

ranges from the atomic to the cosmic—it certainly should have been possible to look beyond the usual suspects. This is a missed opportunity.

Nevertheless *Climate, Chaos and COVID* is an authentic and engaging entry point into the use of mathematical models in the real world. The book feels anything but academic, even though its main audience is likely to be other educators. I would recommend it to anyone who teaches mathematics or a mathematics-rich science at a high school level. In it, you will find many real-world examples of the ways in which mathematical models inform and enrich our society, explained in a friendly, down-to-earth manner.

SHAUN HENDY
Te Herenga Waka
Victoria University of Wellington

An Introduction to the Numerical Simulation of Stochastic Differential Equations.

By Desmond J. Higham and Peter E. Kloeden. SIAM, Philadelphia, 2021. \$79.00. xvi+277 pp., hardcover. ISBN 978-1-61197-642-7. <https://doi.org/10.1137/1.9781611976434>.

Stochastic differential equation (SDE) models play a central role in a vast range of applications from mechanics and microelectronics to economics and finance, from biology and chemistry to epidemiology. Although a deep understanding of the theory of SDEs requires background and knowledge in advanced probability and stochastic processes, it is feasible to know how to simulate SDEs numerically based on some understanding of random variables and Euler's method for deterministic ordinary differential equations. In a previous article by one of the authors [1], an accessible and practical introduction to numerical methods for SDEs has been provided. That paper has become a classical reference for the introduction of the important theory of numerical SDEs.

Following the style of [1], the current book is a self-contained, elementary text that puts across the fundamentals of all topics in numerical methods for SDEs. Like [1], the book is written in a highly pedagogical way in which the important material, well-constructed examples, and perfectly

designed exercises are presented. Thus, the book is suitable for graduate students and young researchers who have a good knowledge of calculus and algebra and some basic understanding about probability theory and numerical analysis. The book not only provides the reader with an intuitive understanding of the essential concepts of the theory of SDEs but also gives excellent instructions on how to efficiently implement and simulate SDEs in various applications.

The proofs of fundamental theorems in the theory of numerical SDEs are also supplied in an accessible way for beginners on the topics. For instance, enough background material is added such that the reader can understand a sketch of the proofs of the key properties of weak convergence (Chapter 9) and strong convergence (Chapter 10) of the Euler–Maruyama method, as well as the relevance of the Itô formula in the derivation of higher-order methods (Chapter 17). Like [1], instead of focusing on technical details, the authors have relied heavily on computational examples and illustrative figures. For this purpose, each chapter has a key computational topic with illustrative figures in MATLAB.

Despite the accessibility to graduate students and young researchers, most of the current important aspects of the theory of numerical SDEs have also been covered by the book. Among those topics are the numerical analysis of mean exit times (Chapter 12), nonsmooth SDE models (Chapter 14), multilevel Monte Carlo (Chapter 15), jump-SDEs (Chapter 16), and applications and simulations of SDEs in chemical kinetics (Chapter 20).

Let us now provide a brief description of the content of each chapter. The first chapter provides an introduction on random variables, including discrete and continuous random variables, expectation and variance, Markov and Lyapunov inequalities, as well as the central limit theorem and strong law of large numbers. Chapter 2 introduces the basic theory of the simulation of random variables, including pseudorandom number generation, the Monte Carlo method, and kernel density estimation. Chapter 3 is an introduction to the theory of Brownian motions. Chapter 4 introduces the basic concepts of stochastic

integrals, and the theory of SDEs is presented in Chapter 5. The most important tool in the theory of SDEs, Itô's formula, is provided in Chapter 6, leading to the Stratonovich form of an SDE discussed in Chapter 7. The introduction to the theory of numerical schemes of SDEs starts with Chapter 8, in which the simplest and most used numerical scheme for SDEs, the Euler–Maruyama scheme, is introduced. A sketch of the proofs of weak and strong convergence of the Euler–Maruyama scheme, mentioned above, is proved in Chapters 9 and 10. The asymptotic stability and mean square of the stochastic θ -method as well as how to use Monte Carlo numerical methods to compute mean exit times are investigated in Chapters 11 and 12. Applications of SDEs in finance are discussed in Chapter 13. Long-time dynamics of SDEs and the multilevel Monte Carlo technique are studied in Chapters 14 and 15. Chapters 16 and

17 introduce jump-SDEs and derive high-order numerical methods for SDEs. Systems of SDEs are also investigated in the book, in particular Chapter 18. Simulations of chemical reactions are discussed in the last chapter of the book.

In my opinion, this book is a marvelous introduction into the theory of numerical SDEs for undergraduate students and young researchers. Senior researchers may also find several discussions and many ideas in this book enlightening.

REFERENCE

- [1] D. J. HIGHAM, *An algorithmic introduction to numerical simulation of stochastic differential equations*, *SIAM Rev.*, 43 (2001), pp. 525–546, <https://doi.org/10.1137/S0036144500378302>.

MINH-BINH TRAN
Texas A&M University