References: 0 |
| From Reviews: 3 |

Of the points in a projective space [the author, Proc. London Math. Soc. (3) 10 (1960), 583--603; MR0120291 (22 #11046)] over the field , 135 points lie on a non-singular ruled quadric . The group of automorphisms of this quadric is of order 2357. It has a simple subgroup of index 2 and order called by Dickson, which fixes each of two systems of 135 solid rulings on . Referred to certain simplexes the equation of the quadric becomes . The points on have, respectively, 1, 4, 5, 8 coordinates not zero. Those 9 with 1 or 8 coordinates not zero form one of 960 enneads that are transitively permuted by . The subgroup fixing an ennead is the

symmetric group of order . All the types of subspaces are described and counted. For example, a -plane is spanned by points, no two conjugate. Given an called there are 70 others in the tangent prime, 2 on each of 35 generators, that are conjugate to , and 64 not conjugate to . Of these 64, there are 28 not conjugate to a chosen one, . Hence there are -planes. The numbers of incidences between pairs of subspaces of each two types are tabulated. Then an exhaustive study is made of the classes of elements according to the types of subspaces that are latent for each. Permutation characters in the classes of the subgroup are given for the representation of degrees 120 and 135 induced by subgroups and , the former the exceptional Lie group and the latter of order having representatives in 55 of the 67 classes which are tabulated. The fact that the group is isomorphic to the central quotient group of a group of monomial matrices was not noted. Finally the matrices of a Sylow subgroup of order 2 are given explicitly, together with their distribution among the classes of .
Reviewed by J. S. Frame

**MR0141708 (25 #5105)**

Edge, W. L.
A second note on the simple group of order .
*Proc. Cambridge Philos. Soc.* **59** 1963 1–9.
20.29

**Citations**
From References: 1
From Reviews: 0

The simple group of order 6048 contains simple subgroups of order 168 and index 36, and can therefore be represented as a permutation group of degree 36. Such a representation was already known, but the author prefers to derive this from a rather complicated study of the geometry of the projective plane of 91 points over the field , in which . The quartic is zero for 28 isotropic points , and not zero for 63 non-isotropic points . Each is the center of one of the 63 conjugate involutions in , each of which belongs to three of the 63 conjugate four-groups , whose 3 involutions have centers at the vertices of one of 63 triangles . Each serves as diagonal point triangle for four quadrangles formed from the points not on the sides of this . Schemes are constructed that consist of pairs of heptads of these triangles, such that each triangle shares a vertex with three members of the opposite heptad. There are 36 such schemes, each invariant under a simple Klein group of order 168. Generators for such a Klein subgroup of are given. The author states that his motivation for this investigation was the known existence of 36 systems of a second kind of contact cubics of a non-singular plane quartic curve, and his desire to study the implication of this for the quartic .
Reviewed by J. S. Frame

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**MR0141708 (25 #5105)** 20.29

**Edge, W. L.**

**A second note on the simple group of order 6048.**

*Proc. Cambridge Philos. Soc.* **59** 1963 1–9

The simple group $U = \mathrm{SU}\,(3, 3^2)$ of order 6048 contains simple subgroups of order 168 and index 36, and can therefore be represented as a permutation group of degree 36. Such a representation was already known, but the author prefers to derive this from a rather complicated study of the geometry of the projective plane of 91 points over the field $J = \mathrm{GF}\,(9)$, in which $\overline{x} = x^3$. The quartic $H_3 = \overline{x}_i x_i = {x_i}^4$ is zero for 28 isotropic points $n$, and not zero for 63 non-isotropic points $p$. Each $p$ is the center of one of the 63 conjugate involutions in $U$, each of which belongs to three of the 63 conjugate four-groups $V$, whose 3 involutions have centers at the vertices of one of 63 triangles $T$. Each $T$ serves as diagonal point triangle for four quadrangles $q$ formed from the $16n$ points not on the sides of this $T$. Schemes are constructed that consist of pairs of heptads of these triangles, such that each triangle shares a vertex with three members of the opposite heptad. There are 36 such schemes, each invariant under a simple Klein group of order 168. Generators for such a Klein subgroup of $U$ are given. The author states that his motivation for this investigation was the known existence of 36 systems of a second kind of contact cubics of a non-singular plane quartic curve, and his desire to study the implication of this for the quartic $H_3 = 0$.

Reviewed by *J. S. Frame*

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**MR0130287 (24 #A153)** 20.20

**Edge, W. L.**

**A permutation representation of the group of the bitangents.**

*J. London Math. Soc.* **36** 1961 340–344

The simple group $\Gamma = \mathrm{PSp}_6(F_2)$ of the 28 double tangents to a plane curve of degree four is represented as a transitive permutation group of degree 120. This representation is derived from a collineation group representation of $\Gamma$ in 6-dimensional projective space over $\mathbf{F} = \mathrm{GF}(2)$, given by the author recently [Proc. London Math. Soc. (3) **10** (1960), 583–603; MR0120291 (22 #11046)]; the permuted objects are certain figures, each consisting of all the 63 points of the quadric $Q$ defined by $\sum_{0\le i<j\le 6} x_i x_j = 0$, of 63 of the 315 lines on $Q$, and of 63 of the 135 planes on $Q$.

Reviewed by *P. Dembowski*

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**MR0122889 (23 #A221)** 20.29
**Edge, W. L.**
**The simple group of order 6048.**
*Proc. Cambridge Philos. Soc.* **56** 1960 189–204

Die einfache Gruppe der Ordnung 6048 wird als Gruppe derjenigen Projektivitäten einer gewissen endlichen Ebene dargestellt, welche die Nullstellen einer ternären Hermiteschen Form invariant lassen. Die neun betrachteten Elemente über dem Galoisfeld GF (3) sind $0, \pm 1, \pm j, \pm j^2, \pm j^3$ und bilden den Körper $J$. Zunächst wird die Gruppe $\mathrm{U}(2, 3^2)$ beschrieben, die aus den zweireihigen Matrizen mit Elementen aus $J$ besteht und die Form $H \equiv \overline{x}_1 x_1 + \overline{x}_2 x_2$ invariant lässt. $x_1$ und $x_2$ werden als homogene Koordinaten einer Geraden $L$ mit zehn Punkten aufgefasst; es liegen auf ihr vier isotrope und sechs nichtisotrope Punkte. In Tabelle 1 wird die Struktur von $\mathrm{U}(2, 3^2)$ festgehalten. Um die Gruppe $\mathrm{SU}(3, 3^2)$ zu erhalten, werden die Elemente von $J$ als homogene Koordinaten $x_1, x_2, x_3$ einer endlichen Ebene mit 91 Punkten aufgefasst. Für die 28 isotropen Punkte verschwindet $\overline{x}_1 x_1 + \overline{x}_2 x_2 + \overline{x}_3 x_3$. Tabelle 2 gibt die Struktur dieser Gruppe an. Zum Schluss wird der Uebergang von der ternären zu der quaternären Gruppe vollzogen.

Reviewed by *J. J. Burckhardt*

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**MR0120291 (22 #11046)** 20.00 (50.00)
**Edge, W. L.**
**A setting for the group of the bitangents.**
*Proc. London Math. Soc. (3)* **10** 1960 583–603

To the quadratic form $\sum_{i<j} x_i x_j$ in seven variables over the field $\mathbf{F}$ of two elements there corresponds a quadric $Q$ in [6], the six-dimensional projective space over $\mathbf{F}$. The group $\Gamma$ of collineations of [6] which leave $Q$ invariant is isomorphic to the simple group of the twenty-eight double tangents to a plane curve of degree four; the order of $\Gamma$ is $1\,451\,520$. The quadric $Q$ consists of 63 points. Of the remaining 64 points one, the kernel of $Q$, is contained in each of the 63 tangent [5]'s (hyperplanes) to $Q$. The author gives a complete description, too complex to be repeated here, of the various configurations formed by the intersections of $Q$ with the subspaces of any dimension in [6]. This description yields geometric interpretations for various properties of $\Gamma$ of which we mention only one simple example: For each tangent [5] to $Q$ there is a unique (necessarily invo-

lutory) elation with this [5] as axis in $\Gamma$. The center of this elation is not on $Q$. Two distinct such elations commute if and only if the line joining their centers is a tangent to $Q$. Other interpretations are given for all elements of orders 2 and 3 in $\Gamma$; also the orders of the normalizers of various elements in $\Gamma$ are determined.

Reviewed by *P. Dembowski*

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From References: 1
From Reviews: 1

**MR0108763 (21 #7475)** 50.00
**Edge, W. L.**
**Quadrics over $\mathrm{GF}(2)$ and their relevance for the cubic surface group.**
*Canad. J. Math.* **11** 1959 625–645

For $n = 2, 3, 4, 5$ the author classifies quadrics $Q$ in projective spaces $[n] = \mathrm{PG}(n, 2)$ over a field of two marks 0, 1, and also classifies linear subspaces of $[n]$ by their relation to $Q$. Tables of incidences between such subspaces when $n = 4$ or $5$ are related to those in C. M. Hamill's paper [Proc. Lond. Math. Soc. (3) **3** (1953), 54–79; MR0055348 (14,1060g)]. A quadric $Q$ in [4] consists of the 15 points $c_{ij}$ of intersection of the quadric $\mathfrak{L}$ in [5] defined by $\sum x_i x_j = 0$, $0 \le i < j \le 5$, with the hyperplane $\sum_0^5 x_i = 0$. The non-ruled quadric $\mathfrak{L}$ in [5] consists of 27 points that correspond to the 27 lines on the cubic surface in such a way that three points on one of the 45 generators of $\mathfrak{L}$ correspond to the three sides of a triangle on the cubic surface. (In [5] if $a_1, a_2, \cdots, a_6$ should denote the columns of the $6 \times 6$ unit matrix $I$, $u$ their sum, and $b_j = u - a_j$, $c_{ij} = b_j - a_i = b_i - a_j$, then the 27 vertices would be assigned the Schläfli labels $a_i$, $b_j$ and $c_{ij}$.) Each of the 36 points $p$ of [5] not on $\mathfrak{L}$ is (like $u$) the center of perspective for two hexads (like the $a_i$ and $b_j$) that correspond to lines of a double six, and each $p$ determines one of 36 involutory matrices $J_p$ that generate a group $G$ of order 51840 isomorphic to the cubic surface group. (If $\tilde{p}$ denotes the row vector for the polar of $p$ with respect to $J_p$, then $J_p = I + p\tilde{p}$.) Matrices $J_{p_1}$ and $J_{p_2}$ commute or not according as $p_1 p_2$ is tangent or skew to $\mathfrak{L}$. The subgroup $S$ of 6! permutation matrices contains representatives of 11 of the 25 classes of $G$, one for each partition of 6. (For $p = a_1 + a_2 + a_3$ the involution $J_p$ serves with $S$ to generate $G$.) The "bitangent" group $\Gamma$ in which $G$ is a subgroup of index 28 is found in a final section to be isomorphic to the $6 \times 6$ symplectic group of matrices $\mu$ for which $\mu' S \mu = S$, where $S$ is a non-singular skew matrix, which may be taken to be the matrix of $\mathfrak{L}$. Under $\Gamma$ a polarity rather than a quadric is left invariant. Cosets of $G$ in $\Gamma$ correspond to the 28 non-ruled quadrics among the 64 quadrics having the same polarity.

Reviewed by *J. S. Frame*

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**MR0099367 (20 #5807)** 20.00
**Edge, W. L.**
**The partitioning of an orthogonal group in six variables.**
*Proc. Roy. Soc. London. Ser. A* **247** 1958 539–549

This paper studies the projective orthogonal group $G$ in six variables over $\mathrm{GF}(3)$. The projective space contains 364 points of which 112 lie on the quadric $Q = {x_1}^2 + \cdots + {x_6}^2 = 0$, and 126 have $Q = +1$, and 126 have $Q = -1$. These points are designated generically as $m, k, l$. This is an extension of two earlier papers [same Proc. **233** (1955), 126–146; Proc. London Math. Soc. (3) **8** (1958), 416–446; MR0076769 (17,941d); **20** #3853].

The conjugate classes of $G$ are partitioned into types depending on the cycle patterns of the points of the geometry distinguishing points of type $m, k, l$. This is done with relative ease using the earlier paper on the similar group in five variables.

Reviewed by *Marshall Hall, Jr.*

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**MR0097384 (20 #3853)** 13.00 (20.00)
**Edge, W. L.**
**The geometry of an orthogonal group in six variables.**
*Proc. London Math. Soc. (3)* **8** 1958 416–446

The finite projective space of five dimensions over the Galois field with the marks $0, 1, -1$ (mod. 3) consists of 364 points. In it are two kinds of non-singular quadric; the paper is interested in the type with 112 points which has no planes on it; the equation of $Q$ may be written as $\sum {x_i}^2 = 0$ $(i = 1, \cdots, 6)$. There are two batches of 126 points off $Q$, points $k$ and points $l$, according as the $\sum$ is 1 or -1. The author studies the simple group $G^*$ of $2^7 \cdot 3^6 \cdot 5 \cdot 7$ projectivities which leave $Q$ invariant, do not transpose the categories $k$ and $l$, have determinant 1 and permute the 126 points of each batch evenly. The group was known to L. E. Dickson, who, however, did not give any geometrical application. The group is transitive on the 567 "positive" simplexes, that is, self-polar simplexes of $Q$ whose vertices are all $k$ points. Led by his geometrical considerations, the author studies the Sylow 2-groups (of order $2^7$) of $G^*$ and the subgroups $S_3$. This paper contains many other noteworthy results in the geometry considered, such as the 540 null systems which reciprocate $Q$ into itself and the 5184 heptahedra that are circumscribed to $Q$.

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**MR0089475 (19,681f)** 14.0X

**Edge, W. L.**

**Baker's property of the Weddle surface.**

*J. London Math. Soc.* **32** (1957), 463–466.

The Weddle surface is the locus of vertices in cones among the $\infty^3$ quadrics through six general points $N_1, \cdots, N_6$ in [3], and the property in question is that the six involutory birational self-transformations of the surface which interchange pairs of points collinear with $N_i$ $(i = 1, \cdots, 6)$ commute amongst themselves, and that the product of all six is identity, so that they generate an abelian group of order $2^5$.

It is shown that the surface is the projection of the surface in [5] common to a net of quadrics having a common self polar simplex and a common line $l$, from $l$. Baker's six involutions are the transforms under the projection of the six harmonic homologies whose invariant elements are a vertex and opposite face of the simplex. The images of $l$ under these homologies meet $l$ and project into the neighbourhoods of $N_1, \cdots, N_6$; its images under the remaining operations of the group project into 10 lines $N_iN_j$ and 15 lines $N_iN_jN_k \cdot N_pN_qN_r$; $l$ itself projects into the cubic through $N_1, \cdots, N_6$.

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**MR0085998 (19,108p)** 01.0X

**Edge, W. L.**

**Obituary: H. F. Baker, F. R. S.**

*Edinburgh Math. Notes* **1957** (1957), *no. 41*, 10–28.

A general and scientific notice, with a complete list of Baker's books and a partial list of his articles.

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**MR0083443 (18,710n)** 01.0X
**Edge, W. L.**
**Obituary: Miss C. M. Hamill.**
*Edinburgh Math. Notes.* **1956** (1956), *no. 40*, 22–25.

A general and scientific biography.

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**MR0081287 (18,377g)** 20.0X
**Edge, W. L.**
**The characters of the cubic surface group.**
*Proc. Roy. Soc. London Ser. A.* **237** (1956), 132–147.

Continuing his previous study of the cubic surface group $G$ of order 51840 [same Proc. **228** (1955), 129–146; **233** (1955), 126–146; MR0069522 (16,1046a); **17**, 941], the author uses a geometrical method, based on the 5-rowed orthogonal representation of $G$ over $\mathrm{GF}(3)$, to obtain the characters of 27 reducible induced representations of $G$, none with more than 3 irreducible components. These are induced by certain geometrically significant subgroups of $G$ of index 27, 36, 40 and 45. From linear combinations of these all but one of the 25 irreducible characters of $G$ are found, and the missing one is obtained from orthogonality. For example, not only are 27 pentagons in $\mathrm{PG}(4,3)$ permuted by $G$, but their vertices suitably ordered are permuted by the operations of the symmetric group $S_5$, six of whose irreducible representations induce "useful" representations of $G$. The characters obtained for $G$ agree with those previously obtained by Frame [Ann. Mat. Pura Appl. (4) **32** (1951), 83–119; MR0047038 (13,817i)].

Reviewed by *J. S. Frame*

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**MR0080311 (18,227d)** 50.0X
**Edge, W. L.**
**Conics and orthogonal projectivities in a finite plane.**
*Canad. J. Math.* **8** (1956), 362–382.

Any non-singular conic in a finite Desarguesian plane can be put in the form $x^2+y^2+z^2=0$ by appropriate choice of the triangle of reference, if there are $q+1$ points on a line, $q$ being a power of a prime $p>2$. Minus one is a square in GF$(q)$ if $q\equiv 1\pmod 4$ and a non-square if $q\equiv 3\pmod 4$. Starting from this the author studies in some detail the relation of the orthogonal group $\Omega(3,q)$ to the geometry of the plane and in particular its relation to the canonical conic above. Of particular interest are the cases $q=5,7,11$, since for these values and no others $\Omega(3,q)$ has a permutation representation of degree $q$. For $q=5$, $7$ the $q$ objects permuted may be considered canonical triangles, these being the whole set for $q=5$, and for $q=7$, the members of either one of two imprimitive systems. For $q=11$ the objects may be taken as the Clebsch hexagons.

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**MR0076769 (17,941d)** 20.0X
**Edge, W. L.**
**The conjugate classes of the cubic surface group in an orthogonal representation.**
*Proc. Roy. Soc. London. Ser. A.* **233** (1955), 126–146.

The cubic surface group of order 51840, whose classes and characters were previously obtained by Frame [Ann. Mat. Pura Appl. (4) **32** (1951), 83–119; MR0047038 (13,817i)], is studied geometrically using its representation as the group of $5\times 5$ orthogonal matrices of determinant 1 over GF$(3)$. For such a matrix $M$ the null spaces of $M-I$ and $M+I$ are denoted by $S_+$ and $S_-$ and have even and odd dimension, respectively. According as the join $\Sigma$ of $S_+$ and $S_-$ is the whole projective 4-space $\alpha$, a plane, or a point, the matrix $M$ is said to be of category $A$, $B$, or $C$. Forty "$m$ points" of $\alpha$ lie on the invariant quadric $\omega$ defined by $x'x\equiv 0\pmod 3$, 36 "$f$ points" have $x'x\equiv -1$, and 45 "$h$ points" have $x'x\equiv 1$. The orthogonal matrices $M$ are classified into 21 types according to the relationships between $S_+$, $S_-$ and $\omega$, and a further refinement produces the 25 classes of conjugates for the group by geometrical considerations. For example, the classes $A_3(h,H)$ and $A_3(f,F)$ contain respectively 45 and 36 involutions determined by an $h$ or $f$ point and its polar solid. The author did not mention the interesting fact that the subgroup $K$ of monomial matrices is of index 27 and corresponds to the subgroup leaving fixed one of the 27 lines $L$ on the cubic surface. Sixteen of the classes are represented in this subgroup. The matrices having four 0's in just one row and four in one column form a double coset of $K$ containing 10 cosets that take $L$ into an intersecting line and contain representatives of all the other classes. Finally the remaining matrices having five 0's, one in each row and column, form a double coset containing 16 cosets that take $L$ into a line skew to $L$. The author illustrates the advantages of studying the cubic surface group by means of the geometry in $\alpha$, but makes no attempt to go beyond the classification into conjugate sets to obtain the group characters, which was the main purpose of Frame's paper.

Reviewed by *J. S. Frame*

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**MR0071033 (17,72f)** 48.0X

**Edge, W. L.**

31**-point geometry.**

*Math. Gaz.* **39,** (1955). 113–121

This is a detailed treatment of various aspects of the projective plane with 31 points. Special properties of cross-ratios, conics, collineations, and polygons are all discussed.

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**MR0069522 (16,1046a)** 48.0X

**Edge, W. L.**

**Line geometry in three dimensions over GF(3), and the allied geometry of quadrics in four and five dimensions.**

*Proc. Roy. Soc. London. Ser. A.* **228,** (1955). 129–146

In this sequel to an earlier paper [same Proc. **222**, 262–286 (1954); MR0061394 (15,818g)] the author describes the line geometry of a projective three space $S$ over a field $K$ of three marks 0, 1, -1, and the related geometry of quadrics in [5] or [4] over $K$. Pluecker coordinates of the 130 lines of $S$ define the 130 points $L$ on a quadric $\Omega$ in projective 5-space defined by $j=0$, where $j = p_{14}p_{23}+p_{24}p_{31}+p_{34}p_{12}$. The 234 points $A$ in [5] not on $\Omega$ are divided into equal "positive" and "negative" sets of 117 points according as $j=1$ or $-1$, each set invariant under $\mathrm{PO}_2(6,3)$. Each $A$ is the pole of a prime $\alpha$ in [5] that intersects $\Omega$ in a quadric $\omega$ whose 40 points $m$ correspond to the 40 lines of a screw (non-degenerate linear complex) $\sigma$ in $S$. The screw $\sigma$ and each of its 4 line reguli are given a sign determined by $A$. Each hyperboloid in $S$ has two complementary reguli of opposite sign. Forty five of the points of $\alpha$ not on the quadric $\omega$ lie on chords to $\Omega$ through $A$. Each of these points is a vertex of three different pentagons $\wp$ in $\alpha$ that are self polar with respect to $\omega$. Each of the 27 pentagons so formed has 10 edges skew to $\omega$, and 10 planes each containing 4 points of $\omega$ that correspond to a regulus in $S$. The configuration of 27 self-polar pentagons with their 45 vertices is shown to be isomorphic with the configuration of the 27 lines and 45 triangles on a cubic surface, and with the configuration built by Baker on the 45 nodes of the Burkhardt primal. Thus in a natural way a subgroup $\omega(5,3)$ of index 117 in $\mathrm{PO}_2(6,3)$ is identified with the cubic surface group of order 51840, and the four classes of involutions in this group are identified with the four types of harmonic inversion in $\alpha$ whose fundamental spaces are polars in regard to $\omega$.

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**MR0067119 (16,672d)** 20.0X
**Edge, W. L.**
**The isomorphism between $LF(2,3^2)$ and $\mathcal{A}_6$.**
*J. London Math. Soc.* **30,** (1955). 172–185

L'isomorphisme (classique) entre les deux groupes indiqués dans le titre est obtenu ici comme conséquence d'une étude approfondie de la géométrie de la droite projective sur le corps $J$ à 9 éléments. L'auteur remarque d'abord que si 4 points de la droite forment une division harmonique quand on les prend dans un certain ordre, il en est encore ainsi quand on les permute arbitrairement. Il décrit ensuite un procédé intrinsèque pour associer en "quintuples" les diverses divisions harmoniques, de sorte que dans chaque quintuple de division harmoniques, un point de la droite apparaisse exactement 2 fois. Ces quintuples sont euxmêmes au nombre de 12, et se divisent en deux classes d'intransitivité pour le groupe projectif; il est facile alors de voir que ce groupe, considéré comme groupe de permutations d'une de ces classes, s'identifie au groupe alterné sur 6 objets. L'auteur étudie aussi les formes hermitiennes à 2 variables sur $J$ et montre que les droites où elles s'annulent correspondent exactement aux points d'une division harmonique. Enfin, il décrit sommairement l'isomorphisme classique entre le groupe projectif de la droite sur $J$ et le “second groupe orthogonal projectif” à 2 variables sur le corps à 3 éléments.

Reviewed by *J. Dieudonné*

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**MR0064050 (16,218c)** 20.0X

**Edge, W. L.**

**The geometry of the linear fractional group** $\mathrm{LF}(4,2)$**.**

*Proc. London Math. Soc. (3)* **4,** (1954). 317–342

Although the isomorphism of the linear fractional group LF $(4,2)$ and the alternating group $A_8$ has been established by C. Jordan [Traité des substitutions, Gauthier-Villars, Paris, 1870], E. H. Moore [Math. Ann. **51**, 417–444 (1898)], and G. M. Conwell [Ann. of Math. (2) **11**, 60–76 (1910)], it is demonstrated anew in this paper through a study of linear complexes or screws in the three-dimensional finite projective geometry $S_3$ over a field $F$ of two marks $0,1$. Using three pairs of Pluecker line coordinates $p_{jk}, p_{i4}$, where $ijk$ is one of three cyclic permutations of $1,2,3$, the equation $\sum a_i p_{jk} + a_i' p_{i4} = 0$, represents a screw if $\sum a_i a_i' = 1 \pmod 2$, or a sheaf if $\sum a_i a_i' = 0$. For $i = 1,2,3$, let us assign the coordinate $0, u, v$ or $1$ to a screw according as $(a_i, ai')$ is $(0,0), (1,0), (0,1)$ or $(1,1)$. Then either all of the 3 coordinates are 1's, or one coordinate is 1 and the other two are arbitrarily chosen from $0, u, v$. The sum of two screws is defined by adding corresponding coordinates $\pmod 2$, noting that $u + v + 1 = u + u = v + v = 1 + 1 = 0 \pmod 2$. If this sum defines the coordinates of a third screw, the three screws form a trio in which each screw is called azygetic to the others. The screws of a trio share a quintuple of skew lines which determines the trio. The seven screws $\sigma_{01} = (10u)$, $\sigma_{02} = (10v)$, $\sigma_{03} = (0u1)$, $\sigma_{04} = (0v1)$, $\sigma_{05} = (v10)$, $\sigma_{06} = (u10)$, $\sigma_{07} = (111)$ form a heptad of mutually azygetic screws and the other 21 screws have coordinates defined by $\sigma_{ij} = \sigma_{0i} + \sigma_{0j}$, $i \neq j$. There are eight heptads of 7 mutually azygetic screws, each consisting of the screws having a fixed subscript (from 0 to 7) in common. The transformations of LF $(4,2)$ correspond in a one-to-one manner with the even permutations on these heptads, and this establishes the isomorphism of LF $(4,2)$ and $A_8$. The 35 lines in $S_3$ correspond uniquely to the separations of the eight heptads into two complementary sets of 4, whereas the 56 trios correspond to the 56 triples of heptads, each pair sharing one screw of the trio. Further studies are made of the geometry associated with certain subgroups of $A_8$.

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**MR0061394 (15,818g)** 48.0X
**Edge, W. L.**
**Geometry in three dimensions over $\mathrm{GF}(3)$.**
*Proc. Roy. Soc. London. Ser. A.* **222,** (1954). 262–286

This is a detailed discussion of the three-dimensional geometry with 40 points. Particular study is made of the quadric surfaces, the hyperboloids, equivalent to $x^2+y^2+z^2+t^2=0$, and the ellipsoids, equivalent to $x^2+y^2+z^2-t^2=0$. This geometry is of particular interest with respect to the simple group of order 360 which is simultaneously the alternating group on six letters $\mathrm{LF}(2,3^2)$ and $\mathrm{PO}_2(4,3)$.

Reviewed by *Marshall Hall, Jr.*

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**MR0042154 (13,62c)** 14.0X
**Edge, W. L.**
**Humbert's plane sextics of genus $5$.**
*Proc. Cambridge Philos. Soc.* **47,** (1951). 483–495

G. Humbert [J. École Polytech. Cahier (1) **64**, 123–149 (1894)] studied a twisted curve $C^7$ of order 7 and genus 5, the locus of points of contact of lines through a fixed point $N_0$ with twisted cubics through five other fixed points $N_1,\cdots,N_5$. H. F. Baker [... Multiply Periodic Functions, Cambridge Univ. Press, 1907, pp. 322–326] obtained the same curve as the apparent contour from $N_0$ of the Weddle surface with nodes at $N_0,N_1,\cdots,N_5$. A distinctive feature, pointed out by both authors, is that $C^7$ lies on five elliptic cubic cones (with vertices at $N_1,\cdots,N_5$ respectively) and is bisecant to the generators of each, so that five of its everywhere finite integrals are elliptic, i.e. have only two independent periods.

The purpose of the present paper is to shew that this curve is the same as that whose canonical model is the intersection of three quadrics in four dimensions with a common self polar simplex. He easily shews that the latter curve has five elliptic integrals, and an abelian group of 32 birational self transformations, and deduces from this many elegant geometrical properties of the projections of the canonical curve from a point, chord, and tangent of itself, which were noted by Humbert of his curve and its plane projections, and some of which he found to be sufficient to characterise the latter. Oddly, the author seems to give no proof that the possession by a curve of genus 5 of

5 elliptic integrals is a sufficient condition for all quadrics through the canonical model to have a common self polar simplex, which seems to be needed to establish the identity of the two curves. It is not hard, however, to fill in this gap in a variety of ways.

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**MR0040690 (12,735b)** 14.0X
**Edge, W. L.**
**A plane quartic with eight undulations.**
*Proc. Edinburgh Math. Soc. (2)* **8,** (1950). 147–162

Two types of plane quartic with 12 undulations are known, the first being Dyck's quartic $x^4 - y^4 - z^4 = 0$, and the second that discovered recently by Edge [Edinburgh Math. Notes no. **35,** 10–13 (1945), p. 11; MR0014731 (7,324h)]. Masoni, in his original paper on quartics with undulations [Rend. Accad. Sci. Fis. Mat. Napoli (1) **21,** 45–69 (1882)], missed the second type because of a surprising flaw in his reasoning. The present paper explains this and throws much new light on the whole question by showing how to construct geometrically quartics with only 8 undulations, and by proving that these can be specialized into quartics with 12 undulations in two distinct ways. The construction derives a unique quartic with 8 undulations from a conic and a line-pair in the plane, and the author obtains the equation of a family of such curves in the form

$$x^4 - y^4 - z^4 + 4fx^2yz + 2f^2y^2z^2 = 0.$$

In this family, excluding the degenerate curves given by $f^4 = 1$, the only values of $f$ which give curves with more than 8 undulations are (i) $f = 0$, which gives Dyck's quartic, and (ii) those given by $f^4 = 81$, which give quartics of Edge's type, also possessing 12 undulations. The geometry of the family is exhibited in detail.

Reviewed by *J. G. Semple*

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**MR0033562 (11,457b)** 14.0X
**Edge, W. L.**
**The Kummer quartic and the tetrahedroids based on the Maschke forms.**
*Proc. Cambridge Philos. Soc.* **45,** (1949). 519–535

If $\Lambda_1, \cdots, \Lambda_5$ are the forms $x^4+y^4+z^4+t^4$, $xyzt$, $y^2z^2+x^2t^2$, $z^2x^2+y^2t^2$, $x^2y^2+z^2t^2$, the equation

$$\Lambda_1+2D\Lambda_2+A\Lambda_3+B\Lambda_4+C\Lambda_5=0$$

represents a linear $\infty^4$ system $\Lambda$ of quartic surfaces which contains, as is well known, an $\infty^3$ system $\Lambda_K$ of Kummer surfaces, represented in the parameter space $S_4(A,B,C,D)$ by the Segre cubic primal $\Gamma$ whose equation is

$$ABC+D^2+4=A^2+B^2+C^2.$$

In part I of the present paper the author indicates the substantial advantages to be gained, in the study of the system $\Lambda$, by writing the equation of this system in the form $\sum_1^6 J_i\Phi_i=0$, $\sum_1^6 J_i=0$, where the $J_i$ are parameters and the $\Phi_i$ are the six Maschke forms $\Lambda_1-6(\Lambda_3+\Lambda_4+\Lambda_5)$, $\Lambda_1+6(-\Lambda_3+\Lambda_4+\Lambda_5)$, $\Lambda_1+6(\Lambda_3-\Lambda_4+\Lambda_5)$, $\Lambda_1+6(\Lambda_3+\Lambda_4-\Lambda_5)$, $-2\Lambda_1+24\Lambda_2$, $-2\Lambda_1-24\Lambda_2$, these being connected by the identity $\sum\Phi_i\equiv 0$. The main points are that any two of the surfaces $\Phi_i=0$ bear the same relation to one another; the vanishing of the differences $\Phi_i-\Phi_j$ give the sets of four planes which form Klein's 15 fundamental tetrahedra; the sums of triads of the $\Phi_i$ give the squares of Klein's 10 fundamental quadrics; and the equations of $\Gamma$ in the space of the parameters $J_i$ are $\sum J_i^3=\sum J_i=0$.

Part II is mainly concerned with the construction of the discriminant $D$ of $\Lambda$; this is obtained in the form $D\equiv(\sum J_i^3)^{16}\prod(J_i+J_j)^4$, each linear factor (associated with a solid which meets $\Gamma$ in three planes) arising from a family of quartic surfaces of $\Lambda$ which have nodes at the vertices of a fundamental tetrahedron. The systems of scrolls of $\Lambda$, each with a pair of opposite edges of one of the tetrahedra as double lines, are also exhibited.

Part III deals with the tetrahedroids belonging to $\Lambda$, a tetrahedroid being a Kummer surface whose 16 nodes lie by fours in the faces of a fundamental tetrahedron. For a surface of $\Lambda$ to be a tetrahedroid it is necessary that two of the $J_i$ should be equal. This gives principally 15 doubly infinite systems of tetrahedroids represented by the 4-nodal cubic surfaces in which $\Gamma$ is met by the solids $\Pi_{ij}$ whose equations are $J_i=J_j$. Multiple tetrahedroids, and, in particular, 30 sextuple tetrahedroids, correspond to points common to several of these cubic surfaces. It appears, however, that $\Lambda$ contains 30 other sextuple tetrahedroids, not represented by points of $\Gamma$, which correspond to intersections of sets of four of the solids $\Pi_{ij}$. One of these has equation $x^4+y^4+z^4+t^4=4xyzt$. The paper concludes with a discussion of the simply infinite systems of triple tetrahedroids, these being represented on $\Gamma$ by nodal cubic curves.

Reviewed by *J. G. Semple*

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**MR0023802 (9,406d)** 09.0X
**Edge, W. L.**
**The discriminant of a certain ternary quartic.**
*Proc. Roy. Soc. Edinburgh. Sect. A.* **62,** (1948). 268–272

The author shows by actual calculation that the discriminant $D$ of the quartic

$$ax_1{}^4 + bx_2{}^4 + cx_3{}^4 + 6fx_2{}^2x_3{}^2 + 6gx_3{}^2x_1{}^2 + 6x_1{}^2x_2{}^2$$

is $abcA^2B^2C^2\Delta^4$, where

$$\Delta = \begin{vmatrix} & a3h3g \\ 3h & b3f \\ 3g3f & c \end{vmatrix}$$

and $A$, $B$ and $C$ are the cofactors of $a$, $b$ and $c$, respectively. In calculating $D$ the author follows a procedure attributed to Gordan by Klein [F. Klein, Math. Ann. **36,** 1–83 (1890)] in which $D$ is obtained as a determinant of order 15.

Reviewed by *J. Williamson*

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**MR0024625 (9,524c)** 48.0X
**Edge, W. L.**
**The Klein group in three dimensions.**
*Acta Math.* **79,** (1947). 153–223

The Klein group is the simple group of order 168, exhibited by Klein as a group of ternary linear substitutions for which a plane quartic $k$ is invariant. An isomorphic group of collineations in three-dimensional space is studied by the author and the geometry of the figure is that of a net of quadrics invariant for the group. His starting points are the original publications of Klein, a note of Baker [Proc. Cambridge Philos. Soc. **31,** 468–481 (1935)] and his own investigations on nets of quadric surfaces [Proc. London Math. Soc. (2) **43,** 302–315 (1937); **44,** 466–480 (1938); **47,** 123–141 (1941); J. London Math. Soc. **12,** 276–280 (1937); cf. MR0004491 (3,15b)]. The left side of the equation of $k$ may be obtained as a symmetrical four-rowed determinant, the elements being linear forms in the coordinates $(\xi, \eta, \zeta)$. This determinant can be regarded as the discriminant of a quaternary quadratic form; this form, when equated to zero, is a quadric which, as $\xi, \eta, \zeta$ vary, varies in a net of quadrics. The locus of the vertices of the cones of the net is its Jacobian, a sextic twisted curve $K$, which is hereby put in one-to-one correspondence with $k$. Six coplanar points of $K$ correspond to six points of $k$ which are the points of contact of $k$ with a contact cubic. There are 36 systems of contact cubics; one of these is so associated with the planes of space and the group of 168 ternary substitutions is associated with a group of collineations in space for which the net is invariant. The net has eight base points and eight base planes; they are permuted by the collineations of the group. There are 24 points $c$ on $K$ where the osculating plane has five-point contact, and the sixth intersection with $K$ is a point $c$ itself. The number of involutions of the group $G_{168}$ is twenty-one; they are all biaxial; the forty-two axes consist of fifty-six triads of lines, the lines of each triad being both concurrent and coplanar; the axes are principal chords of $K$. There is one and only one quartic surface invariant for $G$; it intersects $K$ in the 24 points $c$. There is one and only one sextic surface invariant for $G$. The invariant octavic surfaces are investigated; when they do not contain $K$ they touch $K$ at each of the 24 points $c$. The author studies the covariant line complexes of the group and makes remarks on the covariant cubic and quartic complexes; this material is obviously relevant to a group of 168 substitutions on six variables.

Reviewed by *O. Bottema*

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**MR0015857 (7,479c)** 14.0X

**Edge, W. L.**

**Conics on a Maschke surface.**

*Proc. Edinburgh Math. Soc. (2)* **7,** (1946). 153–161

This is a sequel to the author's paper on Maschke quartic surfaces [same Proc. (2) **7,** 93–103 (1945); MR0012776 (7,71e)]. These surfaces have equations

$$\varphi_1 \equiv \sum x^4 - 6(y^2z^2 + x^2t^2 + z^2x^2 + y^2t^2 + x^2y^2 + z^2t^2),$$
$$\varphi_2 \equiv \sum x^4 + 6(-y^2z^2 - x^2t^2 + z^2x^2 + y^2t^2 + x^2y^2 + z^2t^2),$$
$$\varphi_3 \equiv \sum x^4 + 6(y^2z^2 + x^2t^2 - z^2x^2 - y^2t^2 + x^2y^2 + z^2t^2),$$
$$\varphi_4 \equiv \sum x^4 + 6(y^2z^2 + x^2t^2 + z^2x^2 + y^2t^2 - x^2y^2 - z^2t^2),$$
$$\varphi_5 \equiv -2\sum x^4 - 24xyzt, \quad \varphi_6 \equiv -2\sum x^4 + 24xyzt$$

and they are closely associated with Klein's configuration of desmic tetrahedra derived from six linear complexes mutually in involution. Each of the fifteen differences $\varphi_i - \varphi_j$ gives one such tetrahedron of four planes. A line $d$ passes through three vertices, one from each of three desmic tetrahedra. Each of eight planes through $d$ touches $M$, any one of the six surfaces, at four distinct points, and meets $M$ in two conics through these points. The same eight planes through $d$ belong to two of the six surfaces. One surface has in all 240 such plane sections; 160 others meet the surface in pairs of conics having double contact.

Various properties are worked out, including those of the Hessian $H$ of $M$. The Hessian contains 40 conics which form the complete intersection of $H$ with the fundamental quadrics of Klein.

Reviewed by *H. W. Turnbull*

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**MR0014731 (7,324h)** 14.0X

**Edge, W. L.**

**A plane quartic curve with twelve undulations.**

*Edinburgh Math. Notes* **1945,** (1945). *no. 35,* 10–13

Among the curves of the pencil

$$x^4 + y^4 + z^4 + \lambda(y^2z^2 + z^2x^2 + x^2y^2) = 0$$

(which are all invariant under an octahedral group of collineations), there exists one, namely Dyck's curve, given by $\lambda = 0$, whose inflections are known to coincide in pairs at twelve undulations. The author shows that another curve of the pencil, namely that for which $\lambda = 3$, has the same property; his subsequent discussion of this curve includes a simple geometrical construction for the configuration of the twelve undulations.

Reviewed by *J. G. Semple*

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**MR0012776 (7,71e)** 14.0X

**Edge, W. L.**

**The geometrical construction of Maschke's quartic surfaces.**

*Proc. Edinburgh Math. Soc. (2)* **7,** (1945). 93–103

The author's mathematical curiosity was aroused by a reference of Burnside to a quartic surface whose homogeneous equation is $x^4 + y^4 + z^4 + t^4 + 12xyzt = 0$ [Theory of Groups of Finite Order, Cambridge University Press, 1911, p. 371] and which is invariant for a group of $2^4$ 5! collineations. The quartic form on the left of the equation appears as one of a set of six associated forms in a paper by Maschke [Math. Ann. **30**, 496–515 (1887)]. The quartic curve $D$ in which this surface intersects one of the reference planes gives rise to Dyck's configuration of 12 flecnodal and 16 other bitangents which are here described. The author then studies the surface itself in relationship to Klein's space configuration, which arises from a set of six linear complexes any two of which are in involution. The fifteen pairs of directrices of this configuration are each a pair of opposite edges for three of the fifteen fundamental tetrahedra. Six different synthematic totals of five tetrahedra exhaust the thirty edges, and with each of the six totals is associated one of Maschke's quartic surfaces, $\Phi_i = 0$, $i = 1, 2, \cdots, 6$, where $\Phi_i$ are homogeneous forms of degree 4 whose sum vanishes. The identity $4\sum \Phi_i\Phi_j\Phi_k\Phi_l = \{\sum \Phi_i\Phi_j\}^2$ defines a quartic primal $\Gamma$ whose 15 nodal lines constitute a famous configuration studied by Segre, Castelnuovo and Baker. A group of $2^6$ quaternary substitutions on $(x, y, z, t)$ corresponding to $2^4$ collineations on projective three-space leaves each $\Phi_i$ invariant. The direct product of this group with the group of permutations of the six $\Phi_i$ is of order $2^6$ 6!, and has $12I = \sum {\Phi_i}^2$ as an invariant of degree 8 in $x, y, z, t$. The subgroup leaving one particular $\Phi_i$ fixed is the group of order $2^4$ 5! with which the discussion began.

Reviewed by *J. S. Frame*

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**MR0011001 (6,102a)** 52.0X
**Edge, W. L.**
**The identification of Klein's quartic.**
*Proc. Roy. Soc. Edinburgh. Sect. A.* **62,** (1944). 83–91

Some forty years ago a paper appeared, the purpose of which was to correct a statement concerning a certain curve [Coble, Trans. Amer. Math. Soc. **4,** 65–85 (1903)]. The correction of this mistake led to further investigation of covariants connected with this curve, including the present paper [Edge, same Proc. Sect. A. **61,** 140–159 (1941); **61,** 247–259 (1942); MR0005660 (3,184f); **4,** 167]. The purpose of this present note is to show that all the properties mentioned in these and in various other papers by different authors are consequences of one fact. It concerns Klein's plane quartic curve belonging to the simple linear group of order 168. In the representation of the Veronese quartic surface $F$ in [5] the quadric polars of $F$ as to the cubic primal generated by its chords were outpolar as to a quadric arising from unravelment. In the present case, this quadric is not only outpolar but also inpolar as to $F$. If this condition is satisfied, all the properties of the Klein quartic follow.

Reviewed by *V. Snyder*

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**MR0008466 (5,10g)** 14.0X
**Edge, W. L.**
**The contact net of quadrics.**
*Proc. London Math. Soc. (2)* **48,** (1943). 112–121

Given a tetrahedron $T$ with vertices at $A$, $B$, $C$, $D$. A contact net of quadric surfaces is defined as a net which touches a fixed tangent line at each vertex. These lines are not arbitrary; the necessary and sufficient condition that the system form a net is that each line meets the opposite face of $T$ in $A'$, $B'$, $C'$, $D'$ which with the vertices of $T$ form a pair of Möbius tetrads. If $T$ is taken as the tetrahedron of reference, the vertices of $T'$ are their reciprocals. The relations between $T$ and $T'$ were discussed in an earlier paper [Edge, Proc. London Math. Soc. (2) **41,** 337–360 (1936)]. Various additional properties are derived in the present paper by algebraic methods. The equations of the defining quadrics of the net are obtained. The base points of a Möbius net are given in terms of the parameters of the net by the rows of a certain matrix and of the reciprocals of these elements.

The transversal lines of $AA'$, $BB'$, etc., are transformed into space cubic curves by the same Cremona transformation; these curves all pass through the vertices of $T$ and $T'$. The Jacobian of the net, locus of the vertices of quadric cones in the net consist of these two cubic curves. The trisecant lines of this composite Jacobian are discussed. Their locus consists of two quartic ruled surfaces, each having one cubic curve as double curve and passing simply through the other. The properties of the net can also be obtained by starting with these two quartic ruled surfaces.

Reviewed by *V. Snyder*

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**MR0008170 (4,254a)** 14.0X
**Edge, W. L.**
**Notes on a net of quadric surfaces. V. The pentahedral net.**
*Proc. London Math. Soc. (2)* **47,** (1942). 455–480

Four earlier papers by the same author, the last of which [Proc. London Math. Soc. (2) **47,** 123–141 (1941); MR0004491 (3,15b)] contains the references to the earlier ones, dealt for the most part with general nets of quadric surfaces in [3]. The present note is concerned with a special net having a self conjugate pentahedron. This condition is poristic; the existence of one such pentahedron involves the existence of a singly-infinite set [Reye, J. Reine Angew. Math. **82,** 75–94 (1887)].

The faces of the pentahedra all belong to a developable $\omega$ of the third class, osculating a space cubic curve $\gamma$. The sets of osculating planes belong to a $g_5{}'$. A plane of $\omega$ may be defined by

$$P \equiv x_0 + \theta x_1 + \theta^2 x_2 + \theta^3 x_3.$$

Given two sets of five values of $\theta$, each defined by a quintic equation $(\theta) = 0$, $g(\theta) = 0$, with constant coefficients; the $g_5{}'$ is given by the pencil $\lambda f(\theta) + \mu g(\theta) = 0$. A net of quadrics is given as functions of $\theta$. The Jacobian curve $\vartheta$, a sextic which is the locus of the vertices of the cones in the net, is then considered, including the ruled surface $R^8$ of its trisecants and various apolarity properties. A $(1, 1)$ correspondence exists between the quadrics of the net and the axes of the developable $F_4$; $R^8$ and $F_4$ have eight common generators.

In the second part, lines of [3] are represented by the points of a quadric primal $\Omega$ in [5]; the generators of $R^8$ are represented by the points in which $\Omega$ is met by a Veronese surface, but in the present case this lies on $\Omega$. In the third part the net is still further specialized; the pencil of binary forms is now $\lambda\theta + \mu\theta = 0$. The net is now defined by

$$Q_0 \equiv {x_2}^2 + 2x_1x_3, \quad Q_1 \equiv 2x_0x_3 + 2x_1x_3, \quad Q_2 \equiv {x_1}^2 + 2x_0x_2.$$

The base consists of $X_0$ and $X_3$ of the reference tetrahedron, each taken twice. The Jacobian curve consists of two space cubics which touch at each base point; $R^8$ is composed of two ruled quartics, having $\gamma_1, \gamma_2$ for double curves, and each containing the other simply.

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**MR0007610 (4,167d)** 14.0X
**Edge, W. L.**
**Sylvester's unravelment of a ternary quartic.**
*Proc. Roy. Soc. Edinburgh. Sect. A.* **61,** (1942). 247–259

In an earlier paper [same Proc. **61,** 140–159 (1941); MR0005660 (3,184f)] the author studied a plane quartic curve $\varphi$ in $(1, 1)$ correspondence with the curve of intersection of the Veronese surface $F$ in [5] and a quadric primal, out polar to $F$. Covariants and contravariants of $\varphi$ are represented by certain associated curves in [5]. In the present paper the method used in both papers makes clear that various results, including those of Clebsch [J. Reine Angew. Math. **69,** 125–145 (1861)], Ciani [Accad. Naz. Lincei. Rend. (5) $\mathbf{4}_2$ 274–280 (1855)] and Coble [Trans. Amer. Math. Soc. **4,** 65–85 (1903)], are all included in the unravelment of Sylvester [Cambridge and Dublin Math. J. **7,** 52–97 (1852) or Mathematical Papers, vol. I, pp. 284–327].

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**MR0007257 (4,110e)** 48.0X
**Edge, W. L.**
**A type of periodicity of certain quartic surfaces.**
*Proc. Edinburgh Math. Soc. (2)* **7,** (1942). 73–80

This paper gives further examples of periodicity in a series of contravariant quartic envelopes and of covariant quartic loci first discussed by Reye [J. Reine Angew. Math. **82,** 173–206 (1877), in particular, p. 198]. Consider a form (point locus) of order $2p$ in $n$ homogeneous variables and its $p$th polar. The matrix $\mu$ of its coefficients, associated with proper multinomial numerical factors, is bordered by a row and column of products of the dual coordinates of order $p$ in such sequence as to make the matrix truly symmetric. For $p = 1$, this is the well-known polar duality as to a quadric primal. When the process is repeated, the original quadric primal is reproduced. When $p = 2$ such recurrence of the original form is not possible except in particular cases. Reye obtained one when the given quartic is the square of a quadric surface $(n = 4)$. For $n = 3$ a similar question was considered by Coble [Trans. Amer. Math. Soc. **4,** 65–85 (1903)] concerning plane quartic curves.

The present paper gives two proper quartic surfaces having the same property. These are

$$x^4 + y^4 + z^4 + t^4 + 12xyzt = 0$$

and the developable formed by the tangents to a space cubic curve. Various properties of polar cubic surfaces of the points of a plane are also given.

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**MR0005660 (3,184f)** 14.0X
**Edge, W. L.**
**Some remarks occasioned by the geometry of the Veronese surface.**
*Proc. Roy. Soc. Edinburgh. Sect. A.* **61,** (1941). 140–159

The paper contains an expository discussion of the Veronese surface [G. Veronese, Mem. Accad. Naz. Lincei (3) **19,** 344–371 (1884)], combining a geometric representation with a new algebraic treatment that brings out correctly the self-dual properties of the surface. Various properties of the secant planes (those meeting the surface in conics) are added, including their relations to quadric primals which either contain the surface or are outpolar to it. A new feature of the paper is the derivation of an invariant and of two contravariants of a ternary quartic by means of the $(1,1)$ correspondence between a linear system of plane quartic curves and these outpolar quadric primals. The treatment is first geometric, followed by a purely algebraic proof of the same theorems. Various errors and unsymmetric discussions in the literature [J. J. Sylvester, Collected Mathematical Papers, Cambridge, England, 1904, vol. 1, pp. 61–65] are replaced, in the present paper, by a consistent and completely self-dual presentation.

Reviewed by *V. Snyder*

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**MR0004491 (3,15b)** 14.0X

**Edge, W. L.**

**Notes on a net of quadric surfaces. IV. Combinantal covariants of low order.**

*Proc. London Math. Soc. (2)* **47,** (1941). 123–141

In a series of earlier papers [Proc. London Math. Soc. (2) **43,** 302–315 (1937); (2) **44,** 466–480 (1938); J. London Math. Soc. **12,** 276–280 (1937)] the author discussed a number of combinantal covariants of a net of quadric surfaces. All those mentioned by earlier writers were of order at least 8. The present paper obtains, largely by geometric methods, various new covariants of orders four and six. The covariant surface $F_4$ is the dual of Gundelfinger's contravariant $\varphi_4$. It has the 28 lines joining pairs of base points for bitangents. No claim is made that the new list of covariants is complete. At the end of the paper a particular case is discussed in which the base points are the vertices of two tetrahedra, each inscribed in the other, thus extending a previous paper on the same particular case [Proc. London Math. Soc. (2) **41,** 337–360 (1936)]. In this case the Jacobian curve of the net of quadric surfaces consists of four skew lines and their two transversals.

Reviewed by *V. Snyder*

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**MR1555413** DML Item

**Edge, W. L.**

**Some special nets of quadrics in four-dimensional space.**

*Acta Math.* **66** (1936), *no. 1,* 253–332.

{There will be no review of this item.}

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**MR1555399** DML Item

**Edge, W. L.**

**The geometry of a net of quadrics in four-dimensional space.**

*Acta Math.* **64** (1935), *no. 1,* 185–242.

{There will be no review of this item.}

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