

EDGE DAYS 2013:
A WORKSHOP ON ALGEBRAIC GEOMETRY
JUNE 7-9TH, 2013, UNIVERSITY OF EDINBURGH

IVAN CHELTSOV AND TIMOTHY LOGVINENKO

1. ABOUT THE CONFERENCE

“Edge days 2013” is a three day workshop on modern methods in classical birational geometry to be held in Edinburgh during the weekend of 7-9 June 2012 (Fri - Sun). Birational geometry, which developed very rapidly in the last few decades, has its roots in the classical problems like the rationality problems for cubic fourfolds and the Noether problems for group quotients. The new techniques developed by birational geometers have been applied to a number of these problems. But many of the classical problems remain unsolved. The main goal of this workshop is to gather a small number of experts mostly from the UK and continental Europe for a weekend meeting to discuss several classical problems in birational geometry from the modern viewpoint, including but not limited to: Fano varieties with mild singularities, birational automorphisms and rationality questions. The priority is given to very recent developments.

The proposed meeting will be dedicated to the memory of William Edge, a Scottish mathematician owing to whom for a large part of the 20th century Edinburgh was at the center of the classical algebraic geometry research in Britain. We hope that this will be a beginning of a beautiful tradition.

William Edge, who worked at the University of Edinburgh for most of his life. Edge graduated from Cambridge University and was a friend of Du Val, Semple, Coxeter, and Todd. Edge died in 1997 in Edinburgh at the age of 92, but there are many in Edinburgh who still remember him as an active researcher. Edges research was linked to the classical Italian school of algebraic geometry. Moreover, his research interests still form an active part of modern algebraic geometry, e.g. many algebraic geometers study varieties with small number of apparent double points that have been introduced by Edge in 1932. Edge is a part of the mathematical history of Edinburgh. Recently the Geometry Seminar at the University of Edinburgh was renamed into EDinburgh GEometry seminar or simply EDGE seminar.

2. SCHEDULE

Friday, 7th June 2013:

- **10:00 - 10:30** David Monk, James Hirschfeld, Colin Aitken,
“*Who is William Edge?*”
- **10:30 - 11:20** James Hirschfeld (Sussex)
“*A connection between finite geometry, algebraic geometry and coding theory: a tribute to W.L. Edge*”

- 11:30 - 12:20 Damiano Testa (Warwick)
“Reconstructing plane quartics with many undulations from their inflection lines”
- 13:30 - 14:20 Caucher Birkar (Cambridge)
“Adjoint divisors with good augmented base locus”
- 14:30 - 15:20 Kevin McGerty (Oxford)
“Localisation for quantum Hamiltonian reductions”
- 16:30 - 17:20 Ann-Sophie Kaloghiros (Imperial)
“Birational geometry of terminal quartic hypersurfaces”
- 17:30 - 18:20 Massimiliano Mella (Ferrara)
“On the automorphisms of moduli spaces of curves”
- 19:00 - Wine reception

Saturday, 8th June 2013:

- 10:30 - 11:20 Angelo Felice Lopez (Roma Tre)
“Nakamaye’s theorem on log canonical pairs and an application to moduli”
- 11:30 - 12:20 Vladimir Guletskii (Liverpool)
“Symmetric group action on the étale A^1 -homotopy type with applications”
- 13:30 - 14:20 Edoardo Sernesi (Roma Tre)
“The Jacobian ring of a singular projective hypersurface”
- 14:30 - 15:20 Yuri Prokhorov (MSU)
“G-Fano threefolds and Cremona groups”
- 16:30 - 17:20 Paolo Cascini (Imperial)
“Base point freeness in positive characteristic”
- 17:30 - 18:20 Jürgen Hausen (Tübingen)
“Computing Cox rings”
- 20:00 - Dinner at *Kalpna* restaurant
2-3 St Patrick Square, Edinburgh EH8 9EZ

Sunday, 9th June 2013:

- 10:30 - 11:20 Ruadháil Dervan (Cambridge)
“Log canonical thresholds and K -stability”
- 11:30 - 12:20 Yoshinori Gongyo (Imperial and Tokyo)
“Finiteness of log pluricanonical representations”
- 13:30 - 14:20 Jihun Park (Pohang)
“Alpha-functions of smooth del Pezzo surfaces”
- 14:30 - 15:20 Mingming Shen (Cambridge)
“Fourier transform on certain Hyperkahler fourfolds”
- 16:30 - 17:20 Yifei Chen (Chinese Academy of Sciences)
“The subadditivity of Kodaira dimension for fibrations of relative dimension 1 in positive characteristic”
- 17:30 - 18:20 Francesco Russo (Catania)
“Some characterizations of Edge varieties”

3. ABSTRACTS

- **Caucher Birkar (University of Cambridge):**

Adjoint divisors with good augmented base locus

I will discuss log canonical divisors with "good" augmented base locus. I try to explain how one might construct minimal models for such divisors.

- **Paolo Cascini (Imperial College London):**

Base point freeness in positive characteristic

I will discuss some recent progress towards the base point free theorem in positive characteristic. Joint work with H. Tanaka and C. Xu.

- **Yifei Chen (Institute of Mathematics, Chinese Academy of Sciences):**

The subadditivity of Kodaira dimension for fibrations of relative dimension 1 in positive characteristic

Let $f: X \rightarrow Z$ be a separable fibration of relative dimension 1 between smooth projective varieties over an algebraically closed field k of positive characteristic. We prove the subadditivity of Kodaira dimension:

$$\kappa(X) \geq \kappa(Z) + \kappa(F)$$

where F is the generic geometric fiber of f , and $\kappa(F)$ is the Kodaira dimension of the normalization of F . We mainly follow Viehweg's approach of proof of Iitaka conjecture $C_{n,n-1}$. This is a joint work of Lei Zhang.

- **Ruadhai Dervan (University of Cambridge):**

Log canonical thresholds and K-stability

To a polarised variety (X, L) one can associate an invariant called the log canonical threshold, which measures singularities of divisors in the linear system associated to L . The log canonical threshold is the algebraic counterpart of Tian's alpha invariant. A theorem of Odaka says that in the Fano case a certain lower bound on the alpha invariant implies that the manifold $(X, -K_X)$ is K-stable (which is the algebraic counterpart to a famous theorem of Tian concerning Kahler-Einstein metrics). We give a criterion in terms of the log canonical thresholds for certain other polarisations to be K-stable.

- **Yoshinori Gongyo (University of Tokyo/Imperial College London):**

Finiteness of log pluricanonical representations

I will explain the finiteness of log pluricanonical representations for projective log canonical pairs with semi-ample log canonical divisor. As a corollary, we obtain that the log canonical divisor of a projective semi log canonical pair is semi-ample if and only if the log canonical divisor of its normalization is semi-ample. This is a joint work of Osamu Fujino.

- **Vladimir Guletskii (Liverpool):**

0-cycles on intersections of quadrics and cubics in \mathbb{P}^4 , and relevant motivic stuff

In the talk we will have yet another look at the correlation between 0-cycles on algebraic surfaces, from one side, and codimension 2 algebraic cycles of threefolds, on the other. Working with a fairly concrete model, the intersections of smooth quadrics and cubics in \mathbb{P}^4 , we will see how the associated Prymians reflect rational equivalence of symplectomorphic 0-cycles on the K3 intersections, and then discuss the motivic view on this phenomenon.

- **Jürgen Hausen (Universität Tübingen):**

Computing Cox rings

We discuss methods for the explicit computation of Cox rings and present recent applications to singular del Pezzo surfaces and cubic elliptic threefolds.

- **J.W.P. Hirschfeld (University of Sussex):**

A connection between finite geometry, algebraic geometry and coding theory: a tribute to W.L. Edge

The Main Conjecture for Maximum Distance Separable Codes asks for the largest length of a linear code that corrects the maximum possible number of errors, given the size q of the field and the dimension k of the code. The known results depend on the geometry of the corresponding projective space and an associated problem in the projective plane over the field. The latter problem gets upper bounds from the number of rational points on associated algebraic curves.

- **Ann-Sophie Kaloghiros (Imperial College London):**

Birational geometry of terminal quartic hypersurfaces

Let X be a quartic hypersurface in \mathbb{P}^4 . The question of determining whether X is rational or not is very classical. Iskovskikh and Manin proved that if X is smooth, it is non-rational in a very strong sense. More precisely, they showed that X is birationally rigid, i.e. that X is not birational to any other Mori fibre space.

When X has isolated hypersurface singularities, its birational geometry turns out to be influenced both by the local analytic type of singularities and by global topological properties. For instance, if it is not factorial, then, in many cases it is rational, even when it only has ordinary double points. In this talk, I consider the birational geometry of terminal factorial quartic hypersurfaces X . In this setting, Mella proved that if X has no worse than ordinary double points, then it is birationally rigid, while Corti and Mella gave an example of a quartic hypersurface with a cA_2 point that is bi-rigid, i.e. that is birational to precisely one other Mori fibre space. I will discuss restrictions on local analytic types of singular points, construct examples of non-rigid quartic hypersurfaces, and state some conjectures about (lack of) rigidity.

This is joint work with Hamid Ahmadinezhad.

- **Angelo Felice Lopez (Università Roma Tre):**

Nakamaye's theorem on log canonical pairs and an application to moduli

Let X be a normal projective variety and let D be a big and nef \mathbb{Q} -Cartier divisor on X . Since its introduction by Ein, Lazarsfeld, Mustaa, Nakamaye and Popa, the *augmented base locus* of D

$$\mathbf{B}_+(D) = \bigcap_{\substack{E \geq 0 : \\ D-E \text{ ample}}} \text{Supp}(E)$$

has appeared as an important tool in the study of the birational geometry of X .

Let now V be a subvariety of X of dimension $d \geq 1$. Then $D^d \cdot V \geq 0$ with strict inequality if $V \not\subseteq \mathbf{B}_+(D)$. Let

$$\text{Null}(D) = \bigcup_{V \subseteq X: D^d \cdot V = 0} V.$$

A beautiful theorem of Nakamaye (2000) asserts that, when X is smooth, we have

$$\mathbf{B}_+(D) = \text{Null}(D).$$

As most birational geometry needs to work on normal varieties with some kind of (controlled) singularities, it is certainly of interest to generalize this theorem when X is singular.

In the talk I will explain a generalization of Nakamaye's theorem in the presence of log canonical singularities. We will also see some nice applications of this result to the birational geometry and the log minimal model program of the moduli space of curves \overline{M}_g .

This is work in collaboration with Salvatore Cacciola (Roma Tre) and Filippo Viviani (Roma Tre).

- **Massimiliano Mella (Università degli Studi di Ferrara):**

On the automorphisms of moduli spaces of curves

The Deligne–Mumford moduli spaces of curves, and its pointed version, is one of the most studied objects in algebraic geometry of the last decades. More recently Hassett proposed a weighted construction assigning rational weight to the labelled points. Beside the interest in moduli theory this could be a way to study the LMMP of the original D-M moduli space. I'll discuss an approach to determine the automorphism group of many of these spaces mainly based on projective techniques and earlier work on D-M spaces.

- **Kevin McGerty (University of Oxford):**

Localisation for quantum Hamiltonian reductions

The classical localisation theorem of Beilinson and Bernstein shows that representations of enveloping algebras can be studied via D-modules on the flag variety and is by now a cornerstone of the subject. Recasting it "micro-locally" as a statement about quantisations of the cotangent bundle of the flag variety suggests a way to generalise the theorem to other contexts. We will discuss recent work with Tom Nevins which provides such a framework for such generalisation in the case of Hamiltonian reductions for actions of reductive groups.

- **Yuri Prokhorov (MSU):**
G-Fano threefolds and Cremona groups

First, we discuss certain classes of singular Fano threefolds admitting “large” groups of symmetries. Then we apply these constructions to classification problem of finite subgroups of the space Cremona group.

- **Jihun Park (Pohang):**
Alpha-functions of smooth del Pezzo surfaces

We define alpha-functions of Fano varieties by considering the alpha-invariants of Tian locally. We demonstrate how to obtain the alpha-functions of smooth del Pezzo surfaces. In addition, their applications are briefly introduced.

- **Francesco Russo (Università degli Studi di Catania):**
Some characterizations of Edge varieties

In 1932 Edge studied birational representations on \mathbb{P}^n and geometrical properties of divisors $X^n \subset \mathbb{P}^{2n+1}$ of type $(0, 2)$ and $(1, 2)$ on the Segre varieties $\mathbb{P}^1 \times \mathbb{P}^n \subset \mathbb{P}^{2n+1}$, generalizing former work of Babbage. The first class is nothing but $\mathbb{P}^1 \times Q^{n-1}$ Segre embedded while varieties in the second class have been dubbed *Edge varieties*.

We shall present two different characterizations of these varieties among *varieties with one apparent double point*, that is varieties $X^n \subset \mathbb{P}^{2n+1}$ such that through a general point there passes a unique secant line to X^n . Moreover we shall discuss some connections between these characterizations and other more general and fundamental questions on the geometry of projective varieties.

- **Edoardo Sernesi (Università Roma Tre):**
The Jacobian ring of a singular projective hypersurface

When passing from nonsingular to singular projective hypersurface most good properties of the Jacobian ring are lost. I will illustrate an alternative approach which leads to a more natural transition.

- **Mingming Shen (Cambridge):**
Fourier transform on certain hyperkahler fourfolds

Using an algebraic cycle obtained from the Poincare line bundle, Beauville defines the Fourier transform on the Chow ring of an abelian variety and shows that the Fourier transform gives a decomposition of the Chow ring which is compatible with the multiplicative structure. In this talk, I will show how a similar decomposition can be obtained for certain hyperkahler fourfolds, using an algebraic cycle representing the Beauville–Bogomolov bilinear form. This is joint work with Charles Vial.

- **Damiano Testa (Warwick):**
Reconstructing plane quartics with at least 8 undulations from their inflection lines

In recent joint work with Marco Pacini we study the question of whether a plane quartic can be reconstructed from its inflection lines. In this talk, I will focus on quartics with at least 8 hyperinflection lines (undulations) and show that they can be reconstructed, with two exceptions over F_{13} . The argument uses the classification of such quartics by Vermeulen and exploits the fact that all these quartics have dihedral groups in their automorphism groups.

REFERENCES

E-mail address: I.Cheltsov@ed.ac.uk

E-mail address: LogvinenkoT@cardiff.ac.uk