An elliptic equation on $\mathbb{R}^N$, bifurcation and decay of bound states

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Abstract

We consider the nonlinear eigenvalue problem,

$$-\Delta u(x) + q(x)u(x) + \gamma \frac{u(x)^2}{\xi(x)^2 + u(x)^2} u = \lambda u(x) \text{ for } x \in \mathbb{R}^N,$$

where $\gamma > 0, q \in L^\infty(\mathbb{R}^N)$ and $\xi \in L^2(\mathbb{R}^N)$ are given and we are interested in eigenvalues $\lambda \in \mathbb{R}$ for which this equation admits a bound state, that is, a non-trivial solution in $L^2(\mathbb{R}^N)$. The formal linearization of this problem is

$$-\Delta u + qu = \lambda u$$

but we find that bound states can bifurcate at values of λ which are not in the L^2-spectrum of this linear problem.

It turns out that all bound states belong to $C^1(\mathbb{R}^N)$ and decay to zero as $|x| \to \infty$. However, for given q and ξ, some bound states may decay exponentially fast whereas others do not. Particular attention is paid to the case where the potential q is either periodic or is the sum of a periodic function and a function which decays to zero at infinity.

Similar conclusions hold for a much more general class of equations.